June 21

Grade 6 Mathematics Lesson Plan
What is the size of the shaded part? (Area of curved figures)

Date: Tuesday, June 21, 2016
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1 Unit Area of curved figures

2 Goals of the Unit

- Students will be able to calculate the area of figures.
- Students will be able to approximate figures in their surroundings and determine their area.

3 Assessment Standards for the Unit

Interest, Eagerness, and Attitude	Students try to determine the area of circles and the approximate area of figures in their surroundings based on the idea of quantifying area in terms of the number of units and ways to calculate the area of basic figures.
Mathematical Way of Thinking	Students think about ways to determine the area of circles and the approximate area of figures in their surroundings based on their prior learning of geometric figures.
Mathematical Skills	Students can determine the area of circles and the approximate area of figures in their surroundings by using grids or approximating the figure as similar to figures that have been learned previously.
Knowledge and understanding	Students understand that to determine the area of circles and the approximate area of figures in their surroundings we can make use of geometric figures that have been learned previously.

4 About the Unit

(1) Purposes

About circles, students learned about the center, radius and diameter in Grade 3. They engaged in a variety of activities to investigate and draw circles. In Grade 5, they learned the meaning of *pi* based on the relationship between the diameter and the circumference and also the way to calculate circumference.

In this unit, the primary focus is on ways to calculate the area of circles. Through Grade 5, students have learned about the meaning and properties of various geometric figures. They have also learned about the meaning of and ways to calculate the area of various figures. They learned to derive area formulas by transforming the given figure into the basic figures for which they already know the formulae. In this unit, we will try to remind students of their prior learning so that they can think about and explain ways to calculate the area of circles.

In addition, students will engage in activities to determine composite figures that include circles (or parts of circles) by making use of the area formula for circles. In this way, students' ways to observe geometric figures will be enriched, and their ability to solve problems while making explicit the rationale for their reasoning. In addition,

we want students to further their ability to communicate their own ideas by relating their ideas with diagrams, mathematical expressions and words.

(2) About the students

In this classroom, there are many students who are eager to explain their reasoning and solution strategies to their peers. However, there are individual differences in their knowledge, skills, and reasoning capacity. Therefore, students are encouraged to keep in mind how they will explain their ideas as they write their ideas in notebooks so that their explanations will be well organized and easy to understand for their peers. As a result, these students are beginning to pay attention to the appropriate uses of figures, words and mathematical expressions in their writing.

(3) Abilities we want to emphasize in this unit

First, we want students to think about ways to make use the fundamental idea of "how many of unit area" as they think about the area of circles and derive the formula. We want them to think about the methods of area preserving transformation that will transform circles to familiar figures.

Second, as an application of area of circles, we will have students figure out the area and perimeter of composite figures that include the combination of circles, semicircles, and squares. We want students to further their ability to orderly explain their ideas by relating diagrams and mathematical expressions after they think about how to make use of familiar shapes.

5 Scope and Sequence

[Grade 4] Various Quadrilaterlas

- Concept and ways to draw perpendicular and parallel lines.
- Concept, properties, and ways to draw trapezoids, parallelograms and rhombi.
- Diagonals.

[Grade 4] Area

- Concept of area and area measurement.
- Area formulae for rectangles and squares.
- Large units of area.

[Grade 5] Area 2-D Figures

- Ways to determine the area of parallelograms, triangles, trapezoids, and rhombi. The area formulae for these figures.

[Grade 5] Regular Polygons & Circles

- Concept of regular polygons, their properties and ways to draw them.
- Relationship between the diameter and the circumference. *Pi* and ways to determine the circumference.

[Grade 6] Area of curved figures

- Concept of area and area measurement.
- Area formulae for rectangles and squares.
- Large units of area.

6 Unit Plan

	Learning Activity	Assessment			
		I	T	S	K
Area of Circles	• Approximate the area of the circle with the radius of 10 cm by using the inscribed and circumscribed squares. • Investigate the area of the circle with the radius of 10 cm by drawing it on a 1-cm grid paper.	○	◎		
Area formula for circles	• Think about various ways to determine the area of a circle. • Re-arrange the sectors obtained by equally partitioning a circle into 16 and 32 equal parts so that the resulting figures will be similar to a familiar shape.	◎		○	
	• Think about the situation when the sectors are arranged like a rectangle and derive the area formula for circles. • Calculate the area of circles using the formula. • Investigate how many times as much is the area of a circle when the diameter is doubled.		○	◎	
Problems to find the area of complex figures	• Calculate the area and the perimeter of semicircles. • Calculating the area of figures composed of squares and quartercircles. Calculate the length of curves.		○	◎	○
	• Think about ways to calculate the area composed of quartercircles. (**Today's Lesson**)			◎	
	• Draw sectors with the radius of 5 cm and the central angle of 120° and 300°. • Think about what happens to the area of sectors when their central angles are doubled or tripled. Calculate the area of the sector with the radius of 4 cm and the central angle of 45°.	○		◎	○
Approximate Area	• Determine the area of irregular shapes by counting the number of grids or approximating their shapes using the basic figures. • Calculate the area of regions on a map by copying them onto grid papers. • Calculating the area by approximating the figure as basic figures.	○	◎	○	
Practice	• Deepen the understanding of the content of the unit.				
	• Check for understanding of the unit content.				
	• By transforming a circle formed by a rope into a triangle, derive the area formula for circles using the area formula for triangles.	○	◎		

