< Mathematics Department Research >

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1. Theme of Research

   Creating Mathematics Lessons that Deepen Learning by Connecting Students' Questions

2. About the Research Theme

   (1) Reasons for Setting up the Theme

   Our department has been conducting research on student questions. In the previous year's research[1] under the research theme of "Creating lessons that students truly understand by connecting to classmates' questions," we saw some progress in developing lessons that helped students truly feel they understand. To achieve this progress, we developed some necessary strategies such as: setting up well-thought-out tasks for the lesson, thinking about questions that should be asking to include in the lesson by considering students' anticipating questions, and anticipating the sequence of students' questions. Earlier research focused on lessons designed so that students naturally think of their own questions[2], lessons which pay attention to how students question each other continuously,[3] and lessons that make and build on students' making connections to other students' questions.[4]

   While we were doing research on students' questions, we realized the importance of continuing our research focus on the relationship of "mathematical views and thinking," "deeper learning," and "talent and ability" to the idea of student questions and how those connections help develop students' learning and skill set/understanding. It is clear that studying what students are doing in classrooms every day is important.[5] For example, we studied students' behaviors/learning/understanding when faced with new problem situations; such as, when students refer back to what they learned previously, when students need to think about how they could use what they have learned in daily life, and when students are asked what they want to learn or explore next after having learned something new from the lesson. In these cases, a survey we administered found students' responses to be lower compared to other items included in the survey. The survey's results indicated that students did not think of how the learning from the lesson overlaps with learning from previous or subsequent lessons, rather they appear to see learning from one lesson as being specific to and ending at the end of a lesson. Students did not appear to think that something they learned in a lesson could be used for

[1] 算数科総括 [Summary of Research of Mathematics Department] (2018)

[2] 山梨大学教育人間科学部付属小学校研究紀要 [Research Summary of University Attached Elementary School, Facility of Education Human Sciences, University of Yamanashi] (2004)

[3] 山梨大学教育人間科学部付属小学校研究紀要 [Research Summary of University Attached Elementary School, Facility of Education Human Sciences, University of Yamanashi] (2008)

[4] 山梨大学教育人間科学部付属小学校研究紀要 [Research Summary of University Attached Elementary School, Facility of Education Human Sciences, University of Yamanashi] (2011)

[5] 山梨大学教育人間科学部付属小学校研究紀要 [Results from the Survey of Student State of Learning: National Scholastic Ability & Learning Achievement Survey] (2018) and 本校算数科アンケート [Math Department Survey] (April 2019)

(applied to) other problem situations, other subject areas, or in their daily life. Furthermore, the results did not indicate that students were motivated or eager to apply what they have learned. Given these observations and findings about the status of the students, we believe we need to create lessons that result in students' utilizing a mathematical view and thinking that connects to the essence of our mathematics department's goals; i.e., that students deepen their learning by considering the state of their own and other students' learning.

The first year of the school's general research theme was "Students Connecting Learning: Providing lessons that approach the essence of the subject." Based on the issues we have discussed here, our present belief and thinking is that fostering students who connect their own and others' learning, we hope to nurture students' view of mathematics in service to cultivating a better future for our society. Therefore, our goal is to continue to do research by utilizing the experience and knowledge gained from our past research and our new view of the importance of looking at "students' questions." For these reasons, we established our current research theme as: "Creating Mathematics Lessons that Deepen Learning by Connecting Students' Questions"

(2) About Creating Lessons that Deepen Student Learning[6]

In order for students to deepen their learning, it is necessary to provide fruitful mathematical activities that enhance their mathematical view and thinking about mathematics. The definition of the "mathematical view and thinking" is, as follows:

> Grasp a phenomenon by paying close attention to its quantities, geometric figures, and relationships; think logically based on evidence; and think comprehensively and expansively.

In other words, develop ways of seeing (viewpoints) and ways of thinking in order to better grasp mathematical concepts/mathematics, and make this essential and the central focus of students' developing the meaning of learning in lower and secondary school mathematics. As students develop talents and abilities, they will connect what they learned in the past with new learning and the activities of daily life, acquire the ability to cultivate their future learning, and develop these habits of learning through persistence and perseverance. Moreover, students' learning will become more deeply ingrained as they acquire active and discussion-oriented learning.

