



Grade 3 Mathematics Lesson Plan < Revised >

Monday, July 1, 2013, Period 5
Matsuzawa Elementary School
Grade 3, Classroom 1
Teacher: Sachiko Kawabata

1. Unit: Division

2. Goals of the unit

- Students will understand the meaning of division and be able to use it.
- Students will be able to represent division situations in expressions/equations and interpret division expressions/equations.

3. Evaluation standards

Interest, Eagerness, and Attitude	Mathematical Way of Thinking	Mathematical Skills	Knowledge and understanding
<ul style="list-style-type: none"> • Students are thinking about the meaning and ways of division calculations by relating them to multiplication and subtraction. 	<ul style="list-style-type: none"> • Students are thinking about ways to calculate division problems with 1-digit divisors and 2-digit quotients. 	<ul style="list-style-type: none"> • Students can fluently calculate division with 1-digit divisors and 1-digit quotients. 	<ul style="list-style-type: none"> • Students understand the meaning of division for both partitive and quotative cases. • Students understand how division relates to multiplication and subtraction.
<ul style="list-style-type: none"> • Students are interested in interpreting and representing situations using division expressions/equations, and they are trying to represent various situations in expressions/equations. 	<ul style="list-style-type: none"> • Students can think about division situations using tools such as concrete objects and diagrams, and they can represent them using division expressions/equations. • Students can relate division expressions/equations to concrete situations. 	<ul style="list-style-type: none"> • Students can represent division situations using expressions/equations, and they can interpret division expressions/equations. 	<ul style="list-style-type: none"> • Students understand the relationships among quantities in situations where division is appropriate by interpreting or representing with expressions/equations.

4. About mathematics in the unit

(1) Partitive and quotative division

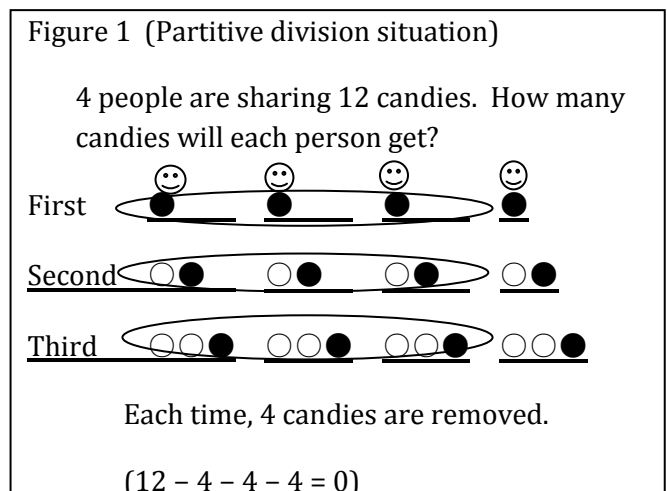
Most of situations in which division is used can be divided into the following two cases.

In one case, division is used to determine how many of one quantity equals the second quantity. This type of division is called quotative division. In the other, division is used to determine how many in one group when a quantity is made into so many equal groups. This type of division is called partitive division.

The idea of partitive division, "sharing equally among \bigcirc people," is common in students' everyday situations, and many students understand how to distribute items equally through their experiences. However, it is difficult to represent those situations in diagrams, and diagrams are often not easy to interpret. Therefore, it is necessary to provide careful explanations and to have students manipulate concrete objects so that they understand how the amount was equally grouped.

The idea of quotative division, "if each person receives \bigcirc items, how many people can share the given amount," is not as commonly encountered in everyday life. Therefore, it is anticipated that not many students have everyday experience of this type. However, the idea of sharing by giving each person \bigcirc items is easier to explain with diagrams. Moreover, when situations are represented with concrete objects, the process of "removing \bigcirc items" repeatedly may be easier to grasp visually.

Comparing partitive and quotative division, it may be better to teach quotative division first because the concrete representation/manipulation is easier to understand. Moreover, the action involved matches the idea of "removing."¹ Moreover, if we start with quotative division, we can explain partitive division situations using the repeated subtraction idea by focusing on "the group of items when one item is given to \bigcirc people." (See Figure 1)



¹ The kanji character used for the formal term, division, is the same character as the kanji character used for "to remove."

