July 3

Grade 6 Mathematics Lesson Plan

Who is the fastest?（Speed）

Wed. July 3, 2013, 5th period
Meguro-ku Sugekari Elementary School Grade 6 (Class 1) 35 Students
Instructor: Koko Morita

Research Theme: "I did it! I understand it!" Designing lessons that students become absorbed. Devising instructions that care about students' questions and provide experiences for students to enjoy thinking and expressing.

１． Name of the Unit: Speed

２． Goals of the Unit

○ Students are able to understand the meaning of speed, how to express it, and how to find it.
○ Students are able to understand the relationship of three quantities: speed, time, and distance.

３． Evaluation of the Unit

Interest, Motivation, & Disposition	Students are applying idea of per unit quantities when they are finding speed. Also they are eager to apply speed in their study and daily lives.
Mathematical Reasoning	Students are utilizing idea of per unit quantities when they are finding speed.
Skills & Procedures	Students are able to find speed, distance, and time, based on the idea of per unit quantities.
Knowledge & Understanding	Students are able to understand how to express speed that is based on the idea of per unit quantities.

４． Structure of the Unit

（１） Goals

The main goals of the unit is that the student to understand speed is expressed using the distance traveled in a unit mount of time, and them to be able to compare speeds and finding distance or time based on the study of "per unit quantity" that students learned in grade 5.

In the sub-unit entitled "1. Speed," a sprint race, which is familiar situation to the students, will be used to introduce this topic. Comparisons of speed in the cases of "different travel distance in the same period of time" and "different travel time for the same distance " are easier to carry out, however, the case with "the different travel distances in different period of time" is very difficult to determine. Therefore, students need to think about how they can compare speed in this case by thinking about making "time" or "distance" the same. Through this exploration of thinking, help students understand the meaning of speed as the distance traveled in a unit amount of time. In addition, the students will learn various ways to express speed (e.g., distance per hour, distance per minute, distance per second) and the relationship among these expressed speeds. Finally, the students will learn about how to solve problems that require them to find distance or time.

In the sub-unit entitled "2. Speed and graphs," students will be representing the relationship of time and travel distance of the cases of walking and bicycling in a table and on a graph, and discuss and interpret them.

（２） Students' state of learning

Many students in this class are serious about learning. They can think carefully and participate enthusiastically in the class. For example, they can hold their own ideas and opinions, explain ideas to their friends willingly, listen to friends' ideas while comparing them with their own ideas, and describe the similarities and differences of ideas that are presented. However, there is a large ability difference among the students. Some students have difficulty explaining their ideas logically in the class and understanding their friends' ideas. If I am not careful, a lesson could be progressed with participation of only a handful of students.

Because of this reason, I would like to think about better ways to pose problems, use instructional materials, develop flow of lessons, and support individual students to match their needs. I would like to apply these ideas to my lessons and help to bring out students' questions and interests. Moreover, I would like to provide problem-solving approach of instruction to the students and promote their active participation to the lessons. Lastly, I would like to support student learning carefully so that each student in the class could understand the importance of thinking on their own logically and expressing their ideas.

（３） Students' abilities that foster in the unit

Speed is related with time and distance. It can be compared by travel distance per unit time or amount of time per unit distance. These ideas for comparing are as same as the idea of unit per quantity (crowdedness) that students have learned before.

Students might have some ideas about speed from their experiences in their daily lives intuitively. However, I will help the students to understand clearly that speed can be express with the relationship between time and distance through engaging them to a mathematical activity. Amount of time is a quantity that is not easily see or feel. This point is different from other kinds of quantities. Also, distance (length) can be obtained by the result of movement of an object in the continuous time. The travel distance per 1 second and the amount of time per 1m that students will learn in this unit are "an average travel distance" and "an average speed." In other words, "speed" is "an average speed" that is thought non-constant speed as an ideal constant speed by theorizing it as "if object is moving as a constant speed."

By helping students to understand the difference between "average speed" and real speed, develop students' awareness that is they are learning about speed by applying the idea of per unit quantity that students have learned before. Clear understanding of speed in this unit becomes a base for future learning such as idea of acceleration speed, instant speed, movement, etc.

