June 22

Grade 7, Mathematics Lesson Plan

Thursday, June 22, 2017
Period 6 (14:20 -15:10)
Grade 7, Classroom B
(20 boys and 20 girls)

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Attached to Tokyo Gakugei University

Location: TGU Conference room

1. The Research Theme and Its Intent

Designing lessons to raise the quality of student understanding of mathematical processes

(1) Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University

This lesson was developed as a part of the activities of the Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University. The purpose and rationale of our research group are as follows: In the Japanese mathematics education, we emphasize not only the mathematical content (results of explorations), but also the processes of exploring mathematical problems and the development of skills and ways of reasoning that are utilized during mathematical explorations.

Even with the emphasis on the process of mathematical explorations in tandem with the emphasis on mathematical content, we are concerned that mathematics teaching overwhelmingly focuses more on mathematical content than process. We are not suggesting that teaching mathematical content should be taken less seriously; nor are we implying that content and process should be considered as separate and distinct from each other. Rather, the concern of our research group results from a question that is critical to mathematics teaching and learning: Are Japanese mathematics lessons indeed emphasizing "mathematical ways of observing and reasoning" or "mathematical explorations/activities?" We have been discussing the importance of processes for a significantly long period of time, but are we seeing a significant emphasis in this domain or do lessons continue to focus primarily on content with process taking a secondary seat?

The idea of emphasizing mathematical processes is well substantiated in the research as the direction of Japanese mathematics education (Nishimura, et al., 2001; Shimizu, 2012; Fujii, 2016). In addition, concrete mathematics learning processes and supporting dispositions and abilities are described in the "Summary of the Discussion" of the elementary and the secondary mathematics working group, which also underscores the importance of mathematical process.

