Unit Title "The area of Geometrical Figures" Grade 5

○Instruction plan (13 lessons*)

- 1st～4th lessons: The area of Parallelogram
- 5th～8th lessons: The area of Triangle
- 9th lesson: The area of Trapezoid **(9/13 this lesson)**
- 10th lesson: The area of Rhombus
- 11th lesson: Figuring out to determine the areas
- 12th/13th lessons: Practice/Power Builder

○About this unit

This unit aims to think, explain and develop the formulas for finding the areas of parallelograms, triangles, trapezoids and rhombuses based on the ways to measure the area of geometrical figures previously learned. In this lesson, below mentioned 3 points will be used as means not only for determining the area but also for developing the formulas.

(1) Focus on effective ideas to develop the formulas

It can be expected that on the outlook stage, students will deliver many ideas by utilizing their previous learning, however, to develop a formula in one hour lesson, only 3 ideas will be presented intentionally.
①An idea that the area is a half of the geometrical figure previously learned
②An idea to split into geometrical figures previously learned
③An idea to move a portion of the geometrical figure without changing its area to a geometrical figure previously leaned
By focusing on ideas that are easy to develop the formula, students will be able to explain and conclude their own ideas more easily. Also, in the individual problem solving, students will develop mathematical expressions based on more than 2 ideas above, compare those with each other and summarize their thinking.

(2) Use worksheets

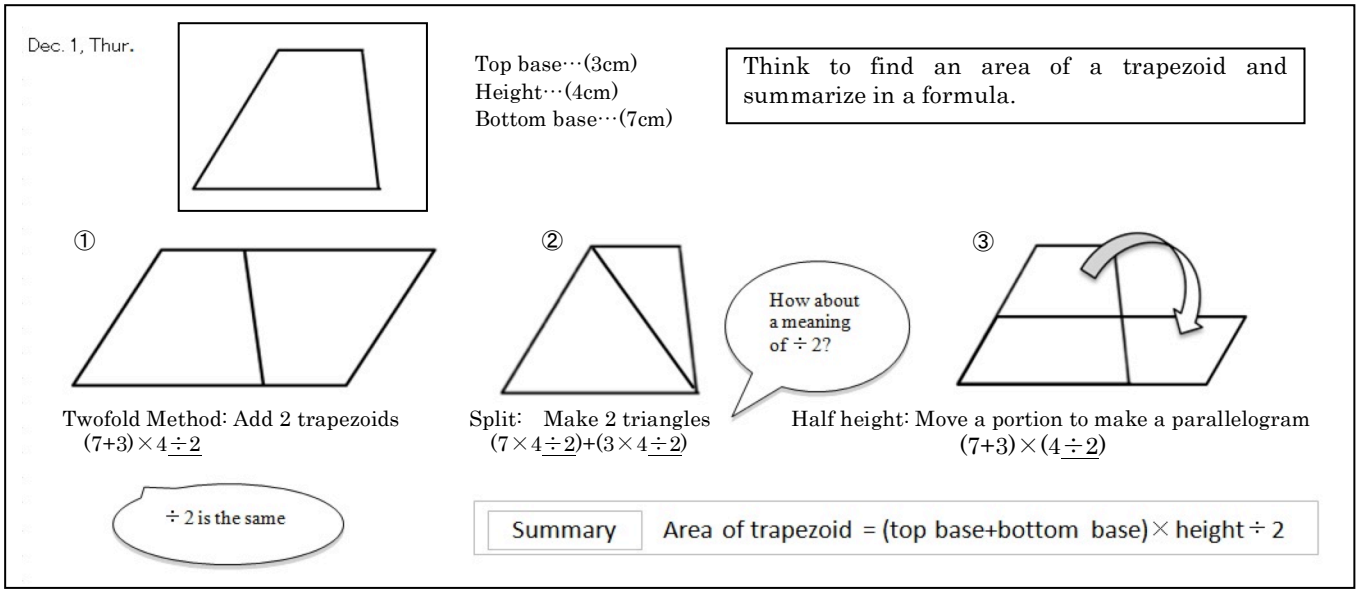
For this unit, worksheets with geometrical figures will be used in the lessons. This is for students to write down diversified ideas or friends' ideas immediately together with geometrical figures and also to acquire ways to represent their own ideas. In addition, when to think about formulas for new geometrical figures, students can go back to the area formulas they previously learned easily and to make use of those for their thinking.

