2012 Grade 9 Mathematics Set A

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URL: https://www.nier.go.jp/English/index.html

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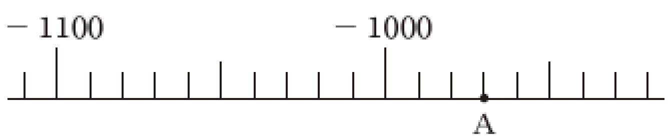
[1] Answer the following questions (1) through (4).

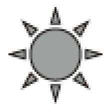
(1) Find the least common factor of 8 and 12.

(2) Calculate 6 – (-7).

(3) The picture below is a part of a number line. What number is represented by point A?

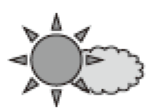


(4) According to a weather forecast, the high and the low temperatures for City A on March 7 are as follows.

Today's Weather (City A) Wednesday, March 7		
	High	15 ℃
Sunny	Low	1 ℃

If we subtract the low temperature from the high temperature, we can determine the temperature difference. The temperature difference for City A is 15 – 1 = 14 (°C).

According to a weather forecast, the high and the low temperatures for City B on March 7 are as follows. Find the temperature difference for City B.

Today's Weather (City B) Wednesday, March 7		
	High	9 ℃
Partly Sunny	Low	– 2 ℃

[2] Answer the following questions (1) through (4).

(1) Simplify $(7x + 5y) - (5x + 2y)$.

(2) Find the value of the expression, $-x^2$, when $x = 3$.

(3) If a is an integer, select all numbers from below that can be represented as $2a$.

0 1 35 78 100

(4) In the algebraic statements (a) through (e) below, there is a statement that represents the relationship, “If you buy 2 items which cost a-yen each, the total cost is less than 1000 yen.” Select the correct one.

(a) $2a \leq 1000$

(b) $2a < 1000$

(c) $2a = 1000$

(d) $2a > 1000$

(e) $2a \geq 1000$

[3] Answer the following questions (1) through (4).

(1) If 6 : 8 = *x* : 12, determine the value of *x*.

(2) Solve the system of equations:

$$\begin{cases} a + b = 8 \\ 2a + b = 11 \end{cases}$$

(3) The linear equation, $7x = 4x + 6$, was solved as shown below.

$$\boxed{\begin{aligned} 7x &= 4x + 6 \\ 7x - 4x &= 6 \\ 3x &= 6 \qquad \cdots\cdots ① \\ x &= 2 \qquad \cdots\cdots ② \end{aligned}}$$

Select from (a) through (d) below the reason we can transform equation ① into equation ②.

(a) If we add 3 to both sides of equation ①, the equation will remain true. Therefore, we can transform equation ① into equation ②.

(b) If we subtract 3 from both sides of equation ①, the equation will remain true. Therefore, we can transform equation ① into equation ②.

(c) If we multiply both sides of equation ① by three, the equation will remain true. Therefore, we can transform equation ① into equation ②.

(d) If we divide both sides of equation ① by three, the equation will remain true. Therefore, we can transform equation ① into equation ②.

(4) Let's think about the following problem.

Problem

A girl left home for the train station that is located 1800 *m* from her house. 15 minutes later, her brother noticed that she forgot to take something with her, so he started chasing her on his bicycle, following the same route. Suppose the girl walks at the speed of 70 meters per minute and the speed of her brother on his bicycle is 220 meters per minute. How many minutes after the brother left home will he catch up with his sister?

This problem can be solved using equations as shown below.

Solution

Suppose that the brother will catch up with his sister x minutes after he left home.

① The distance the brother travel before he catches up with his sister can be represented as $220 \times m$, and the distance the sister travels until her brother catches up with her will be represented as $70\,(15 + x)$.

Since these distances are equal,

$$200x = 70\,(15 + x)$$

If we solve this equation,

$$220x = 1050 + 70x$$
$$150x = 1050$$
$$x = 7$$

When $x = 7$, the values of the left hand side and the right hand side of the equation are 1540. Therefore, $x = 7$ is the solution for the equation.

