June 28

Grade 6, Mathematics Lesson Plan

Date & Time: 5th period (13:35 – 14:20), Wednesday, June 28, 2017
Students: Grade 6 Students at Ohta Ward Kojiya Elementary School (62 students)
Teacher's Name: Hiroki Shibata, head teacher (Advance Class, 30 students)
Yukiko Kobayashi, head teacher (Regular Course: 24 students)
Koichi Hashimoto, head teacher (Basic Course: 8 students)
Place: Gymnasium, Grade 6 Class 3 classroom, and No. 3 study room

2017 Research Theme of Ohta Ward Mathematics Education Research Group
"The Instruction and Assessment that Activate Mathematical View and Reasoning, and Deepen Learning"
~ Focused on Mathematical Activities ~

1. **Name of the Unit:** Let's Think about How to Divide by Fractions (Division of Fractions)

2. **Goals of the Unit:**
   - Students understand the meaning and calculation process of division of fractions when the divisors are fractions, and develop an ability to apply their knowledge.

3. **Assessment Criterion of the Unit:**

	A. Interest, Disposition, Motivation	B. Mathematical Reasoning	C. Skills and Procedures	D. Knowledge and Understanding
Assessment Criterion of the Unit	Students show interest in the meaning and calculation processes of division of fractions when the divisors are fractions. They try their best to figure out solutions by connecting division with fractions to whole number division calculations and properties of calculations they learned previously.	Students are reason about ideas or strategies for how to solve division when divisors are fractions. Their reasoning is based on the property of division and proportional relationships. They express and describe the calculation process using the double number line model and math sentences (expressions)	Students demonstrate they can successfully calculate division problems when the divisor is a fraction and know how to apply strategies.	Students understand the meaning of division with fractions, specifically division when the divisor is a fraction.

Assessment Criterion that Corresponds To Learning Activities	① Students are interested in the meaning and calculation processes of division of fractions when the divisor is a fraction, and they show their best effort to solve these problems by making connections to the calculations and properties of calculations they learned before. ② Students notice the merit of reducing fractions in the process of calculations, realizing that reducing fractions allows them to calculate simply and easily.	① Students are able to use a number line and words to describe, represent, and understand why the expression is fraction ÷ fraction. ② Students are able to explain the process of calculation when problems are fraction ÷ fraction, using the property of division and proportional relationships. They accomplish this by using double number lines, diagrams, and expressions. ③ Students are able to recognize the merit of converting all numbers to fractions when multiplication and division involve calculations with a combination of fractions, decimals and whole numbers; students can also describe why this calculation method is better. ④ Students are able to explain -- using a double number line and words -- the reason why they need to establish a division expression that corresponds to a specific problem situation.	① Students are able calculate fraction ÷ fraction problems that do not involve reduction of fractions. ② Students are able to simplify calculations by reducing fractions in the process of calculating. ③ Students are able to do division of fraction calculations involving mixed numbers. ④ Students are able to do multiplication and division of fractions involving three numbers. ⑤ Students are able to do multiplication and division calculations involving a combination of fractions, decimals, and whole numbers. ⑥ Students are able to find “times as much” (rate) by division, even when the comparison quantity and the base quantity are fractions. ⑦ Students are able to find the comparison quantity from the base quantity and “times as much” (rate) when “times as much” is a fraction. ⑧ Students are able to represent the relationship between quantities with expressions that include division using *x*; they are able to find the value of the base quantity.	① Students understand the meaning of fraction ÷ fraction problems. ② Students understand the calculation process of fraction ÷ fraction. ③ Students understand that reducing fractions in the process of calculating fraction ÷ fraction helps simplify the calculations. ④ Students understand that the properties of calculations that work for whole number calculations also apply to calculations for division of fractions. ⑤ Students acquire basic knowledge and skills related to division of fractions.

4. Rationale of the Unit:

(1) About the Unit

The Course of Study (Notification) describing the content related to this unit:

The Content of Grade 6: A.) Numbers and Calculations

(1) Help students acquire the following by providing mathematical activities related to multiplication and division of fractions:
A. Students acquire these knowledge and skills:
(A) Students understand meanings of multiplication and division of fractions including cases when the multipliers and divisors are fractions.
(B) Students are able to calculate multiplication and division of fractions.
(C) Students understand that the same multiplicative relationships and properties for whole numbers apply to multiplication and division of fractions.
B. Students acquire the following thinking (reasoning), judging, and expressing ability.
(A) Students pay attention to the meaning of numbers and expressions, the properties of calculations, and are able to think about the process of calculation from multiple points of view.

In this unit, students will think about how to calculate division of fractions when the divisors are fractions; and they are able to successfully complete the calculation. Based on decimal division they learned in Grade 5, students understand that the meaning of division as the inverse operation of multiplication. Lastly, teachers help students to understand that division has two meanings: division that solves to find "how many times as many (as much)" (quotitive) and division that solves to find "what is the unit" (partitive).

In addition, teachers help students examine the properties of division applied to fraction calculations, i.e., (a) the identity property, or "when multiplying or dividing the dividend and the divisor by the same number, the quotient remains the same;" (b) the commutative property; (c) the associative property; and (d) the distribute property of fraction calculations. Examining the properties of division applied to fraction calculations leads students to see how the whole number calculations and properties that students learned in the past work also with fraction calculations.

In this lesson, we will help students think about and understand the calculation processes and meanings of division with fractions in multiple ways. Students will engage in mathematical activities that help them understand the process of calculation of division with fractions by applying the properties of calculations, and support students understanding of division by fraction visually using area models. By connecting the expressions and diagrams, helping students to use mathematical view and thinking, develop deeper understanding of calculation process of division with fractions instead of just establishing a shallow understanding such as "multiply the inverse of the divisor."

(2) About the Students

We administer a readiness test to the students and decide what course the students need to talk. We usually administer the test at the beginning of every unit, however, we decided to keep the same group of students in each course and provide continuous instruction for the previous unit.

