

Grade 5 Mathematics Lesson Plan

Let's think about ways of expressing the volume of rectangular prisms:
Through *Mondai Kaiketsu Gakushu* utilizing ICT equipments

Date: Friday, June 24, 2016

Ryuou Elementary School, Kai City, Music Room

Grade 5 Homeroom 1, 24 students

Teacher: ITOH, Yoshitsugu

1 About the Unit

In the Grade 2 unit, "Length and Capacity (Volume)," students learned about the concept of length and capacity as well as the relations among units through activities to measure objects using the units such as centimeter, meter, deciliter and liter. In the Grade 4 unit, "Ways to Measure and Express Area," students learned the concept of area. They also learned that, since the area can be expressed using the number of unit squares, the area can be calculated using the length of sides of the figures. Concerning the "Basic Solids," students have learned about rectangular prisms as the basic solids and the size of a rectangular prism is determined by the length of their sides, - length, width and height for rectangular prisms and a side for cubes. On the other hand, there are students who can calculate the area of rectangles and squares using the formula even though their understanding of the concept of area is rather weak and lack quantity-sense.

In mathematics lessons, I have been emphasizing individual problem solving by providing students with ample time for independent problem solving during *Mondai Kaiketsu Gakushu* style lessons. As a result, an increasing number of students are beginning to use their prior learning and express their ideas accordingly as they engage in problem solving.

In this unit, students are expected to develop the concept of volume through activities to measure the volume of rectangular prisms by selecting appropriate units. The aims of the unit also include developing students' ability to measure volumes and enriching their quantity-sense with respect to volume. Students are expected to develop the ability to calculate the area of volume based on their understanding that the size of solids are determined by the lengths of their edges.

The process of deriving the formula to calculate volume of rectangular prisms will be emphasized in teaching of this unit. For that purpose, lessons will be developed based on *Mondai Kaiketsu Gakushu* format. It is intended that students will recognize the merits of generalization and derivation of the formula on their own. When they do, their understanding of the meaning of the calculation of volume will be solidified. At the same time, by incorporating activities to empirically determine the volume of solids by filling the figures with unit cubes, students' quantity-sense with volume will be enriched.

2 Goals of the Unit

- Develop students' understanding of the units of volume as well as the formulae for calculating the volume of cubes and rectangular prisms.
- Develop the interest in volume of solids and nurture the disposition to think about ways to determine their volumes.
- Extend students' ability to solve problems independently by making use of their prior learning.

3 Assessment Standards for the Unit

Interest, Eagerness, and Attitude toward Mathematics	Students are interested in the volume of objects. They think about ways to determine the volume and try to calculate it.
Mathematical Way of Thinking	Based on their prior learning about area and multiplication, students can think about expressing the volume in terms of the number of units. They can think about ways to calculate the volume using the length of edges.
Skills Related to Quantity and Geometric Figures	Students can calculate the volume of cubes and rectangular prisms using the formulae.
Knowledge and Understanding about Quantity and Geometric Figures	<ul style="list-style-type: none"> • Students understand the meaning of units and volume measurements as well as the fact that the volume of cubes and rectangular prisms can be calculated. • Students poses quantity-sense with volume.

4 Relationship to Research Theme

- (1) About "Abilities We Want Students to Master" necessary to nurturer students who can master and apply.

Based on the research theme, "Nurturing students who can master, apply and explore," we have conducted our research focusing on *Mondai Kaiketsu Gakushu*. In *Mondai Kaiketsu Gakushu*, Our goal is for students to explore ways to solve the given problem using their prior learning during the independent problem solving time. Moreover, during the whole class discussion time, we want students to connect and make use of mathematical expressions, figures and diagrams, and words so that they can explain their ideas in an easy-to-understand way.

In this unit, we address the Grade 5 objective, "(2) Students will understand the meaning of the unit and process of measuring volume and that volume can be calculated." Through the activities in this unit, students will think about ways to determine volume using concrete materials, words, numbers, mathematical expressions and diagrams. Through group discussion, we want students to further develop their ability to express their ideas mathematically as they try to explain their ideas in ways that are easy for their peers to understand. Particularly in this lesson, we want to nurture students' ability to apply their learning as they discuss a variety of solution strategies used during independent problem solving time based on their prior learning.

- (2) About strategies to nurture students who can master and apply

As a strategy to help students to have their own strategies, we will use a physical model to motivate their desire to find the solution to the problem. Also, during the whole class discussion, we will utilize iPad to make the discussion more efficient.