② Since the distance the brother travel in 7 minutes, 1540*m*, is less than the distance between the house and the train station, it is possible for him to catch up with his sister before she reaches the station.

Therefore, the brother will catch up with his sister 7 minutes after he left home.

Answer: 7 minutes

(For some reason I can't insert a comment inside the text box above, but it looks like the first equation has an error, "200" should be 220." I am also unable to fix this.)

In the solution above, inside box labeled ①, the quantities in the problem situation are represented by expressions with a variable.

In the box labeled ② in the solution, the person who solved the problem is checking something. Which of the following statements describes correctly what it is that this person is checking? Select one from (a) through (d) below.

(a) The person is checking whether or not the equation is set up using two quantities that are equal to each other.

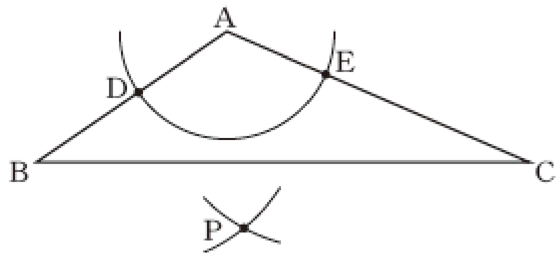
(b) The person is checking whether or not the value obtained from the equation is indeed the solution for the equation by substituting it to both sides of the equation.

(c) The person is checking whether or not the solution for the equation is appropriate as the answer for the problem.

(d) The person is checking whether or not the equation was solved correctly using the properties of equalities.

[4] Answer the following questions (1) through (3).

(1) In △ABC shown in the figure below, line AP was constructed following steps ①, ②, and ③.



① Construct a circle centered at A so that it will intersect with sides AB and AC. The points of intersection with AB and AC are labeled points D and E, respectively.

② Construct circles centered at points D and E with the equal radius so that the circles will intersect. Point P is the point of intersection of the two circles.

③ Construct a line by connecting vertex A and point P.

For any triangle ABC, one of statements a through d below will always be true about line AP constructed following steps ①, ②, and ③. Select the correct one.

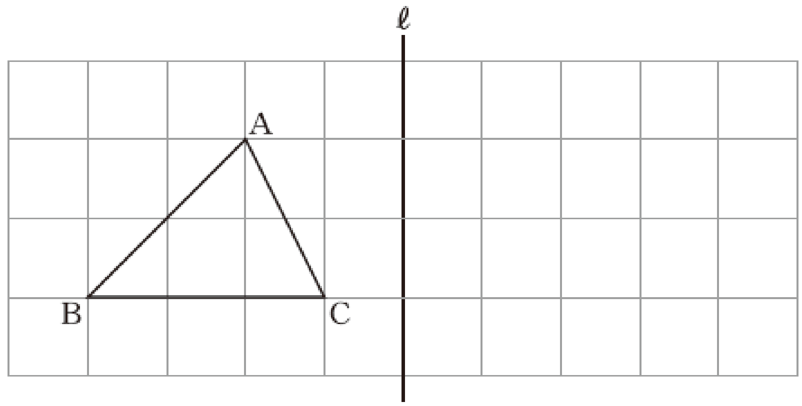
(a) Line AP passes through vertex A and is perpendicular to line BC.

(b) Line AP passes through vertex A and the mid-point of side BC.

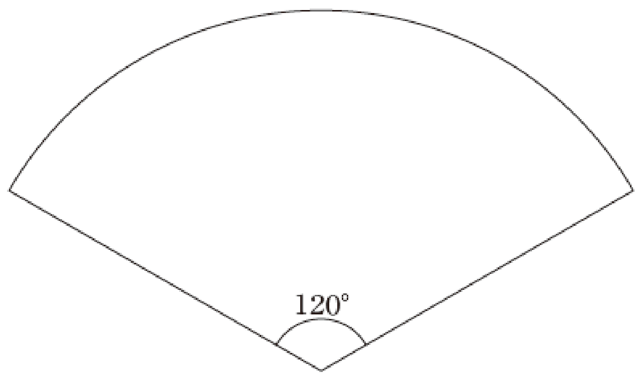
(c) Line AP is parallel to line BC.