I: Interest, Eagerness, and Attitude; T: Mathematical Way of Thinking;
S: Mathematical Skills; K: Knowledge and understanding

7 About the Lesson (Lesson 5 of 10)

(1) Goal of the lesson

- Students can think about variety of strategies to determine the area of a composite figure involving circles and calculate the area.

(2) Points of the Lesson

In Grade 5, students learned the meaning of *pi* and the way to calculate the circumference based on the relationship between the diameter and the circumference. In addition, in this unit, they have studied ways to calculate the area of circles and developed the area formula.

In today's lesson, we will study a complex shape that does not suggest an obvious method of calculating its area. In order to determine the area of the region bound by curves, students must be able to figure out ways to make use of a square or a triangle and a quartercircle. Therefore, at the introduction step of the lesson, we will use the "mystery box" to reveal problems starting with previously learned figures so that students may be able to tackle the task with possible solution approaches.

In the "deepen/heighten" step of the lesson, students will be asked to interpret mathematical expressions using diagrams. By intentionally incorporating re-voicing and pair sharing, students will be asked to think about each of the various approaches.

We hope to nurture students' abilities to observe geometric figures flexibly and reason logically.

(3) Specific strategies to address the research theme

① Engaging students with the task and drawing out the question (Grasp)

In the introduction step, we will pose the question, "What is the area of colored region?" using the "mystery box," starting with a circle and a semicircle, which students have previously learned, and we will verify the mathematical expressions and the area of the figures. It is anticipated that some children will guess that the shape after the semicircle will be a quartercircle. However, contrary to their guesses, it will be a figure looking like a leaf. We designed this way of introducing today's task so that we can draw questions from students. Students will be puzzled to find the way to determine the shape that is different from their guesses. Because many students will think that they can find the area of a quartercircle, they may have possible ways to approach today's task.

② Experiencing the joy of solving a problem independently through solving a genuine problem (Explore)

During the independent problem solving step of the lesson, students will be encouraged to record their ideas in their notebooks in an orderly manner using mathematical expressions, words, and diagrams. Students who are using only mathematical expressions will be encouraged to think about ways to use both diagrams and mathematical expressions so that their ideas can be more easily understood by others. For those students who cannot initiate their solution approach, a small group discussion period will be set up, and hints will be given through questions such as what shape(s) they can see in the given diagram and what the area of the shape(s) so that they can independently find the solution to the question.

③ Through the activity of interpreting other students' ideas, students will answer their questions and deepen their reasoning (Deepen/Heighten)

During the whole class discussion, we will engage in the activity of interpreting mathematical expressions using diagrams. In order to maintain students' intellectual curiosity, we will vary how each of the three ideas will be discussed. For example,

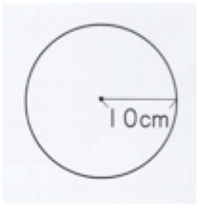
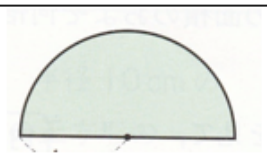
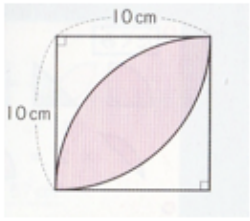
asking questions about each step of calculation in a strategy, or stopping a student's explanation in the middle and have other students figure out the rest of the solution. By intentionally incorporating revoicing and pair sharing during the discussion, we will try to engage all students with each of the approach so that they can explain their own ideas in an orderly manner.

