

Grade 4 Mathematics Lesson Plan

Saturday, June 22, 2019
Grade 4 Classroom 2 (34 students)
Teacher: YAMAGUCHI, Kuniyuki

Let's think about ways to express the size of angles

1 About the unit

Prior to this unit, students have learned about rectangles and squares are special geometrical figures with right angles at all of their vertices in Grade 2. In Grade 3, they have learned about the definitions of isosceles and equilateral triangles and their properties, as well as how to construct those triangles using drawing tools. Moreover, they learned about the definition of angles as a geometrical figure and compare their sizes.

Building upon these prior learning, students will extend their understanding of angles as a geometrical figure to angles as the amount of openings created by turns. In addition, they will understand how to express the size of various angles using a right angle as a unit and express the size of angles as 1 right angles, 2 right angles, 3 right angles, 4 right angles, etc. However, since this way of expressing the size of angles can only be used with specific angles but not others, students will realize the need for a unit which is much smaller. The unit of degree ($^{\circ}$) is then introduced, and students learn to use a protractor to measure angles. In order to measure angles greater than 180° , students will learn about using addition and/or subtraction to figure out their measurements. They will also learn about how to draw angles and angles on their set squares.

In this lesson, students will make a variety of angles by combining the angles on their set squares. We will set up an opportunity for students to interpret their friends' ideas that have been expressed using words, diagrams, and equations. By making various angles through this activity, students will learn to represent angles flexibly and develop the capacity to interpret their friends' ideas.

2 Goals of the unit

- Students will be able to grasp the size of angles as the size of turns. [**Knowledge and understanding**]
- Students will know the unit of angle measurement degree ($^{\circ}$) and be able to measure angles. [**Knowledge and understanding**]
- Students will be able to represent the size of angles in flexible manners and apply it to examine geometrical figures while focusing on the size of angles in the figures. [**Capacity to reason, judge, and express themselves**]
- Students will be able to grasp geometrical figures in their surroundings and make use of them in their daily life while focusing on the size of angles. [**Disposition toward learning, personal characters**]

3 The relationship between the unit and the school research theme

(1) About “the image of approaching the essence of mathematics” and the unit

The mathematical ways of observing and reasoning that we want students to apply in this unit are representing the size of angles flexibly and making use of them in examinations of geometrical figures while focusing on the size of angles. The angles discussed in this unit include those that will be measured as 1, 2, 3, and 4 right angles as well as the angles in the set squares, 30° , 45° , 60° and 90° . We can also measure other angles using a protractor. Angles greater than 180° cannot be directly measured using a protractor. However, by considering those angles flexibly, for example, 210° is 30° more than 180° or 150° less than 360° , they can represent them using their prior learning. By considering angles that they want to measure in diverse ways, they can judge which way be a more efficient way to measure than others.

The mathematics department believe that by deepening students’ learning through the use of mathematical ways of observing and reasoning, students can approach the essence of mathematics. We also believe that contents being learned in this unit will allow students to examine geometrical figures such as parallelograms and rhombuses from diverse perspectives while focusing on the size of angles. Also, when students learn about scaled drawing, they may also focus on the size of angles in the drawings.

(2) About strategies necessary to “approach the essence of mathematics”

① Devise lessons that are centered around questions that must be asked

The Department of Mathematics value lessons that are centered around “questions.” In this unit, we try to design lessons centered around “questions,” so that “questions about prior learning,” “questions on similarities and differences,” and “questions about merits” will naturally arise during the lessons.

② Devise lessons that are based on mathematical ways of observing and reasoning

The types of mathematical ways of observing and reasoning involved in this unit are as follows: like length, liquid volume, and area, we can quantify the size of angles using a unit, for example, if we use a right angle as a unit, a full turn will be expressed as 4 right angles; also like length, liquid volume and area, angle measurements are additive.

③ Investigate the nature of activity that promote students’ reflection on their own learning

We want to examine carefully students’ learning processes such as while they are grasping the main learning task, devising a plan on how to tackle a problem, solving a problem independently, comparing and contrasting various solution approaches, and reflecting on their own learning.

