Grade 3, Mathematics Lesson Plan

Place: Showa City Oshihara Elementary School
Teacher's Name: Yuji Ishikawa

1. **Name of the Unit:** Let's think about Division! (Division with Remainders)

2. *About the Unit:*

In this grade, students learned about division as a new kind of calculation. They have learned that division has two meanings, partitive and quotitive, and both cases can be represented with division math sentences. They also learned that division is the inverse calculation of multiplication, and that answers to divisions can be found by using the multiplication facts if they think about the number in dividend. These are main concepts they have learned already about division.

In this unit, students will learn about division problems with remainder that can be solved by using multiplication facts once. The main concept that students are learning in this unit are: the meaning of division with remainders; their calculation method; the meaning of remainders; and how to use remainders.

A remainder can be found when we maximize the size of the quotient and subtract the product of the divisor and the quotient from the dividend. Although students have learned division without remainder (remainder is 0) previously, what they learned was a special case of division. By helping students understand this, they will be able to integrate the concepts of division with remainders and division without remainders. In addition to what they have learned before, I would like to help students become better at calculations of division with remainders and be able to use division in both partitive and quotitive situations just like they did with the previous unit on division without remainders.

Through the instruction, students will be introduced to division with remainders in the context of a quotitive division situation and understand the meaning of remainders. I would like to help students to grasp the idea of remainders visually by engaging them in an activity using counters and based on ideas such as repeated subtraction and the inverse calculation of multiplication. By comparing these ideas with what they learned previously about division without remainders; and through manipulation of counters, use of diagrams, and use of multiplication facts; students will think about how to find a quotient and understand how to calculate division that is not evenly divisible. Manipulation activities and diagrams will be also used later when students learn partitive division situations to help them understand that they can also use division in this case.

In the discussion on the relationship between size of the remainder and the size of the divisor, a quotitive division situation will be used. Students will discuss the size of remainders by considering a problem situation and through a manipulation activity. By changing the size of the dividend without changing the size of the divisor, students will be able to observe how the size of the remainder changes. Through this observation, I

would like help them notice that the size of the remainder is always less than the divisor. In addition, I would like to help them understand how the result of a calculation can be checked by using a diagram to connect the relationship between the dividend, the divisor, the quotient, and the remainder; and help them understand that the checking process can be done by (divisor) x (quotient) + (remainder) = (dividend).

At the end of this unit, students will solve story problems that involve decision making about how to interpret the remainder. Through problem solving, students will deepen their understanding about the meaning of remainders and how to deal with them. I would like students to not only be able to calculate and find answers, but also be able to interpret the reminder based on the problem situation, and make decisions about how to deal with the remainders.

There are 4 ways to deal with the remainders: ① use the quotient and remainder as they are; ② round up the remainder and add 1 to the quotient (quotient +1); ③ discard the remainder and use only the quotient; ④ continue to divide the reminder and find a quotient. The meaning of number ①, "use the quotient and the remainder as they are" is when you solve a division problem you are answering to something like "so many bags can be made and so many of them are left" and "quantity of 1 person becomes so many and so many of them are left."

The problems that students learn at the beginning of this unit such as "② round up the remainder and add 1 to the quotient (quotient +1)" and "③ round down the remainder and use only the quotient" are the type of division problems that cannot simply use the calculation results to answer to the problem question appropriately. By making connections between the diagram and math sentence and comparing this with the problem situation, I would like to help the students to be able to manipulate/process the calculation result by carefully examining and interpreting questions involving situations in their daily lives.

3. **About this Lesson:**

In this lesson, which comes at the end of the unit, students will deal with a problem situation that involves ④ "continue to divide the reminder and find a quotient."

(1) Reasons why the "continue to divide the reminder and find a quotient" method is often not taught:

In the various elementary textbook series published by six different publishers in Japan, the idea of continuing to divide the reminder and find a quotient is not included in the Grade 3 division with remainder unit. In Grade 3, the main goals of learning are: to understand how to calculate division with whole numbers using multiplication facts once; be able to perform the calculation; understand the meaning of remainders; and find a whole number quotient and remainder. When we continue to divide the remainder the quotient becomes a decimal number. However, students will not learn decimal numbers until later in this grade level. The process of continuing to divide the remainder occurs in partitive division problem situations and depending on the quantities involved in the problem, there are cases in which you can divide the remainder farther. This complicates the understanding of remainder

and students could become confused. That is why idea of continuing to divide the remainder is not taught in the Grade 3 unit on division with remainders.