(d) Line AP is the bisector of ∠CAB.

(2) Draw the image of △ABC shown below when it is reflected across line *l* using the grid provided.



(3) There is a sector with the central angle of 120° similar to the one shown below. How does the area of this sector compare to the area of a circle with the same radius? Select the correct answer from (a) through (e) below.



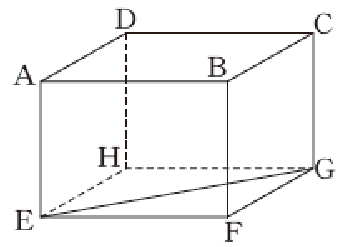
(a) $\frac{1}{6}$ times the area of the circle (b) $\frac{1}{3}$ times the area of the circle

(c) $\frac{1}{2}$ times the area of the circle

(d) $\frac{2}{3}$ times the area of the circle (e) $\frac{5}{6}$ times the area of the circle

[5] Answer the following questions (1) through (4).

(1) There is a rectangular prism like the one shown on the right. EG is a diagonal of rectangle EFGH. In this situation, what can we say about the size of ∠AEG? Select the correct statement from (a) through (d) below.



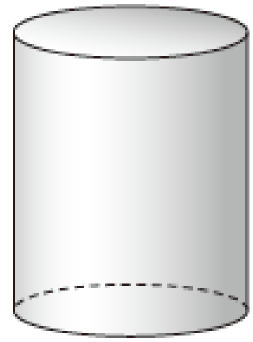
(a) The size of ∠AEG is greater than 90°.

(b) The size of ∠AEG is less than 90°.

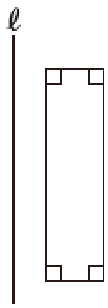
(c) The size of ∠AEG is 90°.

(d) The size of ∠AEG relative to 90° cannot be determined by the given conditions.

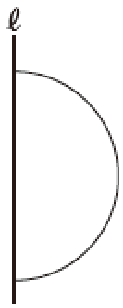
(2) The cylinder shown on the right can be thought of a solid of revolution obtained when a certain plane figure is rotated around a line. One of the figures (a) through (d) will result in this cylinder when it is rotated around line *l* as the axis of rotation. Select the correct one.



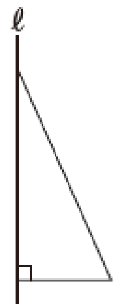
(a)



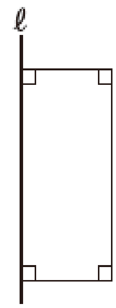
(b)



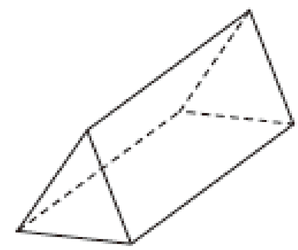
(c)



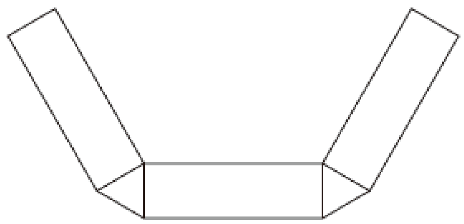
(d)



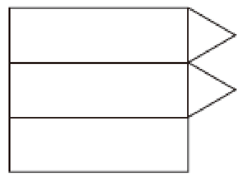
(3) There is a solid like the one shown on the right. One of the figures (a) through (d) below can be folded to make this solid. Select the correct one.