５． Relationship and Expansion of Content

【Grade 4】

Multiplication and Division of Decimal Numbers

- (Decimal Number)×(Whole Number)
- (Decimal Number)÷(Whole Number)

【Grade 5】

Per Unit Quantity

- Per unit quantity
- Average

Multiplication of Decimal Numbers

- (Decimal Number)×(Whole Number)
- (Decimal Number)×(Decimal Number)

Division of Decimal Numbers

- (Decimal Number)÷(Whole Number)
- (Decimal Number)÷(Decimal Number)

Multiplication and Division of Fractions

- (Fraction)×(Whole Number)
- (Fraction)÷(Whole Number)

【Grade 6】

Multiplication of Fractions

- (Whole Number)×(Fraction)
- (Fraction)×(Fraction)

Division of Fractions

- (Whole Number)÷(Fraction)
- (Fraction)÷(Fraction)

Speed

- Meaning of speed, how to find it (per hour, per minute, per second)

6. Unit Plan (A total of 8 lessons)

Sub-Unit	Lesson	Learning Activities	Evaluation View P			
			IMD	MR	SP	NU
1. Speed	1	● Think about speed in a sprint race. ● Think about what quantities are related to speed. ● Think about how to compare speed.	◎	◎		
	2	● Understand how to find speed. ● Understand distance per hour, distance per minute, and distance per second, and compare speed using formula for speed.		○		◎
	3	● Understand the relationship among distance per hour, distance per minute, and distance per second; and learn how to find them.		○	◎	○
	4	● Investigate how many seconds a person takes to walk 50m, and find the speed for distance per hour, distance per minute, and distance per second.	○		◎	
	5	● Investigate how distance change when the time becomes twice or three times as much, and think about how to find distance. ● Think about how to find time when speed and distance are known from the math sentence for finding distance.		○	◎	
2. Speed & Graphs	6	● Representing the relationship between time and distance using a table and a graph.			◎	
Practice	7	● Check the understanding of content student have learned.			◎	◎
	8	● Find speed and distance from a graph.		◎		

* The first lesson of the 8 lessons is this research lesson.

** IMS (Interest, Motivation, Disposition), MR (Mathematical Reasoning), SP (Skills & Procedures), NU (Knowledge & Understanding)

7. Instruction of This Lesson

（１） Goals of this lesson

• Students are able to think about how to compare the speed of running and explain about it by paying attention to the two quantities involve, the distance a person run and the amount of time a person run.
• Students are able to recognize the merit for finding per unit quantity and utilizing it willingly.

（２） Rational of this lesson

The task of this lesson is for the students to think about how to compare the running speed of two children when both the distance and the time they run are different. The comparison can be done mainly using the following four methods:

① Finding a common multiple of distance to compare
② Finding a common multiple of time to compare
③ Finding amount of time per 1m to compare
④ Finding distance per 1 second to compare

All these methods are correct because when times or distances of two children are made the same, the students can compare them. However, this lesson does not end at this point. The goal of the student discussion of this lesson is for them to search “more effective ideas.”

By helping students to have viewpoints such as “concise,” “clear,” “accurate,” and “general,” guide student to search “more effective ideas.” Then help them to understand the meaning of speed, that is “speed is expressed using the distance traveled in a unit amount of time” with their agreement.

（3） Concrete measures for achieving the research theme

① Wrestle with the problem and bring out students’ questions --- (grasp)

First, show the students only the three students’ records of time of a sprint race (insufficient information), and ask them “who is the fastest?” The students might say, “the smallest value is the fastest.” But at the same time, other students might say, “we can’t tell who is the fastest because we don’t know if they run the same distance.” By pick up such students’ voices, bring out idea such as to compare the running speed of these children, the distance (travel distance) need to be set the same. Then provide information of the distance of those children run and help them to recognize that speed of those children can be compared when either the distance or the time they run is made the same. However the problem the students dealing with in this lesson tells two children from the three children run differently both the distance and time, so it is not easy to compare the speed of the three children. At this instance, students would come up with a question, “when both the time and distance of running are different, how can we compare them?” In this lesson, using the question that are rose from the students and use it to establish a main problem, I believe it would help the students to engage in autonomous learning through problem solving.

② Through solving students’ own question, help students to feel the joy of solving the problem on their own (pursue)

In problem solving where students think about problem on their own, helps students to think about how to resolve the problem by utilizing the clue that is if either the distance or the time they run is made the same we can compare the speed of the children. For the students who are having difficulty coming up with a solution idea, provide a small group instruction. Discuss with them by providing hints in question format such as “Why can’t we compare the speed of A and B?” and “I wonder if we could make one of them (time or distance),” and help them to find a clue to solve the problem. Support students in the way that they can feel joy of learning such as “I did it! I understand it!”

③ Through activity involves interpreting friend's ideas, students answer to the question and deepen their thinking (deepen and rise)

First, look at the math sentences of student solution methods that utilize common multiple to make the distances or the times the same. Then help the students to recognize that "when both distance and time are different, if one of them is made the same, they can be compared."