(2) The value for learning about continuing to divide the remainder in this unit:

① Taking away students' unnatural feelings about remainders related with their daily life experiences.

Students have been engaged in situations that involve dividing things during lunchtime at school or at home. I wondered what kind of image they have about dividing as well as what they are thinking about when they are dealing with remainders? Therefore we conducted the following student survey about this

I.   You must have experiences with distributing (dividing) snacks or lunches at home or school. How do you distribute (divide) them? Please describe.
II. What do you think about if something is left over after you distribute (divide) something?
III. There are 3 colored papers. When 2 people divide them, 1 paper is left. If you want to have the 1 paper that's left, what do you think would be the best way to do this?
IV. There are 3 (   ). When 2 people divide them, there is 1 left. If you want to have the 1 left, what do you think would be the best way to do this?*
     * For this item we asked students to choose something they like in the (   ) in order for them to feel like the situation is similar to their daily experiences.

The survey results are as shown below:

I. You must have experiences for distributing (dividing) snacks or lunches at home or school. How do you distribute (divide) them? Please describe.		
Possible choices	Distribute them in a way that everybody has same amount.	Change the amount depending on the person.	I don't think about it particularly.
Results (%)	22 students (85%)	1 student (3%)	3 students (12%)

II. What do you think about if something is left over after you distribute (divide) something?		
Possible choices	I don't think about it particularly.	I think it is better not to have anything left over.	I think it is better to have something left over.
Results (%)	9 students (35%)	16 students (63%)	3 students (12%)

III. There are 3 colored papers. When 2 people divide them, 1 paper is left. If you want to have the 1 paper that's left, what do you think would be the best way to do this?		
Possible Choices	Play Rock Paper Scissors and the winner gets it.	Just leave it as it is.	Divide the paper into halves.
Result (%)	3 students (12%)	3 students (12%)	20 students (76%)

IV. There are 3 ( ). When 2 people divide them, there is 1 left. If you want to have the 1 left, what do you think would be the best way to do this? *For this item we asked students to choose something they like in the ( ) in order for them to feel like the situation is similar to their daily experiences.		
Possible Choices	Play Rock Paper Scissors and winner gets it.	Just leave it as it is.	Divide the paper into halves.
Result (%)	14 students (54%)	1 student (3%)	11 students (43%)

Shown below is our interpretation of the results:

< Item 1>
    Nearly 90% of students are consciously thinking about dividing things equally (dividing things in the same way) in their daily lives.
< Item II>
    About 60% of students think that it is better not to have anything left over after dividing something. A very small number of students have a positive image about leaving something left over.
< Item III>
    In the situation about dividing materials that students might not easily associate with their own experience, about 70% of students thought they it is better to equally divide what is left.
< Item IV>
    In the situation about the dividing materials that students want to have, about 50% of students wanted one of the two people to have what is left. About 40 % of the students said they want what is left to be divided equally between the 2 people.

In the survey, we used all partitive division problem situations. In the case of partitive division problems, depending on the kinds of quantities involved in the problem, left over material can be equally divided. Cookies and buns (discrete quantities), and juice (continuous quantity) are some of the examples. If we insist that students must show remainders when we use these kinds of quantities in problems, there will be discrepancy between their daily life experiences and the problem situation. For example, when there is a bun left at the lunchtime, we can think about leaving the bun as a leftover or dividing it among several people equally. Because students have experienced these situations often in their daily lives, I believe it may seem unnatural to leave something left over for some students. Students who responded, "It is better not to have left over," and "divide what is left by 2 halves," in survey items II, III, and IV may have an unnatural feeling about having something left over but proceed with the learning anyway. Therefore, by touching on the idea of continuously dividing the remainder in this unit, I believe students will be able to interpret the remainder based on their daily life experiences.

In addition, I believe that students' unnatural feelings can be reduced by helping them understand that there are situations when they can divide what is left and others when they cannot.

② Fostering students' ability to make decisions and process based on appropriate situations and goals

Section B of the 2015 National Assessment of Academic Ability includes problems that evaluate students' ability to comprehend problem situations and process/make decisions. One of the problems from this section is a situation involving shopping at a market. There are various packs of tomatoes. The prices of each different pack is indicated. There are prices for packs of 1 tomato, 2 tomatoes, and 3 tomatoes. From these different prices, students are asked to combine them to find the best price for purchasing 7 tomatoes.

