4^{th} Grade Mathematics Lesson Plan

June 30

Period 3, Tuesday, June 30, 2015
Tokyo Gakugei University Koganei Elementary School
Grade 4, Class 2, 35 students
Instructor: Takeo Takahashi (seal abbreviated)

1. Title of the Unit

"Investigating Changes"

2. Goals of the Unit

◆ Students develop the ability to determine the relationship between two quantities that change together by investigating the relationship and representing it using a math sentence.

- Students recognize the merits of investigating the relationship between two quantities that change together using a table, and representing this relationship using a math sentence with □ and ○. Moreover, they are eager to apply this knowledge to their daily lives and future learning. [Interest, Eagerness, and Attitude (IEA)]
- Students are able to grasp the relationship between two quantities that change together by investigating the relationship using a table and representing it using a math sentence with variables such □ and ○. [Mathematical Way of Thinking (MWT)]
- Students are able to represent the relationship between two quantities that change together using a table, interpret the characteristics of the changes, and express the relationship using a math sentence with □ and ○. [Mathematical Skills (MS)]
- Students understand how to find the relationship between two quantities that change together using a table and express the relationship using a math sentence with □ and ○. [Knowledge and Understanding (NU)]

3. Reasons for setting this Unit

The goal of this unit is fostering students' "functional thinking."

There are many phenomena in our daily lives that show the relationship of two quantities when one of the quantities is given the other quantity is determined. We are able to predict a phenomenon and solve a problem by using the relationship of "when one the quantity is given the other quantity is determined."

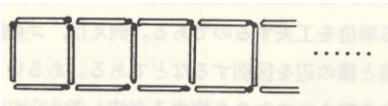
Therefore, "functional thinking" means thinking about the regularity of changes and correspondences between two quantities and paying attention to them to carry out problem solving when studying quantities and geometric figures.

Related to "functional thinking," students have experiences looking at the relationships between two quantities such as: seeing a quantity as the sum of two quantities; the difference of two quantities; the product of two quantities; and when a multiplier increases by 1 the product increase by the size of the multiplier.

By utilizing these prior experiences, in this unit, students will examine changes in quantities by investigating and clarifying the relationships between two quantities by representing the relationships using tables and math sentences.

In this unit, students have learned about relationships between two quantities such as: the sum is constant (The Mysterious Clock), the difference is constant (lining up triangles horizontally), and the quotient is constant (stacking up squares).

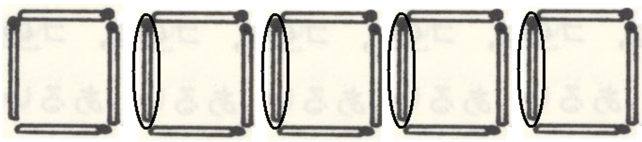
In this unit, students will engage in a problem solving activity, “A Problem with Matchsticks,” based on these past experiences.

> Use matches to construct squares connected side by side, as shown below. How many matches are used when there are 8 squares?
>
> 

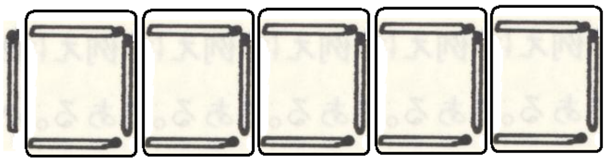
The goal of this lesson is to solve a problem by paying attention to changes in two quantities and discovering patterns of change in corresponding quantities.

There are several ways to find the pattern.[1] (In the case of 5 squares)

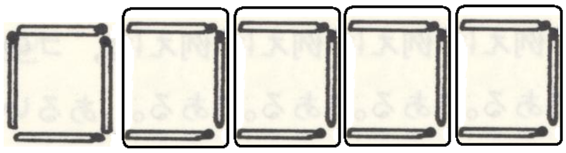
① Using each square as a unit to be counted and subtracting the overlapping matchsticks:
$4 \times 5 - 4 = 20 - 4 \rightarrow 4n - (n - 1)$



② Using units of 3 sides (top, right, and bottom edges of each square) and adding one side at the left end:
$1 + 3 \times 5 = 15 + 1 \rightarrow 3n + 1$



③ Using units of 3 sides (top, right, and bottom edges of each square) and adding one square at the left end:
$4 + 3 \times 4 = 16 \rightarrow 3(n-1) + 4$



④ Adding vertical matches and horizontal matches separately:

Pairs of horizontally placed matches: $2 \times 5 = 10$. There are 5 + 1 + 6 vertically placed matches, giving the total number:
$10 + 6 = 16 \rightarrow 2n + n + 1$

⑤ Drawing 5 squares and counting the number of sides one by one.