Next, bring up the math sentences of student solution methods that incorporate "per 1m" and "per 1 second" ideas, and discuss and interpret these ideas. This time also help the students to understand "when both the distance and time are different, if one of them is made the same, they can be compared." In addition, help students to recognize that "the former methods used idea of common multiple and the latter methods used idea of per unit quantity to make one of the quantities the same."

After all four methods are discussed, confirm that all the methods are in fact correct, and all of them used idea of "making one of the quantities the same." Then ask the students "If you need to find out the order of speed of the following six children what method do you want to use?" The common multiple method is easier to understand but the calculation is complicated and the method is not appropriate when the number of children that you need to compare increased. Thus, the discussion becomes focused on which one of the methods, "per 1m" and "per 1 second," is better. In the case of "per 1m," the smallest value obtained from the calculation is the fastest. In the case of "per 1 second" the largest value obtained from the calculation is the fastest. In our daily life, in general, "the distance run per 1 second" is used. By recognizing the strength of both methods and thinking about how these ideas are used in our daily life, help students to understand that "speed is expressed using the distance traveled in a unit amount of time."

④ Through activities such as summarizing, utilizing, and expanding, bring out students' new questions. (Summarize, expand)

By helping students to reflect on the process of learning of this lesson along with their thinking process, ask students to summarize what they have learned in their words by having viewpoints such as "I see!," "discovery," "question," and "challenge."

8. Flow of the Lesson (1/8)

	Learning content (Main *hatumon* and students' anticipated responses)	☆Measures to achieve the research theme ○Support and points to remember ◎Evaluation
Grasp	**1. Grasp the goal of the learning.** Problem: Let's think about the order of speed of these 3 children.	☆ Pose the problem with an insufficient information (providing only the time), and bring out students' questions. ○When we think about distance in the context of a sprint race of the sports day, the use of distance is more common. So in this lesson, we use distance instead of travel distance to pose the problem.

	Time (second)
A	6
B	6
C	5

T Order the speed of a sprint race of 3 children from the fastest.
C C is the 1st place, and A and B are the 2nd place
C We can not tell only with this information.
T Why do you think so?
C Because we do not know how far they run.
C We can not compare the speed if we don't know the amount of time and the distance they run.

T The distance they run were like this.

	Distance (m)	Time (Second)
A	40	6
B	30	6
C	30	5

C C's time is short but the distance C run is also short so we can't tell if C is the fastest.
C But we can tell that C is faster than B.
T Is that true? Why can ○○ tell about it by just looking at it?
C Because B and C run the same distance but B is taking longer time to run.
C I think so, too.
T So we can say that C is faster than B.
C Yes, I can tell more. A is faster than B.
C A and B run the same time but A run longer distance, so we can say A is faster.
T Well, we have not compare A and B, who is faster?
C …

We cannot compare speed using only with the time.

○Open the table and show the information of distance.

Both the distance and the time are different for A and B. What can we do?

○By comparing 2 persons at a time, help all students to understand the main question of the lesson.

We might be able to compare them if I make one of them the same.

	T Why can't you tell about it? C Because both the distance and the time are different. C Well, we can compare them if we make them the same. C We can compare them if we make the same. C If we make one of them the same we can compare. T If you make the same…? Can you compare them? C Yes, we can! Yes, we can!	◎Students are clear about the main question and trying to solve the problem enthusiastically. 【 Interest, motivation, disposition】
Pursue	**2. Comparing Speed of A and B by making one of them the same.** ⑤ Finding common multiple of distance to compare The smallest common multiple of 30 and 40 →１２０ A １２０÷４０＝３　　６×３＝１８ C １２０÷３０＝４　　５×４＝２０ A is faster. ⑥ Finding common multiple of time to compare The smallest common multiple of 6 and 5→３０ A ３０÷６＝５　　４０×５＝２００ C ３０÷５＝６　　３０×６＝１８０ A is faster. ③ Finding distance run per 1 second to compare A ４０÷６＝６．６６６６６・・・ C ３０÷５＝６ A is faster. ④ Finding time per 1m to compare A ６÷４０＝０．１５ C ５÷３０＝０．１６６６６６・・・ A is faster. ⑤ I don't know	◎ Students are able to come up with ideas to compare speeds by making one of time of distance the same and write their thinking in their notebooks. 【Mathematical Reasoning】 ○Ask students who are having difficulty to come to in front of the blackboard for small group discussion. By providing hints in question sentence format, help them to notice the method ③. 【Examples of hints】 ・C can run 30m for 5 seconds. If he runs for 1 second, how many m can he run? ・How about the case of A? ・I wander what we can do to find distance a person can run in 1 second.

