June 27
Grade 5

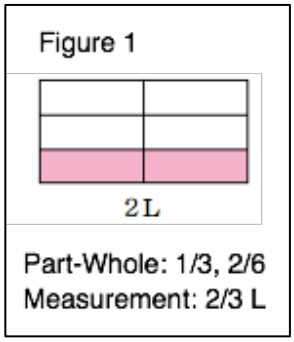
Mathematics Lesson Plan
Division and Fractions: How many times as much is it?

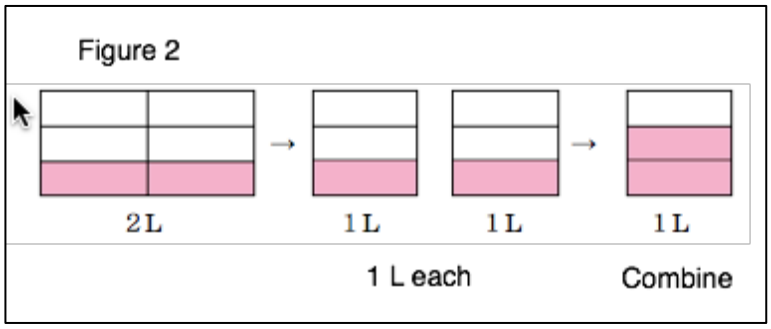
University of Yamanashi Model Elementary School
Date: Saturday, June 27, 2015
Time: 9:00 - 9:45
Class: Grade 5 Homeroom 2 (29 students)
Teacher: KASAI, Sayuri

1 About the Unit

Division may be interpreted in two ways: partitive division and quotitive division. Partitive division is used to determine the base quantity while quotitive division is used to determine a rate of two quantities.

In the 2011 Research Open House at our school, I taught a lesson titled, Division and Fractions. In that lesson, we discussed a partitive division problem, "3 children are going to share 2 L of juice equally. How much juice will each child receives?" As I approached the lesson, I considered the following two challenges: many children's conception of fractions was part of a whole, and many teachers often teach the relationship a ÷ b = $\frac{a}{b}$ procedurally. Through the research lesson, it become clear that the difference in the ways we represent a part-whole fraction and a measurement fraction is a source of difficulty to make sense of fractions (Figure 1). As a result, we learned that a key point is to help students realize that 1 L is the base quantity (Figure 2).





In the context of (whole number) ÷ (whole number) = (fraction), it is easier to capture the meaning through partitive division. In the example above, because we are partitioning 2 L into 3 equal parts, 2 ÷ 3 = ⅔. Therefore, each person will receive ⅔ L. In a quotitive situation, we divide 2 L by 3 L. If students have already learned the relationship, a ÷ b = $\frac{a}{b}$, then they can write an expression from the numbers given in the problem and find the answer. However, because of the difficulty in understanding what this division means, I wonder if too often the meaning of dividing 2 L by 3 L or the meaning of the quotient (⅔) are often glossed over. Therefore, in this research lesson, we are going to focus on a quotitive situation involving (whole number) ÷ (whole number) = (fraction).

In the case when a whole number is not evenly divisible by another whole number, students have learned to express the result with the quotient and the remainder or continue dividing to get a decimal number as the quotient, rounding to a particular place if necessary or appropriate. As for times as much with decimal numbers, students have learned that a decimal number may be used to express times as much and the meaning of times as much with a

decimal number such as "1.5 times as much means, if 20 m is considered as 1, 30 m will be considered 1.5." Finally, about fractions, students have learned that a fraction may represent a way to equally partition a whole or a specific measured quantity. They have also learned that some fractions represent equal number/quantity and how to add or subtract fractions with like denominators.

In this unit, we will learn that the results of whole number division may be expressed as fractions. By noting a ÷ b as $\frac{a}{b}$ or $\frac{a}{b}$ as a ÷ b, this unit will try to unify ideas that have been studied separately so that students can see that these two expressions indeed represent the same idea. Then, based on the understanding that the results of division may be expressed as fractions, we will expand ways of viewing times as much with decimal numbers to times as much with fractions and eventually to the study of multiplication and division of fractions in Grade 6. Moreover, by expressing whole numbers and decimal numbers as fractions or fractions as whole numbers or decimal numbers, students will develop the understanding of the relationship among whole numbers, decimal numbers and fractions.