The appearance of students who deepen their learning in this way includes the following characteristic behaviors: Students who don't stop learning after they have an answer to a problem; reflect on their own thinking; compare their ideas to other ideas and seek and find relationships among ideas; think about a better idea; generalize ideas to grasp the primary learning idea of the problem; and seek to expand their learning, at times connecting previous to novel ideas/solutions. In addition, these students can see the connection between what they have learned in one unit to other related units in mathematics, the connection between what they have learned across units and in different domains of mathematics, the connection between other or related topics in different subject areas, the connection with the activities of daily life, and the desire to

[6] Tokyo Shoseki. 新学習指導要領のポイントと授業づくり vol 1 & 2 [The Key Points of the New Course of Study and Developing Lessons]. P.18

utilize what they have learned. Finally, by conducting this expansive learning process, our department is working toward and looking for evidence that our students will continuously come up with their own questions, contribute to their learning and that of their peers', and the learning becomes even deeper for all students.

Next, as we think about the essence and the approach to lessons, the key important ingredients include the students, instructional materials, and the teacher. It is important that these three elements are connected and function together seamlessly. Therefore, lessons should not be teacher-led (driven by the teacher's thinking), nor should the lessons follow or be led by students' interests without following the purpose of the curriculum. Lessons should be conducted by students coming up with their own questions and engaging in problem solving that is aimed at and reflects the mathematical goals. In order to design these lessons, we need to study and analyze problems that are familiar and interesting to students. In our department, we studied Nakamura's research (1989) which revealed the importance of organizing the conditions of problems and activities that facilitate students' questions and question-posing. These conditions include questions that do the following:

- can be solved by students on their own using previously learned knowledge,
- produce disagreement and agreement or conflict and consent/understanding,
- present students with a common task,
- produce new tasks,
- and can be solved in multiple ways.

Considering these ideas, we designed lessons that are: equipped with instructional materials that produce and encourage students' questions; clearly indicate how teachers can orchestrate students' learning to achieve the goals; and consider ways that unify students, teacher, and instructional materials.

In the process of the lesson, students will use mathematical views and thinking, the essence of mathematical learning, to deepen their understanding. Students' questions will show evidence of their mathematical view and thinking.

(3) About "Student Questions"

There are several types of questions that students ask during a lesson. Among those questions, the questions that relate to a mathematical view and thinking we will call "students' questions that should be asking." Nakamura'[7] (1993) organized "questions" into the following categories:

- asking about previously learned content
- asking about a different way to solve
- asking about evidence
- asking about a commonality or similarity
- asking about differences
- asking about a generalization

[7] Takashi Nakamura (1993). 自ら問う力を育てる算数授業 [Mathematics Lessons that Foster Students Who Ask Questions of Their Own]. pp. 15 – 16.

- asking about expandability, and
- asking about merit

These student questions represent students' thinking and are the driving force of learning during lessons. Also included in these questions are students' questions that should be asking because they are closely related to the mathematical view and thinking to be developed by the lesson. In regard to a person who is striving to deepen his thinking, Sugiyama[8] (2006) said:

> The person who is thinking is not thinking about the question that he was asked but is also asking questions that should be asking to himself and he is answering the question that he asked himself. He advocates the importance of thinking inside the individual that helps to deepen thinking.

In addition, Sugiyama said that educators need to have a view of the lesson that is, as follows:

> Lessons (classroom practice) are formed by thinking which occurs inside an individual person that are then expressed outside into the class; and the questions, answers, discussion that is facilitated in the class will foster thinking that goes directed back inside the individual person. Lessons are the places where the process of thinking (continuous questioning via the "students' questions that should be asking") appears. When a student answers a question, the teacher should ask a question in response, and the teacher could be accepting the role for asking the question, so that the "students' questions that should be asking" are asked. The asking of these questions can be done by students also. In the end, both teacher and students together do the process of questioning and deepening their thinking.

As you can see, Sugiyama also talked about the importance of the teacher's role as well as interaction of students, in addition to individual thinking, for acquiring deeper thinking.

When we think about "students' questions" from the point of view discussed above, a "student question" is not just an act of an individual student's thinking. A "student's questions" appear in other students' minds as well as in the classroom or between student and teacher or one student to another student. This means the teacher's role is vital and of utmost importance. We feel it is critical that we foster the connections created by "students' questions" to deepen students' learning and create a rich learning environment for mathematics and for our learners.

(4) About Connecting "Students' Questions"

① What it means to connect *students' questions*

To our research group, the meaning of "connecting students' questions" refers to students' questioning during a lesson; i.e., students ask questions about evidence, previously learned knowledge, and the similarities and differences among solution ideas. Students establish an understanding of what they discussed together with

[8] Yoshishige Sugiyama (2006) 確かな算数・数学教育をもとめて [Seeking Certain Mathematics Education]. Pp.73 – 79.

others and connect this understanding to other students' questions. Students continue to connect the series of questions to solve the problem posed in the lesson.

