June 21
Ver.2

Mathematics Public Research Lesson 1

"Utilizing Mathematics" Activities in Which Students' Express and Think about Phenomena Mathematically

Mathematics Lesson Plan

Date & Time: 10:00 a.m. to 10:50 a.m., Saturday, June 21, 2014
Students: Tokyo Gakugei University International Secondary School, Grade 7, Class No. 4 (26 students: 11 boys and 15 girls)
Teacher's Name: Hiroko Uchino

The International Baccalaureate Middle Years Programme (MYP) published the *Next Chapter* in May of this year with a newly revised curriculum and evaluation. Because of this change, beginning in the next school year, our school will enforce the *Next Chapter* fully. To make this transition, I will incorporate the *Next Chapter* in my plan of instruction and classroom practice as a forward-thinking research project.

1. Unit Planning considering "Backward Design"

1.) MYP Statement of Inquiry:

Organizing patterns, properties, trends, and relationships of phenomena with numbers, mathematical expressions, tables, and graphs, and using these to grasp the phenomena mathematically, helps us to plan, implement, problem solve, make projections and conscious decisions related to the phenomena.

2.) MYP Global Context: Globalization and Possibility of Sustainability

3.) Name of the Unit in Our Curriculum: TGUISS Mathematics 1: Chapter 2, "How to Look at Phenomena"

The educational goals of our school include fostering Mathematical Literacy; that is, students think about how to make a real-life phenomena better, problem solve, and expand ideas using mathematics. Based on these goals, our textbook was created by incorporating real-life phenomena as exploratory topics that include important mathematics content. Students think about these topics mathematically; they problem solve and practice with selected problems to be sure they utilize their learning within a real world situation context. This structure and flow of learning is represented in the diagram below, "Teaching and Learning in the IB," described in MYP's *Next Chapter*. (See Figure 1.) Our school's vision and plans for instruction follow MYP's vision of education.

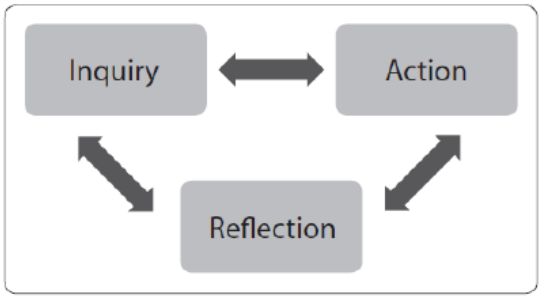


Figure 1

In Chapter 2 of our Grade 7 curriculum, we have included a unique chapter called "How to Look at Phenomena." In the regular curriculum under the guidelines of the Japanese Course of Study, Grade 7 students are expected to study the concept of functions developed through and including proportional relationships, algebraic expressions using letters to denote variables or constants, and equations. However, our curriculum takes a more comprehensive view of this content and considers the mathematical concepts students learn as very effective and clever methods for organizing, examining, and grasping real-life phenomena. For example, in regard to functions, the learning is focused on fostering students' ability to express the relationships between two quantities associated with real-life phenomena using tables and graphs. The learning stops at the point where students are using mathematical expressions to represent the reflexive relation of two discrete quantities. Actual learning about proportional relationships and function expressions is explored in Grade 8. In addition, the focus of studying algebraic expression using letters and equations aims at grasping the mathematical relationships of quantities in the real-life phenomena studied. Thus, Chapter 2 of our curriculum is a unique chapter that reflects our school's vision of mathematics education, fostering mathematical literacy. In other words, Chapter 2 in the first trimester of the first year at our secondary school sets up student's exploration for grasping real-life phenomena mathematically over six years of secondary school mathematics learning. Grade 7 students examine numbers from many different points of view and apply the concepts they learn in real contexts in Chapter 1. For example, students have learned the property of prime numbers, merits of prime factorization, relationships between prime factorization and the greatest common factor or the least common multiple, merits of Euclidean Algorithm, residue class (classifying numbers by remainders in division), and merits of using positive and negative numbers in the context of exploratory real-life situational problems.

In this lesson, I considered that the students' learning about positive and negative numbers will almost be finished by the end of June. Consequently, I have decided to provide a topic of study that connects and bridges the content of Chapters 1 and 2. Students will think about the phenomena mathematically by utilizing positive and negative numbers, quantifying and representing the phenomena and its elements, and grasping the conditions and trends of the elements by organizing them in structures, such as tables. My goal here is to set up a problem-solving activity that becomes the initial journey for learning in Chapter 2. In this problem solving activity, I decided not to use quantities that show change over time in the phenomena. Rather, I decided to help students focus on examining, grasping, and representing the phenomena mathematically. By making this decision, the focal pint of learning is moved from "how to view numbers" to "how to view phenomena" in a more expansive, inclusive manner that (I hope) will make the transition a smoother one.