Modern society is filled with much information. Because of this, people need to be able select pertinent information from multiple informational sources based on evidence, and then make a decision and carry out a procedure appropriately based on the situations and goals. These skills are very important for students to function in society. During this unit along with the other units, I would like to help students foster these skills. In this unit, students will think about how to deal with remainders based on the problem situation. By viewing the result of the calculation in the context of their daily experiences, students can grasp how to process remainders appropriately based on the situation. In order to connect division situations to students' daily lives, quotitive division situations are not the only ones that are familiar. Partitive situations are also familiar to children. I believe that by dealing with partitive division situations I can help students to understand that there are situations when we might want to continue to divide and foster students' abilities for making decisions and processing information appropriately when they face many different problem situations.

③ Considering the connection to learning in Grade 4

The content of this unit is connected to division with an algorithm and division with decimal numbers in Grade 4. Continuing to divide is introduced in Grade 4. Looking at the Grade 4 textbook published by Tokyo Shoseki and examining the connection concretely we can see the following:

In the unit on division with an algorithm, a partitive division problem situation involving dividing 73 sheets of colored paper by 3 people, $73 \div 3$, is introduced. In the process of dividing, 7 bundles of 10 sheets of colored paper among 3 people and 1 bundle of 10 is left, the bundle of 10 is broken and 12 individual sheets are created and are divided among 3 people. In this process, when 1 bundle of 10 is broken into individual sheets, a similar thinking process to continuing to divide the remainder is performed.

In the unit on division with decimal numbers, students will think about (whole number) ÷ (whole number) = (decimal number) in a partitive division situation involving dividing juice equally. 6 liters of juice is divided by 4 people equally. It is $6 \div 4 = 1$ with remainder of 2. Students will think about how many 0.1 liters are in 2 litters, which they learned in the decimal unit before. By incorporating this idea, students notice they can continue to divide.

Later in the year In Grade 3, students learn about decimal numbers in Unit 11 and

fractions in Unit 14. The idea of dividing 1 into equal parts and expressing the parts using decimal numbers and fractions is related to the idea of continuing to divide.

By considering the points described above, I believe that I can help students develop a fundamental idea connected to future learning, namely, that there is a case in which the remainder can be continuously divided. Based on this explanation, in this lesson I would like students to notice that there are problem situations where remainders can be continuously divided after they clearly understand how to calculate division with whole numbers and the meaning of remainders through the learning in this unit. I would like the students to able to understand the problem situations carefully and deepen their understanding of remainders in order to connect the learning to future leaning.

4. Goals of the Unit:

⊙ Students understand the case of division that is not divisible, deepen their understanding of the meaning of division, and are able to utilize it.
   • Students are eager to grasp the meaning of division that is not divisible and it's calculation process based on the previous learning of division that is divisible, the relationship between multiplication, and manipulation of concrete materials. [Interest, Eagerness, and Attitude (IEA)]
   • Students are able to generalize the cases of divisible and non-divisible division as one division concept, and able to express the meaning of division and the calculation process using concrete materials, diagrams, and math sentences. [Mathematical Way of Thinking (MWT)]
   • Students are able to perform division that is not divisible and find the quotient and remainder. [Mathematical Skills (MS)]
   • Students understand division by knowing the meaning of remainders and the relationship between the size of remainders and divisors. [Knowledge and Understanding [KU]

5. Plan of the Unit (Total of 10 Lessons):

Sub-Unit (No. of Lessons)	Lesson No.	Goals	Learning Activities	Main Evaluation Points
1 (6)	1 & 2	Students understand how to calculate division whose divisor and quotient are 1-digit and not divisible.	・Students think about how to find the answer to 14 ÷ 3 in a quotitive division situation. ・Students learn it is represented as 14 ÷ 3 = 4 R2 ・Students understand the meaning of remainders.	• Students are eager to think about division with remainder based on the understanding of division learned previously. [IEA] • Students explain how to calculate non-divisible division using concrete materials, diagrams, and math sentences based on the divisible division they learned previously [MWT]
	3	Students understand the relationship between remainders and divisors.	・Students investigate the relationship between remainders and divisors of □ ÷ 4.	• Students understand remainders are less than divisors [KU]
	4	Students understand that the idea of division with remainder applies to the partitive division situations.	・Students grasp the problem situation, represent the situation with a math sentence 16 ÷ 3, and think about how to find the answer.	• Students are able to think about the process of division with remainders in the context of a partitive division situation based on the previous learning of divisible division, and they explain their thinking using concrete materials, diagrams, and math sentences. [MWT]
	5	Students understand the calculation process for checking the answers of divisions with remainders.	・Students think about the calculation process for checking the answers of divisions with remainders.	• Students are able to understand the procedure for checking the answers of divisions with remainders. [KU]
	6	Students practice and become proficient at calculating divisions with remainders.	・Students solve division problems with remainders.	• Students are able to calculate divisions with remainders and check the answers [MS]
2 (2)	1	Students deepen their understanding of how to deal with reminders.	・Students grasp the problem situation, represent the situation with a math sentence, and find the answer by calculating 23 ÷ 4. ・Students discuss how to deal	• Students are able to think about how to process the remainder (quotient + 1) based on the understanding of the problem situation and they explain their ideas using diagrams and math sentences. [MWT]

