

Grade 5 Mathematics Lesson Plan
How can we draw congruent quadrilaterals (Congruent Figures)

Date: Wednesday, June 24
Sugekari Elementary School
Grade 5 Homeroom 1
23 students
Teacher: KUDO, Noriko

1 Name of the Unit

Congruence and Angles of Geometric Figures

2 Goal of the Unit

Students will understand the meaning of congruence of geometric figures and properties of congruent figures. They will deepen their understanding of plane geometric figures through drawing them.

3 Evaluation Standards for the Unit

- Students will re-examine properties of geometric figures using congruence as a point of view and try to use their new insights in the study of geometric figures such as classifying geometric figures based on their diagonals. [Interest, Eagerness, and Attitude]
- Students will think about factors that determine the shape and the size of geometric figures using congruence as a point of view. Students will be able to summarize their observations as properties of geometric figures and grasp geometric figures from a more unified perspective. [Mathematical Way of Thinking]
- Students will be able to distinguish geometric figures using the lengths of corresponding sides and the measures of corresponding angles. They will also be able to draw congruent figures. [Mathematical Skills]
- Students will understand the meaning and properties of congruent figures. [Knowledge and Understanding]

4 Structure of the Unit

(1) Rationale

Students have learned about positional relationships and constituent parts of plane geometric figures. They have defined the basic plane figures and explored their properties. As an activity that is related to the study of congruence, students have verified that if we cut a square or a rectangle along a diagonal, the two parts can be made to overlap completely. Although students have yet to learn the term, congruence, they recognize in their daily life some shapes are of the same shape and the same size.

In addition, in Grade 3, students learned to draw isosceles triangles and equivalent triangles using the lengths of sides since they had not learned angle measurements. Then in Grade 4, they learned to draw triangles using a side and angles on the ends of the side after

they learned angle measurements. These experiences will be the foundations for drawing figures in this unit.

Based on these ideas previously discussed, the current unit aims to help students understand congruent figures.

First, we will let students have experiences of "2 figures that can be made to overlap completely," and define that they are "congruent figures" when they match up completely. Then, focusing on the constituent parts, "vertices," "sides," and "angles," we will investigate properties of congruent figures.

We want students to pose the question, "how can we draw congruent figures efficiently, using the minimum number of information?" on their own. We want them to discover the "3 conditions for congruent triangles" through their explorations to answer this question. Based on their observations, we will extend the exploration to quadrilaterals and pentagons.

(2) About students

The students in this class have generally mastered appropriate ways to use tools of drawing such as rulers and compass. They also want to explain their ways of reasoning or solution strategies to their friends. They have been encouraged to record their reasoning and strategies in own notebooks and use the notebooks to facilitate their communication to their peers. As a result, students have begun to pay attention to incorporate diagrams, words, and mathematical expressions appropriately so that their writing is more easily understood by their friends. In this unit, we want students to be able to point out commonality or better ideas as they discuss each other's ideas after the independent problem solving time.

(3) Abilities to develop through this unit

As noted earlier, students have experiences of figures that "overlap completely." We want them to develop the ability to summarize this relationship by paying attention to the constituent parts of geometric figures.

The second ability we want to develop in students is the ability to use the definition of congruent figures deductively to figure out ways to draw congruent figures and explain why their methods work logically. We also want to emphasize the efficiency of methods by minimizing the number of factors to use in the process of drawing.

Finally, we want to develop the disposition to extend their learning. For example, they may conjecture whether or not the "3 conditions for congruent triangles" can be applied to quadrilaterals or how we can generalize the conditions for congruent figures. This research lesson will focus on the last two abilities.

5 Scope and Sequence

【Gr. 3】 Triangles

- Isosceles triangles and equilateral triangles
- Angles in geometric figures



【Gr. 4】 Angles

- Size of angles
- How to measure angles
- How to draw angles



【Gr. 4】 Various Quadrilaterals

- Perpendicular and parallel lines, how to draw
- Definitions, properties, and drawing of trapezoids, parallelograms, and rhombi
- Properties of diagonals of quadrilaterals



【Gr. 5】 Congruent Figures

- Concept of congruent figures
- How to draw congruent triangles and quadrilaterals; correspondence of sides and angles