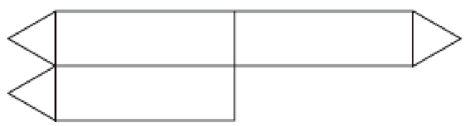
(a)



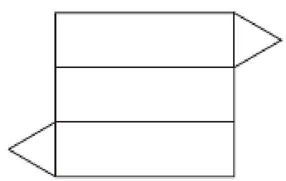
(b)



(c)

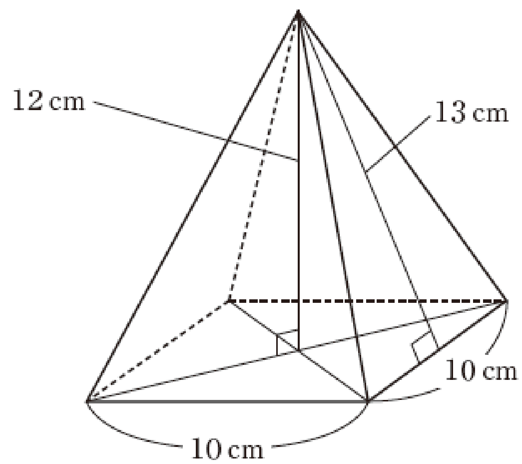


(d)



(4) There is a square pyramid like the one shown in the figure below. The base of this pyramid is a square with sides of 10 *cm*. The height of the square pyramid is 12 *cm*, and the height of the triangle on its lateral side is 13 *cm*.

From (a) through (d) below, select the one showing the correct calculation to determine the volume of this square pyramid.



(a) $10 \times 10 \times 12 \times \frac{1}{2}$

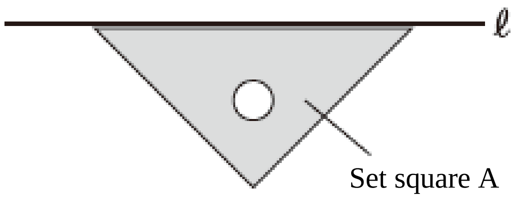
(b) $10 \times 10 \times 13 \times \frac{1}{2}$

(c) $10 \times 10 \times 12 \times \frac{1}{3}$

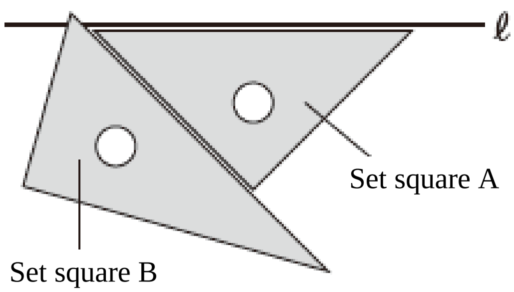
(d) $10 \times 10 \times 13 \times \frac{1}{3}$

[6] Answer the following questions (1) through (3).

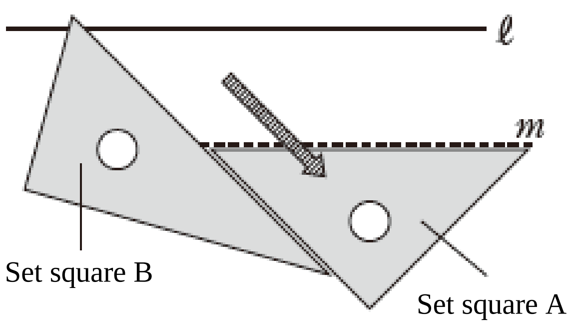
(1) Line *m*, which is parallel to line *l*, will be constructed following steps ①, ②, and ③ below.



1 Place set square A so that its side is aligned with line *l*.



2 Place set square B along a side of set square A.



3 Keeping set square B in its position, slide set square A along the side of set square B, then draw line *m*.

What is the reason that line *m* constructed following steps ①, ②, and ③ above will be parallel to line *l*? Select the correct reason from (a) through (d) below.

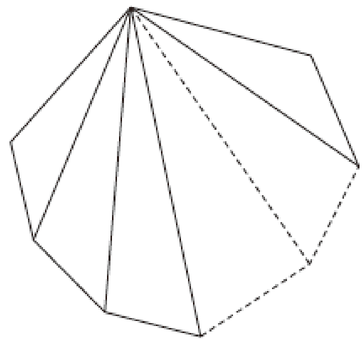
(a) Given a line intersecting a pair of lines, if corresponding angles are congruent, then the pair of lines are parallel.

(b) Given a line intersecting a pair of lines, if alternate interior angles are congruent, then the pair of lines are parallel.

(c) If a pair of lines are perpendicular to a third line, then they are parallel to each other.