To help students explain their ideas more easily to their peers, we will project students' ideas from their iPad's to a large-screen TV monitor using the edutab system. We hope that this will be a time for collaborative learning by developing shared understanding. It is also hoped that this will lead to deeper and solidified understanding.

5 Unit Plan (Total of 10 Lessons)

#	Goals	Learning Activities	Assessment Standards
(1) How to express volume [5 lessons]			
1	Understand the meaning of "volume," and the unit of volume, "cubic centimeter (cm ³)."	<ul style="list-style-type: none"> Think about ways to compare the size of cubes and rectangular prisms based on the prior learning of length and area. Explore the size of cubes and rectangular prisms using 1-cm unit cubes. 	<ul style="list-style-type: none"> [Interest] Students are trying to think about ways to compare volume by relating to their prior learning of length and area. [Thinking] Students are thinking and explaining ways to compare volumes by using the number of units as they did with length and area. [Knowledge] Students understand that the volume of rectangular prisms can be expressed in terms of the number the unit 1 cm³.
2		<ul style="list-style-type: none"> Express the size of a cube using the number of 1-cm unit cubes. Learn the term, "volume," and the unit of volume, "cubic centimeter." 	
3	Understand and use the formula for calculating the volume of cubes and rectangular prisms.	<ul style="list-style-type: none"> Think about ways to calculate the volume of the rectangular prism with length of 4 cm, width of 6 cm, and height of 5 cm, and the cube with an edge of 5 cm. Investigate how many 1 cm³ cubes will fit in each. 	<ul style="list-style-type: none"> [Thinking] Students are thinking and explaining the meaning of the volume formulae using concrete objects, diagrams and mathematical expressions. [Skill] Students can calculate the volume of cubes and rectangular prisms using the formulae.
4		<ul style="list-style-type: none"> Summarize the formulae for calculating the volume of cubes and rectangular prisms. Determine the volume of cubes and rectangular prisms using the formulae. 	

5	Research Lesson Think about diverse ways to determine the volume of figures composed of rectangular prisms and actually determine the volume.	<ul style="list-style-type: none"> Think about different ways to determine the volume of the figure composed of rectangular prisms. Share and discuss students solution strategies. 	<ul style="list-style-type: none"> [Interest] Students recognize that all methods make use of cubes and rectangular prisms, and they realize the merits of using their prior learning. [Thinking] Students think about and explain ways to determine the volume of a composite figure such as subdividing and adding on to the given figure.
(2) Various units of volume [3 lessons]			
6	Understand the unit of volume, "cubic meter (m^3)" and the relationship, $1 m^3 = 1000000 cm^3$.	<ul style="list-style-type: none"> Know the unit of volume, "cubic meter," and determine the volume of the rectangular prism with the length of 3 m, width of 4m and height of 2 m. Investigate how many cm^3 is in $1 m^3$ and summarize the relationship, $1 m^3 = 1000000 cm^3$. Physically experience $1 m^3$ by building a cube with the volume of $1 m^3$. 	<ul style="list-style-type: none"> [Thinking] Students think about and explain the need for m^3 based on their prior knowledge of the relationship among measurement units. [Knowledge] Students understand the relationship $1 m^3 = 1000000 cm^3$.
7	Understand the meaning of and ways to determine "inside measurement" and "capacity." Understand the relationships, $1 L = 1000 cm^3$ and $1 mL = 1 cm^3$.	<ul style="list-style-type: none"> Think about the volume of water that fits in the 1 L measuring cup whose length, width and height are all 10 cm. Know the meaning of the terms, "inside measurement" and "capacity." Explore the relationships, $1 L = 1000 cm^3$ and $1 mL = 1 cm^3$. Read "Mathematical Story" and understand the the volume of complex figures can be determined by immersing them in water. 	<ul style="list-style-type: none"> [Knowledge] Students understand the relationships, $1 L = 1000 cm^3$ and $1 mL = 1 cm^3$.
8	Deepen the understanding and increase interest in the topic through a mathematical activity.	<ul style="list-style-type: none"> "Let's try" - Using a square construction paper with the side length of 12 cm, build different open box and determine their capacities. 	<ul style="list-style-type: none"> [Interest] Students are trying to use and incorporate their learning in the activity.
(3) Summary [2 lessons]			
9	Apply their learning to solve problems.	<ul style="list-style-type: none"> Solve problems in "Power Builder." 	<ul style="list-style-type: none"> [Skill] Students can solve problems by applying what they learned.
10	Check for the learning and solidify learning.	<ul style="list-style-type: none"> Solve problems in "Mastery Problems." 	<ul style="list-style-type: none"> [Knowledge] Students mastered the contents.