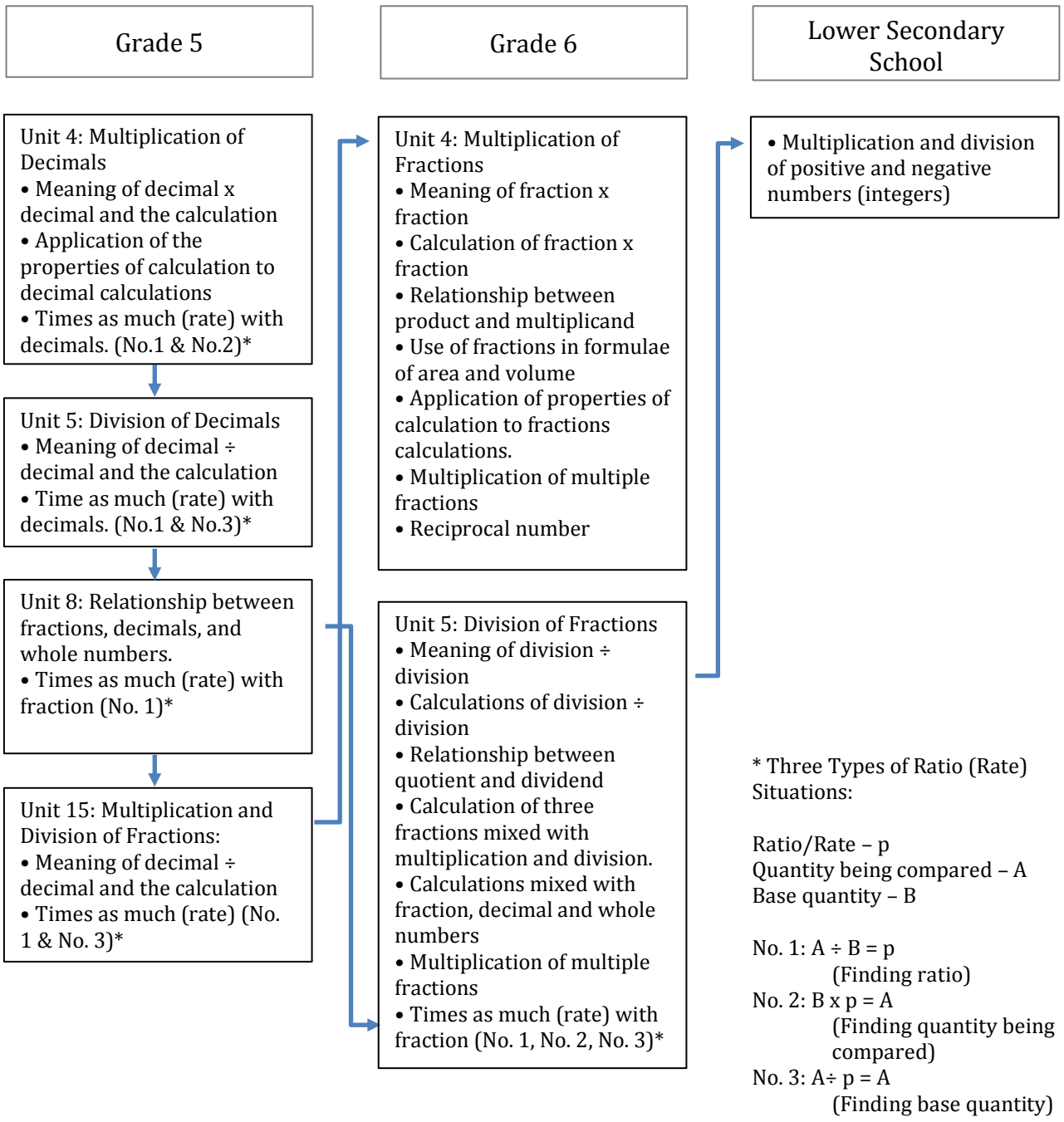
Regarding the advanced course: There are many students who grapple with mathematics problems willingly relatively to others, however, there are only a few students who could express his/her own thinking with basis and present his/her ideas in classroom. Because only a few students presenting ideas regularly, we incorporate pair learning and small group learning and provide more opportunities for the students to practice

describing their ideas to others intentionally. From the analysis of the readiness test we found out that 36% of the students (11 students out of 30 students) already know how to find the quotients of the division of fractions. In the previous unit, "multiplication of fractions," we found out that there are 76% of students (23 students out of 30 students) who knew how to find the products of multiplication of fractions, there are only few students who understand the meaning and able to describe the calculation processes with understanding. Therefore, we believe that there are almost none of the students who understand the meaning of division of fractions.

Regarding the regular course: the students are using notebooks actively, and studying willingly. However, there is a large gap in mathematical knowledge and skills among the students in this course. Because of this circumstance, we use a cooperative learning approach of learning that the students who understand fast help others, or asking the students who do not understand gather near that front of the room where the teacher provide small group instruction or support.

Regarding the basic course: Although there are many students who could come up with expressions from the problem situations, many of the students cannot solve the problem completely because of the lack of basic calculation understanding and skills. However, these students often ask questions that could be the center of the learning, and there are many students who could admit they do not know and be able to say "I don't know" frankly. Because of this circumstance, we identify the stumbling blocks of their learning and provide appropriate supports. In order to help the students we also use ICT equipment in this course so that we could support students understanding.

5. Scope and Sequence of Related Topics:



* Three Types of Ratio (Rate) Situations:

Ratio/Rate – p
Quantity being compared – A
Base quantity – B

No. 1: A ÷ B = p
(Finding ratio)
No. 2: B x p = A
(Finding quantity being compared)
No. 3: A÷ p = A
(Finding base quantity)

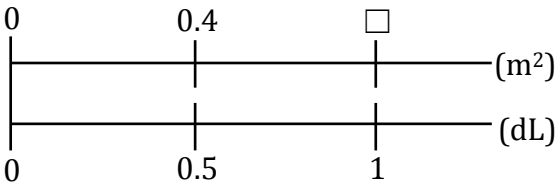
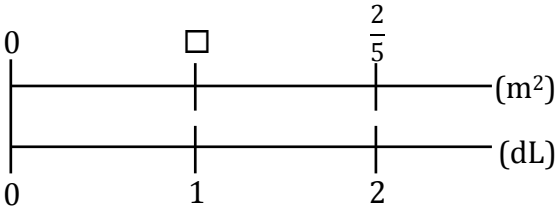
7. The Overview of the Previous Lesson (The 1st lesson of the 11 lessons)*

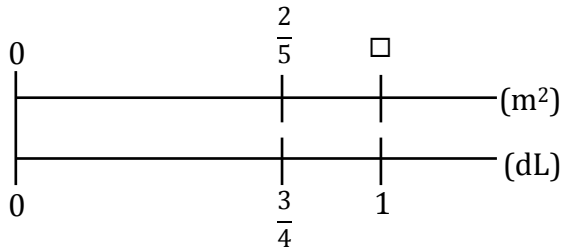
* Differentiate instruction courses: Advanced, regular, and basic courses are indicated in the parenthesis in the text below.