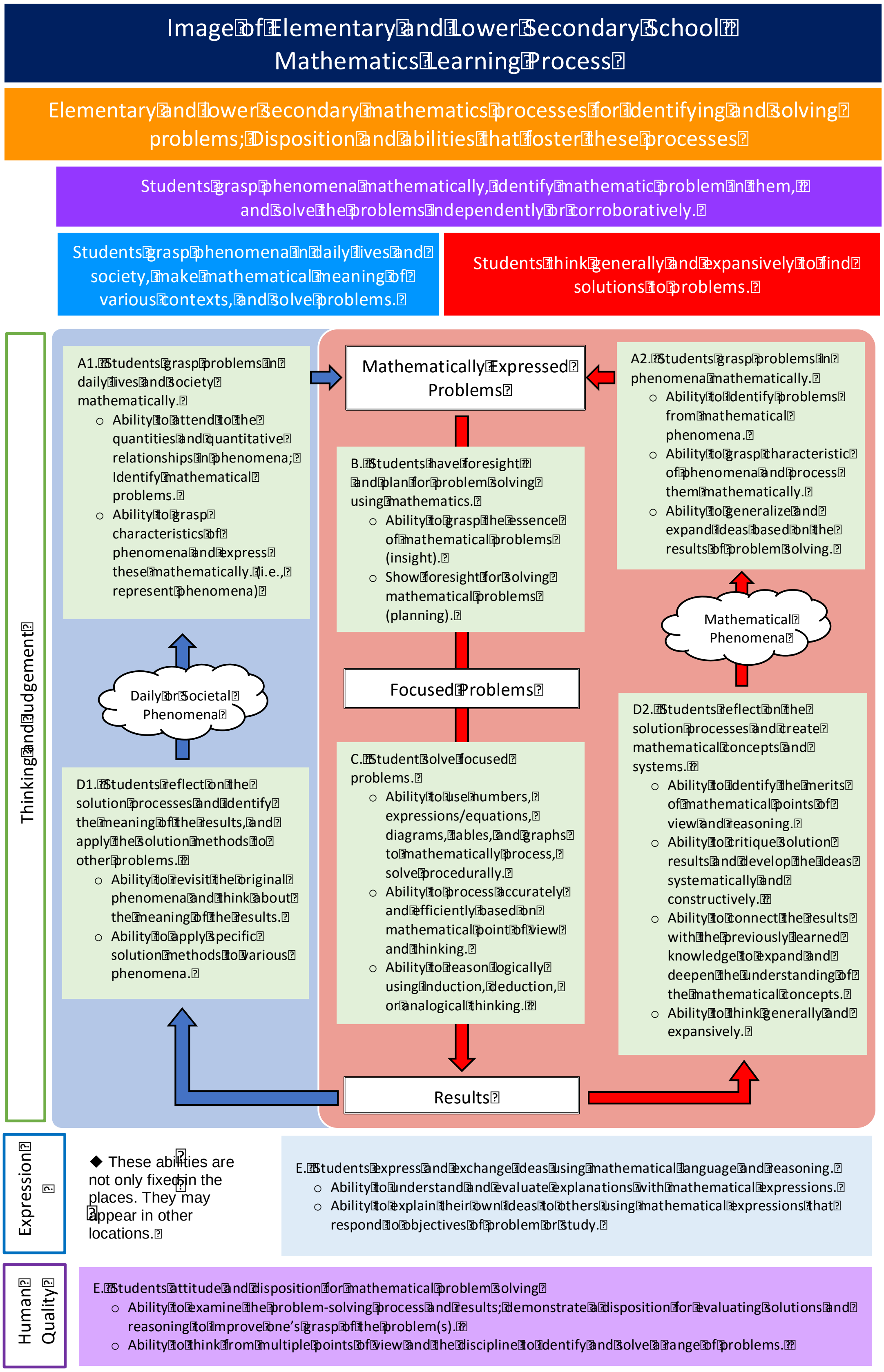


Image of Elementary and Lower Secondary School Mathematics Learning Process
Elementary and lower secondary mathematics processes for identifying and solving problems; Disposition and abilities that foster these processes
Students grasp phenomena mathematically, identify mathematic problem in them, and solve the problems independently or corroboratively.
Students grasp phenomena in daily lives and society, make mathematical meaning of various contexts, and solve problems.
Students think generally and expansively to find solutions to problems.
Thinking and Judgement
A1. Students grasp problems in daily lives and society mathematically.
Ability to attend to the quantities and quantitative relationships in phenomena; Identify mathematical problems.
Ability to grasp characteristics of phenomena and express these mathematically. (i.e., represent phenomena)
Mathematically Expressed Problems
A2. Students grasp problems in phenomena mathematically.
Ability to identify problems from mathematical phenomena.
Ability to grasp characteristic of phenomena and process them mathematically.
Ability to generalize and expand ideas based on the results of problem solving.
B. Students have foresight and plan for problem solving using mathematics.
Ability to grasp the essence of mathematical problems (insight).
Show foresight for solving mathematical problems (planning).
Daily or Societal Phenomena
Mathematical Phenomena
Focused Problems
C. Student solve focused problems.
Ability to use numbers, expressions/equations, diagrams, tables, and graphs to mathematically process, solve procedurally.
Ability to process accurately and efficiently based on mathematical point of view and thinking.
Ability to reason logically using induction, deduction, or analogical thinking.
D1. Students reflect on the solution processes and identify the meaning of the results, and apply the solution methods to other problems.
Ability to revisit the original phenomena and think about the meaning of the results.
Ability to apply specific solution methods to various phenomena.
D2. Students reflect on the solution processes and create mathematical concepts and systems.
Ability to identify the merits of mathematical points of view and reasoning.
Ability to critique solution results and develop the ideas systematically and constructively.
Ability to connect the results with the previously learned knowledge to expand and deepen the understanding of the mathematical concepts.
Ability to think generally and expansively.
Results
Expression
◆ These abilities are not only fixed in the places. They may appear in other locations.
E. Students express and exchange ideas using mathematical language and reasoning.
Ability to understand and evaluate explanations with mathematical expressions.
Ability to explain their own ideas to others using mathematical expressions that respond to objectives of problem or study.
Human Quality
E. Students attitude and disposition for mathematical problem solving
Ability to examine the problem-solving process and results; demonstrate a disposition for evaluating solutions and reasoning to improve one's grasp of the problem(s).
Ability to think from multiple points of view and the discipline to identify and solve a range of problems.

Emphasizing the importance of mathematical process means caring about key aspects of engaging in mathematical processes, such as identifying mathematical problems from phenomena, using mathematics to solve problems, and creating and applying mathematics. As a multi-faceted endeavor, we can consider these processes as “mathematical activities” and the observations and reasoning used in these processes as "mathematical ways of observing and reasoning." Therefore, our research group uses the inclusive term of "mathematical processes" to represent the totality of the processes involved in identifying problems from phenomena, using mathematics to solve problems, and the processes of creating and applying mathematics.

The objective of our research group is to find and map the direction for how to create (materialize) lessons that intentionally raise the quality of mathematical practices. Our research group is pleased to take advantage of this research conference where lower and upper secondary school teachers of Tokyo Gakugei attached schools and regular lower secondary school teachers are here together to observe and discuss the lesson that was designed to focus on raising the quality of “mathematical processes." From this important opportunity as a community of educators, our research team looks forward to investigating a direction for designing lessons that raise the quality of mathematical practices.

(2) The Intention of the Lesson

Our intention with this lesson is to create and foster students who can conduct high quality mathematical processes, namely their ability to demonstrate the processes they couldn’t conduct well in the past. However, achieving high quality mathematical processes does not mean only that students can solve problems they couldn’t solve before. The deeper meaning is that students can revisit the problem-solving processes after having gained a new point of view or purpose, and they generalize and extend the ideas gained from their discussions.

To improve the quality of the mathematical processes, it is necessary to teach the related mathematical content, as well as the process skills. Thus, to plan such lessons, we need to set up problem-based instructions that require students’ learning about problem solving in general. But, we also need to prepare problem based instructions that are particular to the mathematical content of the lesson.

Therefore, in the process of *kyozaikenkyu* (investigation of instructional materials), we must carefully focus on discussions of the mathematical processes, including (a) thinking about students’ anticipated responses for the problems/tasks/materials, (b) identifying necessary student skills for conducting mathematical processes and setting these as one of the objectives of the lesson, and (c) thinking about what supports we can provide to students so they will successfully conduct or demonstrate high quality mathematical processes. In this lesson, *kyozaikenkyu* focused on mathematical processes. I will anticipate students’ mathematical processes based on the topic I select, identify students’ necessary skills for conducting mathematical processes, set these skills as objectives, and think about what support(s) I need to provide in to develop such skills. By doing so, I would like to show that I can plan a lesson that provides evidence of high quality mathematical processes.