(3) Ideas for presentation/discussion

・Help students to think and present that mathematical expressions they wrote during individual problem solving are connected with which one of 3 geometric figures presented. Also, not to have a student to present all, but to have another to tell the rest or to explain.
・Help students to realize numbers of 2, 3, 4, 7 and ÷2 are used in all the mathematical expressions. Especially about ÷2, help students to be aware of difference in meaning then to explain.
・If no idea to be presented by the students when to summarize ideas presented, ask the students "are there any similarities to the shape of ①?" or similar questions.

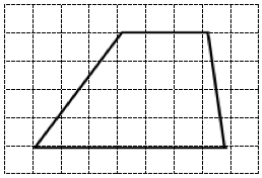
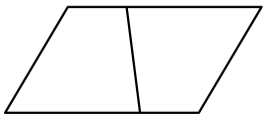
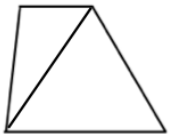
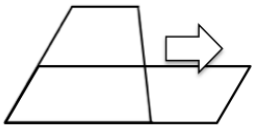
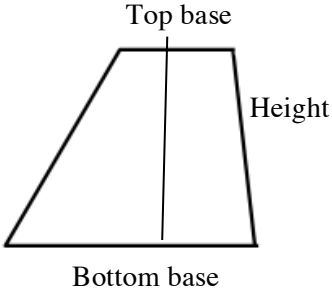
○Examples of anticipated responses by the students and instructional supports

In this lesson, 3 ideas will be discussed to develop the formula for finding the area of trapezoid by using the properties of operations. However, the students might want to discuss the following ideas too;
・Sprit the trapezoid into a rectangle and triangles.
・Sprit the trapezoid into a parallelogram and a triangle.
・Deform the shape of trapezoid without changing its area using triangles.
Since it is difficult for the students to integrate these ideas into a formula, these ideas will be left for the teachers to introduce and display.



○ **About the lesson (9/13)**

Objective: Determine the area of trapezoids and develop the formula for finding the area of trapezoid by using prior learning.

<table>
<tr><td>Problem</td><td colspan="3">T: Find the area of a trapezoid on the right and explain your idea.
(Prepare a diagram with grid so that the students can investigate lengths by themselves if they think its necessary.)
(Terms such as top base, bottom base and height to be taught in advance</td><td>  </td></tr>
<tr><td>Outlook</td><td colspan="4">T: To use ideas you have learned, to what shape the trapezoid should be changed?
C: Combining two trapezoids forms a parallelogram.
C: Drawing a line can make two triangles.
C: Draw a line and move it seems to form a parallelogram.</td></tr>
<tr><td>Individual-Problem -Solving</td><td>①
 
(7+3)×4÷2</td><td>②
 
(7×4÷2)+(3×4÷2)
=(7+3)×4÷2</td><td>③
 
(7+3)×(4÷2)</td><td></td></tr>
<tr><td rowspan="3">Presentation/Review</td><td colspan="3">T: Let’s explain 3 ideas.</td><td></td></tr>
<tr><td>C: ① is a parallelogram combining 2 trapezoids. A segment of the bottom base of the parallelogram is equal to the sum of the top and bottom sides of the trapezoid and the height can be used as it is. This area is equal to the area of two trapezoids so carry out ÷2.
(Twofold method)</td><td>C: ② is split into 2 triangles at the diagonal line. I drew the diagonal line to make triangles. I used the formula to determine the area of each triangle, then add answers delivered.
(Split method)</td><td>C: In ③, the height is halved to split into two trapezoids, turn upper trapezoid upside down to attach to the lower trapezoid, then it turns to be a parallelogram. I cut the trapezoid in half to make it a parallelogram. Turn upside down and attach each other makes a long and thin parallelogram.
(Half height method)</td><td></td></tr>
<tr><td colspan="4">T: Are there similarities in mathematical expressions or something you have noticed?
C: There is a similarity that the top and the bottom sides are added.
C: Properties of mathematical expression are used, split method of ② becomes the same to other algebraic expressions.
C: ( ) is taken away, twofold and half height methods have the same mathematical expressions.
C: There are ÷2, but meanings are different.
C: Twofold method of ① has ÷2, because two geometric figures are combined so ÷2 is carried out to find the area of the one.
C: Split method of ② represents ÷2 in the formula for triangles.
C: In ③, the height is halved so becomes ÷2.
C: (3+7)×4÷2 is the simplest.</td></tr>
<tr><td>Summary</td><td colspan="4">○Look back on the lesson.
T: Height is a side perpendicular to the bottom base.
Make a mathematical expression with words to find the area of trapezoids by using terms.
(top base + bottom base)×height÷2
○Write a learning essay. (Things understood today/things want to do next)
・By combining 2 trapezoids, the formula for parallelograms can be used.
・I understand meanings of ÷2 are different.
 