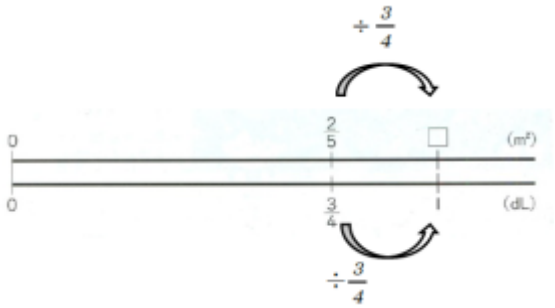
(1) The Goals of the Previous Lesson

- Students understand the meaning of "dividing by fractions" and are able to explain why the expression of the problem is fraction ÷ fraction.

(2) The Flow of the Previous Lesson

	○ Learning Activities	**Instructional Points to Remember [Evaluation], ○ Support**
Grasping Problem	○ Reviewing Division (Textbook p. 58) Review Problem 1: Please write a math sentence and find the answer. With 0.5 dL of paint, we could paint 0.4 m² of boards. What is the area of boards that we can paint with 1 dL of this paint? < Number Line >  < Math Sentence> 0.4 ÷ 0.5 = 0.8 Answer: 0.8 m² Review Problem 2: Please fill in △ with a whole number in, write down the math sentence, and find the answer. With △ dL of paint, we could paint $\frac{2}{5}$ m² of boards. What is the area of boards that we can paint with 1 dL of this paint? < Number Line >  	(Basic & Regular Courses) Provide the number line and help students to recall what they learned in the unit "Division of Decimals" in grade 5. (Advanced Course) Student do not solve this problem. • By covering the numbers in the problem, help student to make transition from whole numbers to fractions. (Basic & Regular Courses) By modeling the number line of Problem 1, draw number line with students in order to help them find the relationships between quantities. Help students to recall that "fraction ÷ whole number" which they learned in grade 5. (Advance Course) Pose only the problem, and see whether students propose using the number line. The should propose and use the number line as a tool to determine the operation. Once the number line is presented by students, it will be shared with other students as part of discussion. • The value for the △ is a fraction that is less than 1. So confirm with the students that the relations/location of the numbers on the number line changed.

	< Math Sentence> $\frac{2}{5} \div 2 = \frac{1}{5}$ Answer: $\frac{1}{5}$ m^2 ○ Students represent the relationships of quantities on the number line and think about the math sentence that finds the value in the □. Problem: Write a math sentence when we put $\frac{2}{3}$ (fraction) in △ and find the answer. < Number Line >  < Math Sentence> $\frac{2}{5} \div \frac{3}{4}$ I don't know how to do this calculation, because this is the first time I've divided a fraction by fraction. Let's explain why the math sentence is fraction ÷ fraction.	(Basic & Regular Courses) The divisor changed from a whole number to a fraction. Draw the number line together with the students. (Advanced Course) Establish the math sentence by asking students to construct the number line and discuss the differences between this problem and the Review Problem 2. [Students are interested in the meaning and calculation process of fraction ÷ fraction and they are reasoning by trying to connect the new learning with previous learning about properties of calculations and the property of division.] (Observation, speaking/presentation)
Independent Problem Solving	○ Students think about why the math sentence is $\frac{2}{5} \div \frac{3}{4}$. (1) The value of △ changed from a whole number to a fraction but the question for finding the area that we can paint in 1 dL has not changed.	○ Help student pay attention to the difference between the problem used at the introduction and this problem.

	(2) Drawing the number line, and add the arrow that shows the relationship of numbers; understands that you need to do division to find the value in the ☐.  • $\square=\frac{2}{5}\div\frac{3}{4}$ $(\square\times\frac{3}{4}=\frac{2}{5})$ (3) If we think about the operation using word math sentence, (Area painted) ÷ (Amount of paint used (dL)) = (Area we can paint using 1 dL), we can write the math sentence as division.	• Ask students to draw a number line. Also help them to recall that they used ☐ to represent the value that they don't know and how they drew an allow pointing toward ☐. • If the arrow starts form ☐, it represents a multiplication math sentence. If this idea comes out from the students, ask the student to share his/her idea. Then, connect the idea of multiplication with the division math sentence. * In the textbook, the arrow start from 1. However, at this school, we instruct students to draw the arrow point toward the ☐. (All Course) O If the idea of (3) does not come from the students, provide three word cards so that students can construct the word math sentence; or create an arithmetic restoration problem using words.
Presentation And Discussion	O Students present their ideas • Identify the students who have the ideas (1), (2), and (3), and ask them to present the ideas in the class. O Students discuss their ideas • Discuss and check the idea of (1), (2), and (3).	• Provide opportunities for the students to explaining ideas by incorporating a pair learning. Also ask student to explain other students' idea. [Students explain why the math sentence is fraction ÷ fraction using diagrams.] (speaking/presentation, notebook)
Summary and Refraction	O Students summarize the lesson • Even if the amount of paint used was expressed in a fraction, to find the area we can paint in 1 dL, we use division similar to what we did with whole numbers and decimals. (Basic and Regular Course) O Students write refraction about the lesson. (Advanced Course) O Students demonstrate foresight for calculation of fraction ÷ fraction. O Ask students to think about how to calculate fraction ÷ fraction problems using the property of division.	• Help students understand that the divisor could be a fraction beside a whole number or a decimal. (Basic & Regular Course) • Help students to establish a task for next lesson that is "how they can calculate the division of fractions whose divisors are fractions" and make connections to the next lesson. (Advance Couse) • Ask students to think about fraction ÷ fraction problems. Inform them that the next lesson will start with some of the students presenting their ideas.

8. Instruction of the today's lesson
(the 2nd lesson of the 11 lessons)

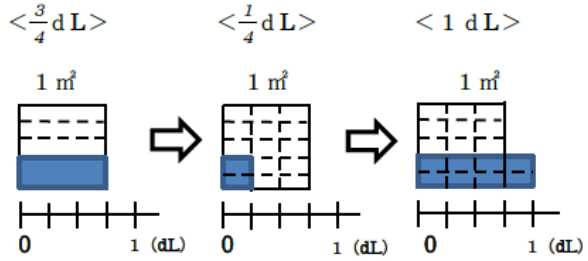
Advanced Course:
Place: Gymnasium
Teacher: Hiroyuki Shibata

(1) The Goals of the Previous Lesson

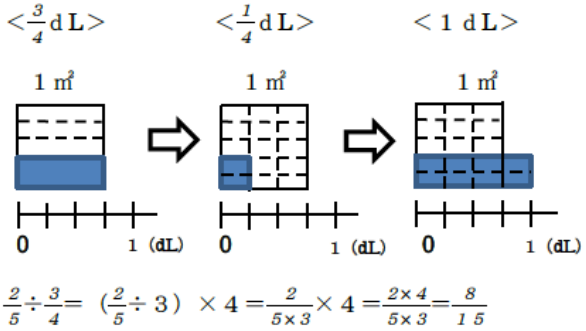
- Students explain and discuss the ideas of the calculations of fraction ÷ fraction and generalize the ideas to establish the generalizable formula.