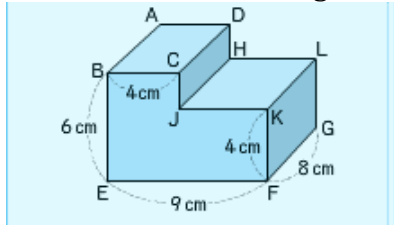
6 Today's Lesson

- (1) Date/Time Friday, June 24, 2016, 14:00 - 14:45
- (2) Location Ryuou Elementary School, Kai City, Music room
- (3) Goal of the lesson
 - Students can apply what they have learned so far and think about ways to determine the volume of complex figures.
- (4) Rationale of the Lesson

In this lesson, we are emphasizing *Mondai Kaiketsu Gakushu* lesson format. Thus, we include an activity where students will engage in independent problem solving. By explaining their own reasoning in ways that are easy to understand and making sense of their peers' ideas, we want to further students' ability to think and reason mathematically.

To utilize ICT equipments, we will distribute the problem to students' iPad's using the edutab system. Students will then record their thinking in iPad so that it can be displayed in a large-screen TV monitor. This way, we want to secure sufficient time for collaborative learning, and students can compare their own ideas with their peers. We want to enhance instruction that allow students to teach and learn from each other.

(5) Flow of the Lesson

Steps	Learning Activity and Content	Anticipated Student Responses	Instructional Considerations Assessment Standards and Strategies
Grasp (Understand the task) 2 min.	<p>1. Review the ways to calculate the volume of cubes and rectangular prisms.</p> <p>2. Grasp the task.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> Let's think about ways to calculate the volume of the L-shape solid. </div> <p>(Problem) Let's calculate the volume of the following solid.</p>  <div style="border: 1px dashed black; padding: 5px; margin: 10px auto; width: fit-content;"> What is different from what we have been studying? </div>	<ul style="list-style-type: none"> • It's like steps. • Not a cube or a rectangular prism. • We can't use the formulae. 	<ul style="list-style-type: none"> • Post the figure and help students grasp the task. • If students have difficulty imaging the figure, remind them about the ways they figured out the area of composite figures as a hint.

Think (Independent Problem Solving) 8 min.

3. Think about ways to calculate the volume using diagrams and mathematical expressions.

Record your idea using diagrams and mathematical expressions.

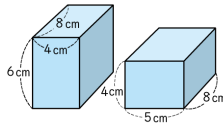
- Summarize their own ideas by marking the figures that have been pasted in their notebooks.
- If students find the volume, encourage them to think about other ways to find the volume.

- Select one of the ideas and record it on their own iPad.

<Subdivide (1) >

Partition vertically to create 2 rectangular prisms and find the sum of their volumes.

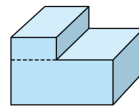
$$8 \times 4 \times 6 + 8 \times 5 \times 4 = 352$$



<Subdivide (2) >

Partition horizontally to create 2 rectangular prisms and find the sum of their volumes.

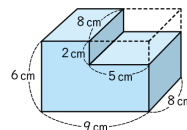
$$8 \times 4 \times 6 + 8 \times 9 \times 4 = 352$$



<Subtract>

From the large rectangular prism, subtract the rectangular prism that is not actually there.

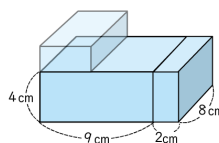
$$8 \times 9 \times 6 - 8 \times 5 \times 2 = 352$$



<Move•Transform>

Cut the solid horizontally and move the top part to the side of the bottom part.

$$8 \times (9 + 2) \times 4 = 352$$

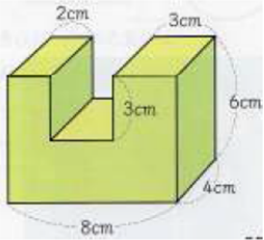


- Distribute a few copies of the figures to students so that they can paste them in their notebooks.
- For those students who cannot get started, prepare the model that can be taken apart.