(d) If a pair of lines are parallel to a third line, then they are parallel to each other.

(2) As shown below, an n-gon can be divided into several triangles by diagonals drawn from a single vertex.



In this situation, the sum of the interior angles of the n-gon can be expressed as $180° \times (n - 2)$.

In this expression, what about this n-gon does $(n - 2)$ represent? Select the correct answer from (a) through (e) below.

(a) the number of vertices

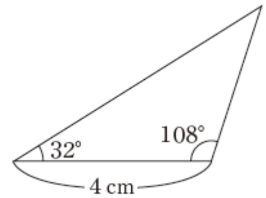
(b) the number of sides

(c) the number of interior angles

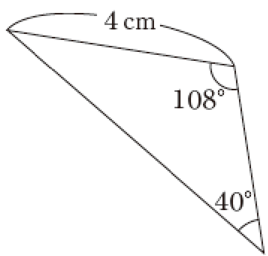
(d) the number of diagonals drawn from a single vertex

(e) the number of triangles created by the diagonals drawn from a single vertex.

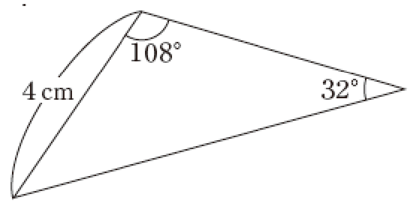
(3) Select the triangle that is congruent to the one shown on the right from (a) through (d) below.



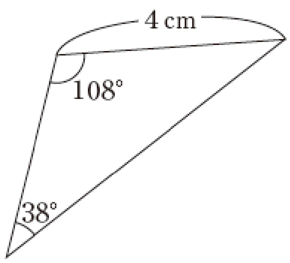
(a)



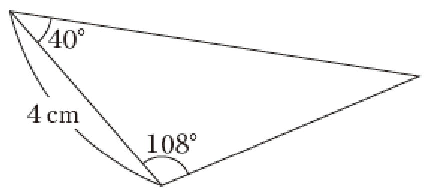
(b)



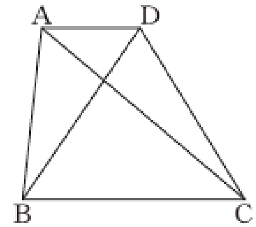
(c)



(d)



[7] In the diagram shown on the right, we can say the following about the areas of △ABC and △DBC.



In quadrilateral ABCD, if AD // BC
then area of △ABC = area of △DBC

We are going to think about the converse of this statement.

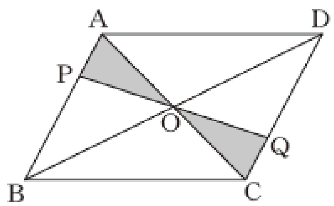
The converse of a statement is the statement obtained by reversing the supposition and conclusion.

Fill in the blanks [1] and [2] below to complete the converse statement.

In quadrilateral ABCD, if [1]
then [2].

[8] In parallelogram ABCD, point O is the point of intersection of the diagonals. P is a point on side AB, and Q is the point of intersection of line PO and side CD. A certain class drew the picture shown in **Figure 1** and proved OP = OQ as follows.

Figure 1



Proof

In △OPA and △OQC,
since the diagonals of a parallelogram intersects at their mid-points,

AO = CO ··· (1)

Since alternate interior angles formed by a pair of parallel lines and a transversal are congruent,

∠PAO = ∠QCO ··· (2)

Since vertical angles are congruent

∠AOP = ∠COQ ··· (3)

From (1), (2) and (3), since two pairs of corresponding angles and the corresponding sides between them are congruent,

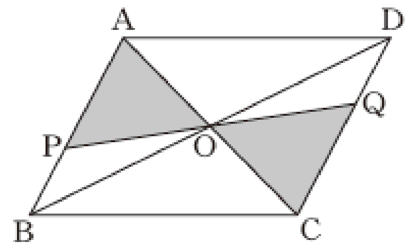
△OPA≅△OQC.