In teaching this unit, we will use a partitive division situation in the introduction. That is because, as stated above, such a situation is easier for students to imagine the act of equal partitioning. The aim is for students to understand that the quotient of whole number division, a ÷ b, can be represented by a fraction, $\frac{a}{b}$, by manipulating diagrams. As students engage in this task, it is essential that they pay attention to the unit amount such as 1 L and 1 m. As students study times as much with fractions, they will use number lines to grasp the relationship among quantities and to justify a calculation necessary. In order to help students understand that the meaning of times as much, "when a particular quantity is considered as 1, the given quantity corresponds to ❐," will apply to fractions just as it did to whole numbers and decimal numbers, we will utilize number lines carefully.

In the second sub-unit, we will try to help students understand that whole numbers, decimal numbers and fractions are all numbers even though we have dealt with them separately up to this point. We will do so by comparing and ordering whole numbers, decimal numbers, and fractions as well as representing them on number lines. We will also touch upon the merits of converting decimal numbers to fractions, and vice versa, so that students can use whichever the form that is appropriate in a given situation.

II Goals of the Unit

- Students will understand ways to think about and represent fractions. They will also understand the relationship between whole numbers, decimal numbers, and fractions and deepen their understanding of fractions.
  - Students will realize merits of representing the quotient of whole number division as a fraction. They will be able to move back and forth among whole numbers, decimal numbers and fractions and try to use them in their study. (Interest, Eagerness, and Attitude)
  - Students will be able to grasp that fractions, whole numbers and decimal numbers are different ways to represent numbers. (Mathematical Way of Thinking)
  - Students will be able to view a ÷ b as $\frac{a}{b}$ or $\frac{a}{b}$ as a ÷ b. They will also be able to convert fractions to decimal numbers or decimal numbers and whole numbers to fractions. (Mathematical Skills)
  - Students will understand that the result of whole number division can be represented as a single number if we use a fraction. They will understand the relationship among fractions, whole numbers and decimal numbers. (Knowledge and Understanding)

III Relationship between school research theme and this unit

1 About characters and abilities necessary for learning toward harmonious living

In our Mathematics Department, we think that the driving force of students' autonomous learning comes from students' questions. When the students are engaged in individual problem solving, they keep in mind the questions as they begin the thinking process. Moreover, through individual problem solving, students develop new knowledge by asking questions on their own. However, there is no guarantee that they will be successful when working independently. Therefore, the important condition for learning is the collaboration with other students. During the discussion of student presented works, students compare others' ideas with their own ideas and become clear about commonalities, similarities, and differences. We believe this process helps students reach and achieve the goals of the lesson.

In this unit, "Can we express the quotient of whole number division precisely using fractions?"will be considered as the main "question." The question in the introductory lesson where a partitive division situation will be considered is "Which is the answer, 1/3 L or 2/3 L?" In quotitive situations, question may be something like, "how many times as much is it?"

2 Dispositions for learning toward harmonious living

In our school's mathematics department we place a great deal of importance on preparing learning tasks/problems that help students generate their own questions, to ask themselves, seek answers, and come up with a solution(s). We want students to develop and apply their newly formed knowledge and ideas to various tasks.

Mathematics is a subject that provides students experience making new knowledge by building on and utilizing what they have learned before. When the students face a new task/problem they create new knowledge and ideas based on previous knowledge. Through this experience, students reflect on their past learning. However, there are some students who focus solely on a solution method. For this reason, during the lesson I would like to provide opportunities for students to discuss the differences among the presented methods, to compare and contrast their own idea with the ideas of others', and recognize the merit of these different ideas. To do this well, we believe students need to think logically. Moreover, in order for students to be able to think logically, it is very important that students engage in activities that require expressing mathematical ideas (using diagrams, tables, math expressions, etc.).

The goal of this unit is for students to understand ways to think about and represent fractions and also the relationship between whole numbers, decimal numbers, and fractions so that they can deepen their understanding of fractions. However, instead of students remembering a ÷ b = $\frac{a}{b}$ procedurally, the emphasis will be for them to understand the meaning of the calculation and the quotient in situations that can be represented in the equation, (whole number) ÷ (whole number) = (fraction), and derive times as much with fractions on their own.