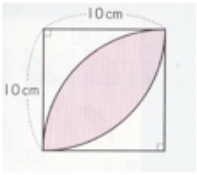
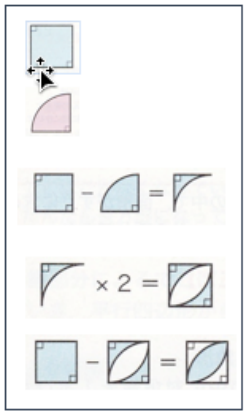
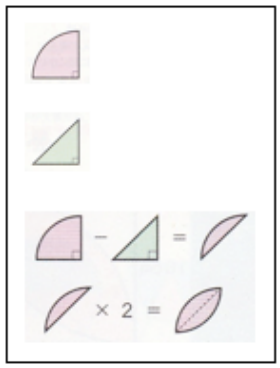
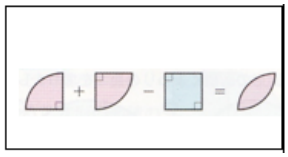
Because the figure is rather complex, there will be a variety of drawing with a square, a triangle, and/or a quartercircle(s). By using different colors for different parts of the drawings, students' explanations will be supported visually.

④ Through the activity of summarizing, applying, and extending, draw out additional questions from students (Summarize/Extend)

In the summarizing step of the lesson, we want students to write that they can figure out the area of a complex figure by making use of the figures they have previously studied. In order to do so, board writing and whole class discussion will be carefully organized. After students write their reflections, we will pose additional problems from the "mystery box" so that students will approach the next lesson with the enthusiasm, "I want to solve those problems" and "I can solve them using what I have already learned."

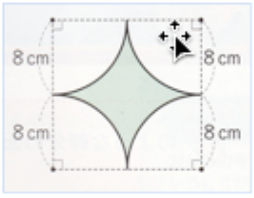
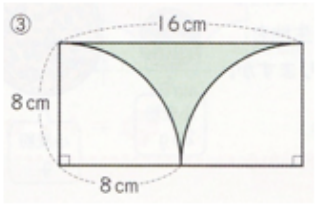
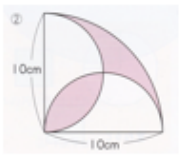
8 Flow of the lesson (Lesson 5 of 10)

	Learning Activity (Main questions and anticipated responses)	☆ Strategy to address research theme ○ Support and points of consideration ◎ Assessment
Grasp	1. Grasp the learning task. T (While drawing a shape from the "mystery box") What is the area of the colored part? Say the mathematical expression. (Circle) Radisu 10 cm  10 × 10 × 3.14 = 314 Answer 314 cm² T What will be the area of the colored part in the next picture? (Semicircle) Radius 10 cm  10 × 10 × 3.14 ÷ 2 = 157 Answer 157 cm² T Wow. You are really good. OK, here comes the next question. (Leaf shape)  C Yes!! C Whoa!? T Why did you say "Whoa!?" C I thought it would be a quartercircle, but it was different. T Can you find the area if it were a quartercircle? C Yes I can. T Do you think you can find the area of this leaf shape? C I think so!! T OK, I will write the problem for today's lesson. Please write it in your notebook, too.	☆ By utilizing the "mystery box," reflect on the prior learning and draw out the question, "How can we calculate the area of the leaf shape?" ○ Review the way to calculate the area of circles. ◎ Students have their own question and try to find the area. [Interest]