Since the lengths of corresponding sides of congruent figures are equal,

OP=OQ.

After this proof was completed, the position of point P was moved as shown in Figure 2. As the class thought about whether or not OP=OQ in this case as it was the case shown in Figure 1, the following ideas (a) through (d) were suggested. Select the correct one.

Figure 2



(a) For the case shown in Figure 2, OP=OQ was proved in the proof shown on the previous page.

(b) We need to prove separately that OP=OQ for the case shown in Figure 2.

(c) For the case shown in Figure 2, we need to verify that OP=OQ by actually measuring their lengths.

(d) For the case shown in Figure 2, it is not true that OP=OQ.

[9] Answer the following questions (1) and (2).

(1) The following statements (a) through (d) describe the relationship between the value of x and the corresponding values of y when y is proportional to x and the constant of proportion is 3. Select the correct statement.

(a) The sum of the value of x and the value of y is always 3.

(b) The difference when the value of x is subtracted from the value of y is always 3.

(c) The product of the value of x and the value of y is always 3.

(d) When the value of x is not 0, the quotient of the value of y divided by the value of x is always 3.

(2) Select a point that is on the graph of a proportional relationship, $y = 2x$, from (a) through (e) below.

(a) (2, 0)

(b) (2, 1)

(c) (-1, 2)

(d) (0, 2)

(e) (1, 2)

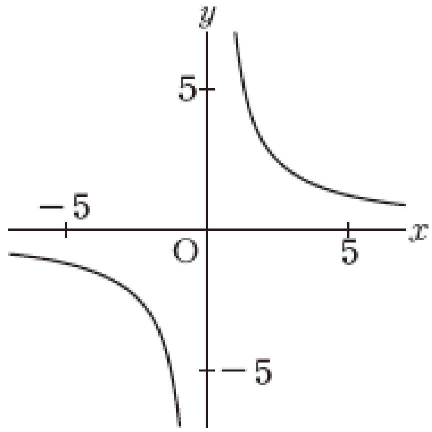
[10] Answer the following questions (1) and (2).

(1) The table below shows a relationship in which y is inversely proportional to x. Determine the appropriate number that goes into $\boxed{}$.

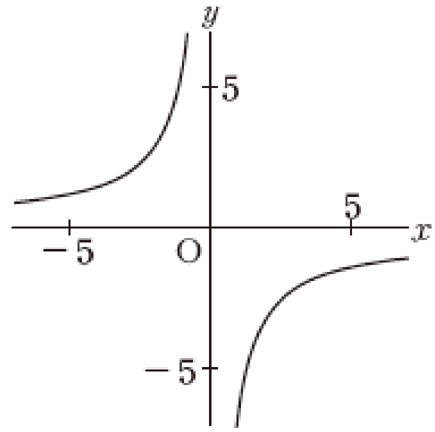
x	$\cdots$	-2	-1	0	1	2	3	$\cdots$
y	$\cdots$	-6	-12	$\times$	12	6	$\boxed{}$	$\cdots$

(2) One of graphs (a) through (e) below is the graph for the inversely proportional relationship $y = \frac{6}{x}$. Select the correct one.

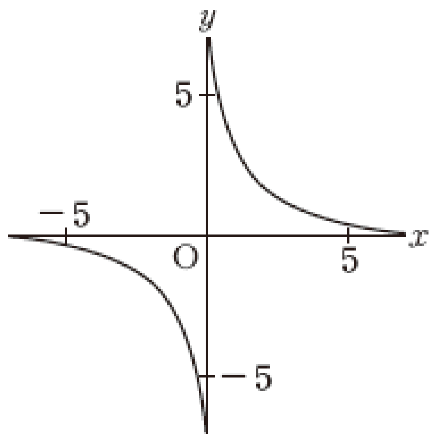
(a)