2. "Mathematical Processes" of this lesson

(1) About the Unit Plan and the "Mathematical Processes"

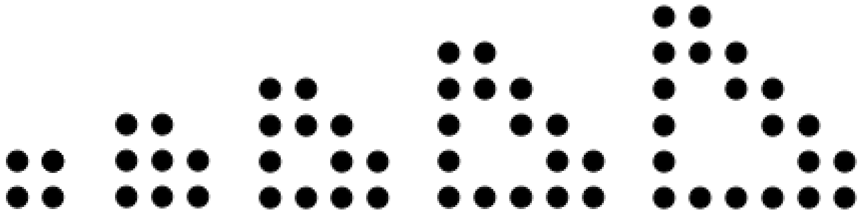
This lesson is in the part of the unit entitled "Letters in Algebraic Expressions." In elementary school, students had used symbols, such as ☐ and Δ in equations; for example, 5 + ☐ = 8 and 3 × Δ = 24. These symbols helped students grasp the relationship between addition and subtraction or multiplication and division. Students learned to represent the relationships among quantities within the context of word sentences, such as (speed) x (time) = (distance) and learned to interpret and understand the meaning of given expressions and equations.

In lower secondary school, as a foundation for the study of algebraic expressions with letter symbols, students learned that letters such as *a* and *x* may be used in place of ☐ and Δ. They also learned how to express direct and inverse proportional relationships using algebraic expressions. Moreover, students have experience using quasi-variables; e.g., thinking about ways to calculate division of fractions or expressing relationships/patterns in numbers. Building on mathematical studies in elementary school, lower secondary school students will learn not only about using letters as representations and manipulating them, but also how to manipulate and interpret letters as variables, unknowns, and representations of a set. Finally, instead of simply introducing letters and studying calculations involving algebraic expressions with letter symbols, we will introduce letter symbols starting with the examination of quasi-variables (numbers that act like variables) and then, through activities of interpreting algebraic expressions and their structures, students will use algebraic expressions to represent mathematical generalizations.

Using letter symbols, students experience numbers not as specific numbers (such as 1, 3, or 0.7), but as a general object of study. They can also express various phenomena as relationships in the mathematical world. Furthermore, by transforming the given algebraic expressions or equations, new interpretations may become possible. The intention here is that through the study of letter symbols, students' mathematical explorations are deepened and become more refined. Such explorations typically take place when students try to prove conjectures and/or utilize the ideas of equations and functions. However, in this unit, instead of simply positioning the current study as the preparation for those future explorations, the main purpose is for students to experience mathematical manipulations and interpret their results by (a) representing a relationship in a real-world phenomenal as an algebraic expression, (b) transforming it to reflect their own thinking, and (c) interpreting and understanding the algebraic expressions and equations that other students express. These are all examples of the mathematical processes that students will experience by the end of the unit.

(2) About *kyozai* (instructional material) and anticipated "mathematical processes"

The instructional material I prepare for this lesson is the go stones where the number of them increase gradually as shown below.

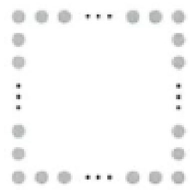


The value of this instructional example is the fact that the actual number of go stones in each stage of the growing figure is the same as the number of go stones when the original figure is transformed into a square whose side is the same as the original side at the base. In other words, the number of total go stones used for the nth number of the figure can be expressed as $4n$, therefore, students may notice that these figures can be transformed into squares. In this case students are not only interpreting the equations and understanding the solution processes within which they are engaged, but they are also interpreting the equations and imagining the transformed figures as squares from the expressions. This additional thinking process helps students to develop deeper understanding of the reciprocal relationship between expressions and phenomena. Although the importance of interpreting expressions and understanding corresponding phenomena has been stressed previously and often, the process itself is not easy for most students to do.

Students have experienced processing expressions formally and they have interpreted expressions to understand the corresponding phenomena; however, they have not experienced transforming the original phenomena into new form based on the interpretation of the expressions. Nor have they transformed the expressions based on the interpretation of the phenomena.

In this unit, the problem asking students to interpret expressions and think about solution ideas is the "application problem" in the section called "utilizing algebraic expressions with letters."

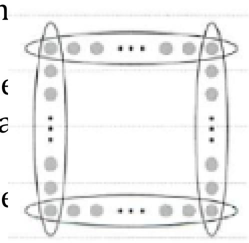
< Application Problem>



We are making squares by lining up n go stones on each side as show to the right. Sakura's idea for finding the total number of the go stones was shown below."

Sakura' idea:

"When I circle in the go stones as shown to the right, each circle contains n number of go stones. There are four such circles, so the total number of the go stones can be expresse as $4n$. However, the go stones at the vertices of the square a counted twice, so the total number of the go stone is 4 less than $4n$. Therefore, the total number of the go stones can be show as $4n - 4$."



Yuto expressed the total number of the go stone as $4(n-2) + 4$. Explain Yuto's idea by drawing circles on the diagram of go stones, just as Sakura had shown.

The textbook includes several problems that ask students to interpret and describe what a given expression is representing and to identify the corresponding phenomenon. This problem, however, is new to students. It asks students to examine a given expression, interpret the structure, then create a diagram to explain the thinking behind the expression. This problem provides an opportunity for the students to think about and understand Yuta's method; yet, I don't think it is enough experience for students to develop a deeper understanding of phenomena like this.

If we look at the result of calculations in today's problem, $4n$, it is not easy for students to define what 4 and n are representing in the phenomenon (original geometric figure). By going back and forth between the phenomenon and the expression, students need to recognize that $4n$ could be representing a square. From this recognition, I want students to realize that the original figure can be transformed into a square.

To interpret expressions and understand corresponding phenomena well, it is necessary for students to interpret and analyze the expressions and understand the corresponding phenomena, and then transform phenomena based on the results and interpretation of expressions.

