

Grade 4 Mathematics Lesson Plan

Teacher: Masaki Tsuruta

- 1 Name of the Unit Let's make quadrilaterals
- 2 About the Unit

In this unit, students will learn about the relationships of lines – perpendicularity and parallelism – and quadrilaterals such as parallelograms, trapezoids, and rhombuses.

The National Course of Study position the topics in this unit, perpendicularity/parallelism and quadrilaterals, as follows.

- (1) Through activities such as observing and composing geometrical figures, to help pupils pay attention to the elements that compose geometrical figures as well as their positional relationships, and deepen their understanding of geometrical figures.
- a. To understand the relationships such as parallelism and perpendicularity of straight lines.
 - b. To get to know parallelograms, rhombuses and trapezoids.

The aim of this unit is that through activities of observing and constructing geometric figures, students will understand the relationships such as perpendicularity and parallelism and quadrilaterals such as parallelograms, rhombuses, and trapezoids. In particular, students will understand the characteristics of geometric figures by using position relationships of their sides (perpendicularity and parallelism) or diagonals (their lengths and the way they intersect with each other).

In Grade 2, students have learned about the foundational ideas for perpendicularity and parallelism in the unit, Triangles and Quadrilaterals, by observing and constructing rectangles and squares. In addition, in the Grade 3 unit, Triangles, they learned about classes of triangles, isosceles triangles and equilateral triangles, by focusing on the lengths of sides in triangles. In those units, students have used “the number of vertices or sides,” “the length of sides,” and “the size of angles” as lenses to observe geometric figures.

In this unit, what is important is for students to use “perpendicularity” and “parallelism” as new lenses to re-examine familiar geometric figures and discover new properties of quadrilaterals. In particular, by focusing on “parallelism,” students will understand that if there is a pair of parallel sides in a quadrilateral, it will be a “trapezoid,” and if there are two pairs of parallel sides in a quadrilateral, it will be a “parallelogram.”

In the 2013 National Assessment, there was a question about how to draw a parallelogram. The question asks students which property of parallelograms was used to draw the parallelogram. In drawing a specific geometric figure, it is important that students grasp the characteristics of the geometric figure and use them to guide their drawing. When the students in this classroom were asked to draw triangles, not many were connecting the characteristics of triangles with their methods of drawing. From this perspective, it is important for these students to learn to draw geometric figures by making use of characteristics of the geometric figures. Through the study of this unit, it is my hope that students will not only understand the relationships of lines such as perpendicularity and parallelism and quadrilaterals such as parallelograms, rhombuses, and trapezoids, but also enrich their sensitivity toward geometric figures.

In teaching this unit, I will have students first use their intuitions to sort different positional relationships of lines instead of immediately using rulers or set squares. Then, by using protractors or set squares, students will more carefully examine the relationship of the given lines. It is my hope that students will be able to use perpendicularity and parallelism as they discuss geometric figures. Moreover, I want students to grasp characteristics of various quadrilaterals by using the relationships of their sides or other constituent parts. To do so, we will incorporate activities of sorting and organizing quadrilaterals. Then, through activities to draw particular quadrilaterals to match the way they sorted and organized quadrilaterals, I want them to understand their characteristics. Moreover, by using those characteristics to draw quadrilaterals, I want them to be able to connect steps of drawing and characteristics of quadrilaterals. In addition, students can discover additional characteristics by manipulating cut out quadrilaterals. Through these concrete activities, I would like them to develop intuitive understanding of congruence and symmetry so that they can be utilized in their future study.

In their study of geometric figures, students have only considered those segments that are visible, that is, sides of polygons. In contrast, diagonals is invisible until students imagine the segment that connects vertices that are opposite of each other. In this way, diagonals involve more abstract aspects and that may create difficulty in some of them. However, when students realize that diagonals can be used to identify properties of geometric figures, I hope that they will understand merits of diagonals.

By studying geometric figures, when students encounter various geometric figures in their daily lives, they may be able to sort them based on the properties learned in this unit or they may recognize the beauty of geometric figures themselves. I hope to enrich students sensitivity toward geometric figures as they realize that there are so many geometric figures around us.

3. Relationship to the school-based research

The research theme for the last academic year was "Elementary school career education that lays the foundation toward autonomy: developing lessons that will raise students ability to reason logically with anticipation and to express themselves." We concluded that we need to incorporate "the ability to reason logically with anticipation and to express themselves" in our lessons and make full use of them.