</td></tr>
</table>

Evaluation Did the students explain the process of finding the area of trapezoid with reasons?

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○About this unit

(1) Find out the area based on prior learning.

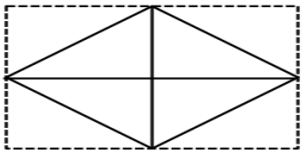
Help students to think based on ways to find the areas of geometrical figures previously learned by moving a portion of a geometrical figure without changing its area to make geometrical figures previously learned or by looking a geometrical figure as a half of the geometrical figure previously learned.

(2) Think meanings of numerical values in mathematical expressions and develop the formula.

Help students to develop the formula by making mathematical expressions based on the ideas previously learned and confirm which side should be looked at, all mathematical expressions have ÷2 and those meanings.

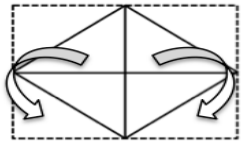
Help students to understand all ideas have diagonal lines as a part of a bottom base or a height.

C1: Split the rhombus into two upper and lower triangles and add both.



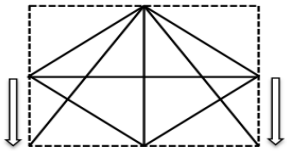
8×(6÷2)÷2×2=24　　24 ㎠

C2: Move upper triangles to the blank areas below to make a rectangle.



(6÷2)×8=24　　24 ㎠

C3: Split the rhombus into right and left and make a triangle by carrying out the peak shift.

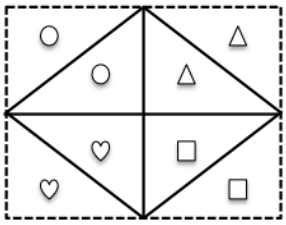


8×6÷2=24　　24 ㎠

These mathematical expressions are expected to be presented. Help students to realize 8 and 6 in those expressions are lengths of diagonal lines. Also, help students to focus on the idea that even meanings of ÷2 are different, it is a common numerical value in all mathematical expressions and to develop the formula.

○Examples of anticipated responses by the students and instructional supports

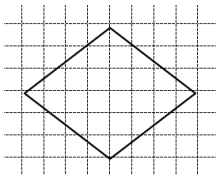
As other ideas, one is to regard there is a rectangle surrounding the rhombus and divide by 2 at the end. In this case, confirm each small triangle is congruence. Then in summary, help students to come down to the idea that diagonal line ×diagonal line in the formula is length×width of the rectangle surrounding the rhombus.



○Board-writing planning

Dec. 1, Thur　Think how to find the area of rhombuses and summarize in a formula

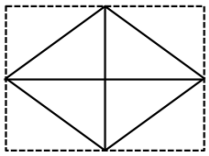
Rhombuses are…4 sides are equal in length.
Diagonal lines intersect perpendicularly.



How much ㎠ is the area of the rhombus?

Since it is doubled and divided by 2, these can be cut out

What does ÷2 mean?

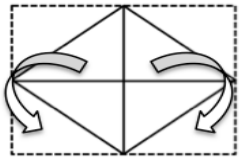


8×(6÷2)÷2×2=24
24 ㎠

Split into upper and lower triangles and doubled. Can be done by splitting left and right.

