June 23

# Grade 4, Mathematics Lesson Plan

Place: Showa City Oshihara Elementary School
Teacher's Name: Toshio Ohma

1. **Name of the Unit:** Pay Attention to Commonalities (Thinking with Diagrams)

2. **Goals of the Unit:**
   - Students recognize the merits of using diagrams that help them visualize the relationships between quantities clearly and easily in the context of word problems.
   - Students are able to solve word problems by paying attention to the difference between two quantities, a difference that results from splitting one quantity into two parts or moving one of the two parts to the other part.

3. **About the Unit:**

The problem of today's lesson is as follows:

> Riko and Kota split 60 sheets of origami paper to make paper cranes.
> Riko has to have 12 more sheets than Kota.
> How many sheets of origami paper will they each have?"

The objective of this task is to foster students' problem-solving skills by helping them understand/see the relationships of quantities involved in a story problem by reading the problem carefully and representing it using a line segment model.

At our school, we have been working on developing students' communication skills through English language lessons and activities developed under the school-based research theme, "Fostering Students' Ability to Thrive in a Global Society." When we develop students' communication skills in Mathematics, we keep in mind the descriptions of the basic educational direction of mathematics education in the 2008 Mathematics Course of Study:

> Mathematical thinking and expression play an important role in rational and logical thinking as well as in intellectual communication. … We will enrich the kind of teaching where students are taught to think systematically, in logical steps, by reasoning, and to understand the connections among words, numbers, algebraic expressions, figures, tables, and graphs. This kind of teaching will also allow students to learn appropriate usage, problem-solving, how to explain one's ideas clearly, and how to express and communicate one's ideas to others.

Active participation of students, such as explaining their own ideas to others and listening to understand their friend's ideas, is the connection to fostering students' communication skills. To develop lessons where the students participate actively, I pay attention and care about the "important questions" which are the students' own questions. That is, questions such as "I wonder if we could use something I learned before" and "I wonder if we could use this thinking to understand and solve other problem situations."

Nakamura (1989) identified and underscored different types of student questions; namely, questions that engage the following:

- a student's previous learning;
- other ideas different from what the student has done;
- the basis for mathematical reasoning;
- commonalities and similarities among solutions, ways of thinking;
- differences among solutions, ways of thinking;
- generalizations;
- expandability/application of reasoning; and the
- merits of a solution pathway.

The questions (wonderings) I care most about in this unit include:

- "If we show it [the relationship] using a diagram, what would the relationship look like?"
- "I wonder if we could solve it by calculating."
- "If we make the amounts of the two quantities the same, I wonder if we could solve the problem."
- "Is there any other way to solve the problem by making the two quantities the same?" and
- "Could we make Kota's quantity the same as Riko's?"

I would like to practice the following instructional moves or decisions:

- During the "grasping the problem" phase of the lesson, I would like to provide opportunities for students to split a quantity (60 sheets) freely into two quantities in order for them to understand the meaning of splitting in two. This activity provides an opportunity for students to understand that although a quantity is split into two, the sum of those two smaller quantities will always be the same (60 sheets). Depending on how a quantity is split in two, the two quantities that result could produce a difference (parts that are different in size). After students understand this, I will propose a difference of 12 sheets between the two quantities. Students will consider this difference of 12 and demonstrate their understanding by drawing a line segment model and using it to solve the problem on their own.
- At the start of the "independent work" phase (when students solve problem on their own), I will ask students to think about what mathematical expressions they need to establish. Then I will ask them to solve the problem freely.
- During the "comparing and discussing" phase, I will first select the incorrect solution that involves students adding or subtracting 12 sheets from the equally split number of sheets, 30 sheets ($60 \div 2 = 30$, $30 - 12 = 18$). When 30 sheets are split in two, each person will receive 30 sheets. Then, when Kota gives 1 sheet to Riko, the difference in the number of sheets between the two students is now 2 sheets. By discussing the wrong solution first, students may discover their misunderstanding and lead themselves to think about and reach the correct solution. During the discussion, I hope students will be able to discuss the misunderstanding not only verbally but also using the line segment model. These diagrams represent the relationship of quantities involved in story problems clearly and easily. Therefore, using diagrams in class discussions helps students see the merit of using this model. The visual clarity and simplicity of diagrams makes them useful for understanding and explaining quantitative relationships and solutions.