Deepen & Rise	**3. Students present their solutions and understand them** Show the math sentences of the method ①. T Look at ○○'s math sentences and think about what he/she was thinking. C I think ○○ used the least common multiple to make those distance the same. C If both of them run for 120m, A's time would be 18 seconds and C's time would be 20 seconds. So A is faster. C I used the common multiple but my method is a little different. T What is that mean? Who used the method with the common multiple but different from what presented. C Yes, I also used the method with the least common multiple. But I made the time the same. T Can you see the method of ○○s' on the blackboard? C Yes, that is the method ②. C If both A and B run for 30 seconds, A would run 200m and C would run 180m. A is faster because he/she would run farther. Show the math sentences of the method ③. T Someone came up with these math sentences. 40÷6＝6．666・・・ 30÷5＝6 Do you understand what this person was thinking? Talk with your partner and describe what you think. T If you your explanation was similar to your partner's, please rise your hands. T If you think your partners' explanations were easy to understand, please rise your hands. C Yes. T Okay, _____. Please explain the meaning of these math sentences. C Yes, I think this person was thinking to find how far each person run in 1 second. C For 1 second, A run 6.66666… and C run 6m. So A is faster. T Why can you say A is faster? Does everybody understand? C Because the time is the same so we can compare with the distance. T What do you mean? C Because you can say someone is faster if the person could run farther in 1 second, so we can say A is faster. T I see.	☆ Show only the math sentences and ask students to think about what the person was thinking. Facilitate an active discussion to promote students' participation. ☆ Ask students to talk with their partners in order to increase each individual student's opportunity to explain their thinking. ○Ask students to write their solution ideas on the portable whiteboards. ◎Students are thinking about the solution methods by recalling how they have compared the crowdedness of rooms in grade 5. 【Mathematical reasoning】 ☆ Ask students to talk with their partners in order to increase each individual student's opportunity to explain their thinking. ☆By asking *hatumon* to students' responses, help each individual student to understand the method clearly.

Deepen & Rise	C I used the ides of per unit quantity. But my math sentences are different. T What are your math sentences. C ④６÷４０＝０．１５ ５÷３０＝０．１６６・・・・・ C Wow, this one also uses per unit quantity. T Do you mean they are the same method? C Well, they are reversed. T They are reversed… Do you understand what is the meaning of reversed, everyone? C Yes, I understand. It is the reverse of the method ③. T What do you mean by the reverse od the method ③? Talk with your partner and describe what you think. If your partner does not understand it please give her/him a hint. C The method ③ is finding the distance run per 1 second to compare but the method ④ is finding the time took to run for 1m to compare. C Both methods use the idea of per unit quantity. But one uses 1 second as the unit quantity and the other one uses 1m as the unit quantity. So that is different. C When we think about the comparison with “per 1 second,” we can say that the greater value means faster just like the method ③. On the other hand, when we think about the comparison with “per 1m,” we can say that the smaller value means faster. T I see, we came up with 4 different methods. What is the similarity among these methods? C All methods use the idea of making either time or distance the same to compare. T The important idea is making either time or distance the same. So which methods used the idea of making time the same? C The methods ② and ③. T How about the methods used the idea of making distance the same? C The methods ① and ④.	☆ Listen to students’ mumbles carefully and proceed the discussion according to student thinking and their pace of thought process. ☆By asking *hatumon* to students’ responses, help each individual student to understand the method clearly.

Deepen & Rise

T Okay, let's say 3 more children run. So we want to decide the order of places of these 6 children. What method do you want to use to determine? Please place your magnet name cards to one of the methods you wan to use on the blackboard.

	Distance (m)	Time (second)	Place
A	４０	6	
B	３０	6	
C	３０	5	
D	３５	5.5	
E	４５	6.5	
F	５０	8	

T Please describe why you chose a particular method.

C I thought it is complicated to find the least common multiples for this.

C Distance÷Time can be used every time and easier to understand (method ③).

C Time÷Distance can be also used every time and easier to understand.

C But it is little difficult to say a person is faster when the value is smaller so even though the method ③ and ④ are similar, I chose the method ③.

○By asking students to place their magnet name cards, each person need to hold his/her own opinion.

○It is not necessary for students to actuary do the calculation to find the order of places but ask students to just think about what method might be the best one to use. Through this discussion, help students to think about the idea of how to express speed such as, distance per second, distance per minute, and distant per hour.

Deepen & Rise

T For example, have you seen the writing showing the speed of the bullet trains? Which one is faster?