In order to raise the ability to reason logically with anticipation and to express themselves," students need to recognize the good points about their own ideas or mistakes contained in them as they try to express their ideas. By repeatedly engaging in such reasoning, students will develop the ability to think logically and further their ideas. Therefore, it is important that students will share their ideas and learn from each other.

In the study of geometric figures, there are many activities in which students explore properties of certain geometric figures. Those activities provide opportunities for students to express their ideas - why did they sort shapes in that way, or why their ideas make sense. It is hoped that students will come to share many ideas that originated from individual students and further develop their sensitivity toward geometric figures.

- (1) How students might "reason logically with anticipation and to express themselves" in this unit

As we progress through this unit, it is hoped that we can observe students doing the following.

- Students understand what it means for 2 lines to be perpendicular or parallel, and they can draw such lines.
- Students can sort quadrilaterals based on the perpendicular/parallel relationships of their sides.
- Students can express their ideas using the properties of quadrilaterals.
- Students can recognize good points of their friends' ideas and try to incorporate them in their own reasoning.

- (2) Strategies to raise students' "ability to reason logically with anticipation and to express themselves"

① How to pose the task

Until now, we have been examining geometric figures by focusing on their constituent parts such as the length of sides. In the "grasp" stage of the lesson, students will learn about parallelograms, trapezoids and rhombuses by including parallelism as an additional view point. Thus, I plan to use questions such as "I wonder if we can use what we have learned so far to sort these shapes" and "What should we focus on?" to help students develop ideas they can use to complete the task. By sharing different ideas uttered by individual students with the whole class, I plan to help students understand the mathematical purpose of the task.

② During independent problem solving

I want students to understand which of the ideas they have learned can be used to complete the task. I will ask students to explain the reason for their ideas through questions like "Why did you decide to use that idea?" as they complete the task. In the same way, I will encourage students to record their ideas in the notebooks so that the reason behind their ideas will be clear. In addition, if students make mistakes or change their ideas, I will encourage them to record their new ideas separately instead of erasing the previous ones and writing the new ideas over them. This way, students can more easily see the changes in their reasoning later.

③ During the whole class discussion

In order to critically compare and contrast various ideas, students must first share their own ideas clearly to other students. To do so, they must first fully understand their own ideas. Therefore, I will ask students to look back on their own ideas and organize their thoughts so that they can present their ideas in a way that will be easy for others to follow. I plan to have students form pairs and share their ideas with their partners. In this way, students will have an opportunity to revise the way they present their ideas to the whole class.

Furthermore, I want students to grasp the similarities and differences between their own ideas and those of their classmates as we engage in the whole class discussion. It is hoped that this will motivate their future learning as well applying what they learned in different contexts.

4. Goals of the Unit

- Through activities of observing and constructing position relationships of lines or various quadrilaterals, students will come to understand the perpendicular and parallel relationships of lines, parallelograms, rhombuses, and trapezoids. They will enrich their sensitivity toward and ways of observing geometric figures.
- Students will pick out 2 perpendicular lines or 2 parallel lines in their surroundings and identify parallelograms, rhombuses, and trapezoids. They will think about situations those figures may be utilized. [Interest, Eagerness, and Attitude]
- Students will be able to identify and represent properties of various quadrilaterals based on the position relationships of their sides and other constituent parts. Students will be able to grasp properties of diagonals for various quadrilaterals. [Mathematical Way of Thinking]
- Students will be able to draw 2 perpendicular lines, 2 parallel lines, parallelograms, rhombuses, and trapezoids. [Mathematical Skills]
- Students will understand the meaning and properties of 2 perpendicular lines, 2 parallel lines, parallelograms, rhombuses, and trapezoids. They will enrich their sensitivity toward geometric figures. [Knowledge and Understanding]

5. Unit Plan (15 lessons)

sub-units [# of lessons]	No.	Main Activity	Assessment Criteria
1 [2]	1	<ul style="list-style-type: none"> ● Investigate how 2 lines can intersect. ● Learn the meaning of the term, "perpendicular." 	<ul style="list-style-type: none"> ● Students are investigating the way 2 lines are intersecting by focusing on the angles formed by them. (Interest, Eagerness, and Attitude) ● Students understand the meaning of lines being "perpendicular."
	2	<ul style="list-style-type: none"> ● Based on the meaning of perpendicularity, students will think about ways to draw perpendicular lines using a pair of set squares. ● Draw perpendicular lines. 	<ul style="list-style-type: none"> ● Students can explain the method of drawing perpendicular lines using a pair of set squares by focusing on the right angles in the set square pieces. (Mathematical Way of Thinking) ● Students can draw perpendicular lines by using a pair of set squares. (Mathematical Skills)

