



Grade 4 Mathematics Lesson Plan

Date: Wednesday, June 22, 2016  
Grade 4 Homeroom 2  
36 students  
Teacher: MIZOSHITA, Takayuki

Research Theme  
**"Mathematics lessons in which students will autonomously reason and create"**  
Valuing "question" and "sharing"

- 1 Name of the Unit  
Let's think about ways to calculate
- 2 Goals of the Unit  
Students can think about ways to calculate  $48 \div 3$  using diagrams and their prior knowledge of division.

3 Assessment Standards for the Unit

	Interest, Eagerness, and Attitude	Mathematical Way of Thinking	Mathematical Skills
Goal	Students are trying to think about ways to calculate $48 \div 3$ using their prior learning.	Students are thinking about different ways to calculate $48 \div 3$ such as decomposing the dividend using concrete materials, diagrams and mathematical expressions.	Students can express their ideas for calculating $48 \div 3$ using pictures and/or diagrams so that others can easily understand.
A	Students are trying to think about a variety of ways to calculate $48 \div 3$ using their prior learning.	Students are not only thinking about different ways to calculate $48 \div 3$ such as decomposing the dividend using concrete materials, diagrams and mathematical expressions but also summarizing how they used their prior learning.	Students can not only record their ideas for calculating $48 \div 3$ using pictures and/or diagrams but also explain their ideas in a way that is easy to understand by others.
B	Students are trying to think about ways to calculate $48 \div 3$ using their prior learning.	Students are thinking about ways to calculate $48 \div 3$ by decomposing the dividend using concrete materials, diagrams and mathematical expressions.	Students can record their ideas for calculating $48 \div 3$ using pictures and/or diagrams.

#### 4 About the Unit

##### (1) From *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics*

The following statements concerning division are found in this document:

**1 Goals**

(1) Students will deepen their understanding of division and use it appropriately.

**3 Content**

(3) Students will deepen their understanding of division of whole numbers and will be able to calculate accurately and reliably. They will extend their ability to utilize division in different situations appropriately.

a. To understand that division of 2- and 3-digit numbers by 1- and 2-digit numbers is based on basic division facts by exploring ways to complete the calculations; to understand how to divide using an algorithm.

b. To be able to divide accurately and reliably and to be able to use divisions appropriately.

c. To explore the relationship among dividends, divisors, quotients and remainders, and to summarize this relationship in the following expression:  

$$\text{Dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

d. To explore properties of division and use these properties to think about ways of dividing or checking answers.

**(Mathematical Activity)**

(1) For the contents of “A: Number and Operations,” “B: Quantities and Measurement,” “C: Geometric Figures,” and “D: Quantitative Relations,” instruction should incorporate mathematical activities such as those listed below.

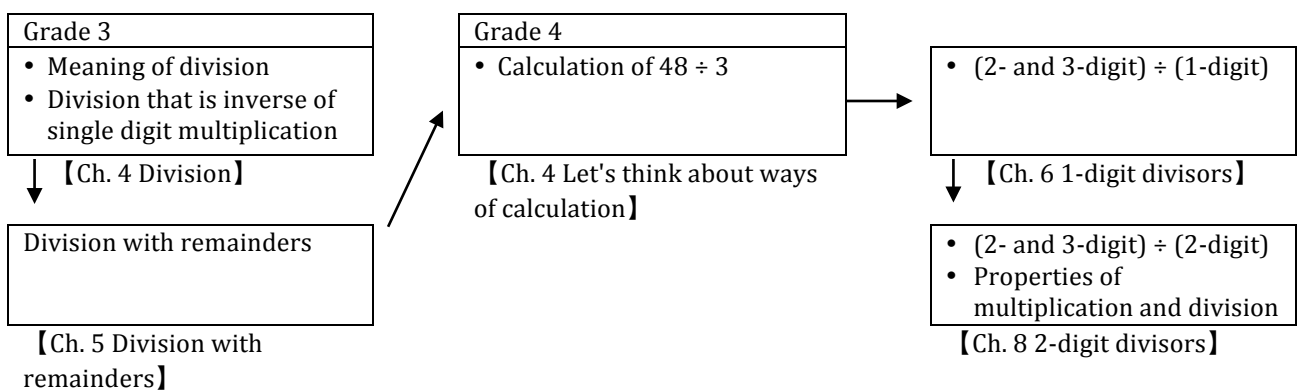
a. Making appropriate estimates for computation, and evaluating the appropriateness of the results of computation.