(2) Relationships between multiplication and division

Division can be considered as the inverse operation of multiplication. It is possible for students to clearly understand the meaning of partitive and quotative division by representing division situations using multiplication equations and noticing whether we are determining the multiplicand or the multiplier. (See Figures 2 and 3)

In this unit, it is important for students to understand the meaning of division equations by relating them to corresponding multiplication equations by using words and diagrams. For example, students should understand that "the divisor is the multiplicand or the multiplier of the corresponding multiplication equation."

The distinction between division to find the group size and division to find the number of groups is also important in students' future learning. For example, in Grade 5 discussion of "division of decimal numbers" and Grade 6 discussion of "division of fractions," this distinction is critical as students determine the appropriate expression based on the word problems or their representations on number lines.

Figure 2

partitive...division to find group size

$$12 \div 4 = 3$$

$$(\text{Total}) \div (\# \text{ of groups}) = (\text{group size})$$



$$\square \times 4 = 12$$

$$(\text{group size}) \times (\# \text{ of groups}) = (\text{Total})$$

quotative...division to find number of groups

$$12 \div 4 = 3$$

$$(\text{Total}) \div (\text{group size}) = (\# \text{ of groups})$$



$$4 \times \square = 12$$

$$(\text{group size}) \times (\# \text{ of groups}) = (\text{Total})$$

Figure 3

Understanding of "division as the inverse of multiplication" → In $A \times B = C$, if either A or B is unknown, it can be determined by $C \div B$ or $C \div A$.

$$\text{group size} \times \# \text{ of groups} = \text{Total}$$

$$A \times B = C$$

$$\square = C \div B \quad (\text{Partitive})$$

$$\square = C \div A \quad (\text{Quotative})$$

(3) Teaching of this unit

As stated in (1), we felt that introducing division using the quotative situations may be easier for students. However, if we treat partitive division as the same operation as quotative division, the distinction between "division to find group size" and "division to find the number of groups" discussed in (2) may become unclear.

Therefore, in this research lesson, we decided to focus on making the distinction between partitive and quotative division based on the relationship between division and multiplication. We will discuss partitive division first, then quotative division.

5. Students' current state

There are many students who eagerly tackle mathematics lessons and try to reason independently in problem solving. On the other hand, there is a wide range in the levels of understanding and mastery of basic ideas. There are some students who require individual support during mathematics lessons.

In a recent class survey, students were asked "When do you feel the enjoyment in a math lesson?" Large percents of the students agreed with the following statements: "When I score well on a test" (94%), "When I understand something I didn't understand before" (69%), and "When teacher or friends understood my idea" (69%). On the other hand, much fewer percents of the students agreed with the following statements: "When I am writing my ideas in notebook," (29 %), "When I am sharing my idea," (31 %), "When I'm doing calculation," (34 %), and "When I am working on application problems" (34 %). From this survey, we can see that the students in this class enjoy and feel satisfaction with mathematics when they understand something or do well on a test. Moreover, although some students do not feel good about writing or sharing their own ideas, they feel good when their ideas are understood by the teacher or their classmates.

The results of the readiness test for the unit, there are several students who have not completely mastered the basic multiplication facts. In particular, 7×6 , 6×7 , 7×4 , 4×7 , 6×8 , and 8×6 were missed by several students. In addition, there are some students who do not completely understand the meaning of multiplication. Those students wrote multiplication expressions/ equations with the multiplicand the multiplier reversed. They also drew pictures of 3×4 for the expression 4×3 .²

Therefore, in this unit, the meaning of "divide" will be carefully taught by utilizing concrete materials and drawings. In addition, as we have been emphasizing throughout this year, we will incorporate activities in which students will represent their own ideas using words, expressions/ equations, or diagrams and explaining their ideas to their classmates. Through those activities, we want students to realize the enjoyment in reasoning. We will also pay close attention to the understanding problem stage of the lesson so that students can tackle

² In Japan, the multiplicand is written first, which is the reverse of the order indicated in the Common Core State Standards. Therefore, 4×3 in Japan means 3 groups of 4.

problem solving with clear strategies. We will also make sure to provide appropriate individual support during the independent problem solving time and secure the sufficient amount of time for whole class discussion. Through these efforts we want all students to experience success.