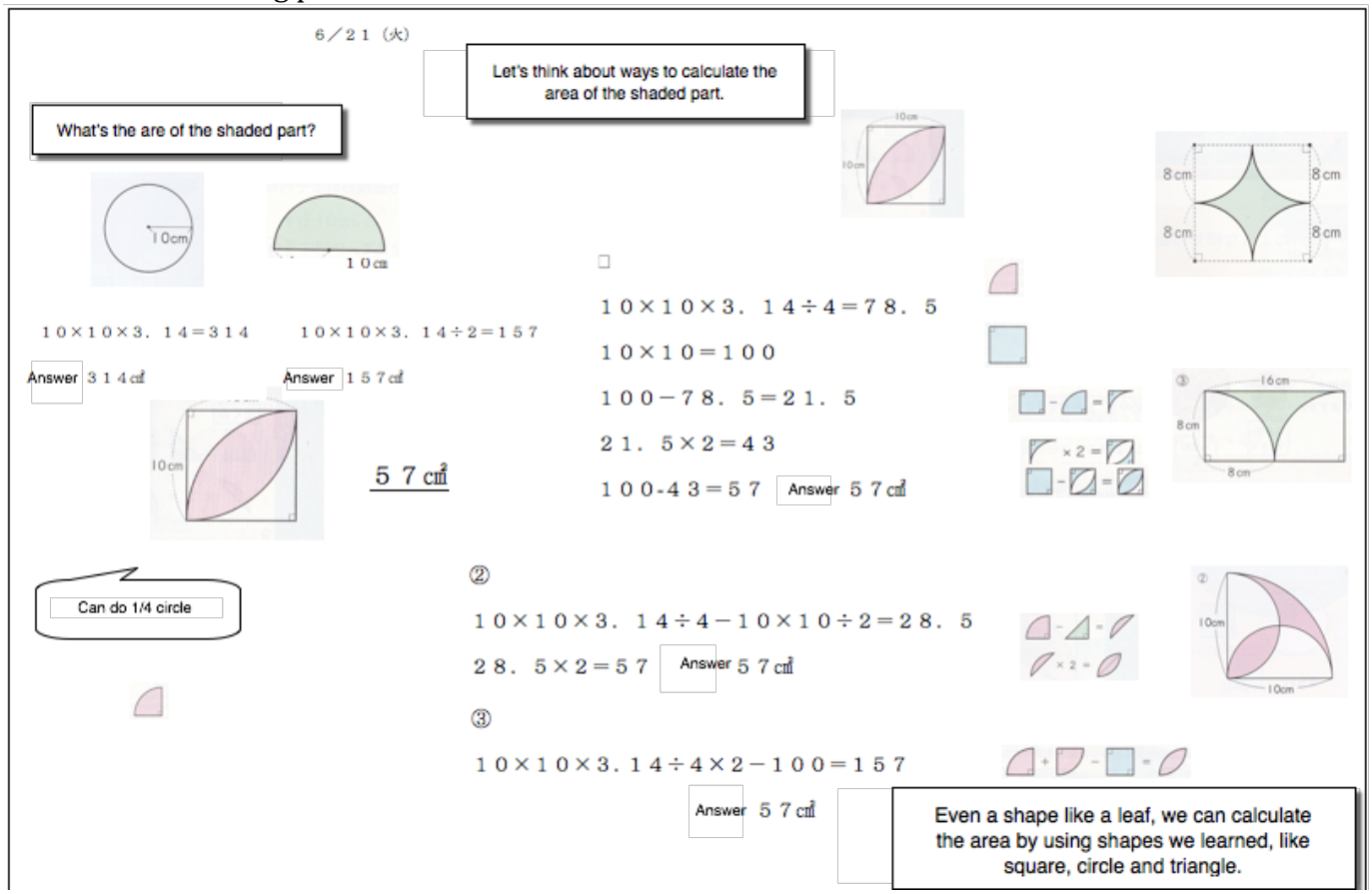
	Let's think about ways to calculate the area of the colored part.  T OK, let's try to find the area of the colored part.	○ Provide the diagram so that students can think more freely. Tell students that the drawing is not to the scale - it is smaller than the actual size.
Explore	2. Think about ways to calculate the area of the leaf shape by using mathematical expressions and diagram. [Anticipated Solutions] ① □ – ¼ circle × 2 10 × 10 = 100 10 × 10 x 3.14 ÷ 4 = 78.5 100 - 78.5 = 21.5 21.5 × 2 = 43 100 - 43 = 57 Answer 57 cm²  ② (¼ circle – ◿) × 2 10 × 10 x 3.14 ÷ 4 = 78.5 10 × 10 ÷ 2 = 50 78.5 - 50 = 28.5 28.5 × 2 = 57 Answer 57 cm²  ③ ¼ circle + ¼ circle – □ 10 × 10 x 3.14 ÷ 4 = 78.5 78.5 + 78.5 = 157 10 × 10 = 100 157 - 100 = 57 Answer 57 cm² 	◎ Students are thinking about ways to calculate the area of the leaf shape by making use of familiar figures such as squares, triangles, and quartercircless. [Thinking] ☆ Conduct small group discussion for students who cannot get started. Hint 1 • For what kind of shapes can you figure out the area? Hint 2 • What kind of shape can you make a square and a quartercircle? Hint 3 • Can you make the leaf shape using the shape you just created? ☆ While circulating among students, observe their reasoning and determine which students to call upon.