The problem in today's lesson provides opportunities for the students to think and discuss where $4n$ is represented in the figure and deepen their understanding of algebraic expressions with letters and their meanings. This is where I believe, the value of the *kyozai* (instructional material) is most evident. The problem helps deepen students' understanding of phenomena and expressions. The experience that students gain from this lesson must help them in the future study of algebraic expressions with letters, through this experience of expressing expressions and phenomena by transforming them appropriately based on their way of thinking. Since the students have been practicing calculating linear expressions, I decided to use this problem in this lesson.

In this lesson, I decided to ask students to think about the number of go stones in the 10th figure instead of asking them to think about the number of go stones in the nth figure. The first reason for that is there are some students who are not used to working with algebraic expressions with letters. Although drawing the 10th figure is cumbersome, it is still possible to draw. So the problem is still accessible for all students. It gives students the chance to produce their own expressions and represent their own ideas, so they are more likely to participate in the presentation and discussions. The second reason involves the number of stones for the 10th figure, which becomes 40 stones. The number is simple, so students may think there is some mystery behind the number. This students' thinking might motivate them to investigate after finding the solution, 4*n*.

I am going to explain the mathematical processes that I anticipate during the lesson. The actual students' mathematical processes are not easily described, because their processes vary greatly and may be complex, including students who go back and forth between ideas and different solution methods. Therefore, what I will describe here is an ideal interaction. The ideal mathematical processes that students think through by trial and error until they resolve the problem.

The first step is to give students time to observe and grasp the phenomena. For example, students observe the diagrams of the geometrical arrangement of the go stones, i.e., the first figure uses 4 stones, second figure uses 8 stones, third figure uses 12 stones. "If you know what number the figure is in the sequence and how many stones increase each time in this function, you can draw the conclusion that with each increment, "There is 4 times as many as the previous number in the sequence." If we apply this thinking (function), the number of stones in the 10th figure can be determined as 40 stones. The process of finding the number of stones in the 10th figure can be expressed as 10 x 4 using an expression. Then if we generalize the method, by replacing 10 with *n*, we can establish an algebraic expression with the letter symbol, 4*n*.

Depending on what counting method a student uses, the algebraic expression with letters can vary. However, when these algebraic expressions are simplified, all the expressions become 4*n*. 4*n* is a simple expression but when you look at the original figure, it is not easy to identify what 4 and *n* are representing in the figure. On the other hand, if you think about what 4*n* might represent without thinking about the original figure, the figure that comes to mind is a rectangle that has 4 stones as its width and *n* stones as its length, or a square that has four equal *n* number of stones in each side. This line of thinking - the interpretations of the expression 4*n*, leads students to realize they could transform the original shape into a rectangle or a square.

In general, a proof using expressions with letter symbols, "expressing," "transferring (processing)," and "interpreting" of an algebraic expression with letters could become an issue for students to conduct. In this lesson, "interpreting the algebraic expressions and understanding the corresponding phenomena" could be an issue. The difficulties of conducting the mathematical processes described above are not explained in figure 1, but these are issues specific to the topic of algebraic expressions with letters. Therefore, it is very meaningful to examine the mathematical processes in the context of algebraic expressions with letters through *kyozaikenkyu*.

3. About This Lesson:

(1) Methods for Raising Quality of Mathematical Processes:

During the Research Conference at Tokyo Gakugei University attached Lower and Upper Secondary School in 2016, I had proposed following four points for the methods for raising quality of Mathematical Processes.

① Having a long-term view for fostering students' quality and ability to conduct the mathematical processes.
② Actualizing the mathematical processes and establishing time for evaluating and improving the mathematical processes.
③ Identifying the ideas that promote the mathematical processes
④ Summarizing the lesson focused on the mathematical processes

Using this as the guide, I have written the lesson plan so that it raises the students' quality and frequency of use of the mathematical processes.

① Having a long-term view for fostering students' quality and ability to conduct the mathematical processes.

Students' quality and ability to conduct the mathematical processes cannot be developed within one lesson. This quality and ability has to fostered by consistent instruction over a long period of time. In our research group, we proposed that "having a long-term view" is the key for developing students' use of mathematical processes.

To show evidence of this key point, I expect to see the following students' behaviors: The students in the class will have practiced describing their own ideas using algebraic expressions with letters. They also interpret other ideas from their algebraic expressions. If students have developed these types of abilities, they should be able to apply the same processes in this lesson. However, even with questioning by the teacher, there will most likely be only a few students who recognize the relationship between the 4n and squares. I think many of the students are capable of interpreting algebraic equations and understanding the phenomena related to them; however, some students will have difficulty conducting and demonstrating this level of understanding.

The actual state of my students reflects an accumulation of students' learning experiences in my everyday instruction. There facility also represents the natural development of skills that are acquired through many experiences of consistent instruction and practice. If I think about this point, I realize also that the mathematical process should be developed over a long time of period; therefore, my instruction needs to be very intentional in regard to planning and practicing instructions that focus on interpreting algebraic expressions with letters and understanding corresponding phenomena. These considerations and careful attention need to be demonstrated, also, in instructions in the previous grade level as well as in future grade levels.

② Actualizing the mathematical processes and establishing time for evaluating and improving the mathematical processes.

In order to develop students' skills to conduct the mathematical processes, it is important to help students think consciously about the importance of the mathematical processes during lessons, evaluate the processes carefully, and improve the process if necessary. Needless to say, in order to do this, we need to make sure that the mathematical processes are realized in everyday practice, observe the lessons with others, and discuss the lessons and students' progress together.