6. Instructional strategies

Through writing division problems and comparing those problems, students will understand that there are two types of division: division to find group size (partitive division), and division to find the number of groups (quotative division).

The relationship between multiplication and division will be carefully taught.

○ Ideas for unit structure

Today's lesson was set up after the meaning and calculation of partitive division (2 hours) and the meaning and calculation of quotative division (2 hours). We use the same numbers in both partitive and quotative division problems discussed in lessons 1 through 4 so that comparison of diagrams and equations will be easier.

○ Manipulation of concrete materials and representations using diagrams

While teaching the meaning of division, we will incorporate manipulation of concrete objects so that students can develop the solid image of "equal sharing" and "making groups of equal size." In addition, we hope to deepen students' understanding by representing manipulation of concrete objects in diagrams or expressions/equations.

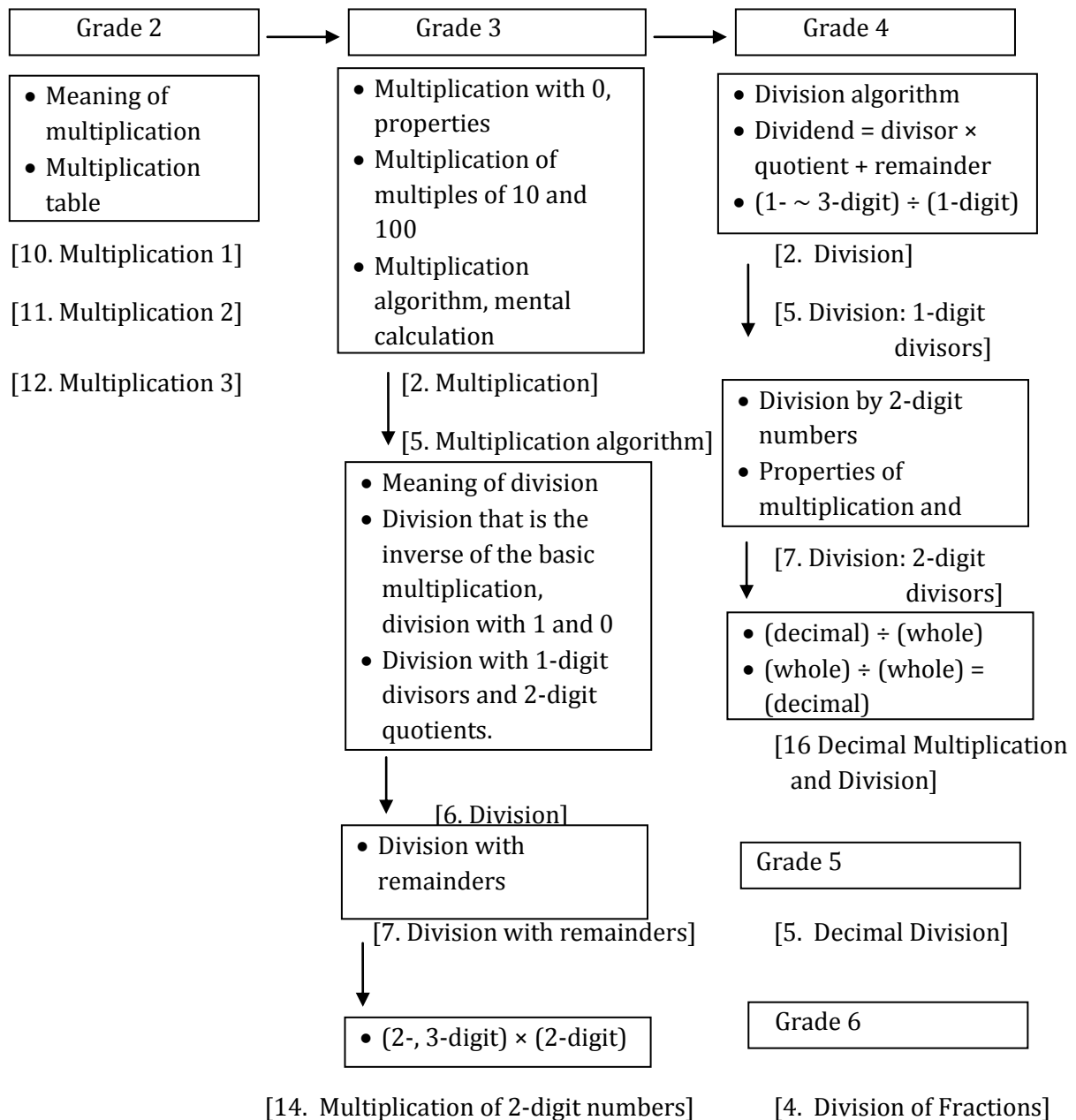
○ Making students conscious of the differences of two types of division through problem writing