[Thinking]
Students are thinking about ways to calculate the volume of the L-shaped solid by partitioning and adding on to the given solid, using diagram and mathematical expressions.
(Notebook)

Push the figure to students' iPad's and have them record their ideas.

<p>Learning Together (Group Reasoning) 15 min.</p>	<p>4. Share their ideas and learn their peers' ideas.</p> <div style="border: 1px dashed black; padding: 5px; margin-bottom: 10px;"> <p>Record your idea using diagrams and mathematical expressions.</p> </div> <ul style="list-style-type: none"> • Share their own ideas. • First confirm that the volume is 352 cm^3. • Think as they listen to other students' ideas. • Think about the mathematical expressions for various approaches. • For the horizontal partitioning method, have them figure out the method from the mathematical expression. <ul style="list-style-type: none"> • Discover the common idea. <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Even shapes like the L-shape, we can find the volume by making use of cubes and rectangular prisms.</p> </div>	<ul style="list-style-type: none"> • It's the same as mine. • What will the mathematical expression be? • Summarizing my idea with diagram and mathematical expression makes it easier to share with my friends. • We can divide the figure either vertically or horizontally. The answers will be the same either way. • I didn't think about <Move• Transform> - we can make a single rectangular prism. • All ideas are making use of cubes and rectangular prisms. <ul style="list-style-type: none"> • I want to try other shapes. 	<div style="border: 1px dashed black; padding: 5px; margin-bottom: 10px;"> <ul style="list-style-type: none"> ♦ Using the eduweb system, grasp students' ideas using the teacher's iPad so that students may be called on intentionally. </div> <ul style="list-style-type: none"> • Organize students' ideas and record them on the black board. • If no one uses <Move• Transform>, post the mathematical expression and have students figure out the method. • Combine multiple students' voices to complete each idea. • Have other students paraphrase one idea so that all students understand each idea. <p>[Interest] Students realize that each idea is based on the shapes they have learned previously, and they recognize the merit of using prior learning. (Oral presentation/ Observation)</p>
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	<p>5. Lets' find the volume of the solid below.</p>  <ul style="list-style-type: none"> • Using one of the 3 methods, calculate the volume. • Students will indicate which of "Subdivide," "Subtract" and "Move• Transform" they will use through the multiple choice feature of the edutab system. • Have students share which option they picked and the mathematical expressions. 	<ul style="list-style-type: none"> • With this solid, I think this method is easier. • We can use Move• Transform with this shape. 	<div style="border: 1px dashed black; padding: 5px;"> <ul style="list-style-type: none"> ◆ Using the eduweb system's multiple choice feature, have students select one of the three methods in response to the diagram of the solid pushed to students' iPad's. </div> <ul style="list-style-type: none"> • Because all students' ideas will be posted, students can autonomously engage in the task. • After observing the distribution of students' choices, summarize students' ideas using the figures displayed on the monitor.
	<p>6. Write the journal.</p>	<ul style="list-style-type: none"> • I could make sense of my friends' ideas. • Some ideas are simpler to use for some shapes. 	

(6)

Points of observation

- ① Were the goals of the lesson achieved?
- ② Could students own their own ideas toward solving the problem.
- ③ Was the use of the ICT equipment effective for students to share their ideas?

(7) Board writing plan/Resources

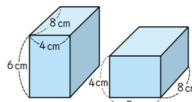
Let's find the volume of the L-shape

Volume of:
Rectangular Prism
Length×Width×Height

Cube
Edge×Edge×Edge

Subdivide 1

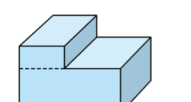
Divide vertically to make 2 rectangular prisms.



$8 \times 4 \times 6 + 8 \times 4 \times 6 = 352$

Subdivide 2

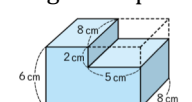
Divide horizontally to make 2 rectangular prisms.



$8 \times 4 \times 2 + 8 \times 9 \times 4 = 352$

Subtract

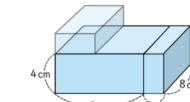
Subtract the small part that is not there from the large rect. prism.



$8 \times 9 \times 6 - 8 \times 2 \times 6 = 352$

Move•Transform

Divide horizontally. Mover the top part next to the bottom to transform the shape into one rectangular prism.



$8 \times (9 + 2) \times 4 = 352$

The volume for shapes like the L-shape can be calculated by making use of cubes and rectangular prism.