			with the remainder based on the problem situation. ・Student summarize that the answer becomes the quotient + 1.	
	2		・Students grasp the problem situation, represent the situation with a math sentence, and find the answer by calculating 30 ÷ 4. ・Students discuss how to deal with the remainder based on the problem situation. ・Students summarize the answer using the quotient as is.	• Students are able to think about how to process the remainder (get rid of the remainder and use the quotient as is) based on the understanding of the problem situation and explain their ideas using diagrams and math sentences. [MWT]
3 (1)	1	Practice and become sufficient the content of this unit	・Student solve problems summarizing the unit.	• Students are able to solve problems using what they have learned appropriately. (MS)
4 (1)	1 This Lesson		・Student grasp the problem situation, represent the situation with a math sentence, find the answer by calculating 9 ÷ 2 ・Student discuss how to deal with the remainder based on the problem situation. ・Student summarize that there are problem situations that quotient can be continuously divided.	• Students are able to think about how to deal with the remainder (continue to divide the remainder) in the problem situation that produces remainder 1 using diagrams and math sentences. [MWT]

6. This Lesson:

(1) Goals of the Lesson:
Students are able to think about how to deal with the remainder (continue to divide the remainder) in a problem situation that produces a remainder of 1 using diagrams and math sentences. [MWT]

(2) Time and Date: 1:50 p.m. to 2:35 p.m. (5th period), Friday, June 26, 2015

(3) Place: Grade 3 Class No. 2, Showa City Oshihara Elementary School

(4) Intent of the Instruction:

Students have studied how to calculate division with remainders and how to deal with remainders based on different types of problem situations. The types of processes for dealing with remainders that students have learned for division with remainders are rounding up the remainder and adding 1 to the quotient (quotient +1) and discarding the remainder and using only the quotient. Both of them were processes for dealing with remainders in quotitive division situations.

This is the last lesson in the unit. In this lesson a partitive division situation will be used for the problem and students will think about what it means to continue to divide the remainder as part of how to dealing with division with remainders. I will set up a problem situation that produces a remainder of 1 and students will discuss how to deal with the remainder. Through this discussion, I will help students to notice that depending on the kinds of quantities used in the problem, the remainder can either be further divided or it cannot be divided. Moreover, I would like to help students to make appropriate decisions about how to deal with the remainders. In order to do this, I will pay attention to the following points as the lesson unfolds:
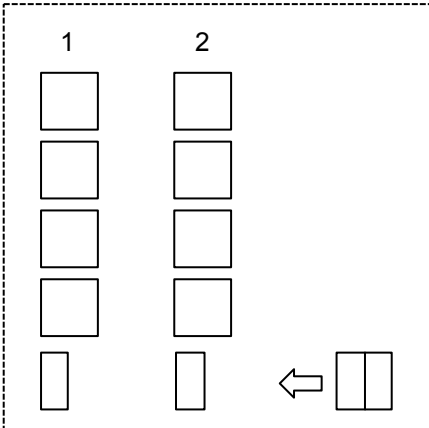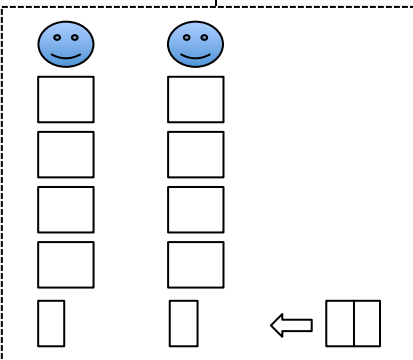
At the introduction, I will devise the lesson in a way that encourages students to think about task of this lesson by connecting to what they have learned before. First I will change the numbers in the problem from 9 ÷ 2 to 9 ÷ 3 and hold a discussion about the problem situation. Then I will check how students think about the answer to the question of the problem, "What would be the amount for 1 person?" After the discussion, students will solve the problem 9 ÷ 3 on their own.