(2) The Flow of the Previous Lesson

	○ Learning Activities	Instructional Points to Remember [Evaluation], ○ Support
Grasping Problem (5 min.)	**1. Reviewing learning from the last lesson and lean about the content of today's lesson.** T: What did we learn from the last lesson? C: We learned that the divisor of the division can be a fraction (as well as a whole number of a decimal). C: We thought about how to do the calculation when the divisor of the division is a fraction. T: Lets' confirm the task for today's lesson: Let's think about and explain how to do the calculation fraction ÷ fraction.	• Help students to recall that they established a math sentence $\frac{2}{5} \div \frac{3}{4}$ that represents the situation of the presented problem. • Talk with the students about how this was the first time they had a fraction as the divisor of a division problem.
Independent Problem Solving (5 min.)	**2. Confirming the process of calculation of $\frac{2}{3} \div \frac{3}{4}$ that was discussed about in the previous lesson.** C1: Make the divisor a whole number $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times 4) \div (\frac{3}{4} \times 4) = (\frac{2}{5} \times 4) \div 3$ $= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C2: Make the divisor 1 by multiplying it by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = (\frac{2}{5} \times \frac{4}{3}) \div 1$ $= \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$	• Select few students for the presentation. Pass out the mini white boards and ask them to write their thinking. • Ask students to put speech bubbles around the math sentences and the process of calculations as a way to make the explanation clear. * In class we write the process of calculation by aligning each part of the calculation at the equal (=) signs. However, in this lesson plan it is not written that way so that we can save space.

	C3: Make both dividend and divisor whole numbers by multiplying both number with the least common multiple 20. $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 20)\div(\frac{3}{4}\times 20)$ $=(2\times 4)\div(3\times 5)$ $=\frac{2\times 4}{3\times 5}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C4: Use the ideas of fractions as division. $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\div(3\div 4)=\frac{2}{5\times(3\div 4)}=\frac{2\times 4}{5\times(3\div 4)\times 4}$ $=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C5: Use invert and multiply the divisor. $\frac{2}{5}\div\frac{3}{4}=\frac{2}{5}\times\frac{4}{3}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C6: Use area model and expressions  $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\div 3)\times 4=\frac{2}{5\times 3}\times 4=\frac{2\times 4}{5\times 3}=\frac{8}{15}$	• The method C5 will not be shared in the class, because those using it are most likely using it as a memorized calculation procedure.
Presentation And Discussion (30 min.)	**3. Present and Discuss ideas.** T: Please bring the portable whiteboards that you wrote your solution to the front of the class and paste them on the board. C1: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 4)\div(\frac{3}{4}\times 4)=(\frac{2}{5}\times 4)\div 3$ $=\frac{2\times 4}{5}\div 3=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ T: Please write C1's idea in your notebook. T: When you finish writing, please talk with your partner. T: What kind of idea C1 is using?	• Select only 3 or 4 student ideas. • Read all students' notes from the previous lesson and select the students for presentation. • Show students the mini whiteboard with writing of math sentences. First students will think about the problem on their own and write down there thinking in the notebooks. Then, each will work with partner to discuss the process of calculations. ○ If I see student pairs having difficulty, I will ask them to come to the board, and think about the problem together. • If students want to add some explanation to the math sentences, ask them to write

	C: Making the divisor a whole number. C: Multiplying the dividend and the divisor by 4. C: The method uses the property of division. T: What is the property of division? C: Even if you multiply the both the dividend and the divisor with the same number, the quotient stay the same. T: Do you have anything you want to add, C1? C1: I think the explanation was great. T: The solution was using the property of division and making the divisor a whole number, wasn't it? T: Next, we see the person used different method. Do you understand the continuation of this math sentence? Please write it in your notebook. C2: $\frac{2}{5}\div\frac{3}{4}=\ (\frac{2}{5}\times\frac{4}{3})\ \div\ (\frac{3}{4}\times\frac{4}{3})\ =$_____ C: $(\frac{2}{5}\times\frac{4}{3})\ \div 1=\frac{2}{5}\times\frac{4}{3}=\frac{2\times4}{5\times3}=\frac{8}{15}$ T: What idea did he/she use? C: he/she used the same idea as the last one. C: To make the divisor 1, inverse of the divisor was multiplied to both the dividend and the divisor. C: The last method, both the dividend and the divisor ware multiplied by 4 but this time, both of them were multiplied by $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$, in order to make the divisor a whole number. T: This method change the divisor $\frac{3}{4}$ into 1. T: How about this one? C3: $\frac{2}{5}\div\frac{3}{4}=\ (\frac{2}{5}\times20)\ \div\ (\frac{3}{4}\times20)$ $=\ (2\times4)\ \div\ (3\times5)$ $=\frac{2\times4}{3\times5}=\frac{8}{15}$	explanation in a speech bubble. • Record student presentations and discussion on the board. • Select a student's work on the board and ask other students to describe his/her thinking. Also ask if students need to add something got the explanation. • Use projection equipment to show student writing in the notebook. Show only a part of the expression and ask other students to think about the continuation of the calculation. ○ Ask students to compare with the method C1 and ask how the ideas different. • Just like before, if students want to add some explanation to the math sentences, ask them to write it and use a speech bubble. • When students finish explaining the calculation process, the mini whiteboard will be posted on the board. Use the whiteboard examples to conduct the class discussion. • Introduce new ideas by asking student to bring their work as written the mini white board. • Select students to explain the solution.