【Gr. 5】 Angles in Geometric Figures

- Concept, properties, and drawing of regular polygons



【Gr. 6】 Symmetry

- Concept, properties, and drawing of line symmetric and point symmetric figures
- Polygons and symmetries



【Gr. 6】 Scale Drawing

- Concept of scale drawings
- How to draw scale drawings

6 Unit Plan (Total of 9 lessons)

	No.	Learning Activity	Evaluation			
			I	T	S	K
Congruent Figures	1	<ul style="list-style-type: none"> • Understand the meaning of congruence through manipulation. • Learn the meaning of the term, "congruence." 	⊙			⊙
	2	<ul style="list-style-type: none"> • Investigate corresponding parts of congruent triangles. • Learn the meaning of the term, "corresponding." • Clarify the size relationship of corresponding sides and angles. 		⊙		⊙
	3	<ul style="list-style-type: none"> • Investigate corresponding parts of congruent quadrilaterals. 		⊙		⊙
	4	<ul style="list-style-type: none"> • Think about ways to draw congruent triangles. • Draw congruent triangles using SSS, SAS, and ASA conditions. 	⊙	⊙	⊙	
	5	<ul style="list-style-type: none"> • Realize that we cannot always draw congruent triangles using any 3 measurements. 		⊙		
	6	<ul style="list-style-type: none"> • Work on practice problems. 			⊙	
	7	<ul style="list-style-type: none"> • Think about ways to draw congruent quadrilaterals. 	⊙	⊙	⊙	
	8	<ul style="list-style-type: none"> • Deepen the learning in the unit. 			⊙	⊙
	9	<ul style="list-style-type: none"> • Solidify the learning in the unit. 			⊙	⊙

I: Interest, Eagerness, and Attitude

T: Mathematical Way of Thinking

S: Mathematical Skills

K: Knowledge and Understanding

7 About the Lesson

(1) Goal of the lesson

Through the activity of drawing congruent quadrilaterals, students will think about the minimum information needed, and they can explain why.

(2) Emphasis in today's lesson

In the previous lessons in the unit, these students have learned about the congruence conditions for triangles through manipulation. They have also explored how to draw congruent figures using as few information as possible. In particular, they have learned to draw congruent triangles.

In this lesson, we aim to deepen students' understanding of congruence by focusing on the constituent parts of geometric figures as we extend our study of congruent figures from triangles to quadrilaterals. In addition, we want to develop the disposition to both broaden and deepen their own mathematical reasoning by generalizing and extending what they have learned. For example, we want students to pose the question, "Can we say the same thing for quadrilaterals as we did with triangles?" on their own and investigate.

The students have experienced that they could draw congruent triangles if they know the lengths of the three sides. Therefore, we will begin by asking them to predict whether or not they would be able to draw congruent quadrilaterals if they know the lengths of the four sides. As they try to draw congruent quadrilaterals, they will soon realize that they would not be able to draw congruent quadrilaterals by knowing the lengths of four sides. We want students to ask themselves, "what additional information would we need in order to draw congruent quadrilaterals?" and pursue that question on their own.

In addition, we want to emphasize the activity to explain own ideas with reasons. For example, we want students to be able to explain "why the lengths of four sides will not define a quadrilateral," or "if we know the measure of one angle, or the length of a diagonal, in addition to the lengths of four sides, we can draw congruent quadrilaterals."

(3) Specific strategies to address the research theme

i. Drawing out questions while engaged in a task (grasp)

In the opening of the lesson, we will review some of the reflections from previous lesson and try to motivate students so that they will think "I can draw congruent quadrilaterals" or "I want to draw congruent quadrilaterals." We will try to draw out from the students the question, "Can we draw congruent quadrilaterals if we know the lengths of four sides?" just like we could draw congruent triangles when we know the lengths of three sides. We anticipate that many students will respond, "yes," instinctively. However, once they try to draw congruent quadrilaterals, they soon realize that knowing the lengths of four sides is not enough. By encouraging students to think about why that is the case, we will help them to think about what additional information is needed to draw congruent quadrilaterals.

(2) Through the use of previously learned ideas, making own ideas clear and enjoy the exchange of ideas with friends (Explore)

Students should have the record of the congruence conditions for triangles and how to draw congruent triangles. During the independent problem solving time, in order to remember what they have learned previously, we will utilize students' own notebooks.