8 and 6 in expressions are lengths of diagonal lines

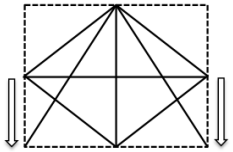
Height of the triangle is a half of 6cm



(6÷2)×8=24
24 ㎠

Move upper triangles to the blank areas below to make a rectangle.

Length of the rectangle is a half of 6cm



8×6÷2=24
24 ㎠

Split into left and right triangles and move each peak to make a large triangle.

All expressions have 8 times 6 and divided by 2

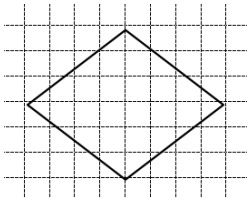
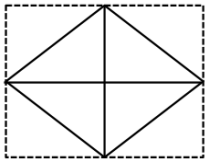
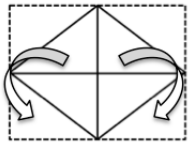
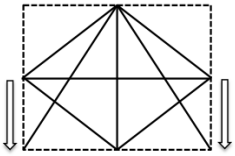
The formula for triangles ÷2

Summary: The area of rhombuses can be found by diagonal line×diagonal line÷2

† Each lesson is for 45 minutes

○ **About the lesson (10/13)**

Objective: Determine the area of rhombuses and develop the formula for finding the area of rhombus by using prior learning.

<table>
<tr><td>Problem</td><td colspan="3">T: What shape is a rhombus? Write down on a grid sheet.
T: How did you write?
C: I wrote diagonal lines intersect at the center at a right angle.
C: I wrote it as all lengths of 4 sides are equal.
T: Think about how to find the area of the rhombus and summarize in the formula.</td><td>  </td></tr>
<tr><td>Outlook</td><td colspan="4">T: Can you find out the area of rhombus by using ways to find the area of parallelograms or triangles?
C: I think it may be a half of a rectangle in area which surrounds the rhombus.
C: The shape is that two triangles are combined.
T: Write down your ideas by utilizing ways to find the area of geometrical figures previously learned..</td></tr>
<tr><td>Individual-Problem-Solving</td><td>①Split rhombus into 2 upper and lower triangles, and add both.
 
8×(6÷2)÷2×2=24 A 24 ㎠</td><td>②Move upper triangles into the blank areas below to make a rectangle.
 
(6÷2)×8=24 A 24 ㎠</td><td>③Divide the rhombus into left and right and carry out peak shift to make a triangle.
 
8×6÷2=24 A 24 ㎠</td></tr>
<tr><td>Presentation/Review</td><td colspan="3">T: In each mathematical expression, what is common?
C: Multiply 8 and 6. Divided by 2.
C: But in the mathematical expression on the left side, there are additional ×2 and ÷2.
C: I think carrying out×2 and then ÷2 means the answer is the same after all.
T: What does ÷2 in each mathematical expression mean?
C: In the first one, ÷2 is the one in the formula for triangles.
C: It is a half of the longitudinal diagonal line.
C: The second one means the length of the rectangle is a half of 6.
C: This is also a half of the longitudinal diagonal line.
C: In the third one, ÷2 is the one in the formula for triangles.
C: It means a half of the rectangle surrounding.
T: Each ÷2 has a different meaning. Now, what kind of mathematical expression is the simplest one?
C: Let's omit ×2 and ÷2 from the mathematical expression on the left side. So that it becomes the same as other two on the right side.
C: 6×8÷2 is the simplest one.</td></tr>
<tr><td>Summary</td><td colspan="3">○Confirm the formula.
C: 6 and 8 are the lengths of the diagonal lines.
T: You are right. Let's make a mathematical expression with words to find the area of rhombus by using a word of diagonal line.
The area of rhombus=diagonal line×diagonal line÷2
○Write a learning essay. (Things understood today/things want to do next)
C: Same as trapezoids, I understand the area can be found by changing it to triangles or rectangles.
C: Even by using any method, it becomes the same mathematical expression after all.
C: Different from geometrical figures previously learned, I understand I can find the area by using lengths of inner diagonal lines.</td></tr>
</table>

Evaluation Did the students explain the process of finding the area of rhombus with reasons?