Next, I will turn the students' attention to the solution that starts with removing the difference (12 sheets) from the total number of the sheets (i.e., 60 – 12 = 48). After subtracting 12 sheets from the total number of sheets, Riko's number is now equal to Kota's number of origami paper sheets. Since 48 ÷ 2 = 24, both boys will have 24 sheets each. I would like to see the students use the line segment model to describe this process. Lastly, I will ask students to think about the method that made the number of Kota's and Riko's sheets the same.

- During the "understanding the problem" phase, students represent the relationship of quantities involved in the problem using a line segment model, think about establishing equations from the diagram, and explain the meaning of these equations using diagrams. In order to develop students' communications skills, I want to hear the students share their ideas with each other using terms they have been using in mathematics class, including the words introduced in this unit, such as "difference" and "make both quantities the same."

**4. About This Lesson:**

(1) Date and Time: June 23, 2017 (Friday)
(2) Place: Showa City Oshihara Elementary School, Grade 4 Class 2 Classroom
(3) Objectives:

- Students recognize the merit of using diagrams to solve word problems, because diagrams help them see the structure of problems easily.
- Students are able to solve word problems by paying attention to the difference of two quantities that results from splitting a quantity into two quantities/parts or moving one of the two quantities/parts to the other quantity/part.

**5. About This Lesson:**

| Time | Learning Content<br>(○ Anticipated Student Questions) | Instructional Points to Remember |
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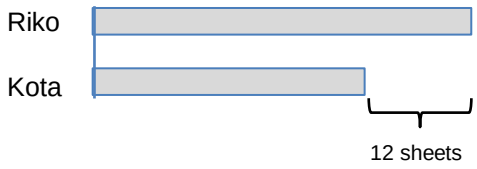
| 15 min. | 1. Grasping the task<br>(1) Teacher pose the problem<br>Riko and Kota split 60 sheets of origami paper to make paper cranes.<br>Riko has to have 12 more sheets than Kota.<br>How many sheets of origami paper will they each have?" | |
| | • Kota gets 30 sheets, Riko gets 30 sheets.<br>• Kota gets 20 sheets, Riko gets 40 sheets. | • In the beginning, the gray-shaded sentence will be covered with a strip of paper, so the students can experiment with splitting the origami sheets freely. Students confirm that the sum of the two sets of origami sheets is always constant (60 sheets). |
| | Riko has to have 12 more sheets than Kota. | |
| | • What do you think about the meaning of the sentence "12 more sheets than Kota?"<br>  • Riko will have more than Kota.<br>  • It is not easy tell.<br>○ "I wonder if we show the problem with a diagram, what will it look like?" | • Peel off the strip of paper to show the sentence, "Riko has to have 12 more sheets than Kota." |
| | (2) Representing the problem situation using a diagram<br>  | • Help students foresee that if they represent the problem situation with a diagram, they can solve the problem more easily. |
| | • The difference in the number of sheets between Riko and Kota is 12 sheets, so if we take 12 sheets away from Riko, the number of sheets for both of them becomes the same.<br>• The difference of the number of the sheets between Riko and Kota is 12 sheets, so if we add 12 sheets to Kota's number, the number of sheets for both of them becomes the same.<br>○ "I wonder if we could calculate and solve the problem." | • Help students to become aware that both ideas ("if we take 12 sheets from Riko" and "if we add 12 more sheets to Kota's") make make the number of Riko's and Kota's sheets the same (equal). |