As students record their ideas in notebooks, we will encourage them to keep in mind that they would be using what they write to explain their ideas to other students later. We want students to think about making connections among terms describing the constituent parts of the figures (such as vertices, sides, and angles), numbers that quantify the lengths of sides or measures of angles, and words that describe the steps of manipulating the figures as they explain their ideas.

For those students who are unable to start with the task, we will have a small group session. As students think about the task using the given hints, we want students to be able to say, "I got it" or "I did it."

- (3) Answer questions and deepen own reasoning while engaged in the activity of interpreting friends' ideas (Deepen/Raise)

During the lesson, the students will be tackling questions such as the following:

- Why is it possible to draw congruent quadrilaterals if we know the measure of one angle (in addition to the lengths of four sides)?
- What is it possible to draw congruent quadrilaterals if we know the length of one of the diagonals (in addition to the lengths of four sides)?

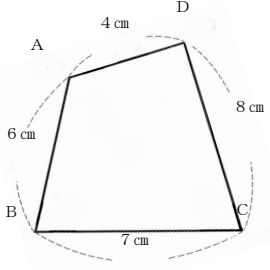
While discussing these questions as a whole class, we want students to be posing and responding to each other's questions. In that way, we hope to deepen students' reasoning collaboratively.

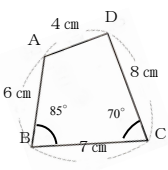
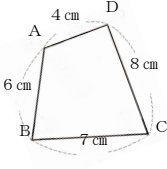
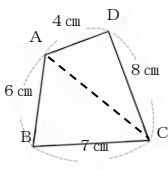
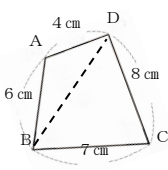
- (4) Draw out new questions as students summarize/apply/extend what they learned (Summarize/Extend)

We will have the students summarize what they learned using their own words in individual notebooks. They will be instructed to include the phrases, "the lengths of four sides" and "we can draw" in their summaries. We believe that we can assess how students were thinking and what they learned in this lesson by analyzing their own words.

If any student has raised the question, "What would happen if the number of vertices kept increasing?" in their reflections, bring it to the attention of the whole class. In this way, we want to develop the disposition to extend their own learning.

8 Flow of the Lesson (Lesson No. 7 of 9)

	Learning Content (main <i>hatsumon</i> and anticipated responses)	★Strategies to address the research theme ○ Support and point of consideration ◎ Evaluation
Grasp	<p>1 Understand the learning task</p> <p>T We have learned how to draw congruent triangles through yesterday's lesson. What information do we need to draw congruent triangles? C Vertices. T In order to draw congruent triangles, how many sides and angle measurements do we need? C 3. T What are the ways to draw congruent triangles? C Use 3 sides. C Use 2 sides and the angle in between them. C Use a side and the angles on the ends of the side. T At the end of the last lesson, you wrote your reflections. Can you read yours, ___? C I wonder if we can draw congruent quadrilaterals like we did with congruent triangles. T I think your wondering of whether or not what we could do with triangles can be done with quadrilaterals is very neat. I wonder if any other would like to try it? Do you think we can draw congruent quadrilaterals just like we did with quadrilaterals? C Yes, I think so. T Let's try. Here is the quadrilateral that I would like you to draw. (Display a quadrilateral with the lengths of four sides shown.) The measurements we know are these four sides. OK, let's try. You have 2 minutes.</p> <p>C Whoa? I can't draw it. C Yes, I got it. T ____, that's great. Can you do it again? C It doesn't work. It doesn't match up.</p> <p>T Raise your hand if you could draw the shape? Who could not? I heard some of you saying, "it doesn't match up," or "I can't find vertex A." Do you mean this? (Demonstrate with manipulative.) C Yes. It keeps moving. T What keeps moving? C Vertices A and D.</p> <div style="border: 1px solid black; padding: 10px; margin-top: 20px; width: fit-content; margin-left: auto; margin-right: auto;"> <p>Let's draw congruent quadrilaterals by thinking about what else we need to know in addition to the lengths of the four sides.</p> </div>	<p>○ Adjust the content of review based on mastery of prior contents.</p> <p>○ Display the congruent conditions for triangles.</p> <p>○ Make sure that the class as a whole have the same wondering as the student: If we use the congruent conditions for triangles, we can draw congruent quadrilaterals. Based on that wondering, lead the class to the question, "If we know 4 conditions (length of 4 sides), can we draw congruent quadrilaterals?"</p> <div style="text-align: center; margin: 20px 0;">  </div> <p>○ To help students visualize the problem of moving angles, use geometric construction model.</p>