As a strategy to assess students' ability to reason creatively, we will utilize their notebooks. We have been helping students to learn to take notes that match the process in problem solving lessons, "independent problem solving" ➔ "critical comparison and analysis" ➔ "reflection." They have been taught to keep records of their own ideas during the independent problem solving time and their friends' ideas during the critical comparison and analysis time. Moreover, instead of just writing the answers, they have been encouraged to write down the reasons and the methods that led to their answers using words, pictures, diagrams and or or mathematical expressions. By writing reflections, students can summarize what they learned. They can also re-examine their own reasoning so that they can utilize their

learning in new problem situations. In this way, their notebook will reflect the development of their thinking processes. Through the examination of the development of students' thinking in their notes taken at the three stages, independent problem solving, critical comparison and analysis, and reflection, we will investigate if we have been able to develop students' ability to reason creatively.

IV Unit Plan (Total of 6 lessons)

No.	Goal	Learning Activity	Evaluation Standard
(1) Division and Fractions (3 lessons)			
1	Students will understand that the quotients of whole number division can be expressed in fractions. (partitive division)	• Think about how much each part will be if 2 L of juice is equally partitioned into 3 parts. • Represent the quotient of 2 ÷ 3 using decimal numbers. • Summarize that the quotients of whole number division can be expressed in fractions.	Do students realize the merit that a fraction can be used to represent the quotient even when a whole number cannot be divided evenly by another whole number? (Interest, Eagerness, and Attitude) Can students use fractions as the quotients of whole number divisions? Can they express fractions using whole number division? (Mathematical Skills) Disposition to generate their own "question" and utilize their own reasoning and that of others.
2		• Work on practice problems.	
3	Students will understand times as much with fractions. (Research lesson) (quotitive division)	• Think about how many times as much 2 km and 4 km are compared to 3 km. • Identify the base and comparison quantities and write mathematical expressions. • Summarize that we can use fractions to express times as much.	Can students think and explain the meaning of times as much with fractions by relating it to their prior learning of times as much with whole numbers and decimal numbers? (Mathematical Way of Thinking) Do students understand the meaning of times as much with fractions? (Knowledge and Understanding) Disposition to generate their own "question" and utilize their own reasoning and that of others.

(2) Relationship among fractions, decimal numbers, and whole numbers (2 lessons)			
4	Students will express the quotients of whole number divisions using both decimal numbers and fractions and understand that they are equal. Students will understand the method of converting fractions to decimal numbers.	• Express the quotient for 3 ÷ 5 using a fraction and a decimal number. Verify that 3/5 and 0.6 are equal using a number line. • Think about ways to represent 3/4 and 2/9 as decimal numbers. • Summarize the method to covert fractions to decimal numbers.	Can students convert fractions to decimal numbers or whole numbers? (Mathematical Skills)
5	Students will understand the method of converting decimal numbers and whole numbers to fractions.	• Think about ways to represent 0.3, 0.29, and 1.57 as fractions. • Summarize the method to convert decimal numbers to fractions. • Think about ways to convert 4 and 12 as fractions. • Summarize the method to convert whole numbers to fractions.	Can students convert decimal numbers and whole numbers to fractions? (Mathematical Skills)
(3) Summary (1 lesson)			
6	Review and consolidate students' understanding.	• Work on problems in the end of unit exercises.	Do students understand the topics discussed in the unit? (Knowledge and Understanding)

V About the Lesson

(1) Goal of the Lesson

- Students will understand times as much with fractions.

(2) Rationale of the Lesson

The goal of this lesson is for students to understand that we can use fractions to express the relationship of two quantities using the idea of times as much. The problem context will be the distance from home to the school.

During the opening, we will display four distances: 4 km, 2 km, 3 km, and 1.5 km. It will be easy for students to see the "2 times as much" between 4 km and 2 km. Pick up students's comment about "2 times as much," ask the class, "The distance between my home and the school is 3 km. Compared to that distance, how many times as much is ____'s house?" This way, we will introduce the task that compare distances based on "times as much" instead of the "difference." The numbers for this lesson were selected because quotients such as 4 ÷ 3 = 1.3333... and 2 ÷ 3 = 0.6666... cannot be expressed in decimal numbers clearly. Moreover, because it may be difficult for students to make sense of "small number ÷ large number" and "'less than 1' times as much," we chose to start with the situation, 4 ÷ 3 = 4/3.

During the independent problem solving time, many students will use the idea "a ÷ b = a/b" we have been studying and calculate 4 ÷ 3 = 4/3. On the other hand, some may calculate 4 ÷ 3 = 1.3333... and cannot use a fraction. There may be some who are stumped and cannot even place 4 km, 3 km, 1 (the base quantity), and □ for "times as much" on a double number line diagram.