**Remarks Concerning Content**

(2) As for the content A-(2)-c, (3), and (4) students should be able to complete simple calculations mentally. Furthermore, students should be able to use mental calculation while carrying out algorithms and estimating.

(3) As for the content A-(3)-d, the property of division that the quotient will remain the same when both dividend and the divisor are multiplied or divided by the same number should be taught.

##### (2) Scope and Sequence



(3) Unit Plan (Total of 2 lessons)

#	Content	Assessment Domains				
		I	T	S	K	Specific Content
1	(Today's lesson) • Explain ways to calculate $48 \div 3$ using diagrams and mathematical expressions based on prior learning.	○	□			• Students are thinking about ways to calculate $48 \div 3$ using concrete materials, diagrams and mathematical expressions.
2	• Think about ways to calculate $56 \div 4$ and summarize own way of calculation so that it is easily understood by others.	○		□		• Students can record their own ways of calculation using pictures and diagrams.

(4) Overview of Instruction

Students have already learned the meaning of division and division problems that are inverse of single digit multiplications in Grade 3. In this unit, students will think about how to calculate division that goes beyond single digit multiplication. They will do so as they represent the calculation using diagrams based on what they have learned about division so far. By drawing out a variety of calculation methods from the students, we want these lessons to be the place where students can identify commonalities and merits of each other's ideas and recognize each other.

(5) About Instruction of the Content

I believe it is important to study this unit, [Ch. 4 Let's think about ways of calculation], with clear distinction from [Ch. 6 1-digit divisors]. Often times, the division algorithm is taught and learned with the sole focus on learning the order of the steps in the algorithm without paying attention to the size of numbers. Instead, in this unit, [Ch. 4 Let's think about ways of calculation], I believe the aim is for students to figure out the quotients using their prior knowledge of division. When students deal with the division that goes beyond the basic multiplication facts - that is, the divisor of 48 - on their own, I believe it is important for them to relate their methods with diagram so that they are not manipulating symbols mindlessly. Students are expected to figure out the quotients using their prior knowledge alone. I also want them to think about what diagrams to use, how to arrange their diagrams, and how to split their diagrams. Moreover, in this unit, students are presented with partitive division situations while the initial phase of [Ch. 6 1-digit divisors] involves quotitive division even though the first lessons in both units start with the dividend of 48. This is another point to which we want to pay attention.

5 About the Students

Many of the students in this class are cheerful and spirited. While many students tackle mathematics learning eagerly, there are some who are struggling with basic skills like the multiplication and division facts. There are some who can calculate but their understanding of the meaning of multiplication and division is rather

insufficient. Some of those students want the teacher to show them "how to" and master the necessary skills through repeated practices using worksheets.

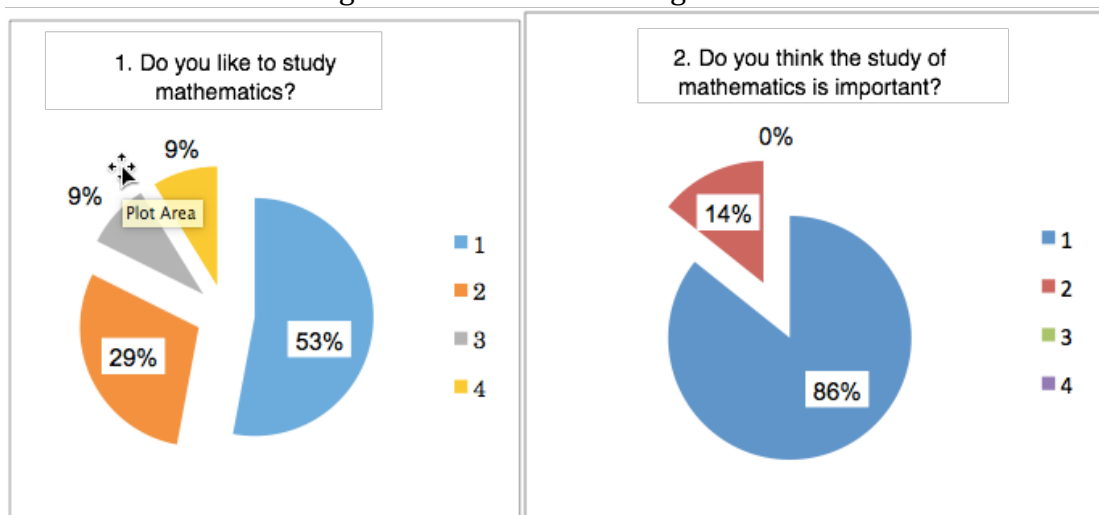
In everyday lesson, I try to base the lesson development on students' ideas. However, occasionally lessons became too open ended because students who were more vocal or who have learned the content outside of the school or who just wanted to say whatever they felt like ended up dominating the class discussion. Some students end up talking for a long time while others are very difficult to understand. As a result, other students get tired of listening to their classmates and give up listening altogether.

