June 25

Grade 7 Mathematics Lesson Plan (Condensed Version)

Tuesday, June 25, 2013, 11:40 ~
Tokyo Gakugei University Affiliated Koganei Junior High School
Grade 7, Classroom A (20 boys & 20 girls) Teacher: Koichi Kabasawa

1. Unit: Letters in Algebraic Expressions

2. Title of the lesson: How many stones (as an introduction of algebraic expressions)

3. About mathematics in the lesson

Today's problem involves determining the total number of stones used in the game of *Go* arranged in patterns. The strength of this problem is that it naturally generates diverse ways of observing and reasoning, which will make it possible to have a rich activity of interpreting algebraic expressions. Sometimes, the same algebraic expression represents different reasoning processes. Therefore, it is particularly suited to naturally discuss the idea of generalization.

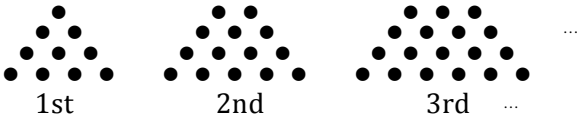
In elementary schools, students have learned about using letters such as a and x in expressions in place of symbols, like □ or Δ, or words that represent numbers and quantities. They have also studied the idea of substituting specific numbers in letters to evaluate expressions. Since the beginning of the lower secondary school, we have been using letters in algebraic expressions frequently. Therefore, students should not have much problem using algebraic expressions with letters to represent their reasoning in today's lesson. However, as it has been reported in the existing literature, understanding variables and the meaning of algebraic expressions with letters may be a significant challenge to students. Therefore, I will approach this topic carefully and thoroughly.

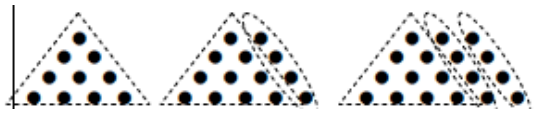
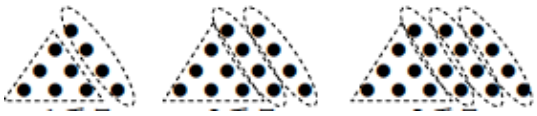
Keeping these things in mind, in today's lesson, we will engage in a mathematical communication activity that requires students to represent own reasoning in algebraic expressions and interpret others' algebraic expressions. Through this activity, it is expected that students will become aware of the idea of variables and understand that an algebraic expression represents both the process and the result.

4. Goals of the lesson

- Students will be able to generalize own ideas and represent them using algebraic expressions, and they understand the meaning of algebraic expressions.
- Students will be able to interpret algebraic expressions in the context of a particular phenomenon.
- Students will solve the problem using diverse ways of observing and reasoning, and they realize the usefulness of using letters.

5. Flow of the lesson

	Learning Activity	Anticipated Responses	Evaluation (O) and Considerations (*)
Opening	• Display the problem "*Go* stones are arranged as shown in the picture. Find the total number of stones in the 5th arrangement."	 1st 2nd 3rd … • Draw the arrangements completely. • Can determine the total number without drawing the complete pictures.	O Students are trying to find ways to determine the total number. * In order not to influence students' reasoning, post the picture instead of drawing these patterns on the board. * For those students who are drawing complete pictures, ask "Do you have to draw them all?"

Development	• Individual problem solving "Write an expression to represent how you figured out the total number of stones."	• Possible student responses 1. $10 + 4 \times 4$ ← There are different reasoning with the same expression $10 + 4 \times (5 - 1)$, $10 + (5 - 1)\ 4$  1st 2nd 3rd … 2. $6 + 4 \times 5$, $6 + 5 \times 4$  1st 2nd 3rd …	○ Students can represent their reasoning processes using expressions. * If a student found a way, encourage him/her to think about other ways and represent them in expressions. * Evaluation points for students' expressions. (1) Is the student paying attention to the difference between the multiplier and the multiplicand? (2) Is the students thinking about variable-like use of numbers? (3) Is the student paying attention to the way the total is increasing (in relationship to methods 1 and 2)? * In particular, focus on (1) and (3). For example, ask students to explain which parts of their expressions represent the position number or the number of stones.

	• Comparing and discussing solutions ○ Have students share their expressions. ○ Have students explain other students' expressions.	3. $3 \times 7 + 5$  1st 2nd 3rd … 4. $8 \times 4 - 6$, $4 \times 8 - 6$  1st 2nd 3rd … 5. $\{5 + (5 + 3)\} \times 4 \div 2$ ➔ trapezoid or parallelogram  1st 2nd 3rd … 6. $\{5 + (5 + 3)\} \times 2$, $\{(5 + 1) + (5 + 2)\} \times 2$ ➔ rectangle  1st 2nd 3rd … 7. Without thinking about generalizing, simply group stones at random. ★ After we study calculations of algebraic expressions, we will know that all of these will be simplified to the same algebraic expression, "$6 + 4x$."	* Do not discuss the benefit of variable-like use of "5" in (5 - 1) here. O Students can explain others' ideas using tools like diagrams. O Students can make sense of others' ideas. * Help students attend to the benefit of keeping "5" (position number) in the expression, like (5 - 1). * Write students' methods on the blackboard so that they can become aware of varying quantities and introduce the use of letters as variables.

	"If we want to find the total number of stones in the 100th arrangement, which expressions would you use? Find the total number of stones in the 100th arrangement." "What characteristics do you notice about the expressions you thought about using?" • Write appropriate expressions with letters	(Example) $6 + 4 \times 100$ ✣ It is possible for some students to argue that simply adding the number of stones in the four rows if we are finding the total number for the 100th arrangement. • We just need to change "5." • The position number is included in the expression. 5^{th} $6 + 4 \times 5$ 100^{th} $6 + 4 \times 100$ position $6 + 4 \times$ (position) x^{th} $6 + 4 \times x$ • Anticipated student responses 1. $10 + 4 \times x$, $10 + x \times 4$, $10 + 4 \times (x - 1)$, $10 + (x - 1) \times 4$ 2. $6 + 4 \times x$, $6 + x \times 4$ 3. $3 \times (x + 2) + x$ 4. $(x + 3) \times 4 - 6$, $4 \times (x + 3) - 6$ 5. $\{x + (x + 3)\} \times 4 \div 2$ ➔ trapezoid or parallelogram. 6. $\{x + (x + 3)\} \times 2$, $\{(x + 1) + (x + 2)\} \times 2$ ➔rectangle	* Depending on student responses, ask "Which expression is easier to use to determine the total number of stones for any position number?" * Have students pay attention to variable-like use of numbers. * Write expressions with specific numbers and expressions with words so that students can become aware of varying numbers, then introduce letters as variables. * Have students find the total number of stones for specific position numbers. * For simple cases, have students substitute the numbers in various expressions to verify that they all result in the same number.
Summary	• Summarize the lesson "What are some of the strengths of using letters in expressions?"	• We can calculate the total number of stones if we substitute the position number in the letter. • We can interpret other people's reasoning from their expressions with letters.	* If there is enough time, ask students which algebraic expression is the best.