Lastly, in this lesson, students will use tables to grasp the trend of a phenomenon, which is not usually practiced in Chapters 1 and 2 of the *Mathematics I* textbook created under our school's unique curriculum.

2. Theme of the Problem Solving Activity and Reason for Setting up the Activity.

"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?":

Mathematically expressing and manipulating a game, and thinking about mathematical strategies for playing the game.

I decided to set up a problem solving activity that makes possible students' expression of the quantitative relationship of a phenomenon by utilizing positive and negative numbers, and helping students bridge the content of Chapters 1 and 2 in our curriculum. In addition, I wanted to set up a problem-solving situation wherein students utilize the table mathematically to make decisions. For these reasons, I concluded that an activity that uses game theory would be advantageous.

Basic game theory, such as zero-sum and non-zero-sum games, requires and thinking about a strategy mathematically using positive and negative numbers to express gain or loss. Because game theory itself is a study for grasping and thinking about phenomena mathematically, including problem solving and decision making, we can think of it as a way to help students foster skills of grasping and thinking about phenomena mathematically. I believe the basics of the game theory fits very well as an activity or context for this lesson and provides a fresh look at a mathematical way of viewing and grasping phenomena.

The rule for playing a zero-sum game is simple and easy for students to understand. It involves either gain or loss and a sum of gains and losses becomes zero (0). Conversely, the non-zero-sum game reveals how the complexity of gain/loss and gains and losses are not equivalent and, importantly, how the rules of this game may apply to many real-life phenomena. For example, in the real life of Grade 7 students when they are facing a conflict, it is not easy to find a situation that is zero-sum even if and when the gain is expressed with numbers. If they consider a sense of value and try to express it using numbers, they will find that it may be possible to do but others will doubt the validity of the result. For this reason and lesson, I decided to set up a game activity that connects to students' daily life and uses a zero-sum.

Given the rules for playing a zero-sum game, there are two mathematical methods to consider: the mini-max and the maxi-min strategies. The basis of both strategies involves a sense of value that both opponents' losses should be kept to a minimum. In other words, the strategies value is that of low risk and low return. If you analyze a table from a data set and this point of view, it is most likely apparent that both opponents have settled on this strategy. It is a commonly held belief this type of strategic thinking is used even if in cases of large social phenomena, such as cases involving the international relations and issues between two countries. Conversely, the strategy can be used in cases of very small data sets and phenomena, such as two individuals competing in a gain or loss situation where the situation dictates that they base their decision on the question, "What should I do if I don't want to lose as much as possible?"

On the other hand, there are situations when people need to think about the decision making from a "What should I do if I want to gain as much as possible?"

strategic mindset. Usually this condition is represented by the knowledge that "if you try to gain as much as possible, your risk will go up." In other words, the strategy is high risk and high return. We can think about the strategy in the large social phenomena (macro) as well as small individual (micro) level phenomena. In this lesson's problem solving activity. In the mathematics of the classroom, students will think about a mathematical strategy for playing the game using a data set and table that students come up with to represent the phenomena. They will grapple with and apply mathematics to either the "low risk and low return" or the "high risk and high return" situations. Through participation in this game activity, students will experience a decision-making process that may apply to their individual micro-level situations, as well as apply to the macro-level social situations that include contexts such as economics and political decision-making. The intent of this lesson is to provide students the opportunity for developing a holistic skill of using mathematics and mathematical tools for thinking about, solving and tackling real-life situations.

3. About the Problem Solving Activity

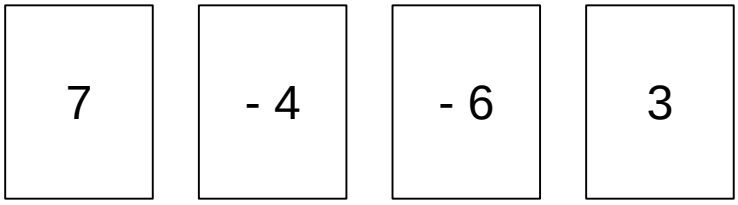
"What should we think about as a strategy for playing this game?"

"In a Grade 7 class, students decided to play the following card game for a recreational activity during a homeroom period. What is the mathematical strategy for playing this card game?"

Playing "Which is lucky?" Card Game

< Game Set Up >

- Play with two players.
- Each player plays the game with the following set of four number cards:



- Players stand on opposite sides of a desk to play the game.

< Rules of the Game >

(1) Players decide who will be Player A or B by using "rock-paper-scissors." The winner chooses who will become Player A and Player B.
(2) Players hold cards in your hands so your opponent can't see the cards, (as when you play the game, "Old Maid."
(3) Two players say, "Which is lucky?" together and simultaneously put down one of the cards on the desk from their hands.
(4) a.) Multiply the numbers on the two cards on the desk. If the product is a positive number, Player A will receive points that are the absolute value of the product calculated. Player B will lose points that are the absolute value of the product calculated.
b.) If the product is a negative number, Player B will receive points that are the absolute value of the product calculated. Player A will lose points that are the absolute value of the product calculated.
(5) The players put down another card from their hand and play the 2nd round. The players play the game for a total of 3 rounds.
(6) Record the process (see Table 1 below) of each round in the game and move on to play with different players.
(7) When a player finishes playing with 3 opponents, students will add the points.