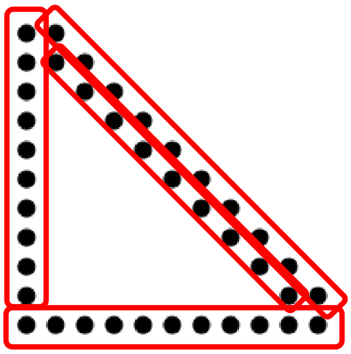
In this lesson, students will find the number of go stones at the 10th figure. Then students will compare the segment when students discuss various different methods and the segment when students find the solution $4n$. Students will realize that although they were interpreting various algebraic expressions and relating them with phenomena (how the stones are arranged to find the number of stones), they have not thought about the calculation results of $4n$ in terms of its relationship to the phenomenon. This realization will help students realize that the algebraic expression 4n can also be interpreted and applied to real phenomena. This discovery naturally will lead the students' motivation to use this new process of thinking. The total process of thinking -- not only thinking about the algebraic expression $4n$, but also the process of thinking about solving for the 10th figure -- will help engage them in reflecting on their own thinking process, as well as provide opportunities for improving their thinking.

In order to carry out the discussion of the mathematical processes, I believe that *bansho* (board writing), not only helps visualize the thinking process, but it is the key to supporting the mathematical processes. *Bansho* provides more than a record of the results of the problem solving. It represents and provides a record of the thinking process, allowing students to jointly share their thinking and understanding; consequently, students can more easily clarify the discussion and state the agreement clearly.

③ Identifying the ideas that promote the mathematical processes.

I have talked about ideal mathematical processes before, but in reality, students will develop the skills necessary to conduct the mathematical processes after many twists and turns over a long period of practice. Therefore, it is important not only to teach the importance of the mathematical practices, but also to teach the ideas for helping to do the process well. These are ideas to carry out problem solving. As I mentioned

before, in order for students to conduct mathematical practices for interpreting algebraic expressions and understand the corresponding phenomenon (ways to find number of the stones with the diagram), it is important to compare different ideas that students represent in their algebraic expressions. In addition, students need to think about making a correspondence between algebraic expressions and phenomena.



For example, students may count the total number of go stones as shown to the left. In this case, by adding the number of stones in the vertical rectangle, horizontal rectangle, inner diagonal rectangle, and outer diagonal rectangle sides, students may represent the counting methods as 10 + 11 + 10 + 9. To get the total 40, students may think of 10 x 4, or 10 + 10 + 10 + 10. In this case, students could also see the expression as moving one stone

from the vertical side of the rectangle to the inner diagonal side of the rectangle, and realize they can make 4 groups of 10 stones as the 10th figure.

To find this solution, the students draw rectangles to create several groups, such as 10, 11, and 9. They use these numbers to create an expression. Then, students interpret the expression and deepen their understanding of the phenomena by connecting the idea represented in both the expression and the diagram. The reason that students can do this is that they have been experiencing this kind of thinking consciously before, including experience in elementary school mathematics. However, they have not had much experience transforming the original phenomenon into a new phenomenon from the expression. For this reason, in this lesson, the students need to do the following: first, recognize the connection between expressions and phenomena; second, discuss and understand the correspondence between expressions and phenomena; and lastly, discuss how to interpret $4n$ which is the result of calculation of an algebraic expression.

Transforming algebraic expressions and understanding of phenomena are the most important ideas students learn in the unit of Algebraic Expressions with Letters. I say this because the practice helps students develop understanding of algebraic expressions and phenomena. I underscore how critical this is by asking you to compare the understanding of a student who has memorized the formula for the composition of trigonometric ratios versus the student who understands the relationship geometrically. It is the latter student who has a much deeper understanding of the phenomena.

④ Summarizing the lesson focused on the mathematical processes.

According to Nishimura (2011), the summary of the lesson should be focused on the mathematical processes. In short, the lesson is summarized by reflecting and highlighting the ideas used to conduct the mathematical processes. In this lesson, for example, the important point of the summary is to discuss how students need to think to conduct a high quality “interpretation of the expressions and the corresponding phenomena;" therefore, a discussion needs to summarize “comparing the cases that only use numbers to solve the problem and those that use variables (letters) to solve the problem and “corresponding the result of algebraic expression to the phenomena." I believe this type of summary helps students realize this important way of thinking and how it can establish deep learning.

The above discussion on points about the methods for rising quality of mathematical processes can be summarized as follows:

(A) Having a long-term view for fostering students' quality and ability to conduct the mathematical processes.
  - Having a stance of long-term view for fostering students' quality and ability to conduct the mathematical processes

(B) Actualizing the mathematical processes and establishing time for evaluating and improving the mathematical processes.
  - Devising questions and *bansho* (board writing) to support the actualization of the mathematical practices.

(C) Identifying the ideas that promote the mathematical processes.
  - Comparing expressions with numbers and algebraic expressions with letters
  - Transforming algebraic expressions and connecting the phenomena.

(D) Summarizing the lesson that is focused on the mathematical processes.
  - Reflection of ideas use in the mathematical processes.