	C: This time it is multiplied by 20. Why it is multiply by 20? C: I see, 20 is the least common multiple of 5 and 4. C: When both the dividend and the divisor is multiplied by 20, both of them become whole numbers. T: I see, 20, the least common multiple of 5 and 4, was used to multiply both the dividend and the divisor. T: These three solution methods used math sentences to explain the process of calculation. Here is another method. This time she/he used something other than math sentences. C: I think the area model. C6: I used an area model and think the process of the calculation.  $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\div 3)\times 4=\frac{2}{5\times 3}\times 4=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ T: Let's look at it and figure out who she/he thought. C: First, the paint $\frac{3}{4}$ dL can paint $\frac{2}{5}$ m^2 of the boards, so it is like the diagram to the left. T: Okay, I will ask another person to describe next. C: The middle diagram shows the part that $\frac{2}{5}$ m^2 of paint can paint. T: Can we show that using a math sentence? C: We are splitting $\frac{2}{5}$ m^2 into three equal parts so it should be $\frac{2}{3}\div 3$. T: Let's pass the baton to the other person. So another person could continue to explain. C: We just find out the amount of boards we could paint with $\frac{1}{4}$ dL of paint. So we could paint 4 of that space for 1 dL of paint. (diagram on the right).	• The number 20 was not randomly selected to use for the calculation. It is important for the students to look into and determine why the number 20 was used. • Make sure to prepare large area models for use on the board. • Provide the students the handout that include the area model and number line. Ask students to study the models, identify what part of the math sentence corresponds with what part of the area diagram. Ask students to use colored pen to highlight the important ides. • Show the area model to the students, and ask them to take turns to describing the presented idea. • Use chalk as baton, students will take turns to explain the calculation idea and develop a cohesive explanation.

	T: So how can we show the part we can paint with 1dL with an expression. C: We need to multiply $\frac{2}{5} \div 3$ by 4. So it should be $\left(\frac{2}{5} \div 3\right) \times 4$. T: Does the area you painted in the diagram matches with the answer you got? C: Wow, the diagram shows 8 fifteenth. So, it is $\frac{8}{15}$. **4. Compare the ideas and find the commonality.** T: This time we came up 4 different solution methods. They appear to be different but can you also find the commonality among the solutions? C: The method presented by C1, C2, and C3 all use the similar idea. They all made the divisor into a whole number. C: The method C3 does not have $\frac{2\times4}{5\times3}$ but that is included in the expression of the methods C1, C2 and C3. C: Wow, the expression right before calculating the answer is the same. T: Which one are you talking about? Let's ask 3 people to draw lines where it shows what she/he said. The method C3 does not have that so it is left out. C: Well, it has it also. You can see $\frac{2\times4}{5\times3}$ in the process of the calculation. If you switch the order of multiplication of the denominator from 3x5 to 5x3. C: Wow, we found $\frac{2\times4}{5\times3}$ in the method C also! So $\frac{2\times4}{5\times3}$ is common to all the methods. C: That means $\frac{2\times4}{5\times3} = \frac{2}{5} \times \frac{4}{3}$. T: Is there anything you notice when you compare the original math sentence and this sentence? C: When you compare $\frac{2}{3} \div \frac{3}{4}$ and $\frac{2}{5} \times \frac{4}{3}$, you can see the second fraction is changed into $\frac{4}{3}$, the reciprocal of $\frac{3}{4}$. C: Also, the operation sign ÷ (dividion) is changed into x (multiplication).	[Students are able to describe the processes of calculating fraction ÷ fraction problems using diagrams, expressions, and words.] (presentation/speaking, notebook, observation) • Although, the ideas and processes of calculation represented with math sentences are different, ask student to look for the commonality among those different solutions. Using the discussion, generalize the calculation methods in to a math sentence. • Ask students to underline the commonality using a colored maker. • It is not easy to identify $\frac{2\times4}{5\times3}$ is in the process of the method C3. Therefore, fist discuss the commonality among the methods C1, C2, and C4. Then establish the commonality with C3 by asking if students could see the similarity. • Ask students to recall the original math sentence, $\frac{2}{3} \div \frac{3}{4}$. Then ask them to look for the math sentence that are similar in the process of calculation.

Summary and Refraction	**5. Summarizing** T: Let's summarize the lesson. Please summarize how to calculate fraction ÷ fraction problem in your notebook. C: When we divide by a fraction, we need to change the divisor into a reciprocal. C: The operation sign ÷ (division) is changed into x (multiplication). D: Can we wright the calculation process using math sentences with letters, just like we did with multiplication? C: Yes. we can. T: Please complete the math sentence, $\frac{b}{a} \div \frac{d}{c}$. When you finish writing, please write your reflection of today's lesson. T: Let's have someone share what they wrote. C: I understand how to do the calculation of fraction ÷ fraction. C: I know that fraction ÷ fraction can be calculated by multiplying inverse of the divisor but I did not know why. Now I understand why … so I am happy! C: Using the area model, I understand how to calculate fraction ÷ fraction. C: There were several different methods for the calculation, but we found the commonality among them.	• Using students' voices and construct the summary. • Using words to describe the process of calculation. • Try to establish the rule of division of fraction with the students using letters and symbols to generalize. • Establish a calculation rule $\frac{2\times4}{5\times3} = \frac{2}{5} \times \frac{4}{3}$. [Students able to generalize and establish the idea of calculation of fraction ÷ fraction problems from several different ideas and create a calculation formula.] (Speaking/notebook/observation)

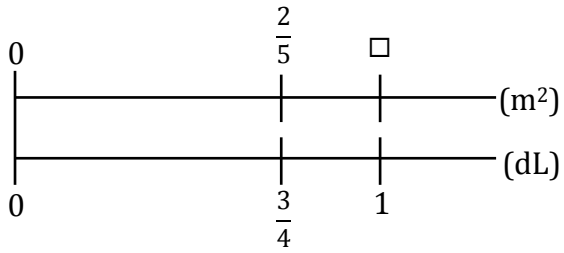
Regular Course:
Place: Grade 6, Class C
Teacher: Yukiko Kobayashi

8. Instruction of today's lesson (the 2nd of 11 lessons)

(1) The Goals of the Lesson

- Students think and explain how to calculate fraction ÷ fraction.