Therefore, I have been providing additional support to those students who are struggling with basic skills outside of the regular mathematics lessons. Also, after reflecting on the way mathematics lessons should be organized, I've been trying to establish mathematics lessons where all students feel safe yet not uncontrollably open ended.

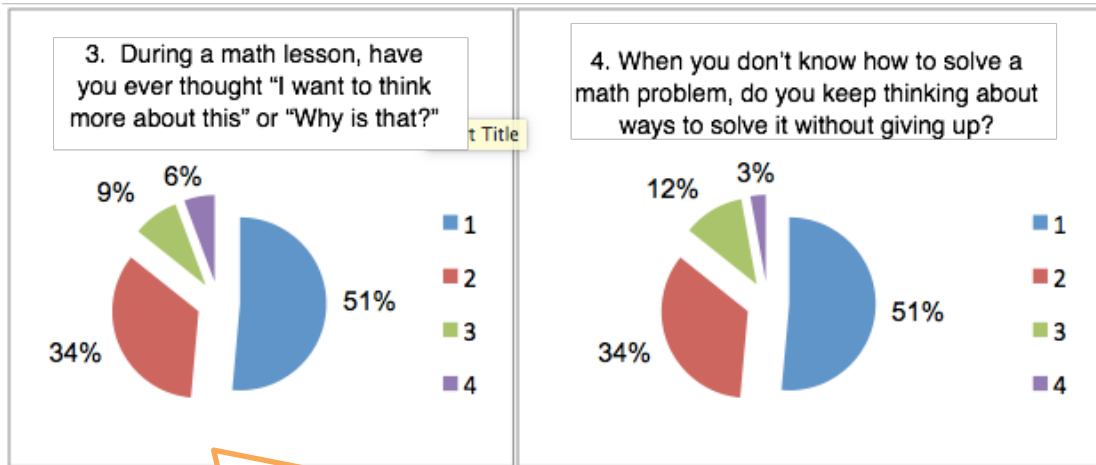
### Results from a classroom survey

Students were indicated their agreement/disagreement on a series of statements. The choices were:

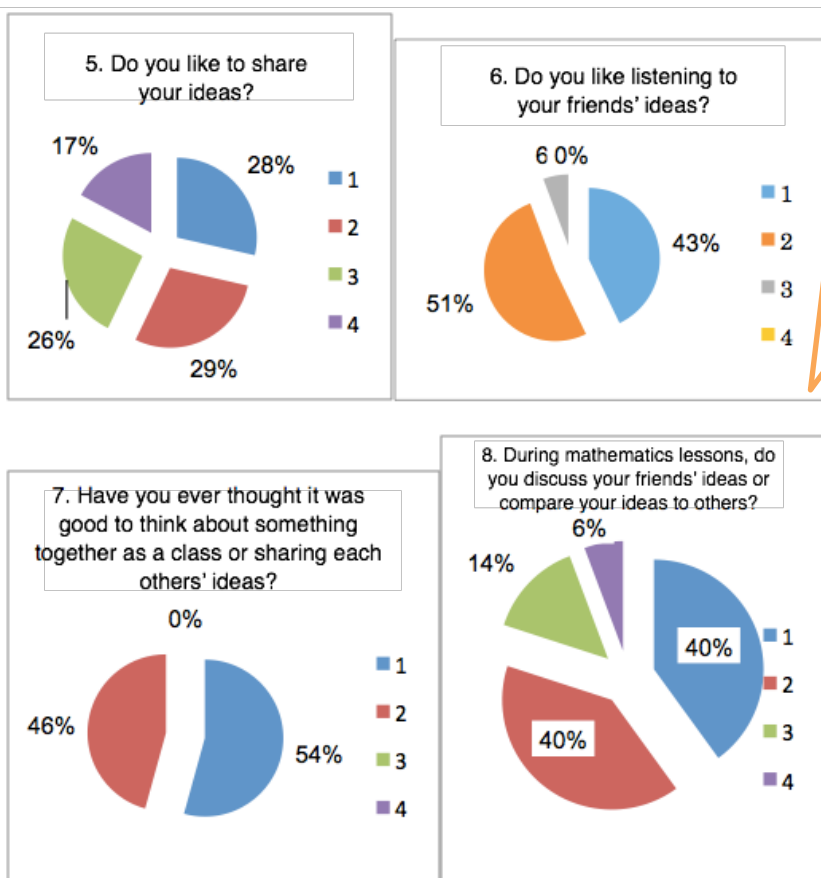
- 1: agree      2: somewhat agree  
3: somewhat disagree      4: disagree