 The player with the highest number of points will become the winner of the class.

Good Luck!

This problem solving activity is developed according to the theme that is described in section 2 of this lesson plan. The following table can be constructed by thinking about how player can gain points mathematically. (See Table 1 below.)

A \ B	−6	−4	3	7
−6	36	24	−18	−42
−4	24	16	−12	−28
3	−18	−12	9	21
7	−42	−28	21	49

Table 1

The number of cards used for this game is 4 cards. If the number of cards is too small, it will be too easy to think about the strategy for playing the game, but if the number of cards is too large, it will be more complicated to think about the strategy for playing the game. Initially, I set up the game so the absolute value of numbers on the cards was small; however, I thought that the students could handle these calculations without much struggle, so I made the difference between gain and loss clearer, and decided to use these numbers. Initially, I also thought about using different sets of cards for Players A and B, but decided to use the same set of cards for both players. If players use different cards, the focus of students' discussion will shift away from thinking about the strategy they want to use to play the game to which set of cards might give a better chance of winning the game. Lastly, I adjusted the total sum of the four cards to be zero (0), so that when the sum of positive products and negative products are calculated from the 16 possible card combinations, the absolute value of the sum of the negative numbers and the sum of the positive numbers become the same. In this way there is not a different advantage for either of the players.

When you think about the strategy for playing this card game, the maximum gain for Player A from one round of game is 49. On the other hand the maximum gain for Player B is 42. It looks like Player B has a disadvantage: however, there are two cases that products become - 42 in the table (that give gain of 42 to Player B), so it is not easy to tell if Player B has a disadvantage. If both players decide they don't want to lose a lot of points, both players show "3" at the same time. In this case the product is positive 9, so students will likely see that Player B has a disadvantage. As you can see, there are many more different ways to think about the strategy for playing this card game: therefore, I think this problem solving activity helps students deepen there mathematical thinking skills.

4. Targets that Establish Using Backward Design and Evaluation Standards

The purpose of this problem solving activity is to help students bridge the content and understanding of Chapters 1 and 2. The Goal of the activity is that students grasp a real-life phenomenon mathematically and engage in deep thinking by utilizing the knowledge they learned previously about positive and negative numbers in Chapter 1. To achieve this goal, we discussed:

- What is the mathematical ability that we want to enhance through "Utilizing Mathematics" activities included in this particular problem-solving activity; and
- Which objectives of MYP's *Next Chapter* align with the mathematical ability we are looking for and want to enhance in these "Utilizing Mathematics" activities?

Objectives stated by MYP's *Next Chapter for Mathematics* are:

Objective A: Knowing and understanding Objective B: Investigating patterns Objective C: Communicating Objective D: Applying mathematics in real-life contexts

In this lesson I would like to enhance students' ability to express the state of a phenomenon mathematically and logically, by reasoning mathematically using data in tables. In addition, the strategies and decision-making processes that students engage in will connect to a real-world setting. I will focus on Objectives C (Communicating) and D (Applying mathematics in real-life contexts) and strive to bring out students' mathematical engagement and the quality of learning activities.

The objectives and the assessment standards of MYP's the *Next Chapter*, are inter-related and interdependent: thus, I will use the Mathematics Assessment Criteria, Year 3 of MYP's the *Next Chapter* to assess the students' learning. Please refer to the school research bulletin for details about the school goals and assessment standards.

5. About the students:

There are 26 students in the Grade 7, No. 4 class. Eleven (11) students were selected from Tokyo Gakugei University International Elementary School, seven (7) students were selected using the Type A Selection criteria, which entails a foreign language essay and a basic Japanese language essay. (Many returning students from countries outside Japan take this exam.) Eight (8) students were selected using the Type B Selection criteria, which entails the administration of an Aptitude Test. An analysis of students' scholastic performance test taken at the time they entered Grade 7 found an average in Mathematics that was 7.4 points below the national average on the 100-point test.

Up to this point in the school year, observations of the students in this mathematics class reveals that most are focused on learning, even though usually only one particular group of students presents their ideas to the whole class. Students, however, actively share their ideas in pairs or small groups without hesitation. The other class I teach (Class No. 2) shows a similar tendency to Class 4 in that students are eager to discuss and present their ideas. The students who

are selected from Tokyo Gakugei University International Elementary School also present and actively participate in the pair and small group discussions.