2 [4]	1	<ul style="list-style-type: none"> Students will explore how lines may be arranged. Learn the meaning of the term, "parallel." 	<ul style="list-style-type: none"> Students understand the meaning of lines being "parallel." (Knowledge and Understanding)
	2	<ul style="list-style-type: none"> Students will explore the width of a pair of parallel lines. Students will summarize that the distance between a pair of parallel lines is constant. Students will explore angles formed by parallel lines and a line intersecting them. Students will summarize that a line will intersect parallel lines forming angles of equal measurements. 	<ul style="list-style-type: none"> Students understand that the distance between a pair of parallel lines is constant. They also understand that a line will intersect parallel lines forming angles of equal measurements. (Knowledge and Understanding)
	3	<ul style="list-style-type: none"> Students will think about ways to draw parallel lines using a pair of set squares. 	<ul style="list-style-type: none"> Students can explain how to draw a pair of parallel lines based on the idea of equal corresponding angles. (Mathematical Way of Thinking) Students can draw a pair of parallel lines using a pair of set squares. (Mathematical Skills)
	4	<ul style="list-style-type: none"> Students will think about ways to identify perpendicular and parallel lines by using the grid lines as a guide. 	<ul style="list-style-type: none"> Students understand how to identify perpendicular lines or parallel lines using the grid lines as a guide. (Knowledge and Understanding)
3 [7]	1*	<ul style="list-style-type: none"> Students will sort various quadrilaterals using perpendicular or parallel sides. Learn the meaning of the terms "trapezoids" and "parallelograms." 	<ul style="list-style-type: none"> Students can sort quadrilaterals using a variety of view points. (Mathematical Way of Thinking) Students understand properties of various quadrilaterals. (Knowledge and Understanding)
	2	<ul style="list-style-type: none"> Investigate characteristics of parallelograms by using 2 pieces of cut out parallelograms. Summarize the properties of parallelograms. 	<ul style="list-style-type: none"> Students can identify and explain properties of parallelograms by focusing on the position relationships of their sides, length of their sides or their angle measurements. (Mathematical Way of Thinking) Students understand the properties of quadrilaterals. (Knowledge and Understanding)

	3 4	<ul style="list-style-type: none"> Think about ways to draw parallelograms. Draw parallelograms using the properties of parallelograms. Work on application problems. 	<ul style="list-style-type: none"> Students can explain how to draw a parallelogram based on the properties of parallelograms. (Mathematical Way of Thinking) Students can construct parallelograms. (Mathematical Skills)
	5	<ul style="list-style-type: none"> Learn the meaning of the term, "rhombus." Summarize the properties of rhombuses and draw rhombuses. 	<ul style="list-style-type: none"> Students can explain properties of rhombuses by focusing on the position relationship of the sides, length of the sides and the measurement of their angles. (Mathematical Way of Thinking) Students can draw rhombuses. (Mathematical Skills)
	6	<ul style="list-style-type: none"> Explore characteristics of lines drawn by connecting vertices of various quadrilaterals. Learn the meaning of the term, "diagonal." Summarize the characteristics of diagonals in various quadrilaterals. 	<ul style="list-style-type: none"> Students are thinking about and understanding the relationships among quadrilaterals based on the characteristics of diagonals. (Mathematical Way of Thinking) Students understand the properties of diagonals for various quadrilaterals. (Knowledge and Understanding)
	7	<ul style="list-style-type: none"> Investigate the two triangles obtained by cutting a rectangle or parallelogram along a diagonal. Make various quadrilaterals by putting together pieces obtained by cutting a quadrilaterals along a diagonal. 	<ul style="list-style-type: none"> Students can make various quadrilaterals by using 2 congruent triangles. (Mathematical Skills) Students understand that the two triangles obtained by cutting along a diagonal are congruent. (Knowledge and Understanding)
4 [2]	1 2	<ul style="list-style-type: none"> Work on unit exercises. 	<ul style="list-style-type: none"> Students can solve problems by utilizing what they learned in the unit. (Mathematical Skills) Students have basic understanding of materials discussed in the unit. (Knowledge and Understanding)

* Today's lesson

6. Today's lesson

(1) Goals of the lesson

- Through activity of sorting quadrilaterals from a variety of view points, students will attend to parallel sides and understand properties of trapezoids and parallelograms.