In today's lesson, we will show a picture of 8 strawberries, and students will be asked to create problems for $8 \div 2$. As we discuss students' problems, we will compare and contrast "division to find group size" and "division to find the number of groups" so that students can think about the differences of the meaning and unknown quantities in these types of division.

In addition, we want to help students realize that the quotients can be determined by using multiplication, and the difference is what quantity is to be found.

Finally, we will ask students to write problems for $10 \div 2$ so that we can assess whether or not students understand the distinction between partitive and quotative division.

7. Scope and sequence of relevant topics



In Grade 2, students first learned about the meaning of multiplication and constructed the basic multiplication table. Then, they tried to master the basic multiplication. By the end of Grade 2, about 90% of students were able to recite the basic multiplication table fluently. As we studied the multiplication algorithm in Grade 3, we checked students' mastery level of the basic facts, and some students have forgotten or confused some facts. In addition, we learned that some students were having difficulty with division in Grade 4 because their mastery of the basic multiplication facts was unsatisfactory.

In this unit, we want students to be able to write appropriate expressions/equations by relating multiplication and division and making clear the type of the unknown quantity. What students study in this unit will relate to other topics studied in later grades.

In Grade 4 unit of "Division: 1-digit divisors," there is a task titled, "What calculation do we need?". In that task, students will examine word problems and represent them on numbers lines to identify how quantities are related and whether the appropriate calculations are multiplication or division. Students must determine which of the quantities in the equation, $\boxed{\text{group size}} \times \boxed{\text{\# of groups}} = \boxed{\text{Total}}$, needs to be found. In Grade 5 unit of "Decimal Division" and Grade 6 unit of "Division of Fractions," the reasoning such as "because division is inverse of multiplication, we can do \div decimal or \div fraction," or "since we are finding group size, the calculation is partitive division" play an important role.

8. Unit and evaluation plan

Sub-Unit	Lesson #	Content	Evaluation Standards			
			Interest, Eagerness, and Attitude	Mathematical Way of Thinking	Mathematical Skills	Knowledge and understanding
Division	1	<ul style="list-style-type: none"> • Discuss the situation in which "12 pieces of chocolate are shared among 4 people." • Understand the meaning of partitive division as the process of determining the number of pieces for one person when "12 pieces of chocolate are equally shared among 4 people" through manipulation of concrete objects. • Understand how to represent situations using division expressions/equations. 	<ul style="list-style-type: none"> • Students are trying to figure out the meaning of partitive division by representing the problem situations using concrete objects or diagrams. 	<ul style="list-style-type: none"> • Students are reasoning about division situations by representing the problem situations using concrete objects or diagrams. 		

	2	<ul style="list-style-type: none"> • Think about ways to find the quotient for $24 \div 6$ in a partitive situation. • Think about the relationship between the quotient and the multiplicand ($\square \times 6 = 24$). • Find the quotients using the basic multiplication facts. 		<ul style="list-style-type: none"> • Students can relate partitive division situations with multiplication. Through diagrams and manipulation of concrete materials, students can think about ways to find the quotients using the basic multiplication facts. 		<ul style="list-style-type: none"> • Students understand how division relate to multiplication and subtraction.
	3	<ul style="list-style-type: none"> • Understand the meaning of quotative division as the process of finding the number of people who will get cookies when "each person receives 4 cookies and there are 12 cookies" through manipulation of concrete objects or representing problem situations in diagrams. • Understand how to represent situations using division expressions/equations. 		<ul style="list-style-type: none"> • Students are thinking about division situations using diagrams and concrete objects. 		<ul style="list-style-type: none"> • Students understand that quotative situations can also be represented by division expressions.