(2) The Flow of the Lesson

	○ Learning Activities	Instructional Points to Remember [Evaluation], ○ Support
Grasping the Problem (5 min.)	**1. Reviewing learning from the last lesson and lean about the content of today's lesson.** T: What did we learn from the last lesson? C: We learned that the divisor in division could be a fraction (we knew from our previous learning that the divisor can be a whole number or a decimal). T: What math sentence did we establish in the previous lesson? C: It is $\frac{2}{3} \div \frac{3}{4}$. C: We have not learned how to calculate when the divisor is a fraction. T: I see, let's confirm the goal of today's lesson: Let's think about and explain how to do fraction ÷ fraction calculations.	• Talk with the students about how it was the first time they had a fraction as a divisor in a division problem. • Confirm the problem situation and the math sentence from the previous lesson.
Independent Problem Solving (5 min.)	**2. Estimate the quotient $\frac{2}{5} \div \frac{3}{4}$ that we thought about in the previous lesson.** T: What do you think the value of the quotient could be?  C1: If you look at the number line, the value should not be less than $\frac{2}{5}$. T: How can we calculate $\frac{2}{5} \div \frac{3}{4}$? C: I could calculate this if the divisor was a whole number. T: If that is the case, if we could change the divisor to a whole number using what we have learned before, you could calculate it, couldn't you?	• Show the number line that the class used in the previous lesson. • Ask students to estimate the quotient. ○ Ask students to share what they are thinking about and having difficulty with this calculation. Confirm with students that the calculation is difficult because the divisor is a fraction.

3. Think about how to calculate $\frac{2}{5} \div \frac{3}{4}$.

C1: Changing the divisor to a whole number by multiplying the divisor by 4.

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right) = \left(\frac{2}{5} \times 4\right) \div 3$$

$$= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

C2: Changing the divisor to 1.

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times \frac{4}{3}\right) \div \left(\frac{3}{4} \times \frac{4}{3}\right) = \left(\frac{2}{5} \times \frac{4}{3}\right) \div 1$$

$$= \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

C3: Changing both dividend and divisor to a whole number by multiplying both by 20, which is the least common multiple of the divisors, 5 and 4.

$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times 20\right) \div \left(\frac{3}{4} \times 20\right)$$

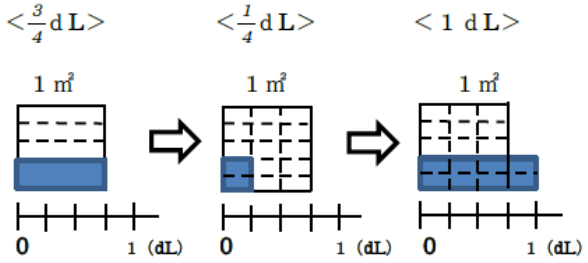
$$= (2 \times 4) \div (3 \times 5)$$

$$= \frac{2 \times 4}{3 \times 5} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

C4: Find the inverse of the divisor and multiply the dividend by it.

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

C5: Using an area model and expression



$$\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \div 3\right) \times 4 = \frac{2}{5 \times 3} \times 4 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$$

C6: I don't know what to do.

- For students who already figured out a solution method: Ask them to think about how to present their ideas to others clearly using a diagram, a math sentence, and words.

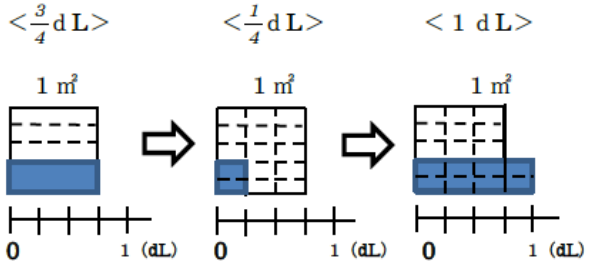
- When students come up with and show a solution method, encourage them to think about other solution methods.

- Do not ask students who used the "inverse and multiply" procedure to present their ideas to the class. Instead, ask them to think about why the quotient can be found by applying this procedure.

- If there is a student who is using the area model to describe the calculation process, provide a worksheet that has blank squares and number lines.

○ Prepare a hint card that says "even if the dividend and the divisor are multiplied by the same number the quotient remains the same." Give the card to students who are having difficulty thinking of a solution/idea.

Presentation And Discussion (30 min.)	**4. Discuss own ideas in small groups.** T: Let's talk about your solution ideas with your friends in small groups. **5. Present and discuss ideas.** T: Please bring your poster papers to the front and past them on the board. T: Let's start the presentation with the group No. 1. C1: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times 4)\div(\frac{3}{4}\times 4)=(\frac{2}{5}\times 4)\div 3$ $=\frac{2\times 4}{5}\div 3=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C1: We wanted to make a whole number divisor so we decided to multiply both the dividend and the divisor by 4. T: Is there anything that you did not understand from this explanation? C: Why did you multiply both the dividend and the divisor by 4? C: If you multiply only the divisor by 4, the answer will be wrong. C: We must use the property of division. T: What is the property of division? C: When you multiply both the dividend and the divisor by the same number, the quotient remains the same. T: I see you used the property to change the divisor to a whole number. T: Okay, how about Group No. 2? C2: $\frac{2}{5}\div\frac{3}{4}=(\frac{2}{5}\times\frac{4}{3})\div(\frac{3}{4}\times\frac{4}{3})$ $=(\frac{2}{5}\times\frac{4}{3})\div 1=\frac{2}{5}\times\frac{4}{3}=\frac{2\times 4}{5\times 3}=\frac{8}{15}$ C2: Our group decided to multiply both the dividend and the divisor by $\frac{4}{3}$. When we did that the divisor became 1 and we could find the answer.	• Walk around the classroom to observe and monitor student work. Select different methods among the groups and ask the students to write their methods on poster paper. • Provide an enlarged copy of the worksheet that has squares and number lines to the group that used the area model to think about the calculation process. • Even if some of the students have not come up with an idea and aren't finished, ask these students to stop their work, so the presentation can be started when the specified work ending time is reached. Use the presentation and discussion to complete the full description and understanding of each presented idea. • Choose only 2 or 3 ideas for the whole group presentation. • Do not show the written poster papers all at once. These are presented one-at-a-time as each presentation is made. • If there are additional explanations, ask students to share these explanations with the class. • Record important points on the board. • Show the hint card to the students and review the property simply.

