July 2

Grade 8, Mathematics Lesson Plan

1. Date & Time: 6th period, Tuesday, July 2, 2013
2. Theme: Explaining with Algebraic Expressions
3. Instructor: Tokyo Gakugei University Affiliated Koganei Junior High School, Hideyuki Kawamura
4. Class: Tokyo Gakugei University Koganei Junior High School, Grade 8, Class D (40 Students)
5. Place: Educational Technology room
6. Name of the Unit: Calculations with Algebraic Expressions
7. About the Theme of This Lesson

The instructional material that I will be providing to the students in this lesson requires the students to describe a statement that is a revers statement of usual statements for determining numbers if they are multiples or not.[1] The objective for dealing with this material is providing an opportunity for the students to have experiences for generalizing and specializing by interpreting transformation process of algebraic expressions and grasping the meaning of them clearly.

About the process of utilizing algebraic expressions, Miwa (1896) describes it using the following diagram (see figure 1).

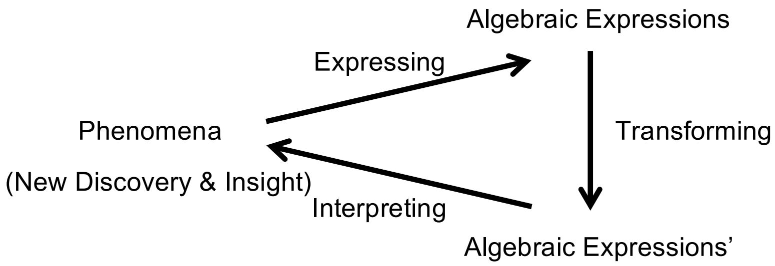

Figure 1: Diagram for Use of Algebraic Expressions (Miwa, 1996)

The diagram shows, when a algebraic expression is utilized, the processes of "expressing," "transforming" and "interpreting" take in the place. In the process of interpreting the algebraic expressions, generalization and specialization can be conducted by reexamining and grasping the meaning of the algebraic expressions. (Miwa, 2001) In other word, we can think that the act of interpreting an algebra expression facilitates an opportunity to create new mathematics. I thought if students could experience this process through a lesson, they would use algebraic expressions more actively and try to interpret them more willingly.

In the article written by Miwa (2001), an example of a process of interpreting algebraic expression for generalization and specialization is discussed by providing an example regarding how to distinguish multiples.

The discussion of the transformation process of the algebraic expression for distinguishing multiples of 9 (If $a + b$ of the 2-digit numbers $10a + b$ are multiples of 9, the numbers are multiple of 9.) describes that $10a + b$

1 Translator's note: Usually the statement for determining a 2-digit number is a multiple of 9 or not is written as "A 2-digit number $10a + b$ is a multiple of 9 when $a + b$ is a multiple of 9." So the reverse statement that the instructor explaining here is "If a 2-digit number $10a + b$ is a multiple of 9, $a + b$ is also a multiple of 9."

can be split into the part shows the multiples of 9 (9*a*) and the remainder part (*a* + *b*). By generalizing this idea, students could create methods for distinguishing multiples of other numbers.

In this lesson, I decided to ask students to think about the reverse of this problem that is "If a 2-digt number is a multiple of 9, (a number in the tens place) + (a number in the ones place) is also a multiple of 9."　I have two reasons for setting up the problem like this way.

The first reason is when the algebraic expression $10a + b = 9n$ was established, because of the goal of the transformation of the algebra expression is $a + b = 9(n - a)$, it might be easier for the students to recognize the process of subtracting 9*a* from the both side of the equation.

The second reason is in the case of distinguishing if a number is the multiple 9 or not, students need to explain a reason why a number is a multiple of 9 or not by determining if the sum of the numbers in each place is a multiple of 9 or not. Therefore, many students would try to examine if the statement is valid or not by substituting the algebra expression with actual numbers that are the multiple of 9. If students examine the statement this way, students might proof this problem using a wrong reasoning that is because the original number is a multiple of 9 so the statement is valid. If this is the case, I thought it might be easier for the students think about the explanation if we make the supposition to "the 2-digit number is the multiple of 9."　In other word, I thought the statement can be reversed and provide it to the students to work on. In this way, it might be easier for the students to explain and understand the logical story of generalizing idea by interpreting the algebraic expressions.