(2) Date/Time Monday, June 23, 2014, 1:50 - 2:35 (Period 5)

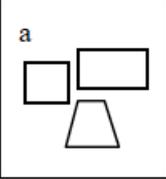
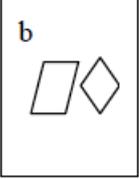
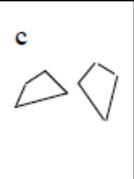
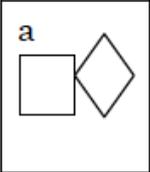
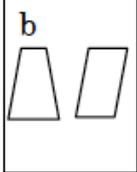
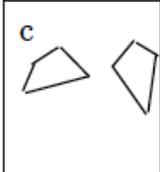
(3) Location Oshihara Elementary School (Showa Town), Grade 4 Room 1

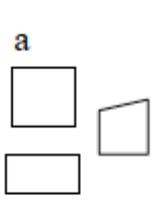
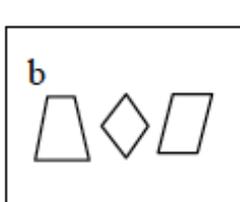
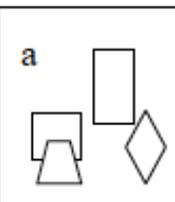
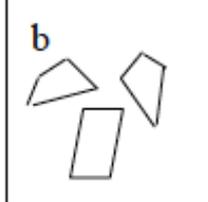
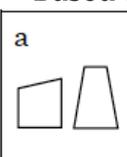
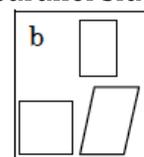
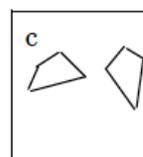
(4) Purposes of the lesson

In previous lessons, students have learned about perpendicularity (2 lines intersect to form right angles) and parallelism (2 lines that are perpendicular to a common line). By understanding these relationships, students have learned to draw lines that are perpendicular or parallel to each other. In today's lesson, using parallelism as a viewpoint, students will learn about trapezoids and parallelograms. Therefore, I want students to make full use of what they have been learning in previous lessons. As students sort quadrilaterals, I anticipate that students will use a variety of criteria. Therefore, as students express how they sorted quadrilaterals, I would like them to use words and diagrams so that they can make their explanation easier for other to understand. Moreover, by incorporating the activity to think about other students' sorting strategies, I want students to recognize the similarities and the differences between their own ideas and those of other students.

(5) Flow of the lesson

Steps	Content and Task	Instructional considerations	Assessment
Grasp (10 min)	<p>1 Understand the task</p> <p>(1) Look at triangles (previously learned)</p> <ul style="list-style-type: none"> • It's an isosceles triangle. • It's an equilateral triangle. <p>(2) Think about the quadrilaterals they created in previous lessons.</p> <p>(3) Have students share what they noticed.</p> <ul style="list-style-type: none"> • There are squares. • There are rectangles. 	<ul style="list-style-type: none"> • Display some of the quadrilaterals students created in the previous lesson. • Make sure students understand that some of them have names while others do not - this will help them plan their strategies. 	

	<p>2 Understand the task for today's lesson</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Let's sort these quadrilaterals and make some groups.</p> </div> <p>(1) Develop a plan.</p> <ul style="list-style-type: none"> • I think I will make a group with sides of equal length. • Some of them have sides that are parallel. • Some have right angles and others don't. 	<ul style="list-style-type: none"> • Remind students to think about what they have been studying so that they will have ideas what to focus on. 	
	<p>3 Independent problem solving</p> <p>(1) Sort quadrilaterals based on own criteria</p> <ul style="list-style-type: none"> • Based on the appearances <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>a</p>  </div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>b</p>  </div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>c</p>  </div> </div> <p>a: They look close to squares. b: They have slanted sides. c: Their shapes are different.</p> <ul style="list-style-type: none"> • Use the length of sides <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>a</p>  </div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>b</p>  </div> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>c</p>  </div> </div> <p>a: The length of sides are all equal. b: Opposite sides are equal length. c: The length of sides are all different.</p>		