When I compare the two classes I have observed a very interesting characteristic. There are great differences between these two classes in regard to students' presentations, responses, and types/kinds of thinking. The students in Class No. 2, are good at listening to and thinking about the questions I pose. They have a tendency to expand and deepen their thinking in a way that goes beyond basic answers to the questions I ask. Thus, I feel they have wider range and depth of thinking and reasoning skills. Students of Class No. 4 are loyal, honest, and hard working. Yet, I sometimes feel their thinking is narrower and more limited. However, ironically, when the students studied the topic of prime factorization and residue (classifying numbers by remainders in division) the characteristic of the two classrooms was reversed. So I think the topics the Type A and Type B selection students learned when they were in elementary school has an influence on the characteristics of the students' learning in class.

6. **Plan of the Unit** (Total of 3 Lessons):

	Content of Instruction	No. of Lessons
1	Students will play the zero-sum-like game, provided within the context of a real-life situation.	1
2	Students will think about how to mathematically organize the different cases of gains and examine the methods. Students will also think about the strategies used to play the game using the table created. **(This Lesson)**	1
3	Students think about multiple different ways to play the game and compare and contrast these strategies. The teacher helps students to make connections about what they have learned in these lessons with the real-life phenomena.	1

7. **Flow of the Previous Lesson** (T: Teacher, S: Student(s)):

Learning Activities	T	Points to Remember for the Instruction
S1: Students understand the learning goals of Chapter 2, "How to Look at Phenomena."	2	T1: Show and explain the Statement of Inquiry and Global Context of MYP's *Next Chapter*.
S2: Students understand the problem.	2	
Problem: "What should we think about as a strategy for playing this game?" "In a Grade 7 class, students decided to play the following card game for a recreational activity during a homeroom period. What is the strategy for playing this card game?"		
S3: Students understand the setting and rules of the card game. • Play the game three times, each time with a different opponent. • Record the process of each round of the game.	2	T2: Distribute the worksheets (No. 1) and post the setting and rules of the card game on the blackboard. T3: Demonstrate how to play the game.
S4: Students prepare for playing the game.	5	T4: Ask students to arrange the desks and prepare pencils and recording sheets (worksheet No. 1) for playing the game. Distribute the game cards.
S5: Students play the game.	10	T5: Monitor the students to determine if they are playing according to the rules of the game, correctly calculating the game points, and recording the process of the game. If students are not playing the game appropriately, point out and correct the issues.
S6: Each student calculates and records game points.	3	T6: Ask the students to check each other's calculations.
S7: Students present their points in the classroom and determine the winner of the game.	10	
S8: Students write their reflections about the game: the strategies they thought about and used when selecting cards, points of improvement for playing the game, and their competence with playing the game. Ask some to share their reflections.	7	T7: Ask the students to write about their strategies for choosing cards. Help students, who appeared to not think much about the strategy while they were playing the game, to recall what they should have done when they put down a card in a specific round or case.
S9: Students learn about what they will be learning in the next lesson.	1	T8: Ask the students to hand in their worksheets.

8. About this Lesson:

1.) The Goals of this Lesson:

Year 3, Objective C: Communicating
Elaboration of the objectives of this lesson:
• Students are able to express the gains using positive and negative numbers, and able to show the gain and loss of Players A and B. • Students are able to understand the different points of views of the players from the table, and are able to read the table effectively. • Students are able to explain how they think about the mathematical strategy for playing this game in an orderly and logical way.

Year 3, Objective D: Applying Mathematics in Real-Life Contexts
Elaboration of the objectives of this lesson:
• Students are able to examine gains in points of each other's decisions or mathematical methods and determine what strategy to use when they need to make a decision about what cards to play to give themselves a mathematical advantage. • Students are able to think about their own strategy by thinking about and identifying the other players' points of view and outcomes by going back and forth between the two player's points of view. • Students think about their own strategies using different senses of value, such as "high risk and high return," and "low risk and low return."

2.) The Goals of this Lesson:

Year 3, Criterion C: Communicating

Level	Level Descriptor	Specific Indicator
0	The student does not reach the Year 3 level described by any of the descriptors across the criteria of levels 1-8 given below.	The student does not demonstrate the criteria described at any of the descriptor levels below.
1-2	The student is able to: i. use limited mathematical language (notations, symbols and terminology) ii. use limited forms of mathematical representation to present/convey information iii. communicate using lines of reasoning that are difficult for others to understand.	The student is able to: • partially express the gains achieved in the game • explain the strategy, but the explanation is ineffective and difficult to comprehend.
3-4	The student is able to: i. use some appropriate mathematical language ii. use different forms of mathematical representation to present and convey information adequately iii. communicate by using lines of reasoning that are able to b e understood, although these are not always clear iv. adequately organize information using a logical structure	The student is able to: • calculate gains of the game • explain the strategy used, so that others can understand it ; however, the explanation is not always clear.
5-6	The student is able to: i. usually use appropriate mathematical language ii. usually use different forms of mathematical representation to present and convey information correctly iii. moves between different forms of mathematical representation with some success iv. communicate through lines of reasoning that are clear although not always coherent or complete v. present work that is usually organized using a logical structure.	The student is able to: • accurately calculate gains of the game and tries to express gains of the both Players A and B • understand the different points of views for interpreting gains recorded in the table. • explain the strategy clearly and coherently, but not consistently is a logical way.
7-8	The student is able to: i. consistently us appropriate mathematical language ii. use different forms of mathematical representation to consistently present information correctly iii. moves effectively between forms of mathematical representation iv. communicate clearly through coherent lines of reasoning that are complete and coherent v. present work that is consistently organized using a logical structure.	The student is able to: • accurately calculate gains of the game and express gains of both Players A and B • understand the different points of views for interpreting gains given in the table and provide an effective interpretation. • explain the game strategy in a clear, coherent, and logical manner.