Considering learning motivation, 100% of the students think mathematics is "important." On the other hand, 18% (6 - 7 students) of the students do not like mathematics. Thus, not a small number of students feel mathematics is important yet dislike the subject.

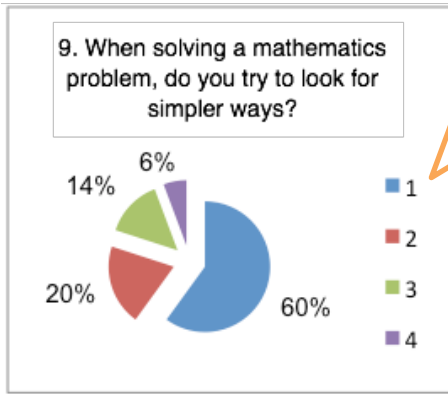


The response patterns to items 3 and 4 are almost identical. In other words, if students think "I want to think more about this" or "Why is that?", they will persist during the mathematics lesson. On the other hand, if they do not have those questions, they do not actively engage in mathematics lessons. This results clearly shows the importance of student thinking "I want to think more about this" or "Why is that?" during mathematics lessons.

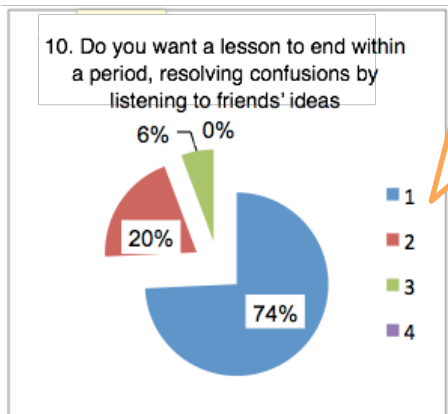


The contrasting responses to items 5 and 6 are most telling outcome in this survey. While 94% of students consider positively about listening to their peers' ideas, about a half of the class feel uncomfortable about sharing their own ideas.

However, 100% of the students responded that it was good to share each other's ideas. Moreover, the responses to item 8 show that many students are comparing their own ideas with their peers' ideas. From these results, many students believe discussing each others' ideas is important but they feel uncomfortable speaking in front of the whole class. Therefore, I want to incorporate more group and pair learning opportunities in lessons.



From item 9, we can tell many students are thinking about simpler ways to solve problems. On the other hand, many students are observed to stop working once they find an answer. I believe that is because many students think there is only one answer for a mathematics problem and one way of calculation. Therefore, I want to encourage students to keep looking for other ways of solving the same problem.



This survey was conducted independently. In the past, when I tried to conduct a lesson aiming at students generating own questions and sharing their ideas, too often the lesson lost focus and ended without clear a conclusion. Sometimes we simply ran out of time. I wanted to see how students felt about those lessons. The responses show that 94% of students would like a lesson to conclude within a period. I will try to make the problem posing more efficient and be more conscious about time allocation for independent problem solving and summarizing so that a lesson will be completed within a period.

## 6 Strategies to Address Research Theme

### Research Theme

Mathematics lessons in which students will autonomously reason and create:  
Valuing "question" and "shared"

Based on this research theme, we developed the image of ideal students with respect to "questions" and "shared," and strategies to achieve the them were devised.