4 Unit plan (Total of 9 lessons)

	Goal	Learning Task	Ways of observing and reasoning	Evaluation Standards
1	Students will understand that they can form many different angles by rotating a ray.	<ul style="list-style-type: none"> Investigate how the size of angles change Express the size of angles using a right angle as a unit 	Just like length, liquid volume, and weight, we can quantify the size of angles by selecting a unit and express the size in terms of	<ul style="list-style-type: none"> Students can grasp the size of a half turn and a whole turn using a right angle as an informal unit. [C]

2	Through the examination of a protractor, students will learn the unit of angle measurement, "degree (°)."	<ul style="list-style-type: none"> Examine the structure of a protractor Know that the unit of angle measurement, degree, and the relationship 1 right angle = 90° 	<p>the number of units.</p> <p>"If an angles is the same as ○ right angles together, we can say the size of the angle is ○ right angles.</p>	<ul style="list-style-type: none"> Students understand the structure of a protractor and the relationship, 1 right angle = 90°. [KU] Students will be interested in the size of 1 degree, and they investigate the structure of a protractor. They try to express the size of angles using a degree as a unit. [D]
3 4	Students can measure angles using a protractor.	<ul style="list-style-type: none"> Know how to measure angles using a protractor Students will measure angles by first estimating if they are greater/less than 90° 		<ul style="list-style-type: none"> Students have a sense of angle measurement such as judging if an angle is greater/less than 90°. [KU] Students can measure angles using a protractor. [KU]
5	Students will understand how to measure angles greater than 180°.	<ul style="list-style-type: none"> Understand how to measure angles greater than 180° Investigate the sizes of vertical angles. 	<p>Express the size of angles flexibly and measure angles based on angles that they have learned previously.</p> <p>"measure based on 180°"</p>	<ul style="list-style-type: none"> Students think about and devise ways to measure angles greater than 180°. [C]
6 7	Students can draw angles and triangles using a protractor.	<ul style="list-style-type: none"> Think about ways of drawing triangles Know ways to draw triangles using a protractor Draw angles of many different sizes Draw equilateral triangles using a compass, then measure their angles and verify that the angles are congruent 	<p>"measuring based on 360°"</p>	<ul style="list-style-type: none"> Students understand how to draw angles and triangles using a protractor. [KU] Students can draw angles and triangles using a protractor. [KU]

8	<p>Today's Lesson</p> <p>Students can make a variety of angles by putting together the angles on set squares.</p> <p>Students can interpret the ways to make angles by putting together the angles on set squares.</p>	<ul style="list-style-type: none"> • Make a variety of angles by putting together the angles of set squares, and think about angles created by others • Interpret the way others made angles and explain how they were made using diagrams and equations 	<p>By focusing on the angles on the set squares, express various angles that can be made by putting together the angles of set squares.</p> <p>"If we put together these angles, you can make a \circ angle."</p> <p>"These angles are getting bigger by 15°."</p>	<p>Students are trying to make a variety of angles by putting together the angles on set squares. [D]</p> <p>Given angles created by others, students can figure out and explain how set squares were put together using diagrams and equations. [C]</p>
9	<p>Verify the mastery of the unit contents and consolidate their understanding.</p>	<ul style="list-style-type: none"> • Tackle application problems 		<p>Students are making use of what they learned in the unit and engaged in a variety of activities.</p>

Evaluation domains:

[KU] Knowledge and understanding

[C] Capacity to reason, judge, and express themselves

[D] Disposition toward learning, personal characters

5 Today's lesson

(1) Date: Saturday, June 22 (10:00 – 10:45)

(2) Location: Yamanashi University Dept. of Education Attached Elementary School
Grade 4 Classroom 2

(3) Goals of the lesson

- Students can make a variety of angles by thinking about different ways of putting together the angles of set squares.
- Students interpret how angles were created by putting together the angles of set squares.