Year 3, Criterion D: Applying mathematics in real-life context

Level	Level Descriptor	Specific Indicator
0	The student does not reach the Year 3 level described by any of the descriptors across the criteria of levels 1-8 given below.	The student does not demonstrate the criteria described at any of the descriptor levels below.
1-2	The student is able to: i. identify some of the elements of the authentic real-life situation ii. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to: • try to think about the gains mathematically in a limited fashion; thus, finding a strategy is also limited.
3-4	The student is able to: i. identify relevant elements of the authentic real-life situation ii. select, with some success, adequate mathematical strategies to model the authentic real-life situation iii. apply mathematical strategies to reach a solution to the authentic real-life situation. iv. describe whether the solution makes sense in the context of the authentic real-life situation	The student is able to: • chose strategies by analyzing both players' gains using a mathematical method *in a limited fashion* when they need to make decisions in the context of playing the game for gains • think about opponent's point of view and try to develop his/her own strategy.
5-6	The student is able to: i. identify relevant elements of the authentic real-life situation ii. select adequate mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation iv. describe the degree of accuracy of the solution v. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: • chose strategies by analyzing both players' gains, applying a mathematical method most of the time when they need to make decisions in the context of playing the game for gains • think about the opponent's point of view and develop his/her own strategy. • think about strategies by considering various value perspectives.
7-8	The student is able to: i. identify relevant elements of the authentic real-life situation ii. select appropriate mathematical strategies to model the authentic real life situation iii. apply the selected mathematical strategies to reach a correct solution iv. explain the degree of accuracy of the solution v. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: • chose strategies by analyzing both players' gains, applying a mathematical method when they need to make decisions in the context of playing the game for gains • think about his/her own strategies by considering both players' points of view, by considering the reasoning behind each move (going back and forth) • think about strategies by considering various value perspectives, such as "high risk, high return" and "low risk, low return."

3.) Focus on Enhancing Students' Ability and Support for Achieving Lesson Objectives

In this public lesson, I would like to focus on the objective "Grasping a phenomenon mathematically, expressing and manipulating the elements of the problem appropriately, and thinking mathematically by utilizing mathematics to solve it," which is part of the criterion of (2) in Heading 6 above. I have thought about how I can provide support in a way that enhances my students' opportunities and chances of demonstrating this objective. Below, the left side of Figure 2 shows the structure for improving students' mathematical activity; the right side of Figure 2 shows the primary support for achieving the objective.