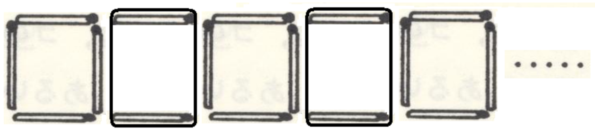
[1] Toshiakira Fujii, in *Teaching of Mathematical Problem-Solving in Japan and in the United States,* ed. by Tatsuro Miwa, Toyokan Publishing (1992), pp. 58—79.

⑥ Counting every other square and then adding the top and bottom sides between those squares:

$4 \times 3 + 2 \times 2 = 16$

→ $4 \times n/2 + 2 \times n/2 + 1$ (if n is even numbers)

→ $4 \times (n + 1)/2 + 2 \times (n - 1)/2$ (if n is an odd number)



⑦ Constructing a table and finding the answer:

Number of Squares	1	2	3	4	5	6
Number of Match sticks	4	7	10	13	16	19

As I described above, there are variety of methods students might come up with, including drawing diagrams, constructing a table, and using calculations. Here, it is important to recognize and appreciate the value of each method. As students draw a diagram, or as they consider how the situation corresponds to the diagram, I would like to help them to notice that each additional square requires 3 additional matches. I also would like to help them notice that the total number of matchstick is 3 times the number of squares plus one. I would like to help the students to consciously recognize that the counting method (counting the number of matchsticks one by one) is too inefficient and time-consuming. Moreover, because of this, I would like to help students to find the pattern (regularity in the relationship of two quantities) in the early process of their investigation. By helping the students recognize the point above, I believe they will see the merit of finding the number of matchsticks using calculations based on defined patterns when the number of squares is large.

Here, the goal is to allow the students to consider various solution methods so that by focusing on the table and the problem structure, they will see the advantage of "functional thinking," the idea of capturing relationships between numbers/quantities.

4. Lesson Teaching Plan (total of five lesson hours, 5/5)

Lesson 1: Investigate the relationship between two quantities that change together (the sum is constant)

Lesson 2: Examining the relationship between two quantities that change together by representing the relationship in a table and with a math expression.

Lesson 3: Investigate the relationship between two quantities that change together (the difference is constant)

Lesson 4: Investigate the relationship between two quantities that change together (the quotient is constant)

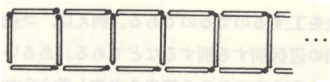
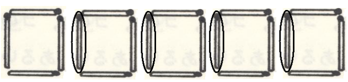
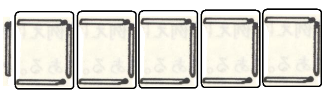
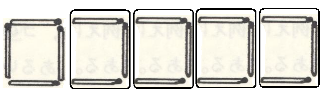
Lesson 5: Investigate the relationship between two quantities that change together ($y = an - 1$) [This Lesson]

5. Instruction of this Lesson

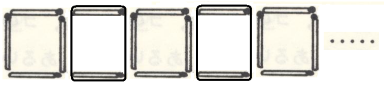
(1) Goals of this Lesson

- Students are able to find the pattern between the number of squares and the number of matchsticks and solve the problem.
- Students are able to understand the generality of the pattern by considering simple cases first and by focusing on the increase in the number of matchsticks in order to solve this problem.

(2) Flow of the Lesson

	Learning Activities and Students' Anticipated Solutions	Teacher's support (●); Points of evaluation (★)
	Use matchsticks to construct squares connected side by side, as shown. How many matchsticks are used when there are [8] squares?  T1: How many matchsticks are used now? C2: 16 matchsticks. T3: What do you think? C4: I think that is good. T5: Let's check. (Count the matchsticks) T6: Well it looks like you need 16 matchsticks for 5 squares. T7: When there are 8 squares, how many matchsticks do we need?	● Post the problem on the board in large letters. Do not show the pupils the actual figure-drawing process. ● Prepare an A3-size hand-out showing the figure in order to explain the thinking process to the class. ● Allow them to keep their sense of having to do cumbersome work while leading them to discover the pattern using a small number of squares.
	When there are 8 squares, how many matchsticks were used?	
	T8: Could you tell us the answer? C9: 25 matchsticks. T 10: Well, we are going to think about whether the answer is right. Let's think about how to find the number of matchsticks that are used to make 8 squares. 【Begin Individual Problem-Solving】 C11：Consider the squares as units to be counted and subtract the overlapping matches. $4 \times 8 - 7 = 32 - 7 \rightarrow 4n - (n - 1) = 3n + 1$  C12：Use units of 3 sides (top, right, and bottom edges of each square) and add one side at the left end. $1 + 3 \times 8 = 25 \rightarrow 1 + 3n$  C13：Use units of 3 sides (top, right, and bottom edges of each square) and add one square at the left end. $4 + 3 \times 7 = 25 \rightarrow 3(n - 1) + 4 = 3n + 1$ 	● If the majority of students use the counting method for finding the answer of 8 squares, ask them to think about another method that does not require counting the matchsticks because counting is cumbersome. ● In individual problem-solving, direct the students to label their own ideas "my idea" and their classmate's ideas ("friend's idea"). ● Direct them to write an explanation as if they were explaining their idea to a classmate. ★ Are they engaged in problem-solving using their own ideas? (IEA) ★ Are they thinking of the relationship between the "number of matchsticks" and the "number of squares"? (MWT)