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| 5 min. | 2. Solving the Problem on Their Own<br><br>Anticipated Solutions:<br><br>(a) First, split the 60 sheets equally among two people, then add 12 sheets to Riko's and subtract 12 sheets from Kota's number (a wrong solution)<br><br>60 ÷ 2 = 30<br>30 + 12 = 42<br>30 – 12 = 18<br><br>Answer: Riko will have 42 sheets and Kota will have 18 sheets.<br><br>(b) First, split the 60 sheets equally among two people, then add 6 sheets to Riko and subtract 6 sheets from Kota (a correct solution)<br><br>60 ÷ 2 = 30<br>12 ÷ 2 = 6<br>30 + 6 = 36<br>30 – 6 = 24<br><br>Answer: Riko will have 36 sheets and Kota will have 24 sheets.<br><br>(c) Subtract the difference of 12 sheets from 60 sheets in order to make the number of Riko's sheets the same as Kota's.<br><br>60 – 12 =48<br>48 ÷ 2 = 24<br>24 + 12 =36<br><br>Answer: Riko will have 36 sheets and Kota will have 24 sheets.<br><br>(d) Add the difference of 12 sheets to 60 sheets in order to make the number of Kota's sheets the same as Riko's.<br><br>60 + 12 =72<br>72 ÷ 2 = 36<br>36 - 12 = 24<br><br>Answer: Riko will have 36 sheets and Kota will have 24 sheets. | |

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| 20 min. | 3. Comparing and Discussing<br>(1) Discussing the incorrect solution method<br><br>(a) First, split the 60 sheets equally among two people, then add 12 sheets to Riko's and subtract 12 sheets from Kota's quantity (a wrong solution)<br>○ The difference between Riko's and Kota's will be 24 sheets.<br>○ The answer is wrong. What was the cause of this mistake?<br><br>○ "I wonder if we could solve this problem by making the number of sheets of Riko's and Kota's the same."<br><br>(2) Review the diagram<br>• When we move 1 sheet from one person to the other person the difference in the number of sheets becomes 2 sheets, not 1 sheet.<br>• Therefore, when we move 6 sheets the difference will be 12 sheets.<br>• We can use equations to show what is happening: 30 + 6 = 36 and 30 – 6 = 24 --- (b) method<br><br>(3) Discuss other solution methods<br><br>○ "I wonder if there are other ways to make the number of sheets of Riko's and Kota's the same."<br><br>Subtract 12 sheets to make the number of Riko's sheets the same as Kota's. --- (c) method<br>• If I use the equations to explain …<br>• If I use the diagram to explain …<br><br>○ "I wonder if there is a way to make the number of sheets of Kota's as same as Riko's.<br><br>Subtract 12 sheets to make the number of Kota's sheets the same as Riko's. --- (d) method<br>• If I use the equations to explain …<br>• If I use the diagram to explain …<br>• The answer became as same as when we make the number of sheets of Riko's as same as Kota's. | • Help students be aware that if they split the total number of sheets into two equal quantities, then subtract 12 sheets from one part and add 12 sheets to the other part, the answer will be wrong. Lead them to look at the diagram carefully and rethink this incorrect solution.<br><br>• Help students to recognize that when 1 sheet was moved from one person's quantity to the other person's, the difference in the number of sheets become 2 sheets.<br><br>• Be sure to confirm with students that when the difference of 12 sheets is subtracted from the total number of sheets, the numbers of sheets for Riko and Kota become the same using not only equations but also using the diagram.<br><br>• Be sure to confirm with students that when the difference of 12 sheets was added to the total number of sheets, the numbers of sheets for Riko and Kota become the same … using not only equations, but also using the diagram. |
| 5 min. | 3. Looking back and summarizing<br>(1) Solving an application problem<br>(2) Writing a "reflection of learning"<br>• I could make two quantities the same by adding or subtraction the difference.<br>• It is easier to think about the solution method using a diagram. | • Using the "reflection of learning", assess students' learning and the lesson. |

**6. Evaluation of the lesson:**

- Did the students understand the merit of using diagrams that clearly show the structure of problem situations?
- Were the students solving word problems by paying attention to the difference of two quantities that resulted from splitting a quantity into two quantities/parts or moving one of the two quantities/parts to the other quantity/part?

**7. Board Planning:**

The board plan will be added here later.