We, as teachers, should not go along with a series of questions that may randomly come up from students. Rather, we need to carefully design questions that appear in instructional materials that are to be used by students and teachers, so that the value of questions asked by students meet the standard of questions that students should be asking. Students, teacher, and instructional materials are all important for raising the value of questions. A student might raise the value of questions by connecting a series of his own questions. A teacher might pick a student's question or statement and use it to raise the value of the discourse to meet the level of what we are calling "students' questions that should be asking." By carefully selecting and designing a problem that facilitates students' asking questions that should be asking, students might naturally come up with questions that raise the value of questions to reach "students' questions that should be asked." Even when students' thinking slows down, the teacher may direct students to reread and think again about the problem, which triggers students' thinking of a new question. This new question may lead to other questions which help deepen students' learning.

At the end of a lesson, when we think about how to evaluate the deepening of student learning, we could evaluate the questions that students asked, the questions used from the instructional materials, and the teacher's questions. For example, some evidence for concluding that the learning was deepened by connecting questions may include the observation that students', teacher's and instructional materials' questions matched, and the series of questions met the criteria of "students' questions that should be asking." Conversely, reasons why questions may not connect well could include evidence of a gap between the questions students asking and the question(s) that the teacher asking or had in mind, which resulted in the teacher being unable to facilitate students' questions that meet the criteria for "students' questions that should be asking."

② From the students' point of view

From an analysis of past National Scholastic Ability Test results, Nakamura[9] (2012) cited a high rate of non-response to questions, so it is important to decrease the number of non-response questions and discuss incorrect solution ideas or misunderstandings and misconceptions during lessons. This practice will help students develop an enriched view and thinking that leads to students' acquiring certain target knowledge and skills. To deepen student learning, it is important that students think of their own questions. Even if a student cannot think of his own questions, by sharing what he doesn't understand and listening to what other classmates don't understand, the student is likely to see more clearly what is wrong and to apply this new insight. This process helps the student to modify his original question or think of an entirely-new question. Moreover, the new question acquired helps lead the student to use the mathematically-valued "students' questions that should be asking" via learning through interactions with other students. A student's

[9] Yamanashi Prefecture's *Mathematics Education Alliance Magazine* (2012)

question might also become a question the whole class engages in and, thus, helps deepen everyone's learning.

③ From an instructional material point of view

Nakamura (1989) describes the characteristics that instructional materials and problems have that produce student questions. These characteristics include problems that can be solved by students on their own by using previously-learned knowledge, materials that produce a common task that students pursue, problems that can be solved in multiple ways and/or produce a range of agreement to disagreement or conflict and consent/understanding, and materials or problems that produce new tasks to pursue.

By designing a task that includes these conditions and engages students in problem solving, students' questions will improve and reflect deeper understanding of the content being studied. These student questions lead to establishing subsequent new questions, and the process continues. This process of student questioning facilitates students' view of and thinking about mathematics, which in turn nurtures the talent and ability of our students to think deeply and mathematically.

④ From the teacher's point of view

Students come up with many different kinds of questions during a lesson. From these questions, teachers need to use the students' voices wisely to orchestrate and formulate a question to pose to the class to consider. By following how the teacher uses the questions and ideas posed by classmates, the students who could not think of their own question(s) will learn and begin to think about questions in the future. In addition, the teacher's move helps foster students who continue to think of and ask questions on their own. However, there are cases when the students' questions and the question the teacher is aiming for do not match. This mismatch may happen even when teachers grasp the students' state of learning and design lessons with well-thought-out instructional materials and problems. In cases like these, the teacher needs to redirect a line of questioning by asking questions that correspond to students' questions but flexibly move student thinking to a place where it will more closely approach the teacher's intended line of questioning.

We would like to foster students who deepen their learning by making the connection among the three important elements[10] that are important for developing lessons and questions.

3. Connection to School's General Research

(1) Image of "Lessons that Approach to the Essence of Mathematics Department Goals"

In order to conduct lessons that approach the essence of mathematics department goals, students have to operate on their mathematical views and think through mathematical activities to increase their talent and ability to think mathematically. In the school's general research document, it defines "Lessons that Approach the Essence

[10] Lecture by Professor Takashi Nakamura at 第５８回山梨県数学教育研究（峡南大会） 58th Yamanashi Prefecture Mathematics Education Research (Kyonan Conference) (2015)

of a subject's goals" as lessons that provide a structure for learning content by connecting individual knowledge and skills by operationalizing "views and thinking" that match the characteristics of a subject/content area, instead of merely increasing the amount of knowledge or skill without a general structure or framework. As we described in (2) in section 2, About Creating Lessons that Deepen Student Learning, students should not stop thinking (or asking questions) when they find an answer in the process of problem solving. It is important for students to continuously ask questions, such as "why does …?" and then answer these questions by asking about and looking for evidence or asking themselves and classmates if there is any other way to solve the problem or an easier way to solve the problem. We believe that problem solving includes connecting and continuing a line of questioning that leads to other new questions in the lessons that approach the essence of mathematics. Students' attitude about interdisciplinary learning, such as "I wonder if we can apply what we learned in the other subject areas?" is also important. Students' attitude about thinking uniformly and expansively is important. Students will ask questions, such as "What previously learned knowledge is connected with this learning?, What is the basis of this learning?, and Where can we apply this idea?" In this way, students are more likely to ask "questions that should be asked" that connect to their own mathematical views and independent thinking by and ultimately result in lessons that deepen learning.