８．Goal of the Lesson

・ Students generalize the statement by interpreting the process of explanation using algebraic expressions and grasp the mechanism of the expression.

＜References＞

三輪辰郎 (1996). 文字式の指導：序説，筑波数学教育研究 15，pp.1-14.
三輪辰郎 (2001). 文字式の指導に関する重要な諸問題，筑波数学教育研究 20，pp.23-38.

9． Flow of the Lesson

Learning Process	Instruction	Anticipated Student Reactions	・ Instructional Pints to Remember 〇Evaluation Viewpoint
1. Introduction	・The last lesson I asked you to explain "If a 2-digt number is a multiple of 9, (a digit in the tens place) + (a digit in the ones place) is also a multiple of 9." Do you remember how you explained about that? ・Is there any part of the statement that you might want to change?	・ If we assign the numbers in the tens place as *a* and in the ones place as *b*, the original number can be expressed as $10a + b$. This number is a multiple of 9 so: $10a + b = 9n$ $9a + a + b = 9n$ $a + b = 9n - 9a$ $a + b = 9(n - a)$ From this algebraic equation, (a digit in the tens place) + (a digit in the ones place) is a multiple of 9. ・ 2-digit number → Increase the number of digits. ・ Multiple of 9 → Multiple of other numbers.	〇 Students are able to understand the explanation with algebraic expressions that they worked on in the previous lesson. ・Ask the students to show all the steps involve for transformation of algebraic equations. 〇 Students are eager to think about what part of the statement can be change.

2. Expansion	・If we change the digits to 3-digit, what should we do to the part, (a digit in the tens place) + (a digit in the ones place)? ・If we change it to (a digit in the hundred place) + (digits less than and equal to the tens place), how should we change the algebraic equation. ・If the statement says multiple of 7 instead of multiple of 9, what do we need to do? "If a 2-digt number is a <u>multiple of 7</u>, (a digit in the hundred place) + (digits less than and equal to the tens place) is also a multiple of 9." Is there any part of the statement you need to change? ・Let's think about the statement "If a 2-digt number is a multiple of 7, (a digit in the hundred place) + (digits less than and equal to the tens place) is also a multiple of 7."	・ (a digit in the hundreds place) + (a digit in the tens place) + (a digit in the ones place), (a digit in the hundreds place) + (digits less than and equal to the tens place), (digits higher than and equal to the tens place) + (a digit in the ones place). ・ If we assign the numbers in the hundreds place as *a* and in less than and equal to the tens place as *b*: $100a + b = 9n$ $99a + a + b = 9n$ $a + b = 9n - 99a$ $a + b = 9(n - 11a)$ ・ Multiple of 9 → multiple of 7	○ Students are thinking enthusiastically about the part that they could change in the statement. ・Make sure to write the statements side by side so that the students could understand clearly about what were changed and what were not changes.

	• What kind of ideas did you use to do the transformation of the algebraic equation?	• If we assign the numbers in the hundreds place as *a* and in less than and equal to the tens place as *b*: $100a + b = 7n$ $98a + 2a + b = 7n$ $2a + b = 7n – 98a$ $2a + b = 7 (n – 14a)$ • The method (a digit in the hundreds place) + (digits lesson than and equal to the tens place) does not work well. • We need to change it to (a digit in the hundreds place) × 2 + (digits less than and equal to the tens place). • For the multiples of 9, 100 is split into 99 and 1. • 99 is a multiple of 9 so it that we can factor 9 out. 1 is what is left.	○ By recalling and utilizing how the algebraic equation was transformed in the case of multiple of 9, students are trying to think about the case of multiple of 7. • By interpreting the explanations using algebra equations carefully, help students understand the necessity of transformation of the equation to reach to the conclusion of this problem solving. • Help students to become conscious about they are thinking about the case of multiple of 7 based on the case they worked on multiple of 9.
3. Summary	What are the commonalities of these three transformations of algebraic equations?	• Leaving *b* at the left side of equal sign as it is. • Splitting 10 and 100 into the number that is the multiple of 9 or 7 and the number that is left. • Transform the right side of the equations into something like 7×(…) or 9×(…).	• By asking the students identify the commonality among the algebraic equations, help students to pay attention to the part of the structure of the algebraic equations that have not changed although the statements were changed.