C14：Add horizontal sides and vertical sides separately. Consider the pairs of horizontal sides (top and bottom): $2 \times 8 = 16$. Then there are $8 + 1$ vertically placed matchsticks, giving the total of: $16 + 9 = 25 \rightarrow 2n + (n + 1) = 3n + 1$



C15：Draw 8 squares and count the sides, one by one.

C16：Count the four sides of every other square and add the two (top and bottom) sides of the other squares.

$4 \times 4 + 2 \times 4 + 1 = 25$

$\rightarrow 4 \times n/2 + 2 \times n/2 + 1$ (if n is even numbers)

$\rightarrow 4 \times (n + 1)/2 + 2 \times (n-1)/2$ (if n is odd numbers)

$\rightarrow 4 \times n/2 + 2 \times n/2 + 1$ (if n is even)

$\rightarrow 4 \times n+1/2 + 2 \times n-1/2$ (if n is odd)

C17：Construct a table and find the answer.

Number of squares	1	2	3	4	• • •	8
Number of matchsticks	4	7	10	13	• • •	25

T18：Let's have someone present his or her idea.

C19 presents his/her method.

Number of squares	1	2	3	4	• • •	8
Number of matchsticks	4	7	10	13	• • •	25

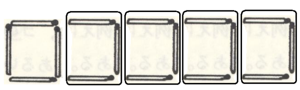
Use the table to find the pattern.

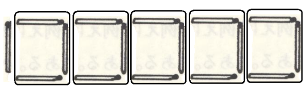
• Presentation of the pattern that changes (+3)

T20：Do you want to add anything?

T21：What does +3 mean? Can you explain why we add 3 using the diagram?

C22：The class adds explanations for C13 and C14.





C23：I think it all comes down to whether you consider the first square as a unit or you begin with the one vertical side at the left end.

If we consider the first square as 4 sides, we add 3 x (Number of squares – 1); if we start with one side, we add 3 x (Number of squares). I think both methods can be thought of as being the same.

● Pick up a student's expression that uses the numbers in the table and ask the class to explain the meaning of those numbers. Connect the ideas of the students to derive the math sentences:

$1 + 3 \times$ (Number of squares)

$4 + 3 \times$ (Number of squares - 1).

Here, put more weight on the students' explanations using the diagrams so that the class will understand that this stricture/pattern will continue if more squares are added.

	T24：Oh, I see. You've just mentioned two relationships between the number of squares and the number of matchsticks. 4 + 3 × (Number of squares - 1) 1 + 3 × (Number of squares) T25：Anything else to add? C26：No. T27：Okay, does anyone else have a different idea? C28：Explanation of the other methods (such as C11 and C16). T29：I see. As I listen to your explanations, even though your diagrams are all different, the answer always seems to be 1 + 3 × (Number of squares). T30: Now, using the math sentence we came up with, if we have 20 squares how many matchsticks will be used? C31: We need 61 matchsticks. T32: How did you figure it out? C33: 3 x 30 + 1. T33: Why do you think we can use the math sentence? C34: Because the number of squares increases in the same pattern up to 8 squares, and from 8 to 20 squares. T32: I see, how many matchsticks do we need? C33: We need 61 matchsticks. C34: I agree. T35: Let's write a reflection of learning in your notebooks and finish today's lesson.	● By returning to the original problem and solving it, we can touch on the usefulness of "functional thinking." ● If the time allows, ask students to find the number of matchsticks needed for 100 squares, how to solve it, and why they can use the math sentence.