During the comparison and analysis stage of the lesson, we will verify that the calculation, 4 ÷ 3 = 4/3, is appropriate representing the relationship of 2 quantities on a double number line. Using students' comment, "we have times as much with a fraction," we will look back on what we have learned about the idea of times as much. We will ask students, "can we use times as much with fractions just as we did with whole numbers and decimal numbers?" We will carefully develop the meaning of times as much with fractions by using double number lines and relating it to the meaning of times as much with whole numbers and decimal numbers we have already studied. We believe that making sense of the meaning of times as much with fractions visually using double number lines will be helpful for students' Grade 6 study of the meaning of multiplication and division of fractions.

At the end of lessons, we have been asking students to write reflections in their notebooks. We hope to see statements that reflect their understanding of the quotient meaning of fractions and the merit of using fractions with times as much such as follows: "I understood that we can use fractions with times as much just as we did with whole numbers and fractions," "times as much fractions is just like times as much with decimal numbers. They are both useful when we cannot use whole numbers," and "we haven't learned how to multiply or divide by fractions, but I think I can do that, too."

(3) Flow of the Lesson

<table>
<tr><th>min.</th><th>Making learning task/content</th><th>Points of consideration/ relationship to the research theme (emphasis)</th></tr>
<tr><td>15
Grasp</td><td>1. Understand the task
• Compare the distances for 4 people.
○ A lives farthest from the school. (D lives closest to the school.)
○ A lives 2 times as far as B does. (C lives 2 times as far as D does.)
○ 2 (km) × 2 = 4 (km)
○ If we look at it other way around, B's distance is a half of A's (D's distance is a half of C's.)
○ 4 (km) ÷ 2 = 2 (km)

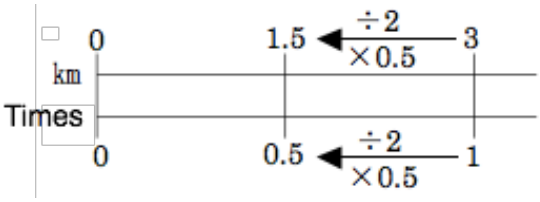
The distance from my (the teacher's) home to the school is 3 km. If we use the distance between my home and the school as the base, how many times as long is the distance for ___'s?

• Compare 3 km and 1.5 km.
○ There are some we already know. D's distance is a half of the teacher's.
○ Because it is a half, it must be ÷ 2.
○ It is 0.5 times as much.
○ If we look at the diagram, because it is 0.5 times as much, it should be less than 3 km.
○ Let's compare others.

How many times as much is 4 km as 3 km?</td><td>• Display the distances from home to the school.
(Distance to the school)

<table><tr><td>Name</td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>(km)</td><td>4</td><td>2</td><td>3</td><td>1.5</td></tr></table>
• Make sure we discuss why it is "2 times as much."
• Based on the comment, "2 times as much," make sure that the students understand that we are using "times as much" to compare.

• Start with the case where we can use times as much with a decimal number.
• If necessary, verify times as much with a decimal number using double number line.