	T: Do you have anything that you did not understand from their explanation? C: Why did you decide to multiply by $\frac{4}{3}$? C: Because they wanted to make the divisor into the whole number 1. C: A fraction that changes a given fraction to 1 by multiplying it (by the given fraction) is called the "reciprocal" of the given fraction. T: What more can we say about the idea this method used? C: I think this method used the same idea as the previous one. C: The number the group multiplied by was different from the last group's, but both methods made the divisor a whole number. T. I see. So you are saying there is a commonality between the two methods. T: How about Group No. 3. C5: We used the area diagram and the math sentence.  $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \div 3) \times 4 = \frac{2}{5 \times 3} \times 4 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ T: Let's listen to how they were thinking carefully. C: First, you could paint $\frac{2}{5}$ m² of the board with $\frac{3}{4}$ dL of the paint, so the diagram is like shown on the left. C: Next, the middle diagram shows the area of the board that $\frac{1}{4}$ dL of the paint can cover in paint. T: How could we represent that part with a math sentence? C: We are dividing $\frac{2}{5}$ m² by three (3) equal amounts. So the math sentence would be $\frac{2}{5} \div 3$.	• If there is anything the students might want to add, ask these students to present. • Record the points of discussion that come from the discussion among students. • Prepare an enlarged worksheet for the board. • Paste the area diagram on the board and ask the students to explain and build their understanding of the calculation method by taking turns to describe the method. • Provide a worksheet that contains area diagrams and number lines to all students, so those who are trying to follow the explanations will have a chance to draw the diagram, also. • The explanation of the method should not end in explaining the methods with the diagrams only. It is important to connect the diagram with the math sentence and clarify what parts of the diagram are represented in the math sentence.

	T: Then what happens next? C: We want to find out the area of the board that we can paint with 1 dL of the paint. So if we shade this 4 of the area that $\frac{1}{4}$ dL of the paint could paint. (See the diagram on the right in the above illustration.) T: How could we express the last part with a math sentence? C: Because we need to multiply [$\frac{2}{5} \div 3$] by 4, so it will become $\left(\frac{2}{5} \div 3\right) \times 4$. T: Could you check if the area shaded on the diagram matches the calculation? C: Wow, it is 8 fifteenths so it is $\frac{8}{15}$! **6. Compare ideas and find the commonality.** T: This time we found 3 different solutions methods. They are different, but is there any commonality among the three methods? C: All of the methods C1, C2, and C3 are changing the divisor to a whole number. C: All of the math sentences include $\frac{2\times4}{5\times3}$ in the process of the calculation. C: Wow, that is true. The last math sentence before finding the answer is the same. T: What part of the math sentence are you talking about? I want three of you to come up and underline where you see $\frac{2\times4}{5\times3}$. C. It is true, that is common to all the math sentences. T: If you take this part of this math sentence, $\frac{2\times4}{5\times3}$, how can we express it? C: $\frac{2\times4}{5\times3} = \frac{2}{5} \times \frac{4}{3}$ T: When you compare $\frac{2\times4}{5\times3}$ with the first math sentence, $\frac{2}{5} \div \frac{3}{4}$, do you notice anything? C: The denominator and the numerator of the divisor in $\frac{2}{5} \div \frac{3}{4}$ is switched. C: The operation sign ÷ is changed to x.	• Make sure to use different colored markers, so the $\frac{8}{15}$ can be seen clearly in the area model. [Students explain the process of calculating fraction ÷ fraction clearly using diagrams, math sentences, and words.] (Speaking/presentation, notebook, observation) • The methods or the ways that the math sentences are written is different among the presented ideas; however, help students to identify the commonality among these ideas. Drawing from this commonality, generalize the different math sentences into a single common math sentence. • By underlining given parts, the similarity is emphasized visually. • Ask students to compare the initial math sentence and the new math sentence. Ask them to identify the part(s) of the math sentence that are different from the original one.

Summary and Reflection (5 min.)	**7. Summarizing** T: Let's summarize the lesson. Please summarize how to do the calculation for fraction ÷ fraction problems in your notebook. C: When we divide by a fraction, we need to change the divisor into its reciprocal. C: The operation sign ÷ is changed into x … division changed to multiplication. T: Let's write the reflection of today's lesson. T: I would like some of you to share what you wrote. C: I understand how to do the calculation of fraction ÷ fraction. C: The area model helped me to understand what we are doing with the math sentence. C: The methods presented were different, but there is a commonality.	• Using students' voices to construct the summary. • Using words to describe the process of calculation, fraction ÷ fraction. [Students understand how to calculate fraction ÷ fraction.] (Speaking/notebook)

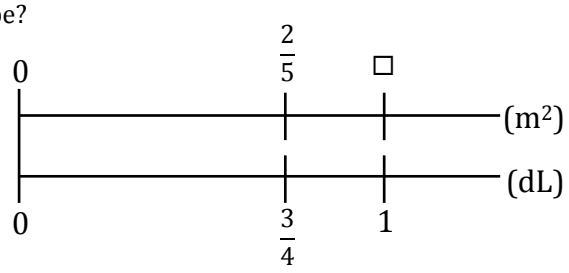
8. Instruction of the today's lesson
(the 2nd of 11 lessons)

Basic Course:
Place: No. 1 Study Classroom (3F)
Teacher: Koichi Hashimoto

(1)The Goal of the Lesson

- Students think about and explain how to calculate fraction ÷ fraction problems.

(2)The Flow of the Lesson

	○ Learning Activities	Instructional Points to Remember [Evaluation], ○ Support
Grasping the Problem (5 min.)	**1. Reviewing learning from the last lesson and learn about the content of today's lesson.** T: What did we learn from the last lesson? C: We learned that the divisor can be a fraction (besides a whole number of a decimal). T: What math sentence did we establish in the previous lesson? C: It is $\frac{2}{3} \div \frac{3}{4}$. C: We have not learned how to calculate when the divisor is a fraction. T: I see, let's confirm the goal of today's lesson: Let's think about and explain how to do the calculation fraction ÷ fraction.	• Discuss with students that it was the first time they had grappled with division where the divisors are fractions. • Confirm the problem situation and the math sentence in the previous lesson
Independent Problem Solving (7 min.)	**2. Estimate the quotient $\frac{2}{5} \div \frac{3}{4}$ that thought about in the previous lesson.** T: What do you think the value of the quotient could be?  C1: If you look at the number line, the value should not be less than $\frac{2}{5}$. T: How can we calculate $\frac{2}{5} \div \frac{3}{4}$? C: I could calculate if the divisor was a whole number. T: If that is the case, if we could change the divisor to a whole number using what we have learned before, you could calculate it, couldn't you?	• Show the number line that the class used in the previous lesson. • Ask students to estimate the quotient. ○ Ask students to share what difficulty they may have had in thinking about how to do this calculation. Confirm with students that the calculation is difficult, because the divisor is a fraction.