#### To generate "questions"

What is students' question...

- What? I think I can do it! I think I can use what I learned!

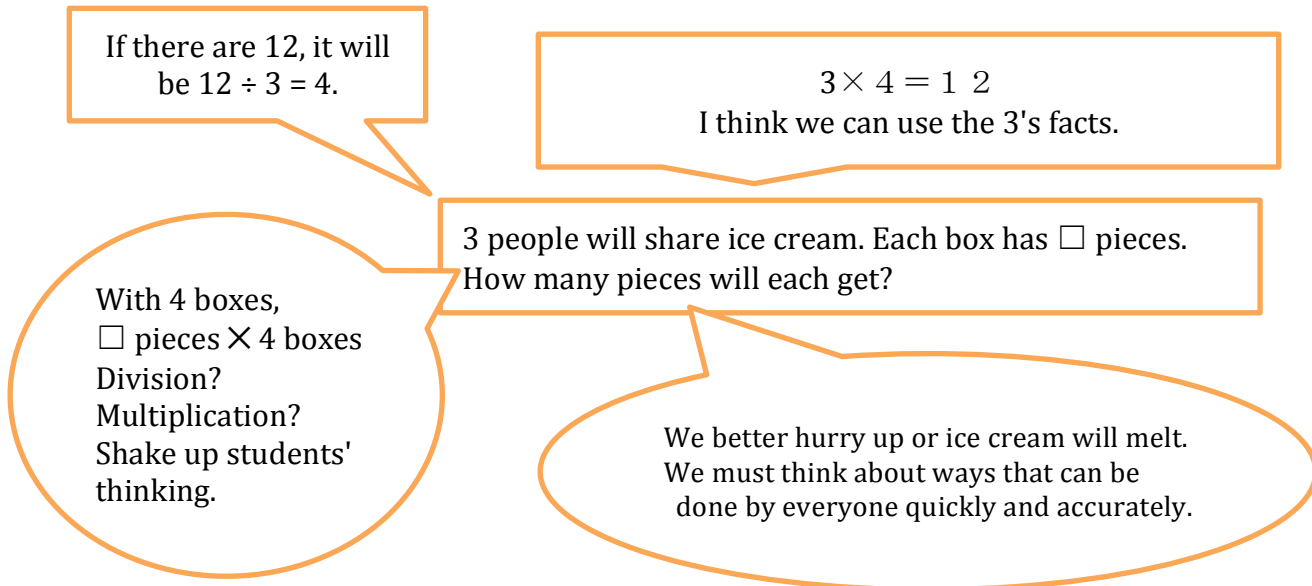
To generate the questions above, we devised the following strategies.

#### ① Modifying *hatsumon*

Sometimes, I paid attention solely on students' ideas and how to connect them and lost focus of the discussion. As a result, students simply express their ideas randomly and the lesson became open ended. Such an approach will not reach the problem solution within the class period. In order to reach a satisfactory resolution of students' questions within a class period, the teacher must carefully draft *hatsumon* and help students generate their questions efficiently by picking up their noticing and ideas. Students' noticing and ideas will be valued by teacher comments or board writing.



In the textbook, the lesson starts with  $48 \div 3$ . However, the research lesson will start with  $12 \div 3$  to set up the situation in which students will eagerly tackle problems. Thus, we will start with a problem that students can readily solve first then a more challenging yet related problem will be given. It is hoped that students will approach the new problem with the idea that they could use what they learned previously.



To promote "sharing"

Sharing will be evident in students:

- So what \_\_\_\_ is saying is \_\_\_\_\_.
- Oh. I got it. That's what he/she meant.
- If you change the order of numbers, you can see different calculations.
- I think we can use that idea with other numbers.

① Carefully devise *hatsumon*

◇ Reflecting the question

"Did you understand what \_\_\_\_ said?" "Did \_\_\_\_'s explanation make sense to everyone?"

Too often, only a certain group of students respond to the teacher's questions. When that happens, a lesson simply becomes a series of teacher question and student answer, and only a part of the class actually participates in the lesson. Thus, by having multiple students answer the same question or reflecting questions, I will try to have more students participate in the lesson.