	4	<ul style="list-style-type: none"> • Think about ways to find the quotient for $24 \div 6$ in a quotative situation. • Think about the relationship between the quotient and the multiplicand ($\square \times 6 = 24$). • Find the quotients using the basic multiplication facts. 		<ul style="list-style-type: none"> • Relating division situations to multiplication, students can explain how quotative divisions can be calculated by using the basic multiplication facts. 		<ul style="list-style-type: none"> • Students understand that quotative situations can also be represented by division expressions.
	5	<ul style="list-style-type: none"> • While looking at pictures of 8 strawberries, make problems for which $8 \div 2$ is the appropriate calculation. • Think about and share the differences between two types of problems (partitive and quotative). • Summarize the two types of division. • Make two types of division problems for which $10 \div 2$ is appropriate calculation. 		<ul style="list-style-type: none"> • Students are thinking about the differences of the meaning of the two types of division problems. 		<ul style="list-style-type: none"> • Students understand the differences of the two types of division, and they can write both types of problems from the same division expression.

	6	<ul style="list-style-type: none"> • Write division word problems and solve each other's problems. 			<ul style="list-style-type: none"> • Students can represent division situations using expressions, and they can interpret division expressions. 	<ul style="list-style-type: none"> • Students understand the meaning of division in both partitive and quotative cases.
	7	<ul style="list-style-type: none"> • Practice division calculations. • Create division books. 	<ul style="list-style-type: none"> • Students are trying to identify situations in their daily life where division may be used. They are enjoying the activity of making division books. 		<ul style="list-style-type: none"> • Students calculate the quotients when both the divisors and the quotients are 1-digit numbers. 	
Division with 0 & 1	8	<ul style="list-style-type: none"> • Think about the expressions and answers for sharing 12, 4, and 0 cookies among 4 people equally. • Summarize division problems with the quotients of 1 and 0. 		<ul style="list-style-type: none"> • Students can make sense of division with the quotients of 1 or 0 based on their previous learning. They can think about ways to calculate such division. 		

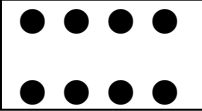
Using patterns of calculation	9	<ul style="list-style-type: none"> • Think about ways to calculate $36 \div 3$ using properties and rules of calculations. • Look for patterns among expressions with the constant multiplier or divisor. 		<ul style="list-style-type: none"> • Students are thinking about ways to find the quotient using diagrams or the relationship between multiplication and division. 		
	10	<ul style="list-style-type: none"> • Think about ways to calculate $80 \div 4$. • Learn about the division algorithm. 		<ul style="list-style-type: none"> • Students are thinking about ways to find the answers by drawing diagrams or using what they have previously learned about multiplication and division. 		
	11	<ul style="list-style-type: none"> • Apply division in everyday situations and think about problems. 		<ul style="list-style-type: none"> • Students can connect division equations to concrete situations. 	<ul style="list-style-type: none"> • Students can write and calculate division expressions based on diagrams. 	

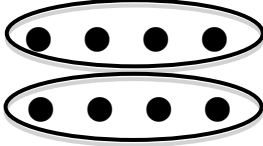
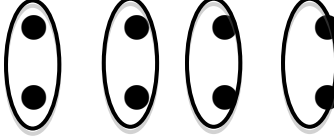
Review	12	<ul style="list-style-type: none"> • Deepen students' understanding. 			<ul style="list-style-type: none"> • Students can fluently calculate division with 1-digit divisors and 1-digit quotients. 	
Exercises	13	<ul style="list-style-type: none"> • Re-affirm what students learned in the unit. 				<ul style="list-style-type: none"> • Students understand the relationships of quantities in situations in which division is used.

9 Goal of the lesson

- By comparing and contrasting partitive and quotative division problems, students will understand that there are two types of division: division to find the group size and division to find the number of groups.