	T: Okay, if the divisor is a whole number, do you remember how to do the calculation? For example, how do you calculate $\frac{2}{5} \div 3$? C: When we are dividing with a whole number, we can multiply 3 to the numerator. T: Okay, let's use what we learned before to make the divisor into a whole number. **3. Think about how to calculate $\frac{2}{5} \div \frac{3}{4}$.** C1: Changing the divisor to a whole number by multiplying it by 4. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times 4) \div (\frac{3}{4} \times 4) = (\frac{2}{5} \times 4) \div 3$ $= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C2: Changing the divisor to 1. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = (\frac{2}{5} \times \frac{4}{3}) \div 1$ $= \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C3: Finding the inverse of the divisor and multiplying with it. $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ C4: Dividing the denominator by the other denominator, and the numerator by the other numerator. $\frac{2}{5} \div \frac{3}{4} = \frac{2 \div 3}{5 \div 4} =$ (Students don't know how to do this calculation.) C5: I don't know what to do.	• Make sure to review how to calculate when the divisor is a whole number. Use a concrete calculation problem example. • For students who already came up with a solution method, ask them to think about how to present their ideas to others clearly using a diagram, a math sentence, and words. ○ Walk around the classroom and grasp what students are doing. Provide a hint card to students who are having difficulty (like in the response C5). • C3: Do not ask the students who used the "inverse and multiply" procedure to present their idea to the class. Instead, ask those students to think about why the quotient could be found by using this procedure. • C4: Students who came up with this idea should be recognized because they are thinking about applying ideas learned from the previous unit, multiplication of fractions. However, the calculation 2÷3 cannot be done, so recommend to the students that they think about another method.

Presentation And Discussion (28 min.)	**4. Discuss ideas with a partner.** T: Let's talk about your solution ideas with your partner. **5. Present and discuss ideas.** T: Please share your ideas with your friends. C: I multiplied the divisor by 4. I have done the solution up to that point, but I could not finish it. T: What is the math sentence? C: $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \div \left(\frac{3}{4} \times 4\right)$ C: If you multiply only the divisor the answer will be wrong. T: That is right. Do you remember the property of division? 200 ÷ 2.5 = 80 x 10 x 10 → The answer is the same! 2000 ÷ 25 = 80 C: Well, I don't remember the property. T: Is there somebody who can explain the property? C: When you multiply both the dividend and the divisor by the same number, the quotient remains the same. T: When you use this property, is it okay to multiply only the divisor by 4? C: I think we need to multiply the dividend also. T: Okay, then what should we do to the math sentence? The sentence will be: $\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \times 4\right) \div \left(\frac{3}{4} \times 4\right)$ T: So what is the divisor going to be? C: It will be 3. C: Wow, the divisor is a whole number now. I think I can do the calculation.	• Walk around the classroom to see what things/ideas students are grappling and struggling with. • Be sure to encourage students to write down their thinking, even if they are still in the process of finding a solution. • Choose two ideas for the presentation and discussion. ○ Even if the students have not completed their work, start the presentation. Through the discussion of methods, try to construct a way that helps students understand each method and completes the task. • Show the property from the textbook using projector. • Make sure to record the important points on the board. • Make sure to carry out the discussion so that as many students as possible participate and complete the calculation.

	C: $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times 4) \div (\frac{3}{4} \times 4) = (\frac{2}{5} \times 4) \div 3$ $= \frac{2 \times 4}{5} \div 3 = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$ T: Well, we could find the answer. What do you think about this idea. C2: $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3})$ T: When you look at this, can you tell what idea she/he is using? C: The last time both the dividend and the divisor was multiplied by 4. But this time both the dividend and the divisor is multiplied by $\frac{4}{3}$. C: Why did you decided to multiply by $\frac{4}{3}$? T: Let's calculate the divisor part of the math sentence. C: Wow, the divisor is 1! T: Wow, is that by chance? Do you remember the name of the fraction that changes a given fraction to 1 when multiplied by the given fraction? C: A fraction that changes a given fraction to 1 by multiplying [to the given fraction] is called the "reciprocal" of the given fraction. T: What can we say about the idea this method used? C: I think the method uses the same idea as the previous method. C: The number the group multiplied was different from the last group's number, but both methods made the divisor a whole number. T: Let's continue and calculate it. $\frac{2}{5} \div \frac{3}{4} = (\frac{2}{5} \times \frac{4}{3}) \div (\frac{3}{4} \times \frac{4}{3}) = (\frac{2}{5} \times \frac{4}{3}) \div 1$ $= \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}$	• If there is a student who came up with this idea, ask the student to present it. • Make sure to record the important points on the board.

	C: We do not need to calculate ÷1. C: The answer is the same as the other one. **6. Compare ideas and find the commonality.** T: This time we found two different solutions methods. They are different, but is there any commonality between the two methods? C: Both methods changed the divisor to a whole number. C: I can see the same math sentence, $\frac{2\times4}{5\times3}$ in the process of the calculation. C: Wow, that is true. The math sentence right before finding the answer is the same. T: What part of the math sentence are you talking about? I want two of you to come up and underline where you could see $\frac{2\times4}{5\times3}$. C. It is true, that is common to the two math sentences. T: If you take a part of this math sentence, $\frac{2\times4}{5\times3}$, how can we express it? C: $\frac{2\times4}{5\times3}=\frac{2}{5}\times\frac{4}{3}$ T: When you compare $\frac{2\times4}{5\times3}$ with the first math sentence, $\frac{2}{5}\div\frac{3}{4}$, do you notice anything? C: The denominator and the numerator of the divisor in $\frac{2}{5}\div\frac{3}{4}$ is switched. C: The operation sign ÷ (division) is changed to x (multiplication).	[Students able to explain how to calculate fraction ÷ fraction clearly using math sentences and words.] (speaking/presenting, notebook, observation) • The methods or the ways that the math sentences were written are different between the presented ideas; however, help students identify the commonality between the ideas. Using the commonality, generalize the idea into a math sentence. • By underlining important parts, the similarity is emphasized visually. • Ask students to compare the initial math sentence and the new math sentence. Ask them to identify the part of the math sentence that is different from the original one.
Summary and Refraction	**7. Summarizing** T: Let's summarize the lesson. Please summarize how to do the calculation fraction ÷ fraction in your notebook. C: When we divide by a fraction, we need to change the divisor into a reciprocal. C: The operation sign ÷ is changed into x. T: Let's write the reflection of today's lesson. T: I would like some of you to share what you wrote.	• Using students' voices and construct the summary. • Using words to describe the process of calculation, fraction ÷ fraction.

	C: I understand how to do the calculation of fraction ÷ fraction. C: The methods presented were different but there is a commonality.	[Students understand how to calculate fraction ÷ fraction.] (Speaking/notebook)