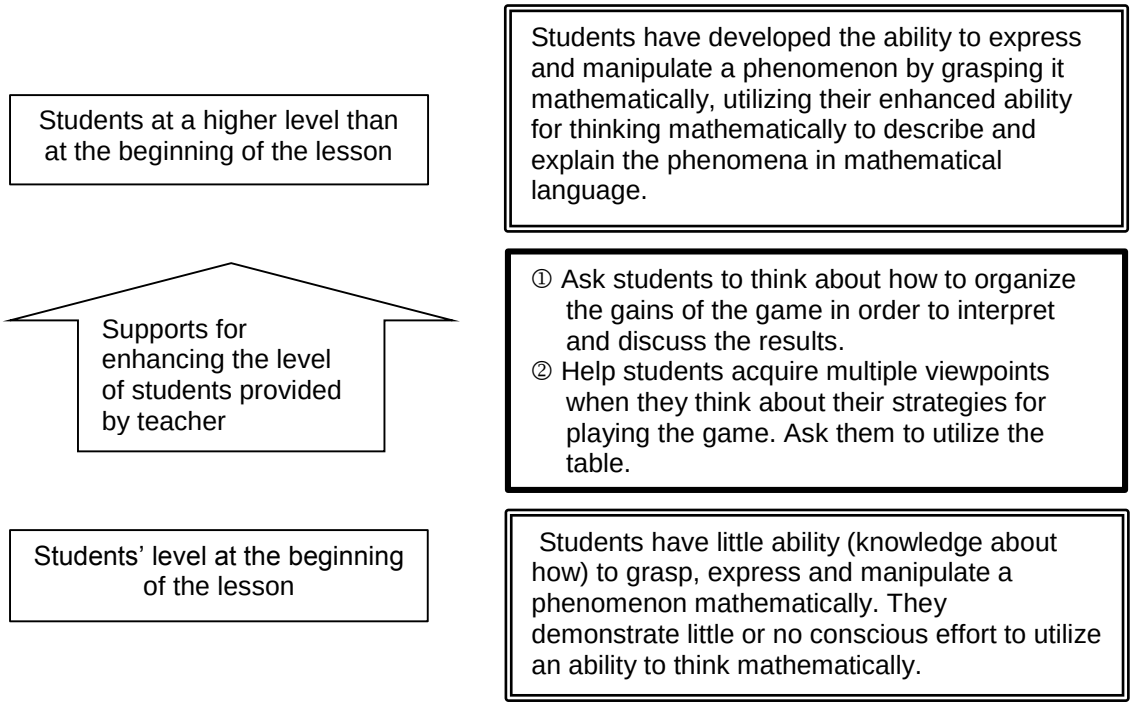


Figure 2: The structure of a lesson that enhances the students' objective

At the beginning of the lesson, it is probable that students demonstrate different levels of understanding and little evidence of having reached the objective of enhanced standards for thinking mathematically. I suspect that many of the students do not have enough knowledge or experience to demonstrate a conscious effort and the requisite mathematical abilities to organize the data of the phenomenon into a table and use the organized table to think about a strategy for playing the game mathematically. Because of the range present in regard to each student's status of learning and ability, I thought about and am including supports ① and ② to raise each student's performance standard to reach the objective above.

Given support ① ("methods that help organize the gains of the game"), I will set up the *hatsumon* for grasping the overall picture of the game and for organizing the data of gains from the game mathematically, I would like all students to experience the first step of mathematical problem solving, "grasping

the total picture and organizing the data." Then, expecting that students will try organizing and expressing the gains of the game in various ways, I will ask them to examine which methods might be better, so that students can enhance their ability to grasp and express the phenomenon mathematically.

Given support ② ("strategies for playing the game"), I will ask students as they are thinking about strategies to use what that they came up with to show the gains of the game mathematically. This way, I can help students realize that mathematically organized data can be helpful for thinking about how their opponent might think about gains of the game. Using this as a base, students will become aware of and reflect on their decision making about which card he/she might want to put down next. By engaging the students in thinking about questions, such as "Do I want to gain as much as possible?" or "Do I want to lose as little as possible?," students experience thinking about many different points of view. This experience helps students to think about strategies mathematically so their demonstration of the standards and higher-level criteria is improved. Students will be thinking about strategies for playing the cards with multiple perspectives, so support ② in not only utilized in this lesson but also continuously and subsequently used through the 3rd lesson.

4.) Other Supports

① Ask students *hatsumon* that help them think whether there is a problem in their way of thinking about strategies or what idea might help them think about how to improve and make their strategies better. By asking these *hatsumon*, students are supported in seeing and understanding how necessary it is to grasp the holistic view of the game and analyze the details of the gains critically.

② Use portable blackboards to gather many different ideas, organize the data collected from students, and organize students' ideas into several categories.

③ Check if students are thinking about the gains of the game from the perspective of Player A, the perspective of Player B, or the perspective of both players. Help them become aware of important points, such as the following question: Given how unwieldy it is to show data in a diagram if we think about Players A and B separately, is it possible to show the gains from both players in one diagram? If students are wondering whether they can show both gains in one diagram, make sure they have an opportunity to share their thoughts or ideas with each other and the class.

④ Ask students to present several ideas about how to organize the gains mathematically. Compare and contrast these ideas, in order to classify and unify across ideas to deepen the analysis of the various methods. Make sure to discuss the ideas that are from the point of view of Player A, Player B, and both Players A and B.

⑤ Examine all the ideas that show the gains for playing the game and help students choose the method on which everyone agreed. Once students agree on an idea, ask them to use it to think further about strategies to play the game.

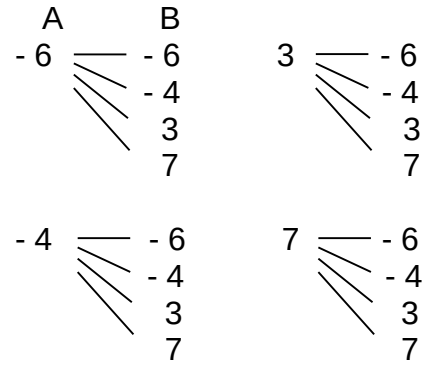
5.) Anticipated Students' Responses

① Methods that Help Organize the Gains of the Game

a.

A	B	
- 6	x (- 6) = 36	3 x (- 6) = - 18
- 6	x (- 4) = 24	3 x (- 4) = - 12
- 6	x 3 = - 18	3 x 3 = 9
- 6	x 7 = - 42	3 x 7 = 21
- 4	x (- 6) = 24	7 x (- 6) = - 42
- 4	x (- 4) = 16	7 x (- 4) = - 28
- 4	x 3 = - 12	7 x 3 = 21
- 4	x 7 = - 28	7 x 7 = 49

b.



c.