• Change the numbers to be compared.</td></tr>
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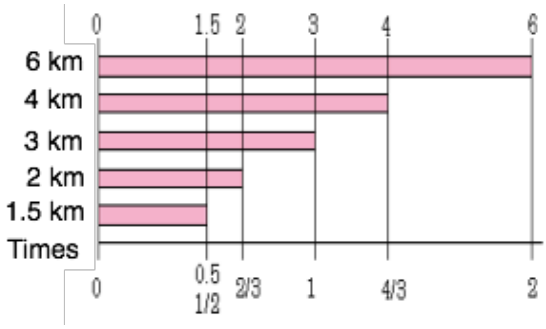
5 Think	2. Independent problem solving (a) Use the quotient fractions  (c) Cannot represent the relationship between two quantities using a diagram. Cannot put numbers in a diagram. (d) 4 ÷ 3 = 1.33... 1.33... times as much.	In the process of learning toward harmonious living: ⊙ the abilities to develop • Have "questions" of their own and make use of own ideas as well as others'. ⊙ desirable dispositions • Try to use prior learning to solve problems. • Explain why we can use 4 ÷ 3 = 4/3 even in the quotitive context by using a diagram. • Make sense of other's idea from a diagram (or a mathematical expression). • Realize a new idea (times as much with fractions) from listening to other students' ideas. ⊙ strategies • "Selecting the task" • "Questioning" "When we used 'times as much' until now ..." "How many times as much is 2 km?" ⊙ Support • Write "questions" on the board. • Set up a time when students try to interpret each other's idea.
20	3. Compare and analyze ideas • Write a mathematical expression from a diagram and confirm the appropriate calculation is 4 ÷ 3. • Re-affirm D's idea. ○ The quotient was 1.33..., and it did not divide evenly. ○ We learned 4 ÷ 3 = 4/3 in the previous lesson. So, we can use the fraction, 4/3. ○ 4/3 times as much. It's times as much with a fraction. • Think about the meaning of times as much with fractions. "When we used 'times as much' until now ..." ○ Compared weights. ○ 2 times as much, 3 times as much, ... ○ 2.5 times as much, 0.6 times as much, ... ○ If we consider a quantity as 1, 'times as	• Discuss (c)'s idea. Make sure everyone understands that 3 km is the base quantity, and we consider the base quantity as 1. • Make sure that the students understand 3 × ☐ = 4 from the number line. • Next, discuss (d)'s idea. Ask students if we can express the relationship precisely. • From the comment "times as much with a fraction," lead to the question that will look back on what we studied about the idea, times as much. • When the idea, "if we consider a quantity as 1, 'times as much' tells us how many of that quantities are in another," comes up from students, connect to the question, "Can we say

	much' tells us how many of that quantities are in another. ○ If we consider 3 as 1, ...	the same thing with times as much with fractions?"
	• Consider the meaning of times as much with fractions. "Can we say the same thing with times as much with fractions?" ○ If we consider 3 km as 1, 4 km corresponds to 4/3. ○ 1 and 1/3. In 4 km, there is one 3 km. ○ It is 1.** times as much. It is like times as much with decimal numbers. ○ We can use the equation, 3 × 4/3 = 4. ○ It means if we multiply 3 km by 4/3, the product will be 4 km.	• Verify that 4/3 times as much means if we consider 3 km as 1, 4 km corresponds to 4/3. • Think about the meaning of quantities division and what 4 ÷ 3 means. • Help students get the sense that 4/3 times is between 1 time as much and 2 times as much. • Plug in 4/3 in the box in the equation, 3 × □ = 4. Although students have not learned to multiply by fractions, I want to show them the equation as a tool to explain the relationship of 2 quantities.
	• Check if we can use times as much with fractions in other situations. " How many times as much is 2 km?" ○ 2 ÷ 3 = 2/3. It is 2/3 times as much. It is times as much with a fraction. ○ If we consider 3 km as 1, 2 km corresponds to 2/3. ○ Because 2/3 is less than 1, it means 0.** times as much. ○ We can again use the equation, 3 × 2/3 = 2. ○ It means if we multiply 3 km by 2/3, the product is 2 km. ○ If we use the teacher's distance (3km) as the base, D (1.5 km) is 0.5 times as much. Since 0.5 is 1/2, ... ○ 3 × 1/2 = 1.5. It means if we multiply 3 km by 1/2, the product is 1.5 km.	• Interpret what 2/3 times as much means as we did with 4/3 times as much. • Think about the question while observing a number line diagram that shows 2 km, 3 km and 4 km altogether. • By adding 1.5 km and 6 km, try to show we can use times as much with whole numbers, decimal numbers and fractions. 
5 Summarize	4. Reflect on today's lesson. ○ I understood that there is also times as much with fractions in addition to whole numbers and decimal numbers. ○ Times as much with fractions is similar to times as much with decimal numbers. It is not exactly like 2 times or 3 times as much, but if we look at number lines, I could tell about how much. ○ There were equations for multiplication by fractions. We haven't learned about it yet, but I think I can do that.	• From reflections, we will evaluate today's lesson.

(4) Evaluation Points

1. Were the characters and abilities this lesson focused on appropriate for developing dispositions for learning toward harmonious living?
2. Was the lesson organization, the selected learning task, and the teaching strategies effective in developing the characters and abilities necessary for harmonious living?
3. Did students generate "new values" by examining "differences between self and others" in this lesson?
4. Were students assessed appropriately? Was the support provided effective?

(5) Resources (omitted)