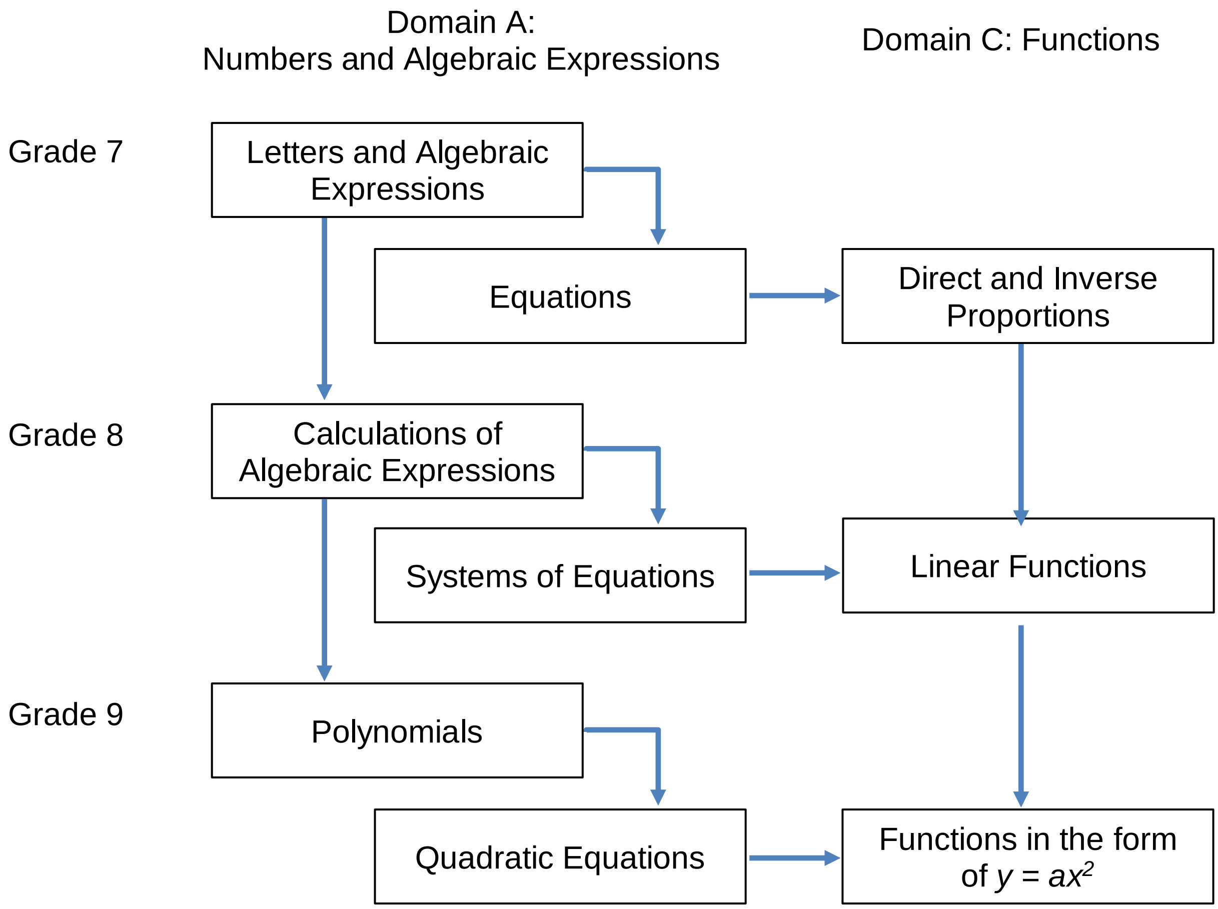
(2) About the students:

This year, students studied positive and negative numbers (integers) in a unit where students learned to view negative numbers in the same way they view whole numbers. Moreover, by studying calculations with integers, including patterns and properties of operations, they studied what numbers are. During these lessons, I tried to help students by asking them to "think about 'what needs to be considered;'" to "reflect on your solution processes;" and to "explain your thinking process in words."

In general, students' mathematical achievement levels are high. For example, when examining calculations with integers, some students were able to use 3 and -2, as quasi-variables. Most students were able to understand their mathematical explanations, indicating that most of them understood the notion of quasi-variables. Therefore, I anticipate that few students will have difficulty generalizing numbers.

Although few students consider mathematics to be difficult, there are some students who find it difficult to explain their ideas. Yet, they have experienced how their mathematical understanding was deepened by clarifying questions other students had. Therefore, I believe there is a classroom culture where students feel safe to admit openly something they don't understand. Moreover, many students are willing to share their ideas in whole class discussions, and they do not hesitate to share even simple ideas.

(3) Scope and sequence in lower secondary school:



(4) Unit Plan:

	Content	Anticipated Process	Main Evaluation Points
I	**<u>Section 1: Algebraic expressions with letters</u>** • Merits of using letters in place of numbers • Representing various quantities using letters • Learning how to write algebraic expressions with letters • Interpreting algebraic expressions with letters • Substituting values in letters of algebraic expressions and the meaning of the value of an algebraic expression	• Grasp the phenomenon and represent it with numbers and symbols. • Identify variables in numbers and represent it with a letter. • Establish an algebraic expression with a letter • Interpret an algebraic expression • Substitute numbers with the letter (variable)	• Students are interested in the necessity and merit of using letters to represent relationships/patterns among quantities generally, and they try to use algebraic expressions with letters to represent relationships/patterns and interpret the given expressions. [Interest, eagerness, and attitude] • Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning]