A \ B	－6	－4	3	7
－6	36	24	－18	－42
－4	24	16	－12	－28
3	－18	－12	9	21
7	－42	－28	21	49

d.

A \ B	－6	－4	3	7
－6	36，－36	24，－24	－18，18	－42，42
－4	24，－24	16，16	－12，12	－28，28
3	－18，18	－12，12	9，－9	21，－21
7	－42，42	－28，28	21，－21	49，－49

- ❖ Some students may have organized the numbers without thinking about the order in which they placed the numbers, such as from smallest to largest. Help students see that considering the order of numbers also helps them analyze the data more easily. These differences in organizing data reflect the difference in levels of mathematical thinking ability.
- ❖ Given the different ways of organizing data listed above, the most helpful organization of the data is shown as "diagram c" above. The most mathematically involved or "elegant" thinking to the least mathematically involved thinking is as follows in order from diagrams c, d, a, to d.

② Strategies for Playing the Game

(1) Thinking about high return

i. Think about Strategies from the point of view of Player A:
I am Player A and I want to gain the most points, so I decide to put down the "7" card. However, Player B does not want to lose the greatest number of points, so Player B won't put down his/her "7" card. If Player B thinks I will put the 7 card down, Player B may put down the – 6 card, so I should put down the – 6 card, also. → I (Player A) will put down the – 6 card.

If I (Player A) continue to think about what Player B might do, I will realize that Player B may think I will put down the – 6 card, so I will put down the 7 card. → I (Player A) will put down the 7 card.

ii. Think about Strategies from the Point of view of Player B:
I am Player B and I want to gain the most points, so I decide to put down the 7 or – 6 cards. However, if Player A puts down his/her 7 card, my loss would become greater; so the – 6 card seems like a better choice for me to put down. But Player A might suspect that I will put down the -6 card and there is a possibility that Player A will put down his/her -6 card. So, I should put down the 7 card. → I (Player B) will put down the 7 card.

If I (Player B) continue to think about what Player A might do, Player A might think I will put down the 7 card, so I (Player B) will put down – 6 to avoid the risk of losing a lot. → I (Player B) will put down the -6 card.

❖ Students will notice that the logic of the decision-making process is circular. They may change their minds and start thinking about and consistently using a low risk scenario or strategy.

(2) Thinking about losing as little as possible

i. Think about Strategies from the Point of view of Player A:
I (Player A) want to lose as little as possible, so I will put down the 3 card. Whatever cards Player B puts down, my loss will be 12 or 18 points; but that is much better than a loss of 28 or 42 points. → I (Player A) will put down the 3 card.

ii. Think about Strategies from the Point of view of the Player B:
I (Player B) want to lose as little as possible, so I will put down the 3 card. Whatever cards Player A puts down, my loss will be 9 or 21 points; but that is much better than a loss of 16 or 24 points that will happen if I put down the – 4 card. → I (Player b) will put down the 3 card.

❖ Students will notice that if they think about risking as little as possible, there is a distinct possibility that both players will put down the 3 card. In this case, students will recognize that being Player A is better. When playing the card game using the strategy of low risk (minimizing risk), if the circumstances become worse for Player A, he/she can always put down the 3 card to gain points; therefore, it is better to be Player A.

❖ In this lesson (2nd lesson in the unit), I suspect there are not many students who will think about or use a "low risk" mindset/strategy.

6.) Flow of the Lesson (T: Teacher, S: Student(s)):

Learning Activities	T	Points to Remember for the Instruction
[Reviewing the Previous Lesson] S1: Revisiting the learning activity that engaged students in the previous lesson.	5	T1: Use a projector to share student reflections from the previous lesson; i.e., about what strategy they thought about while they were selecting what cards to play, points of improvement for playing the game, and their competence for playing the game.
[Focusing on Learning] S2: Students will understand today's problem.	3	T2: Distribute worksheets (No. 2). T3: Project the worksheet and help students understand the problem. Today's Problem (Support for achieving the objective) If we change the rule of the "Which is lucky?" game to playing the game one time only and choosing 1 card only from the 4 cards, which card should we play? Let's think about the strategy and explain about it. [Support for S2] ◆ Demonstrate how the game will be played to be sure students understand the new rules clearly.
S3: Students will think and write about the strategy on their own.	5	[Support for S3] Monitor the students and identify if: i. Students are writing their strategies using diagrams, tables, mathematics expressions, and words. ii. Students are writing their strategies in words only. iii. Students are writing their strategies using tables and mathematics expressions only. iv. Students are having difficulty coming up with a strategy. And provide appropriate support, such as: ◆ For students ii, ask them "What did you think about to come up with your strategy? Please write down the math expressions, diagrams, and tables that you used to come up with your idea." ◆ For students iii and iv, tell them "Be sure to listen to your friends' ideas during group sharing. On your own, spend some time to think about the strategy you liked and thought was the best one."
S4: Students will present their strategy ideas to the group. They will discuss each idea one by one. □ Students will write and organize the strategies they discussed, being sure to record what they discussed on the mini-whiteboard. Then post the whiteboard on the blackboard.	12 (6)	T4: Directions "Share your strategies with your friends in the group. Then discuss each strategy with your friends." T5: Directions "Please organize your group's discussion and be ready to present to the class the strategies your group discussed.