10 Flow of the lesson (Lesson # 5/13)

	<ul style="list-style-type: none"> ■ Learning Activity (T) <i>Hatsumon</i> (question) (C) Anticipated responses 	<ul style="list-style-type: none"> ⊙ Points of consideration □ Evaluation ❖ Teaching strategy
Posing the problem/Plan strategies	<ul style="list-style-type: none"> ■ Understanding the problem and planning solution strategies T1 Today, we will be making division word problems. T2 There are 8 strawberries. We want to share these strawberries so that it will match the expression, $8 \div 2$. How can we share these strawberries?  <ul style="list-style-type: none"> C1 2 people share strawberries. C2 Put 8 strawberries on 2 plates. C3 Make groups of 2 with 8 strawberries. T3—What kinds of words should be used in your problems? C4—Strawberries. C5—2 and 8. C6—By 2. C7—Share. T3 OK, let's write word problems that can be solved by $8 \div 2$. Draw a picture or diagram for the problem situation. Also, write an equation and the answer, too. 	<ul style="list-style-type: none"> ⊙ By showing a picture of strawberries, help students understand the problem situation more easily. ⊙ By discussing ways to share and some key words to be included in problems, help students develop solution strategies. ⊙ To make it easier to compare and contrast problems, clearly mark spaces for the problem, diagram, equation, and answer on the worksheet.

Individual Problem Solving	<p>A: Division to find the group size (partitive division)</p> <p>2 people are sharing 8 strawberries. How many strawberries does each person get?</p>  <p>Equation $8 \div 2 = 4$</p> <p>Answer 4 strawberries</p>	<p>B: Division to find the number of groups (quotative division)</p> <p>We are going to give 2 strawberries to each person. If there are 8 strawberries, how many people will get strawberries?</p>  <p>Equation $8 \div 2 = 4$</p> <p>Answer 4 people</p>
	<p>☉ Make sure each students will write one problem.</p> <ul style="list-style-type: none"> ❖ Set up a "Hint Time" for students who are having trouble. Remind them the sharing strategies discussed earlier in the lesson and different division word problems we have studied previously so that they might be able to write their own word problems. ❖ For those students who completed the task, encourage them to write a second word problem. Also, ask them how they found the answer and write their steps down. 	
<p>■ Compare and contrast 2 problems</p> <p>T4 Let's look at the problems Students A and B. Which of these two problems is the next problem look like?</p> <p>C4 It looks like Problem A.</p> <p>T5 Why did you think so?</p> <p>C5 Because they both "share equally between 2 people."</p> <p>C6 If you look at the diagrams, they both have made groups of 2.</p>	<p>☉ Select several partitive and quotative problems and have students compare and contrast.</p>	

<p>T6 Now, let's look at the problems you wrote. Is your problem like Problem A or Problem B? Please put your name plate under the problem.</p> <p>T7 What is similar between the two problems, A and B? What is different? Let's share what you noticed.</p> <p>C7 They both have $8 \div 2$.</p> <p>C8 The answers are both 4.</p> <p>C9 Even though they both calculate $8 \div 2$, the "2" in A is "2 people," and in B, it is "2 strawberries." So, it means something different.</p> <p>C10 If you look at the diagram, the way ● are circled are different. In A, there are groups of 4, and in B, there are groups of 2.</p> <p>C11 The answers are both 4, but A's answer is 4 "strawberries" and B's answer is 4 "people." So, the units are different.</p> <p>T8 So, why are the answers, "4 strawberries" and "4 people," different even though the problems have the same equation?</p> <p>C12 Problems A and B are asking for different things. In A, "how many does each person get," and in B, "how many people will get strawberries."</p> <p>T9 OK, so the quotients are both 4, but the meanings are different. How did you find the answer, 4?</p> <p>C13 I used multiplication.</p> <p>C14 2×4.</p> <p>T11 Do A and B both use 2×4?</p> <p>C15 For A, it's 4×2. In multiplication, "(group size) \times (number of groups) = (total)." In A, the group size is 4 (4 strawberries each) and the number of group is 2 (people).</p>	<p>☉ By having students identify their own problems with A or B, help students attend to the problem situations.</p> <p>□ Mathematical Way of Thinking Students can compare 2 problems and think about the difference in the meaning of the types of division.</p> <p>❖ Have students think about the difference between "sharing between 2 people" and "making groups of 2" from the diagrams and word problems.</p> <p>❖ Encourage students to think about ways to explain the differences of the problem so that others can understand it easily.</p> <p>☉ If the relationship of multiplication and division or the meaning of multiplication, "(group size) \times (number of groups) = (total)" are not raised by students, ask questions to help students think about these ideas.</p>
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	<p>C16 For B, it is 2×4, because the group size is 2 (2 strawberries each) and there are 4 (people).</p> <p>T12 OK, let's look at Problems A and B once again.</p> <div style="display: flex; justify-content: space-around;"> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p>A (partitive division to find how many in one)</p> <p>$[1] \times 2 = 2$</p> <p>$[2] \times 2 = 4$</p> <p>$[3] \times 2 = 6$</p> <p>$[4] \times 2 = 8$</p> <p>$8 \div 2 = [\quad]$</p> <p>$[\quad] \times 2 = 8$</p> <p style="text-align: center;">group size # of groups Total</p> </div> <div style="border: 1px solid black; padding: 5px; width: 45%;"> <p>B (quotative division to find how many groups)</p> <p>$2 \times [1] = 2$</p> <p>$2 \times [2] = 4$</p> <p>$2 \times [3] = 6$</p> <p>$2 \times [4] = 8$</p> <p>$8 \div 2 = [\quad]$</p> <p>$2 \times [\quad] = 8$</p> <p style="text-align: center;">group size # of groups Total</p> </div> </div>	
	<p>From the same expression $8 \div 2$, we could write two different types of division word problems. In Problem A, what is the division calculation finding?</p> <p>C17 Problem A is finding "how many in one."</p> <p>T12 How about B?</p> <p>C18 Problem B is finding "how many groups."</p> <p>T13 OK, please write a journal on what you learned today.</p> <p>In A, you were asked to find "how many does each get," and the answer was "4 strawberries." Therefore, we were trying to find the "group size" in multiplication using division. In B, we were asked to find "how many people will get strawberries," and the answer was "4 people." So, we were trying to find the "number of groups" in multiplication using division.</p>	<p>© For Problems A and B, use the labels students invent to make it easier to distinguish the two types.</p>