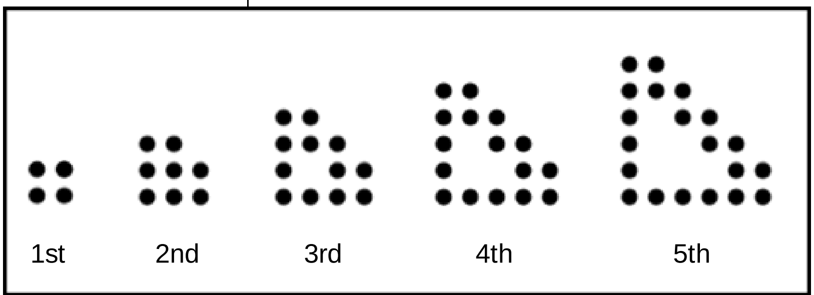
II	**Section 2: Calculations of algebraic expressions** • Relationships between terms and coefficients • Combining like terms • Addition and subtraction of linear expressions (distributive property) • Multiplying and dividing linear expressions (commutative and associative properties)	• Find the properties of calculations of linear expressions. • Identify preconditions (properties) for calculating four operations in the domain of linear expressions.	• Students know how to represent multiplication/division within algebraic expressions with letters and they try to use them to manipulate expressions. [Interest, eagerness, and attitude] • Students are able to think about ways to calculate algebraic expressions with letters by realizing that calculations with algebraic expressions as analogous to calculations with numbers. [Mathematical ways of observing and reasoning] • Students can use algebraic expressions with letters involving multiplication and division by following conventions appropriately. They can add and subtract simple linear expressions. [Mathematical representations and manipulations]
III	**Section 3 Applications of algebraic expressions with letters** • Quantities represented by algebraic expressions • Algebraic expressions that represent relationships	• Grasp the phenomena and represent phenomena with numbers and symbols. • Identify variables in numbers and represent it with a letter. • Establish an algebraic expression • Interpret an algebraic expression and understand corresponding phenomenal • Using letters to represent the relationship of quantities with an algebraic expression	• Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning] • Students are able to represent quantities and relationships/patterns among quantities in phenomena using algebraic expressions with letters, and they can interpret given algebraic expressions. [Mathematical representations and manipulations] • Students understand that by using letters quantities, and patterns, quantitative or functional relationships can be represented generally or interpreted from the given algebraic expressions with letters. [Knowledge and skills about numbers, quantities, and geometric figures]

(5) About this Lesson:

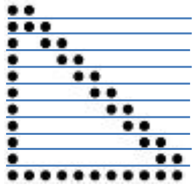
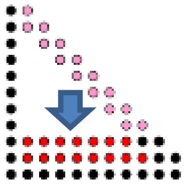
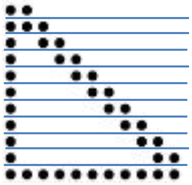
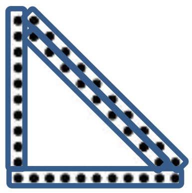
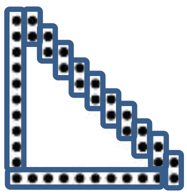
① Goals of this Lesson

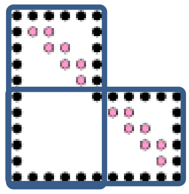
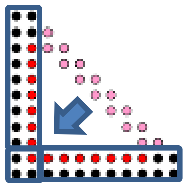
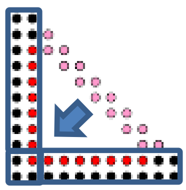
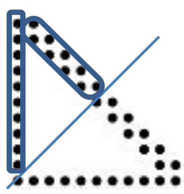
- Students are able to express the quantities, relationships among quantities, and rule of patterns in a phenomenal using algebraic expressions with letters. In addition, they are also able to interpret the algebraic expressions with letters and understand the quantities and quantitative relationships of the phenomena.
- Students understand the importance of comparing and discussing quantities and letters in algebraic expressions, and making connections between/among the transformation of algebraic expressions and phenomena. Because these actions help them conducting high quality mathematical processes.

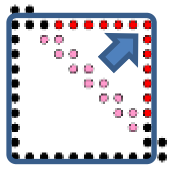
② Flow of the Lesson

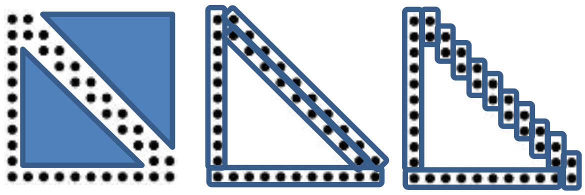
Time	Main Learning Activities	Students' activities and their anticipated responses	◆ Things need to remember ○ Evaluation
5 min.	**[Introduction]** Posing problem		
			◆ Paste the poster of the problem on the board.
	"How many stones are there in each figure?"	"The first one has 4 stones." "The second one has 8 stones." "The third one has 12 stones. "It looks like the number of stones increases 4 stones each time. "	◆ Try not to discuss how the number of stones increases because it will have and effect on students' problem solving.
10 min.	Hatsumon: How many stones are in the 10th figure? Show how you can find the number of stones using an expression.		◆ Walk around the classroom and monitor how students are solving the problem.
	[Solving the problem on their own]	(1) Count by putting check mark. (2) Use table	