<p>Explore (10 mi)</p>	<ul style="list-style-type: none"> Based on angles <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> a  </div> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> b  </div> </div> <p>a: There are right angles. b: There is no right angle.</p> Based on similarity <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> a  </div> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> b  </div> </div> <p>a: The parts match up when these are folded. b: The parts don't match up when these are folded.</p> Based on parallel sides <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> a  </div> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> b  </div> <div style="border: 1px solid black; padding: 5px; width: 100px; height: 100px; position: relative;"> c  </div> </div> <p>a: A pair of opposite sides are parallel. b: Two pairs of opposite sides are parallel. c: None of the sides are parallel.</p> 		
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<p>Deepen (15 min)</p>	<p>4 Whole class discussion</p> <p>(1) Explain what they used as criteria to sort.</p> <p>(2) Compare the shared ideas and discuss which previously learned ideas are being used to sort the quadrilaterals.</p> <ul style="list-style-type: none"> • If we look at the length of sides, the square and the shape of a diamond have 4 sides that are equal. • When we grouped based on right angles, we could make several groups. • We also got several groups when we sorted based on parallel sides, but the groups we made were different from those based on right angles. <p>(3) Think about the groups based on parallel sides using the ideas we already learned.</p> <ul style="list-style-type: none"> • We call quadrilaterals with a pair of parallel sides "trapezoids," and quadrilaterals with two pairs of parallel sides "parallelograms." 	<ul style="list-style-type: none"> • Have students other than the one that shared their groups to explain how the quadrilaterals were sorted. • Have students think about why we cannot make good groups by simply focusing on the length of sides of angle measurements. • Let children know that because we use parallel sides as the criterion, we call them parallelograms or trapezoids.¹ 	<ul style="list-style-type: none"> • Students can explain the reason for their sorting in an easily understandable way. (Mathematical Way of Thinking) [notebook]
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¹ The Japanese word for trapezoids literally means the shape of a footstool/pedestal.

Summarize (10 min)	<p>5 Summarize the lesson</p> <p>(1) Using the words for summary, distinguish the given trapezoids and parallelograms.</p> <p>(2) Write a journal entry</p> <ul style="list-style-type: none"> • I understand what we focused on to sort the quadrilaterals. • I understand the difference between trapezoids and parallelograms. • I didn't think parallelism we learned will be useful in today's lesson. I want to see how else we can use that idea. 	<ul style="list-style-type: none"> • Make suggestions so that students can incorporate the following point in their journals. <p>⌘ About ways of explanation that were easy to understand.</p> <ul style="list-style-type: none"> • We will discuss the inclusion relationship in the next lesson. 	
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(6) Assessment for today's lesson

- Did the students attend to parallel sides and come to understand properties of trapezoids and parallelograms through activity of sorting quadrilaterals from a variety of view points?

References: *Omitted*

Results of the Readiness Test for the Unit

Problem	Score	Main incorrect answer
[1] Draw an isosceles triangle whose side lengths are 5 cm, 6 cm, and 6 cm.	84 %	No response
[2] Write the numbers in the []. (1) One right-angle measures ² [] degrees. (2) Two right-angles measures [] degrees. (3) One whole turn = [] degrees.	(1) 100 % (2) 96 % (3) 92 %	(2) 18 degrees (3) 36 degrees
[3] Name the following figures (not included). a: rectangle b: square c: isosceles triangle d: equilateral triangle	a: 68 % b: 68 % c: 84 % d: 76 %	a: quadrilateral ³ b: quadrilateral c: equilateral triangle d: isosceles triangle
[4] Name the triangles we can make by putting together a pair of set squares as shown below (figure not included). a: equilateral triangle b: isosceles triangle	a: 64 % b: 80 %	a: triangle b: triangle
[5] What are the measurements of angles <i>a</i> and <i>b</i> ? <i>a</i> : 60 degrees <i>b</i> : 120 degrees	<i>a</i> : 68 % <i>b</i> : 48 %	<i>a</i> : 120 degrees <i>b</i> : 60 degrees
[Question on topics not yet discussed] [6] Answer the following questions based on the figure below (figure not included). (1) Which lines are parallel to each other? (<i>c</i> & <i>d</i>) (2) Which line is perpendicular to <i>a</i> ? (<i>e</i>)	(1) 12 % (2) 24 %	(1) <i>a</i> and <i>e</i> ; no answer (2) <i>b</i> ; no answer

A Readiness Test was administered before the unit. Although students were able to do well on drawing an isosceles triangle or identifying the degree equivalences of one and two right-angles (Problems 1 & 2), they were not as successful in naming shapes or problems involving vertical angles (Problems 3, 4, and 5). Therefore, in this unit, as we will re-examine isosceles triangles and equilateral triangles by clearly focusing on the measurements of their sides and angles. In addition, I want to make sure students understand that the measurements of vertical angles are equal by actually measuring them, as well as calculating to determine missing angles.

² "Right-angle" here is used as a "unit."

³ The Japanese word for quadrilaterals literally means "four-angle shape."