	<p>If $\bigcirc \times \Delta = \square$, we can use division to find \bigcirc (group size), OR Δ (number of groups).</p> <p>C19 So, there are 2 different types of division problems, aren't there?</p> <p>■ Solve the application problem T13 OK, now let's write word problems for which we use $10 \div 2$ to solve. This time, please write two types of problems. You don't have to use strawberries for things to be shared. Use your own ideas. For each of the two types of the problems, write your problem, the equation, and the answer. If you finish, also draw diagrams.</p>	<p>◎ By using the application problem, evaluate students' understanding. (Compare what students wrote initially and what they write here to capture changes in students' thinking.)</p> <p>□ Knowledge and understanding Students understand the difference of the meaning of the two types of division problems.</p>
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11 Evaluation

- Do students understand there are two types of division based on the meaning of multiplication, "(group size) \times (number of groups) = (total)": division to find the "group size," and division to find the "number of groups."

12 Board writing plan

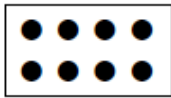
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There are 8 strawberries

Way to share that matches

$8 \div 2$

- 2 people share 8 strawberries
- Make groups of 2 strawberries with 8 strawberries.



Problem A

Problem B

...

$8 \div 2 = []$

$[] \times 2 = 8$

...

...

$8 \div 2 = []$

$2 \times [] = 8$

...

Summary

Division may be used

- * to find "how many in 1," OR
- * to find how many groups.

Goal

Let's write word problems that can be solved by $8 \div 2$. Write an equation and solve the problems.

Key words

~~2~~ ~~8~~ ~~Strawberries~~

~~Share~~

What is different you noticed when you compared

Same	Different
=	• 4 strawberries and 4 people
equation	• Shared between 2 people, or give 2 strawberries to each
answer	• (diagram) groups of 4 or 4 groups
we can use	• (what we have to find)
How many for 1 person?	OR
How many people will get strawberries?	

Multiplication

~~A $4 \times 2 = 8$~~

~~Ans. 4 strawberries~~

⇒

~~$8 \div 2 = 4$~~

⇐

~~B $2 \times 4 = 8$~~

~~Ans. 4 people~~

~~group-size × # of groups = Total~~

If $\bigcirc \times \Delta = \square$, we can use division to find

~~\bigcirc (group size), OR~~

~~Δ (number of groups).~~