Deepen/Heighten	3. Discuss ways to calculate the area. T Let's first check what the answer is. What is the area? C 57 $cm^{2.}$ T Please share your idea. (Call on a student who used strategy 1. Have the student share his/her mathematical expressions, and the teacher will write them on the board.) 10 × 10 x 3.14 ÷ 4 = 78.5 100 - 78.5 = 21.5 21.5 × 2 = 43 100 - 43 = 57 Answer 57 cm^2 T Let's think about his/her reasoning together. We will start with the first equation. What did he/she do that can be expressed as 10 × 10 x 3.14 ÷ 4 = 78.5? C I think the first equation is the calculation of the area of a quarter circle. T What does 100 in the second equation represent? C I think 100 is the area of the square. 10 × 10 = 100. T What does it mean to subtract the quartercircle from 100? C From the area of the square, 100 cm^2, subtract the quartercircle. T What does 28.5 × 2 represent? C Because there are 2 copies of the shape he/she created, he/she is doing × 2. T There is another 100 in the next expression. What does it mean to subtract 43 from 100 to get 57? C 100 is the area of the square. By subtracting the area of 2 copies of the shape he/she created, he/she figured out the area of the colored parts.	☆ By figuring out the reasoning process from mathematical expressions, help students experience the joy of reasoning and expressing own ideas. ☆ By having students explain one calculation at a time, involve more students in discussion. ☆ For the parts that may be difficult to explain, use revoicing and pre-made parts of the figure so that students can explain their ideas more easily. ○ Pre-make parts of the figure that students might use in their reasoing processes to support students' explanations. ○ If a student's reasoning involve an unwritten mathematical expression, add the expression on the board. ◎ Students can share their own ideas and figure out their peers' reasoning from their mathematical expressions and diagrams. [Thinking]

	(Call on a student who used strategy 2. Have the student share his/her mathematical expressions, and the teacher will write them on the board.) T Let's think about his/her reasoning together. Can you say the first expression only? C 10 × 10 x 3.14 ÷ 4 − 10 × 10 ÷ 2 = 28.5. 10 × 10 x 3.14 ÷ 4 − 10 × 10 ÷ 2 = 28.5 28.5 × 2 = 57 Answer 57 cm^2 T The first part is the same as the first one - calculating the area of a quartercircle, right? But, what does the second part, 10 × 10 ÷ 2, mean? C I think it is the area of the triangle - he/she calculated the area by 10 × 10 ÷ 2 = 50 cm^2. Then, subtracting the triangle from the quartercircle, he/she got a half of the leafe shape, 28.5 cm^2. T So, what did he/she do next? 10 × 10 x 3.14 ÷ 4 − 100 = 57 Answer 57 cm^2 C The leaf shape is made up of two copies of that shape, so we must do × 2. That's the equation, 28.5 × 2 = 57. (Call on a student who used strategy 3. Have the student share his/her mathematical expressions, and the teacher will write them on the board.) T So, he/she used this calculation to find the answer. Can you see a part of the equation that is the same as somethign we have already seen? C The first part is the same as before - finding the area of the quartercircle. C 100 is the area of the square. T What does it mean to double the area of the quartercircle and subtract the area of the square? Why don't you discuss it with your partner? C When you double the quartercircle, there will be an overlap. The overlapped part is the area of the leaf shape. T Where is the overlapped part?	○ If students look confused after reading the equation, tell students to discuss the equation with their partners. ☆ It is anticipated that students will have more difficulty interpreting this equation. Therefore, conduct pair discussion.

	C When you overlap the two quartercircles, the middle part will overlap. C Because the area of the square is 100 cm², we subtract the square from the 2 copies of quartercircle to find the area of the leaf shape. T Today, we learned that there are different ways to calculate the area of the leaf shape, didn't we. What idea may be common in all of the methods? Please discuss it with your partners. C All of them are using the shapes we have studied before. C If we change the shape into something we already learned, we can figure out the area.	☆ To discuss the common feature, have pairs discuss it first so that their reasoning can be deepened.
Summarize	4. Reflect on today's lesson and summarize. T We learned today that we can calculate the area of a leaf shape by using the shapes we have already learned like circle, square and triangle. Please write in your journal a friend's idea that you liked. Also, write what you learned and what you want to study next. T (While pulling out other pictures out of the "mystery box") Oh, there are more problems! Do you think you can find the area of the colored part of these pictures?    C Yes, we can. T You did really good job goday. In the next lesson, let's use what we learned today and tackle more problems.	☆ Have students write friends' ideas that they liked. Encourage them to specify the parts they thought were good. Have them write their own ideas and friends' ideas separately. ☆ At the end of the lesson, display the shapes that are different from today's and have them think about whether or not they can use the ideas from today to find their areas to connect to the next lesson. If time allows, let them try to solve the problems independently.

9 Board writing plan



Points for observation

- ☐ Was the introduction that presented today's task after showing them shapes students previously learned effective in drawing out students' own questions and motivating them to engage in the task eagerly?
- ☐ Did students' writing their ideas in the notebooks using diagrams and mathematical expressions lead to the activity in which students would express and explain their ideas in a logical manner?
- ☐ Did the activity to interpret mathematical expressions to draw out various strategies lead to students' deepening/heightening of their reasoning and their autonomous learning?
- ☐ Was it effective to incorporate pair discussion in situations where students got confused effective?