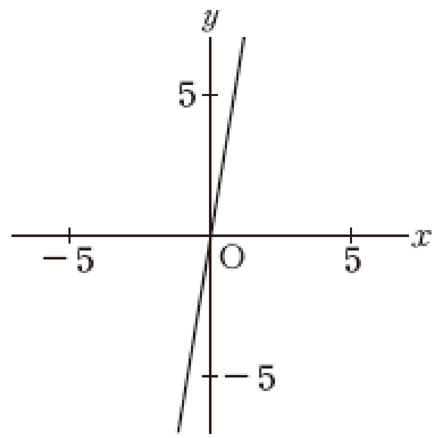
(b)



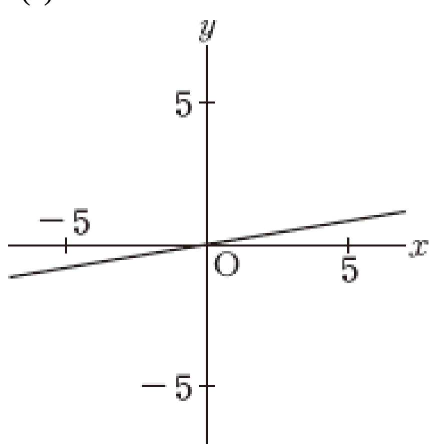
(c)



(d)

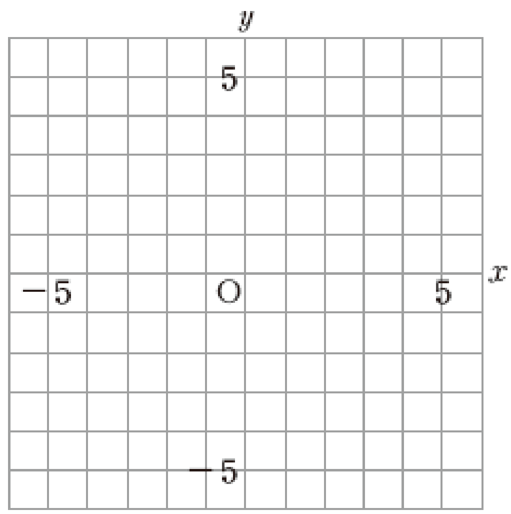


(e)

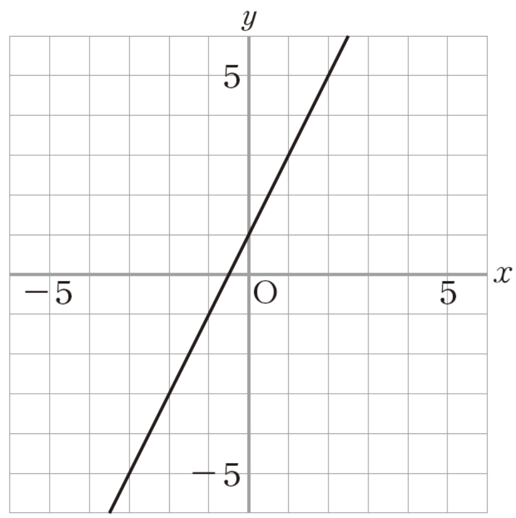


[11] Answer the following questions (1) and (2).

(1) Show point (-1, -4) as • on the figure provided.



(2) From (a) through (e) below, select the equation that shows the relationship between x and y shown in the graph below.



(a) $y = 2x + 1$

(b) $y = 3x + 1$

(c) $y = x + 2$

(d) $y = 2x$

(e) $y = 3x$

[12] From (a) through (e) below, select the case where y is a linear function of x.

(a) In the rectangle with the area of 60 cm^2, the length, x cm, and the width, y cm.