Explore	<p>2 Independent problem solving with clear understanding of the challenge</p> <p><Anticipated Responses> [4 sides and 2 angle measurements]</p> <p>C1 Use angle C to draw. Draw side BC. Measure angles B and C. Then locate vertex A 6 cm from B and vertex D 8 cm from C. Join A and D.</p>  <p>[4 sides and 1 angle measurement]</p> <p>C2-1 Use 4 sides and 1 angle to draw. Use angle B to draw. Draw side BC. Measure angle B and draw side AB. Using a compass, draw an arc (centered at A) with the radius AD and an arc (centered at C) with the radius CD. Make the point of intersection vertex D. Connect D with A and C.</p>  <p>C2-2 (We can also use angle C instead of B)</p> <p>[Using length of a diagonal]</p> <p>C3-1 Use 4 sides and a diagonal. Draw diagonal AC. Then, using a compass and lengths AB and BC, draw arcs to locate vertex B and draw triangle ABC. Using lengths AD and CD, locate vertex D and connect it to A and C.</p>  <p>C3-2 Draw diagonal BD. Then using a compass and lengths of BC and DC, draw triangle BCD. Then using lengths DA and BA, locate vertex A and connect it to B and D.</p> 	<p>○ Have students mark the additional parts they measured in red.</p> <p>☆ For students who are stuck, conduct a small group session.</p> <p>[Hints]</p> <ol style="list-style-type: none"> 1. Why can't we find vertex A or D? 2. What additional information do we need to find where vertex A will be? 3. Now we found vertex A. Can we find where vertex D should be using the lengths of the sides? <p>☆ For C2, ask if "one angle" can be any angle in the quadrilateral.</p> <p>☆ For C3, point out that there are 2 diagonals, and ask if either one can be used and why. I would like to draw out the idea "because there are 2 congruent triangles" from students in their own words.</p> <p>◎ Students are drawing figures accurately using what they have learned previously.</p> <p>[Mathematical Skills]</p> <p>◎ Students are thinking what additional information is needed to draw congruent quadrilaterals.</p> <p>[Mathematical Way of Thinking]</p>
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Deepen/Raise	<p>3 Discuss as a whole class what information in addition to the lengths of 4 sides is needed to draw congruent quadrilaterals.</p> <p>T Let's hear from some people how they drew congruent quadrilaterals.</p> <p>C In addition to the four sides, I used this measurement (have the student mark in red).</p> <p>T How do you think ____ drew the congruent quadrilateral?</p> <p>C First, draw side BC. Next, draw angle B using the measurement. Then, we know where vertex A is.</p> <p>C Then, we will draw angle C and draw side DC using the measurements. Finally, we connect A and D.</p> <p>T Good. We now have a congruent quadrilateral. But, did we use the lengths of the four sides?</p> <p>C Yes.</p> <p>C No, we just connected vertices A and D, and we didn't need to use the length of side AD.</p> <p>T OK, so we have to use 2 angles in addition to the 4 sides, don't we?</p> <p>C No, we can draw using only angle.</p> <p>T Really? Can you come and make the part you used to draw a congruent quadrilateral?</p> <p>C Mark angle B.</p> <p>T What do you think? Do you think we can draw a congruent quadrilateral? We will start with side BC, right? Next, just like we did before, we will draw angle B and locate vertex A. Now we have to find vertex D. But, we already used the one angle. What should we do now? Do you think we can draw a congruent quadrilateral? Why don't you discuss with your neighbors?</p> <p>C (Pair-wise discussion)</p> <p>T Can someone who thought we could draw using only one angle measurement explain how we can complete the drawing?</p> <p>C Using a compass, draw an arc centered at A with the radius AD.</p> <p>T OK. What do you all think? Do you think we can draw a congruent quadrilateral?</p> <p>C Next, we draw an arc centered at C with the radius DC. Then, the point where the arcs intersect is vertex D. Finally, we connect vertex D to vertex A and vertex C.</p> <p>T Wow, we thought we had to use 4 sides and 2 angles, but we can draw using only one angle, can't we? We looked at 2 angles then 1 angle. I wonder if it is possible to use 0 angle? Is there a way to draw without using an angle? We know we can't draw a congruent quadrilateral using only 4 sides. I wonder if there is a way that does not use an angle?</p> <p>C We can draw using a diagonal.</p> <p>T Diagonal? Where? Can you come up front and mark what you used?</p>	<p>☆ Instead of one student explaining all the steps, have multiple students explain different parts or incorporate pair-wise sharing.</p> <p>○ Verify quickly that we can also use angle C as the one angle. Also, in case some students wonder whether or not we can use either angle A or angle D, have additional <i>hatsumon</i> ready.</p> <p>○ Change the orientation of the diagram and confirm that if we know the measurement of one angle, the two sides get fixed.</p> <p>☆ When a student suggest "we can draw using a diagonal," ask the students if it is really possible and have the class think if it is possible to draw congruent quadrilaterals using a diagonal.</p> <p>○ Adjust the flow of the lesson depending on the number of students who used a diagonal.</p> <p>○ Use the manipulative we used earlier to verify.</p> <p>◎ Students are explaining their ideas with reasons and integrating own ideas and friends' ideas from the perspective of congruent figures. [Mathematical Way of Thinking]</p>
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	<p>C (Draw diagonal AC)</p> <p>T Well, ____ drew a segment here. How do you think ____ drew a congruent quadrilateral using this segment?</p> <p>C (Pair-wise discussion)</p> <p>T Do you think we can draw congruent quadrilaterals is we use a diagonal?</p> <p>C I think so.</p> <p>C I don't think so.</p> <p>T Well, let's talk about it in tomorrow's lesson.</p>	
Summarize/Extend	<p>4 Summarize the lesson</p> <p>T Today, we thought about what information in addition the lengths of sides to draw congruent quadrilaterals. To draw a congruent quadrilateral, we had to find the locations for vertices A and D, didn't we? We then learned the method to draw a congruent quadrilateral using the measurements of 2 angles in addition to four sides, and also the method to draw using only one angle. We will think about whether or not we can draw a congruent quadrilateral is we know the length of a diagonal in addition to the lengths of sides. Please write your reflection in your notebooks. Please include whether or not you think we can draw a congruent quadrilateral using a diagonal.</p> <p>T Can someone read the reflection?</p> <p>C Today we drew congruent quadrilaterals. At first I thought we could draw just using the length of 4 sides, but we couldn't. Then, we learned that we can draw a congruent quadrilateral if we use the measurement of one angle in addition to 4 sides. I want to try to draw a congruent quadrilateral using a diagonal.</p> <p>C I understood that we can draw congruent quadrilateral using 4 sides and 1 angle. I wonder if we can really draw one using a diagonal.</p>	<p>○ Verify that to draw a quadrilateral, we need to fix the positions of the four vertices.</p> <p>☆ By having students write reflections, draw out new questions.</p>