A

Super Komachi 300km per hour

B

Hayabusa 320km per hour

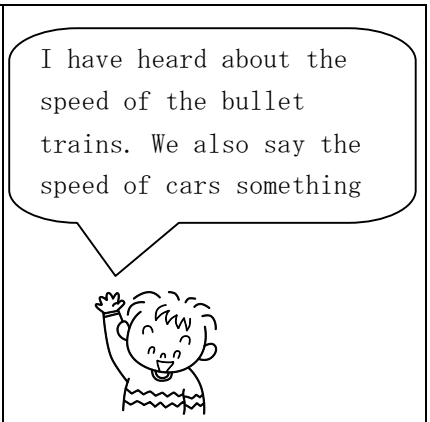
C Hayabusa is faster.

C I think the larger value (number) means faster.

C I wonder if “per hour” means how far the train can run in 1 hour.

T Both methods ③ and ④ can be used to compare the speed but when we show a larger value (number) for indicating faster speed, it is easier to imagine so in general, we show the speed of something using distance per time (e.g.,1 second, 1 hour).

T In the next lesson, let’s compare the speed of these 6 children and other objects that use expression of speed such as distance per hour.



Summarize & Expand	**4. Summarize the learning** T Today, we have thought about ways to compare speed. When both distance and time are different, we learned that we can make one of them the same to compare. We came up with these four methods to compare. ① and ② use the idea of the least common multiple to make one of them the same. ③ and ④ use the idea of per unit quantity such as per 1 second and per 1m to compare. All the methods are similar because they are making something the same such as "distance a person run" and "time a person run." T However, when you have many things to compare all at once, using ideas of "per unit quantity" is the best. In addition, in general, it is easier for people to have image of a larger value means faster. So the method ③ is usually used for expressing speed. However, both ideas using the least common multiple and per unit quantity can be used to solve this problem. So we can say that both methods are correct. T At last, let's write down the points for comparing speed and the easy ways to compare based on what we learned from today's lesson. C "I See!" --- We can compare speed if we make time or distance the same. C "Discovery" --- I think the idea of using the least common multiple is a good idea but when we compere many things all at once the method is not easy to do. So I would like to use the idea of distance per 1 second. C "Question" --- We talked about "distance per hour" but for the case of the sprint race, I wonder id we can call it as "distance per second." Is there "distant per minute" also? C "Challenge" I want to compare speed of cars, trains, airplanes. I also want to compare speed of many other things.	It is easier to compare speed if we use Distance ÷Time to find distance  It is better to use the method that is easy to understand and be able to use  ◎Students are thinking about new questions and try to solve problems willingly. 【Interest, motivation, disposition】

9. Blackboard Plan

Problem

Let's think about the order of speed of these 3 children!

① Make distance the same using common multiple to compare

The least common multiple of 30 and 40 → 120

A -- 120 ÷ 40 = 3　6 x 3 = 18

(Shorter time means fast)

C -- 120 ÷ 30 = 4　5 x 4 = 20

	Distance (m)	Time (second)
A	40	6
B	30	6
C	30	5

Easy to

A is faster.

② Make time the same using common multiple to compare

The least common multiple of 6 and 5 → 30

A -- 30 ÷ 6 = 5　40 x 5 = 200

(Longer distance means fast)

C -- 30 ÷ 5 = 6　30 x 6 = 180

	Distance (m)	Time (second)
A	40	6
B	30	6
C	30	5

Easy to

A is faster.

③ Find distance traveled in 1 second to compare

A -- 40 ÷ 6 = 6.666…

(Longer distance means fast)

C -- 30 ÷ 5 = 6

Distance ÷ Time = Distance per 1 second

	Dista e (	ime (s ond)
A	40	6
B	30	6
C	30	5

Be able to use the numbers as they are for the

A is faster.

④ Find time took in 1m to compare

A - 6 ÷ 40 = 0.15

(Shorter time means fast)

C -- 5 ÷ 30 = 0.166666…

Time ÷ Distance = Time per 1m

	Dist e (	me (s nd)
A	40	6
B	30	6
C	30	5

Be able to use the numbers as they are for the

A is faster.

１０. Viewpoints for looking at the lesson

① When the students are solving the problem on their own and discussing solution methods, did the teaching methods, such as providing hints with question sentence format, asking students to explain each other with partner, and asking *hatumon* to students' responses help the students to deepen their thinking as well as engaging them learning autonomously.

② Did the student discussion help to achieve the goal of discussion, "search more effective ideas."

③ Other points that observes noticed during the lesson. (e.g., How main goal/task was established during the lesson?).