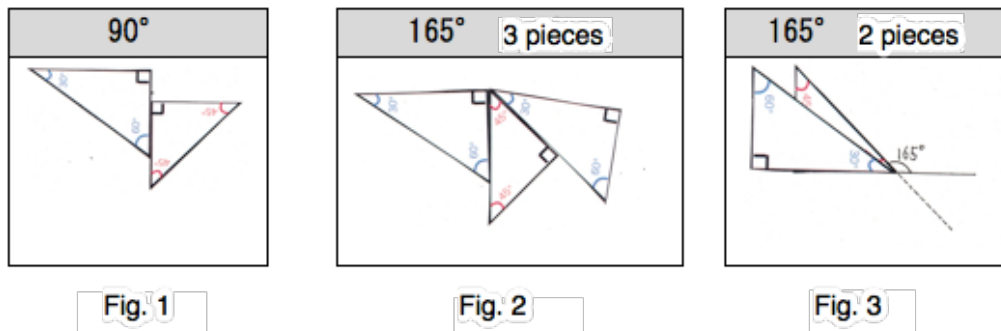
(4) Rationale of lesson design

Students have already learned about the measures of the angles in set squares (30° , 60° , 90°) and (45° , 45° , 90°). In today's lesson, students will engage in creating angles by using the angles from a pair of set squares, such as $30^\circ + 45^\circ$ or $45^\circ - 30^\circ$. The angles you can make by putting together the angles from a pair of set of set squares are: 15° , 30° , 45° , 60° , 75° , (90°), 105° , 120° , 135° , 150° , (165°), and 180° . These angles may be created either the sum or the difference of the angles on a pair of set squares.

Sum	30°	60°	90°
45°	75° 	105° 	135°
90°	120° 	150° 	180°

Diff.	30°	60°	90°
45°	15° 	15° 	45°
90°	60° 	30° 	0°

The table above does not show how a 90° angle can be made. However, one can be made outside of set square pieces as shown below (Fig. 1). In addition, a 165° angle can be made if we use 3 pieces of set squares (Fig. 2). A 165° angle cannot be made by using only a pair of set squares, however, if you can draw an auxiliary line, it is possible to create a 165° angle.



During the independent problem solving time, students will express the angles they made using words, diagrams, and equations.

During the whole class discussion, we will start with those angles that can be made by adding two angles (75°, 105°, 120°, 135°, 150°, and 180°). Then, the teachers will ask, “I wonder if it is possible to make an angle that is less than 75°?” This question will lead students to consider angles that can be made by looking at the difference of two angles on a pair of set squares. For both types of angles, we will represent the angles using words, diagrams, and equations. In this way, students will try to interpret others’ ideas and deepen their own understanding.

When we summarize the angles created in a table (and ordered from the smallest to the greatest angles), they realize that the angles are getting bigger by 15°. By organizing the angles in a table and examining it from a function perspective, students will realize that a 165° angle is missing. Students will then naturally be asking, “is it possible to create a 165° by combining set squares?” Based on that question, they may realize that $165^\circ = 180^\circ - 15^\circ$, but there is no 15° angle on a set squares. Another possibility is $90^\circ + 45^\circ + 30^\circ = 165$, thus, they realize that we can make a 165° only if we can use 3 pieces of set squares. We would like students to extend their learning by asking, “Is it possible to make an angle that is greater than 180°?”

We want to extend students’ capacity to reason and express themselves by engaging them in activities to communicate their problem solution processes and results using words, diagrams, and equations and to interpret other people’s ideas. Students should make use of these capacities in the study of other units.

(5) Flow of the lesson

Min.	<ul style="list-style-type: none"> • Main learning activity • Content ○ Anticipated students' responses 	<ul style="list-style-type: none"> • Instructional consideration ○ Strategies to "approach the essence" 																																								
5	<p>1. Grasp today's problem</p> <ul style="list-style-type: none"> • Verify the angle measures on a pair of set squares. ○ 30°, 60°, and 90° ○ 45°, 45°, and 90° <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Let's make different angles by putting together the angles from your set squares.</p> </div> <ul style="list-style-type: none"> • Verify that the students understand what it means to put together the angles from set squares. 	<ul style="list-style-type: none"> • Verify the angle measures on a pair of set squares. ○ Select a task that have a variety of solutions or a variety of solution processes. • Discuss an example of putting together the angles from a pair of set squares, help students have some ideas on how to approach the task. • Grasp students' ideas while circulating around the classroom. • Have students express their ideas in words, diagrams, and equations. • Provide students with small set squares so that students can possibly paste them in their notebooks to show how they created angles. 																																								
5	<p>2. Independent problem solving</p> <ul style="list-style-type: none"> • Make angles by putting together the angles from a pair of set squares. 																																									
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3. Whole class discussion

- Examine the angles that can be made by adding two angles.