When a variety of students express their ideas, once we reach the main task to be considered in a lesson, I will try to crystalize the question and ensure that everyone has the shared understanding of the task.

◇ Predict/interpret peers' ideas

"Do you understand what \_\_\_\_ wrote?" "Do you understand why \_\_\_\_ thought this way?"

Have students write only mathematical expressions and diagrams so that other students can think about how those students who shared their ideas thought about the task.

② Make use of visual representations.

When we share our ideas, if we just communicate orally, even when the audience is attentive, they may have difficulty understanding our ideas. Thus, by encouraging students to incorporate diagrams and mathematical expressions as they explain their ideas, they can make their sharing more effective and deepen other students' understanding.

③ Make use of hand signals.

From the classroom survey, we know that many students hesitate to share their ideas in class. To help these students engage in a lesson more actively, we feel it is important for them to make their ideas more public. Therefore, we will ask them to use hand signals (paper-rock-scissors) to make their ideas explicit. Even if they are not comfortable sharing their ideas orally, they can still make their ideas known to others and their ideas can be shared by others in the classroom.

④ Provide opportunities for pair and group discussion.

From the class survey, we know that there are students who know the importance of sharing yet uncomfortable about sharing their ideas in public. In order to support these students, we will set up opportunities for these students to share their ideas in pairs or in small groups. By providing them more opportunities to express their ideas orally, they can deepen their understanding.



7 Flow of the Lesson

<p>Learning Activity T: Teacher <i>hatsumon</i> C: Anticipated responses</p>	<ul style="list-style-type: none"> <li>• Instructional consideration [ ] Assessment</li> </ul>
<p>1 The problem will be presented and students grasp the task.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>There are <input type="checkbox"/> ice creams in each box. If 3 people share the ice creams fairly, how many ice creams will each person receive?</p> </div> <p>C: It will be easy if there are 6 in each box. C: We can do it even if there are 12. T: So, what if there were 12 ice creams in a box? C: <math>12 \div 3 = 4</math> T: How did find the answer? C: I used the 3's facts.</p> <p>T: OK, I'm going to challenge you. (Increase the number of boxes from 1 to 4.)</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>There are 12 ice creams in each box, and there are 4 boxes. If 3 people share the ice creams fairly, how many ice creams will each person receive?</p> </div> <p>T: Can you write the calculation expression? C: <math>48 \div 3</math>. T: Since this goes beyond the 3's facts you learned, let me find the answer. (Deal one ice cream at a time.) C: You should give more each time. C: This takes too much time.</p> <p>T: It seem like you have many ideas. Let's think about ways to calculate, not just finding the answer.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0; text-align: center;"> <p>Goal: Let's think about ways to calculate <math>48 \div 3</math>.</p> </div> <p>T: The answer will be greater than what number? Any idea? C: 10. Because <math>30 \div 3 = 10</math>. So, it must be greater than 10.</p>	<ul style="list-style-type: none"> <li>• Start with the numbers that all students feel comfortable with.</li> <li>• Remove the poster with the problem and have them think about the division expression.</li> <li>• Make sure that students understand that this division goes beyond the 3's facts they have learned.</li> <li>• Have students make an estimate of the answer.</li> <li>• Plant the idea of splitting the dividend into simpler numbers to calculate.</li> </ul>

## 2 Independent problem solving

C1: Each box has 12. We share those among 3, and there are 4 boxes.

Calculations:

$$12 \div 3 = 4$$

$$12 \div 3 = 4$$

$$12 \div 3 = 4$$

$$12 \div 3 = 4$$

C2: Split 48 into 30 and 18

$$30 \div 3 = 10$$

$$48 \div 3 < \quad > = 16$$

$$18 \div 3 = 6$$

C3: Split 48 into 24 and 24

$$24 \div 3 = 8$$

$$48 \div 3 < \quad > = 16$$

$$24 \div 3 = 8$$

C4: Distribute one box to each, then share the final box among three.

$$12 \div 3 = 4$$

$$12 + 4 = 16$$

C5: Get stuck within the 3's facts.