[Neriage] (*Discussing and Kneading Ideas*) S5: All groups present the strategies they discussed in the group to the class. Students will grasp many different strategy ideas.	15	[Support for S5 -- A] ◆ Monitor each group's discussion to grasp their strategies and ideas about how to explain the strategies. Decide what order the strategies should be presented. [Support for S5 --- B] ◆ Presentation should begin with the groups that does not have well-constructed strategies. Help students clarify what issue impeded their ability to construct a good strategy and what is needed to explain the idea(s) better. ◆ After finishing all the presentations, reflect on the issues they discussed, the questions raised, and the answers they constructed. ◆ Working with the class as a whole, organize the strategies by looking at commonality among them. [Support for S5 --- C] Lead the discussion so that students see the merit of organizing the data of gaining points. ◆ If some students (or groups) state something like "I am not sure how I can think about a strategy," bring up a good example that was presented and ask the students (or group) to think about what might be a good idea or way to start thinking about his/her own strategy. ◆ Or ask *hatsumon,* such as: "How should I start thinking about a strategy?," "Which group's strategy was easier to understand? Why was it easy to understand?" Help students to understand that organizing the gains of the game in some form or structure helps us think about strategies. ◆ Introduce the word "gain."
S6: Students discuss the way to organize the gaining points of the game and think about what method is helpful to make a strategy by comparing and contrasting ideas presented in the class. And come up with the best method by choosing from what was presented or developing a new method.	5	
S7: Students present the best method that they discussed in the group.	5	[Support for S7] ◆ If students are not thinking about the different view points of Players A and B, ask students if their methods analyze both player's point of view and way of coming up with the best strategy.
[Ending] S8: Students understand what they will be studying in the next lesson.	1	T6: Tell students that students will think about the strategy individually based on the strategy they came up with in groups. T7: Ask students to hand in the worksheet.

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Objective

"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?"

Problem Solving

What should we think about as a strategy for playing this game? (No. 1)

Problem: "In a Grade 7 class, students decided to play the following card game for a recreational activity during a homeroom period. What is the strategy for playing this card game?"

Playing "Which is lucky?" Card Game

< Game Setup >

- Play with two players.
- Each player plays the game with the following set of four number cards:

7	- 4	- 6	3

- Players stand on opposite sides of a desk to play the game.

< Rule of the Game >

(1) Players decide who will be Player A or B by using "rock-paper-scissors." The winner chooses who will become Player A and Player B.
(2) Players hold cards in your hands so your opponent can't see the cards, (as when you play the game, "Old Maid."
(3) Two players say, "Which is lucky?" together and simultaneously put down one of the cards on the desk from their hands.
(4) a.) Multiply the numbers on the two cards on the desk. If the product is a positive number, Player A will receive points that are the absolute value of the product calculated. Player B will lose points that are the absolute value of the product calculated.
b.) If the product is a negative number, Player B will receive points that are the absolute value of the product calculated. Player A will lose points that are the absolute value of the product calculated.
(5) The players put down another card from their hand and play the 2nd round. The players play the game for a total of 3 rounds.
(6) Record the process of each round in the game and move on to play with different players.
(7) When a player finishes playing with 3 opponents, students will add the points.

 The player with the highest number of points will become the winner of the class.

Good Luck!

Record the Results of the Game:

Results for Playing "Which is Lucky?" Game				
Grade 7, Class No. ___, Student No. ____, Name_____				
Opponent Name: _____				
Your Position (Players A or B)	Rounds	My Number	Opponent's Number	Gaining Points
Player A or Player B (Circle one of them)	1st			
	2nd			
	3rd			
Opponent Name: _____				
Player A or Player B (Circle one of them)	1st			
	2nd			
	3rd			
Opponent Name: _____				
Player A or Player B (Circle one of them)	1st			
	2nd			
	3rd			

Total Points gained:
Points

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Objective

"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?"

Problem Solving

What should we think about as a strategy for playing this game? (No. 2)

Today's Problem: If we change the rule of the "Which is lucky?" game as playing the game only once by choosing one card from the 4 cards, which card should we play? Let's think about the strategy and explain about it.

Let's think about the strategy of the game!	
1. My Strategy	
2. Strategies came up in the group	

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Objective

"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?"

Problem Solving

What should we think about as a strategy for playing this game? (No. 3)

Problem: Let's think about the strategies for each Player A and Player B using the table of gaining points of the players we created.