No. of figure	1	2	3	4	5	6	7	8	9	10
No. of stones	4	8	12	16	20	24	28	32	36	40

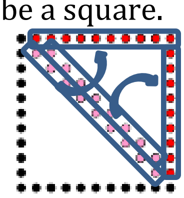
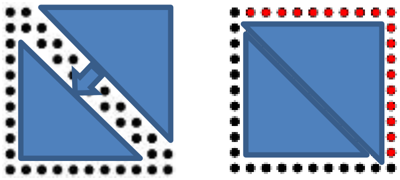
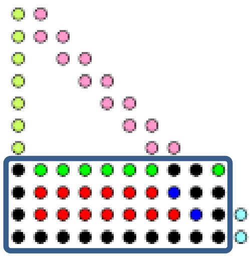
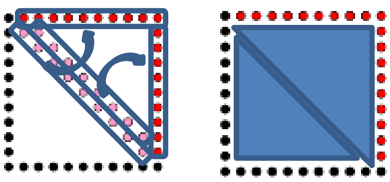
		(3) Slice horizontally and add 2+3+3+3+ … +3+11 	◆ If see students who are not showing their work, ask them to think about how they counted the stones and if they can represent the counting method into an expression. Or at least, write about their thinking.
		(4) Move stones to the bottom 11+3+3+3+ … +3+2 	◆ The stones could be mover to the left side instead of to the bottom. The expression remains the same.
		(5) Slice horizontally and use multiplication at the middle part 2+3 x (6-1) +7 	◆ Confirm with the students that the methods (3), (4), and (5) are basically the same method.
		(6) Split vertical, horizontal and inner and outer diagonal parts 10+11+10+9 	◆ It is easier for students to make a group of 10 when counting. Be sure to notice this method. Even if none of the student show this idea, the teacher will share the idea.
		(7) Split vertical, horizontal and diagonal parts 10x2+2x10 	◆ It is easier for students to understand to transfer from 10 groups of 2 (2x10) to 2 groups of 10 (10x2), therefore, the method (7) will be shared to the students, even if none of the students came up with this method.

		(8) Subtract overlapping square part form the two rectangles. (wrong solution)  (11x2+4x2)x2-5x4 (9) Move stones from the diagonal part into the vertical and horizontal rectangles.  (9x2)x2+4 (10) Similar to method (9). Subtracting the overlapping square part.  (11x2)x2-4 (11) Cut the figure into two halves. Add vertical and diagonal parts, then double it. At last add two stones that are not in the boxes.  (10+9)x 2+2 (12) (10+9)x2 (incorrect solution) Using the same method as (11), but forgetting to add two remaining stones at the end.	◆ The method (8) shows an incorrect solution. If none of students came up with the idea of transforming the shape into rectangle or square, this method will be shared even though it is an incorrect solution. ◆ If students do not come up with the method (13), the methods (9) or (14) will be sheared.

		(13) Make a square by filling with stones, then subtract the two triangular parts. 11x11-9x9 	◆ The method (13) will be shared because it will help students to notice the methods related to the square. If none of the students came up with this method and students do not think about changing the shape into a rectangle or square, the teacher will share the method.
		(14) Adding 2 extra stones at the top and at the side 9x4+4 	
		(15) Count by two's (This method cannot be generalized) 2 x 20	◆ This method cannot be generalized using algebraic equations with letters.
		(16) 4+4x10 (wrong solution) Using the table to solve the problem, but miscounts the number of increases.	
		(17) Wong solution of 11+11+10+9. Using the method (6) but counts the left bottom corner stone twice. (18) (10x2+8x2)x2-9x4: an incorrect solution. Using the method (8) but makes mistakes counting the number of stones in the vertical side and horizontal side.	◆ If the method (6) did not come from the students, use this method. However, this is an incorrect solution, so it is important to make sure to explicitly discuss what the misunderstanding is.

10 min.	**[Sharing and Discussing]** "What is happen to the number of stones when it is in *n*th figure?"	 • It will be 4n • It will be n+(n-1)+n+(n-1) • It will be $(n+1)^2(n-1)^2$ • If we calculate all of them, they all become 4n.	◆ It is important to focus on students' interpretations of the expressions. Thus, the student presentation will be done by sharing expressions. Then, ask other students to explain the presented solutions. (I use this practice in everyday classroom.) ◆ Confirm with students that the result of calculations is 4*n*. The students do not know how to calculate method (13) to get 4n since they have not learned factorization. At this point, simply tell students the result of calculations should be 4n.
5 min.	"When we discussed the expressions that use numbers, what did you do?"	• I thought about where the numbers are represented in the diagram. • I looked at the expression and thought about how the person counted the stones. • I wondered if there is any other way to write the presented expression. • I thought about whether or not the presented expression is correct. • I wondered if I could extend the presented expression.	◆ I'm hoping that students will come up with the anticipated responses to the left. If there is no response, I will ask the students what they did in order to understand other students' expressions.

Hatsumon: Where 4n is represented in the diagram? Let's think about what 4n mean in the diagram.

10 min.	**[Solving problem on their own]**	• If we move the stones at the diagonal part to the top and to the right, the figure will be a square.  • If we move the triangle part that do not have stones, I can make a square.  • Move the stones to create a rectangle whose height is 4. Move pink stones to the red circles Move yellow-green stones to the green circles Light blue stones to the blue circles 	◆ If I see students who are not working, I will ask them to think about where the 4n is coming from and how we are getting the 4n. ◆ If it is necessary, I will ask the same questions to the class, even if students could not finish the solution completely.
5 min.	**[Sharing and Discussing]**		◆ Make sure to bring up these two methods on the left. ◆ If the idea of square does not come from the students, ask students if they could they make a rectangle with four groups of *n* stones. ◆ If students came up with the idea of using rectangle, make sure to choose and discuss the method.

5 min.	**[Summarizing]** "What did you think about? What did you learn?"	• When we simplify the algebraic expressions, we can get the simple algebraic expression. When we think about the simple algebra expression with the original figure, we can see that we are able to transform the original figure to the different figure. • First, I did not understand how people find the number of stones from the expressions, but when I look at the diagram and think about how the numbers are represented in the diagram, I felt it is possible to understand.	◆ While students write their reflections, walk around and read there writing. Identify good responses and ask these students to share what they wrote.