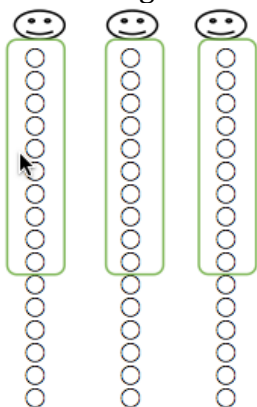
$$48 \div 3 = 10 \text{ remainder } 18$$

C6: First share among 6, then then make 3 groups.

$$48 \div 6 = 8$$

$$8 \times 2 = 16$$

C7: Use a drawing.



C8: Cannot get started.

[Thinking]

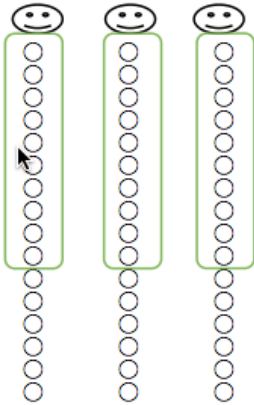
Decompose the large number and make use of the 3's facts.

- Thinking 48 as 12, 12, 12, and 12 - 4 boxes.

- I want to help this idea to  $30 \div 3$  and  $18 \div 3$ .

- I want to connect this to  $30 \div 3$  and  $18 \div 3$  as well.

- If a student can't get started, allow them to go look at how other students' are doing (spying time).

<p>3 Group discussion</p> <p>T: OK, show me your progree. If you have found the answer, show me a "rock." If you are in the middle of figuring it out, show me a "scissors." If you are not sure, show us a "paper." People with a "paper" is really important people for the lesson because you make the lesson meaningful. It will be great if we can help each other get to a higher level.</p> <p>(Form 4-member groups for group discussion.)</p>	<ul style="list-style-type: none"> <li>• Make sure that each group has at least one person who shows a "rock."</li> </ul>
<p>4 Whole class discussion</p> <p>T: I want those of you started with a "scissors" or a "paper" and understood something to share. Please share the wonderful ideas you learned.</p> <p>C2' Split 48 into 30 and 18.</p> $30 \div 3 = 10$ $48 \div 3 < \quad > = 16$ $18 \div 3 = 6$ <p>C7 Draw a picture.</p>  <p>C4' Distribute one box to each, then share the final box among three.</p> $12 \div 3 = 4$ $12 + 4 = 16$ <p>C1': Each box has 12. We share those among 3, and there are 4 boxes.</p> <p>Calculations:</p> $12 \div 3 = 4$ $12 \div 3 = 4$ $12 \div 3 = 4$ $12 \div 3 = 4$	<ul style="list-style-type: none"> <li>• Make sure to include C2, C4, and C1 in the discussion.</li> <li>• Start with the idea with the fewest number of students.</li> <li>• As they listen to other students' ideas, have them show their copenhension using hand signals: "rock" - I understand; "scissors" - sort of understand; "paper" - hard but I'm trying to make sense.</li> <li>• Stop in the middle of the explanation, and have students discuss the rest of the idea with their neighbors.</li> <li>• Highlight the important points in each idea on the board.</li> </ul>

<p>5 Summarize</p> <p>T: Which idea made sense the most? Why? C: (Have students raise their hands for the idea that made sense the most.) T: A large number like 48 .... C: ... can be split so that we can use the multiplication table.</p> <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p>Summary If we split a large number, we can still use the multiplication table to calculate division.</p> </div>	<ul style="list-style-type: none"> <li>• If time allows, have students write journal entries.</li> <li>• The prompt for the journal entry will be, "A large number like 48..."</li> <li>• Have students think about ways to calculate division that goes beyond the multiplication table.</li> <li>• Try to use students' words to summarize the lesson.</li> </ul>
<p>6 Consolidate learning through an additional problem.</p> <p>T: What should we do with 54 to find the answer for <math>54 \div 3</math> by using the multiplication table? C: <math>30 \div 3</math> and <math>24 \div 3</math>. C: <math>27 \div 3</math> and <math>27 \div 3</math>.</p>	<p>[Thinking] Students are using the multiplication table to calculate <math>54 \div 3</math> by decomposing 54.</p> <ul style="list-style-type: none"> <li>• I want to emphasize how the large number is decomposed instead of just focusing on the correct answer.</li> </ul>

#### Evaluation Points

- Based on prior learning, students are explaining ways to calculate  $48 \div 3$  using mathematical expressions and diagrams.
- Student can decompose large dividends into appropriate numbers so that they can use the multiplication table.