During the individual problem solving part, I will encourage students to show their work; not only the math sentence but also with diagrams and words if necessary. During the whole class discussion, we will look at the math sentences, diagrams, and words carefully and grasp the meaning of "divide all of them equally" and the situation that the remainder 1 can be divided. To express the amount that equally divided parts of remainder, students may use an expression such as ½ or a half that they have learned previously. Students will understand that the remainder can be equally divided. Lastly, toward the end of the whole class discussion, I will touch upon cases where we can divide the remainder and where we cannot divide the remainder, and I would like to deepen students' understanding of remainders.

For the evaluation of the lesson, I would like to assess if the goals of the lesson were met by examining student's solutions, presentations, discussions and participation, and reflections on learning in their notebooks.

(5) Flow of the Lesson:

Flow	Learning Activities and Content	Instructional Points to Remember	Evaluation
Grasping 5 min.	1. Grasping the task (1) Students learn the task of this lesson.	• Post colored papers on the board. • Write the problem on the board.	
	There are 9 sheets of colored paper. 2 people will divide all of them equally. What would be the amount for 1 person?		
		• Think about what amount 1 person gets by the case of 3 people dividing 9 sheets.	
	(2) Write the math sentence together. • 9 ÷ 2		
Investigating 7 min.	2. Solving the problem on their own (A) 9 ÷ 2 = 4 R1 The amount for 1 person will be <u>4 sheets and 1 sheet will be left.</u> (B) 9 ÷ 2 = 4 R1 The amount for 1 person will be <u>4 sheets and a half.</u> (C) 9 ÷ 2 = 4 R1 The amount for 1 person will be <u>4 sheets and ½.</u> (D) 9 ÷ 2 = 4 R1 4 + 1 = 5 The amount for 1 person will be <u>5 sheets.</u> (E) 9 ÷ 2 = 4 R1 The amount for 1 person will be <u>4 sheets.</u>	• Tell the students that they can solve the problem not only using a math sentence but also using diagrams and words if necessary. • If I find a student who is having a difficulty coming up his/her own solution, ask the students the reason why he/she is having the difficulty. • Walk around the class and monitor the progress of student work, and grasp several different students' ideas.	

Deepening 28 min.	3. Discussing the solutions with the class (1) Students present their solution ideas and understand each other's ideas. • 9 ÷ 2 = 4 R1 (A) The amount for 1 person will be 4 sheets and 1 sheet will be left. (B) The amount for 1 person will be 4 sheets. (C) The amount for 1 person will be 4 sheets and a half. (D) The amount for 1 person will be 4 sheets and ½. (2) Students compare and discuss the solution ideas. ① Students will discuss what they need to do with the remainder by looking at the problem situation. • I have not written it down in my notebook but I think we can keep it as a remainder. • The problem does not say "how many sheets will be left?" so I think we can divide the remainder. • We need to divide all of them so we need to divide the remainder. ② Students will think about how to express the amount for 1 person. • 4 sheets and a half • 4 sheets and ½ 	• Record students' presented ideas on the board. Discuss the relationship among math sentences, diagrams, and words used in the problem. • Distribute 9 sheets of colored paper to 2 people and cut the remainder (1 sheet) into two equally. • Ask students to recall what they have learned before and bring out that they have learned that a half can be expressed using fractions. • Ask students to identify what part of the diagram shows ½ and help them understand the difference between 1 sheet and ½ sheet. • Check with students that ½ sheet cannot be counted as 1 sheet. • Recognize the merit of using a diagram to think and understand.	 Students are able to think about how to deal with the remainder (continue to divide the remainder) in the problem situation that produces a remainder of 1 using diagrams and math sentences. [MWT] (Notebooks, Presentations)

	(3) Students learn how to deal with remainders based on the problem situation. • When you need to divide all equally, you can divide the remainder 1.		

	4. Think about whether we can divide the remainder 1 into halves always.
	⭘ Yes, we can divide
	  <Reason>
	    • Just like what we did today, when we have a remainder we can split it into halves.
	