○ 75° $30^\circ + 45^\circ = 75^\circ$



○ 105° $45^\circ + 60^\circ = 105^\circ$



○ 135° $45^\circ + 90^\circ = 135^\circ$



- Look back on the angles students have created.

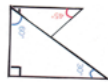
“ I wonder if we can make angles less than 75° .”

- Confirm that angles can be made by using the differences of the angles on the set squares.

○ 15° $45^\circ - 30^\circ = 15^\circ$



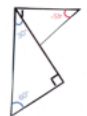
○ 30° $90^\circ - 60^\circ = 30^\circ$



○ 45° $90^\circ - 45^\circ = 45^\circ$



○ 60° $90^\circ - 30^\circ = 60^\circ$



- Start with those angles that can be made by adding two angles as they are easier to understand.
- Select the angle that is most common first.
- Although the same angle can be made in many ways, select one.
- Express own ideas using words, diagrams, and equations, and try to figure out other students' thinking.
- Think about commonalities and differences between own ideas and other people's ideas.

○ Students focus on the variety of angles that can be made using the angles on a pair of set squares and represent them. [Ways of observing and reasoning]

- If no students make an angle by looking at the difference of angles during the independent problem solving time, ask if it is possible to make an angle less than 75° .
- If a student creates an angle by looking at the difference of the angles, ask the student to share the angle.
- Ask if other angles can be made.
- Verify where the angle created by the difference of two angles by using diagrams.

<ul style="list-style-type: none"> Organize the angles they have made in a table. <p>“Let’s order all of the angles you have created so that it is easy to see them.”</p>	
<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, (90^\circ), 105^\circ, 120^\circ, 135^\circ, 150^\circ, (165^\circ), 180^\circ$ </div>	
<ul style="list-style-type: none"> It’s increasing by 15°. <ul style="list-style-type: none"> Verify if the angle measures are indeed increasing by 15°. <p>“Are the angles really increasing by 15°?”</p> <ul style="list-style-type: none"> $45^\circ - 30^\circ = 15^\circ, 60^\circ - 45^\circ = 15^\circ$ There is no 165°. (There is no 90°.) <ul style="list-style-type: none"> Explore if it is possible to make a 165° angle. <p>“How can we make a 165° angle?”</p> <ul style="list-style-type: none"> $180^\circ - 15^\circ = 165^\circ$, but there is no 15° angle on a set square. So, we can’t make 165°. $90^\circ + 45^\circ + 30^\circ = 165^\circ$, so we can make a 165° if we can use 3 set squares. <ul style="list-style-type: none"> Think about if it is possible to make other angles. <p>Are there other things you want to try?”</p> <ul style="list-style-type: none"> I want to make angles that is greater than 180°. I want to use 3 pieces of set squares. I want to make angles using 2 pairs of set squares. 	<ul style="list-style-type: none"> By looking at the angle measurements from a function perspective (looking at how the measurements are changing), students will try to make an angle that is not in the table. [Ways of observing and reasoning] <ul style="list-style-type: none"> Build this exploration on students’ desires to create a 165° angle, which is not in the table. Verify what combinations of angle measurements can make 165° using words and equations. <ul style="list-style-type: none"> Pose the question so that students can extend their reasoning. Ask if we can make other angles, for example, by changing the number of set squares.

5	<p>4. Looking back</p> <ul style="list-style-type: none"> ● Write the summary of the lesson. ● Write the learning reflection. <ul style="list-style-type: none"> ○ The measurements of the angles we can make with a pair of set squares increases by 15°. ○ I want to make angles using 3 set squares. ○ I wonder if we can make angles greater than 180°. ○ When we organized the angles in a table, we could see that the measurements were increasing by 15°, and we can also find an angle that was not in the table. 	<ul style="list-style-type: none"> ● Look back based on the board writing. ● Assess the lesson by observing students' learning reflections.
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(6) Assessment of the lesson

- Students can make a variety of angles by thinking about different ways of putting together the angles of set squares.
- Students interpret how angles were created by putting together the angles of set squares.
 - Were we able to pull out the ways of observing and reasoning we wanted in this lesson through questions that must be asked?
 - Were students using words that related to mathematical ways of observing and reasoning on their own?
 - Were the reflections conducted during the lesson effective?

(7) References

Omitted