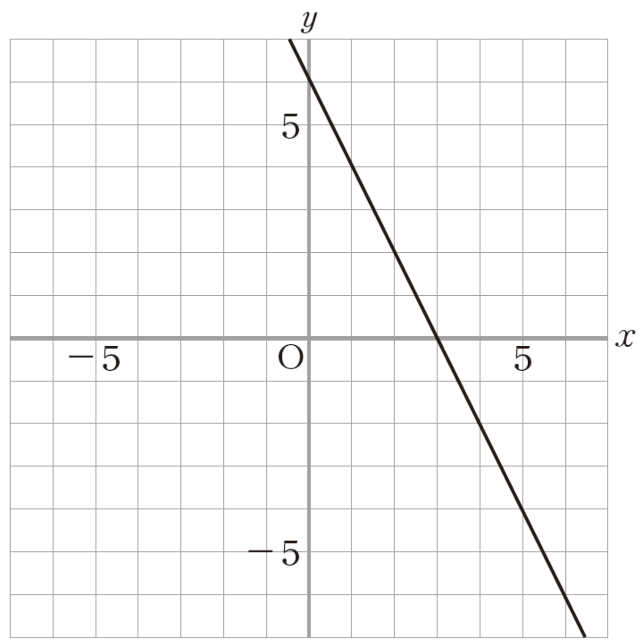
(b) On a path that is 1500 m long, the distance remaining, y m, after one has already walked x m.

(c) The height of a person, x cm, and the weight of that person, y kg.

(d) The length of a ribbon, y m, each person will receive when a 6 m ribbon is equally divided among x people.

(e) The temperature, y°C, at a certain location at x o'clock in the afternoon.

[13] The line below is the graph of the linear equation $2x + y = 6$. The statements (a) through (e) are about a point whose coordinates are a pair of values of x and y that is a solution of the equation. Select the correct statement.



(a) There is no point whose coordinates are a pair of values of x and y that is a solution of this equation.

(b) There is exactly one point whose coordinates are a pair of values of x and y that is a solution of this equation.

(c) There are exactly two points whose coordinates are a pair of values of x and y that is a solution of this equation.

(d) There are infinitely many points whose coordinates are a pair of values of x and y that is a solution of this equation, and the values of x and y are always integers.

(e) There are infinitely many points whose coordinates are a pair of values of x and y that is a solution of this equation, and the values of x and y are not limited to integers.

[14] Answer the following questions (1) and (2).

(1) There is a fair coin—that is, if the coin is tossed, it is equally likely to land heads as tails. When this coin was tossed three times, it landed heads each time. We are going to toss the coin a fourth time. From (a) through (d) below, select the correct statement about the probability of the coin landing heads on the fourth toss.

(a) The probability of the coin landing heads is greater than the probability of the coin landing tails.

(b) The probability of the coin landing heads is less than the probability of the coin landing tails.

(c) The probability of the coin landing heads is equal to the probability of the coin landing tails.

(d) We cannot determine whether the probability of the coin landing heads is greater or less than the probability of the coin landing tails.

(2) There are three number cards 1 through 3 like the ones shown below. The cards will be shuffled well and 2 cards will be drawn from them simultaneously. What is the probability that both cards will be odd numbers?

$$\boxed{1} \quad \boxed{2} \quad \boxed{3}$$

[15] Answer the following questions (1) and (2).

(1) A survey was conducted with 9th grade students who attend A Lower Secondary School and B Lower Secondary School about their commuting time. The frequency distribution table below summarizes the results of the survey for each school.

Commuting time (min.)	A Lower Secondary School	B Lower Secondary School
	Frequency(persons)	Frequency(persons)
Greater than or equal to 0 ～ Less than 10	4	1
10～20	9	2
20～30	16	8
30～40	23	14
40～50	22	17
50～60	16	12
60～70	10	6
Total	100	60

Based on this frequency distribution table, we are going to investigate which of the two schools has a greater ratio of students whose commuting time is less than 30 minutes relative to the total number of students surveyed at the school. Select the correct statement from (a) through (e) that describes the method of comparison.

(a) For each school, determine the total frequency for less than 30 minutes. Then, compare the total frequencies.

(b) For each school, determine the relative frequency for less than 30 minutes. Then compare the sums of relative frequencies.

(c) Compare the frequencies for greater than or equal to 20 minutes but less than 30 minutes.

(d) Compare the relative frequencies for greater than or equal to 20 minutes but less than 30 minutes.

(e) Since the total numbers of students surveyed are different in these two schools, we cannot make this comparison.

(2) The members of a basketball team at a certain lower secondary school each attempted 10 free throws. The graph below shows the relationship between the number of successful free throw attempts and the number of players. Determine the mode of the successful free throw attempts.