	⭘ No, we can't divide
	  <Reason>
	  ① If the problem is asking for "How many sheets of the colored paper does each person get?," then we need to show the remainder.
	  ② If we know how much each person gets and are trying to find how many persons can get paper (quotitive division), we show the remainder.
	  ③ You can fold an origami paper into halves. But there are things that cannot be divided into halves.
	
	  ④ There are things that can be and can't be divided into halves.
	    • Things that can be divided:
	      a bread
	      a pancake
	
	    • Things that can't be divided:
	      a pen
	      a seed of a flower
	      an insect
	
	5. Students solve an application problem.
	
	There are 11 pancakes.
	2 persons will divide all of them equally.
	What would be the amount for 1 person?
	
	Math Sentence:
	    $11 \div 2 = 5$ R1
	    <u>5 pancakes and 1/2</u>
		• Tell the students that they can discuss it with their neighbors.
		
		• Ask students to think about why they think so.
		
		• Response ② may not come out. In that case, we will not discuss it.
		
		• Remainder of some things/materials can't be divided. Ask students to discus what things can't be divided.
		
		• Discuss the math sentence and diagram in the class and check the answer.	

| Summ arizing 5 min. | 6. Summarizing
(1) Summarize today's lesson.
　• When we need to divide all of them equally we can also divide the remainder.
　• For division with remainders, there are cases when we can divide the remainder and ones when we can't divide the remainder.

(2) Students write their reflection on the lesson in their notebook
　• I learned for the first time that there are cases where we can divide the remainders.
　• I noticed that we can divide the remainder 1 into halves by looking at _____'s idea (or diagram). | ・First, ask students to explain what they have learned from today's lesson with their own words. | |

(6) Evaluation:
   • Students are able to think about how to deal with the remainder (continue to divide the remainder) in a problem situation that produces a remainder of 1 using diagrams and math sentences. [MWT]

References

Toko Shoseki Editorial Committee (2014), Teachers Manual, Instruction Volume, New Mathematics Grade 3 Volume A, pp.106 – 118, Tokyo Shoseki
新しい算数3上　教師用指導書　指導編（2014）　pp.106-118　東京書籍

Toko Shoseki Editorial Committee (2014), Teachers Manual, Research Volume, New Mathematics Grade 3 Volume A, pp.198 – 211, Tokyo Shoseki
新しい算数3上　教師用指導書　研究編（2014）　pp.198-211　東京書籍

Ministry of Education, Culture, Sports, Science and Technology (MEXT) (2008), Elementary School Teaching Guide for The Couse of Study: Elementary Mathematics, Toyokan Publishing Co., Ltd.
文部科学省（2008）「小学校学習指導要領解説　算数編」　東洋館出版社

Sugiyama, Y. (2012) Toward achieving certain Elementary and Secondary Mathematics Education, Toyokan Publishing Co., Ltd.
杉山吉茂（2012）「確かな算数・数学教育をもとめて」　東洋館出版社

Nakamura, T. (2008), Elementary Mathematics Lessons aim for developing Mathematical thinking and expressive abilities, Meijitosho Shuppan Corporation
中村享史（2008）「数学的な思考力・表現力を伸ばす算数授業」　明治図書

Tsukuba University Attached Elementary School, Elementary School Research Group (2011), Elementary School Lesson Study, Vol. 75, Toyokan Publishing Co., Ltd.
筑波大学附属小学校算数研究部（2011）「算数授業研究　VOL.75」　東洋館出版社

Tsukuba University Attached Elementary School, Elementary School Research Group (2013), Elementary School Lesson Study, Vol. 89, Toyokan Publishing Co., Ltd.
筑波大学附属小学校算数研究部（2013）「算数授業研究　VOL.89」　東洋館出版社

Koike, K. (2008), Division with Remainders, Kofu Ward Lesson Implementation Report
小池孝二（2008）「あまりのあるわり算」　甲府地区実践発表レポート

Ohma, T. (2014) Think about Division, University of Yamanashi Attached Elementary School Public Research Lesson Materials
大間俊男（2014）「わり算を考えよう」　山梨大学附属小学校初等教育公開研究会資料

Yamaguchi, K. (2014) Think about Division, University of Yamanashi Attached Elementary School Public Research Lesson Materials
山口国之（2014）「わり算を考えよう」　山梨大学附属小学校初等教育公開研究会資料