4. Content of Research

(1) Measure of "Lessons that Approach the Essence of Mathematics Department Goals"

In order to conduct lessons that approach the essence of mathematics department goals, it is important students experience "questions that should be asking" during lessons and reflect on the result and process of problem solving, reconsider the result, come up with a new problem, etc. to think uniformly and expansively. Here are the three ways of measures we include in such lessons.

① Structure lessons that focus on "questions that should be asking"

In order to create lessons that connect questions to deepen student learning focus on students, instructional materials, and the teacher. Think about the questions that should be asking (questions students need to think about and what kind of "*hatsumon* (question)" the teacher needs to ask his/her students in order for them to recognize the questions that should be asking; i.e., the question the teacher is thinking about/has in mind. Moreover, think about what kind of instructional material would help students come up with/think of the questions that should be asked to deepen understanding and grasp the mathematics of the problem. Consider these points when we plan the lesson.

② Creating lessons that consider mathematical views and thinking clear to students

In order to foster students' talent and ability for mathematical thinking, clarify the mathematical views and thinking that are common in a unit and across units. By doing so, we gain foresight into students' learning and can better identify the mathematical views and thinking that students need to operationalize during the day's lesson. It is important to, first, analyze and organize the mathematical views and thinking in the unit plan and be explicit about the teaching notes and anticipated

students' responses, such as how students will express their views and think in words or writing. Then, when observing actual lessons, see if there are places during the lesson that students demonstrated the mathematical views and thinking the teacher anticipated and expressed in the lesson itself.

Based on reflections about what was observed, we engage in and investigate the following: a.) the mathematical views and thinking that are common between and across grade levels; b.) a plan to connect units across different domains using mathematical views and thinking, and c.) connections of the learning in the unit to the student's daily life. An investigation of these three things increases the likelihood that we will be able to conduct lessons that approach to the essence of mathematics.

③ Investigate activities that reflect students' own learning

As described in the general research document, in order to foster students' ability to "connect learning" it is necessary for students to reflect their own learning and thinking. During what situations in lessons do students reflect their own learning? For example, in the learning process of a lesson when students are solving problem on their own, they try to reflect their own learning in order to establish foresights into problem solving by thinking about how they could use what they previously learned to solve the problem they are tackling now. In addition, in the process of discussing and comparing solution ideas, students reflect their own learning when they identify the similarities and differences among the different ideas posed by classmates. In the process of writing their reflection of learning in the section of lesson looking back, students reflect their own learning of the past lessons including the lesson they just learned.

When conducting classroom practice and reflecting on a lesson in this way, teachers can investigate the situations students use to reflect their own learning. Moreover, teachers can analyze the quality of students' reflections and consider how effective the lesson was in its ability to approach the essence of mathematics department goals.

(2) Assessing "Lessons that Approach the Essence of Mathematics Department Goals"

There are several different ways to assess lessons. We use student reflection (writing in notebook) to understand student understanding. The reflection students write at the end of a lesson has four phases (Nakamura, 2002)

The first phase:

Students write their own feelings such as "Learning was fun." and "I want to learn again." There is no writing about the content of the mathematics they learned. This level of reflection writing could be seen in any other subject area students are learning.

The second phase:

Students write about the mathematical content, such as what part of the mathematics the student understands and what part the lesson student finds difficult to understand. In this phase students start to reflect on and write about their own thinking.

The third phase:

Students write about what they thought about other friends' ideas. They include friends' names in the writing, therefore, connecting specific ideas to a specific friend's thinking about the problem or task.

The fourth phase:

Students write a reflection about their own thinking. The writing also reveals their attitude about pursuing mathematically better ways to think about a problem or task by reflecting on their own learning.

By analyzing students' reflections about lessons, the teacher grasps what phase and where each individual student appears to be functioning and can better understand how students' writing about learning is changing. As students move up to the fourth phase of written reflection, their writing includes mathematical views and thinking that approach the essence of mathematics. By grasping how students' thinking changed and how their learning changed across recorded reflections, we can better assess if our lessons are getting closer to meeting the standard of "lessons that approach the essence of mathematics department goals."

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