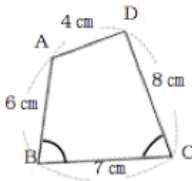
9 Board Writing Plan

Wed. June 24 Congruence of Geometric Figures

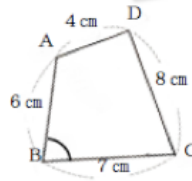
Let's draw congruent quadrilaterals by thinking about what else we need to know.

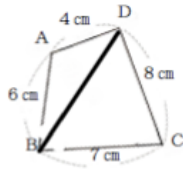
Let's find A and D with as efficiently as possible.

Did you use the length of AD? Did you just connect them?



Angle B tells us where vertex A is.





4 sides and a diagonal?

Summary

If we know 4 sides and an angle, we can draw congruent quadrilateral.
Can we draw one using 4 sides and a diagonal?

10 Today's Board Writing

1. Was the introduction (after reminding the students that we could draw congruent triangles if we know the lengths of 3 sides, give them the lengths of 4 sides and challenge them to draw a congruent quadrilateral) effective to draw out questions from students and motivate them to draw congruent quadrilaterals?
2. Was asking students to write their own reasoning for drawing congruent quadrilaterals in their notebooks effective to promote students to think logically and express their ideas?
3. Was the use of diagram to share own ideas or to interpret other students' ideas effective to deepen students' level of thinking? Was the teacher's questioning effective and timely?
4. Others