Grade 7 Mathematics Lesson Plan

Tuesday, June 17, 2014
Period 6 (14:20 ~ 15:10)
Grade 7 Classroom B
(20 boys and 20 girls)

Teacher's Name: Shou Shibata
Koganei Lower Secondary School
attached to Tokyo Gakugei University

Location: Educational Technology Room (3rd Floor)

0. Research theme and its intent

Designing lessons to raise the quality of mathematical processes

(1) Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University

This lesson was developed as a part of the activities of the Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University. The purpose and rationale of the research group are as follows. In the Japanese mathematics education, we have emphasized not only the mathematical content (results of explorations) but also the processes of exploring mathematical problems and the development of skills and ways of reasoning utilized in the process of explorations. However, in spite of this emphasis, we are concerned that mathematics teaching overwhelmingly focuses on the mathematical content. We are not suggesting that we should treat teaching of the mathematical content less seriously, nor are we suggesting that processes and contents should be treated separately. However, we wonder if Japanese mathematics lessons are indeed emphasizing "mathematical ways of observing and reasoning" or "mathematical activities" even though we have been discussing their importance for a long time.

To emphasize mathematical processes means to emphasize the processes of creating and applying mathematics. As an activity, we can consider those processes as "mathematical activities," and the ways of observing and reasoning utilized in those processes can be considered "mathematical ways of observing and reasoning." Therefore, in our research group, we call the totality of the processes involved in creating and applying mathematics as "mathematical processes," and the purpose of our group is to continuously examine the nature of mathematics lessons that raise the quality of "mathematical process." Therefore, our research theme has been established as "designing lessons to raise the quality of mathematical processes."

(2) Proposal in today's lesson

As I planned this lesson, I have chosen the following as the working definition of "raising the quality of mathematical processes." Students are engaged in a higher quality mathematical process when they re-engage in a mathematical activity with a new perspectives or purposes gained internally or externally after the initial engagement with the mathematical activity. In a mathematics lesson, this rise in quality of mathematical processes should take place when students re-engage in the problem for which they had previously found a solution independently with a new idea or purpose gained internally through reflecting on own problem solving processes or externally through their peers or the classroom teacher's comments. Therefore, as we plan a lesson, it is necessary to clearly articulate mathematical processes that could surface in the selected mathematical activity and to describe strategies to raise the quality of mathematical processes. This idea is shown visually in Figure 1. Figure 2 shows the rise in the quality of mathematical processes and strategies employed in today's lesson.

The hypothesis being proposed in today's lesson is "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer."

Based on the above, the research objective for today's lesson has been established as follows.

> Research objective
> To demonstrate the viability of the hypothesis, "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer" through a lesson that introduces algebraic expressions.

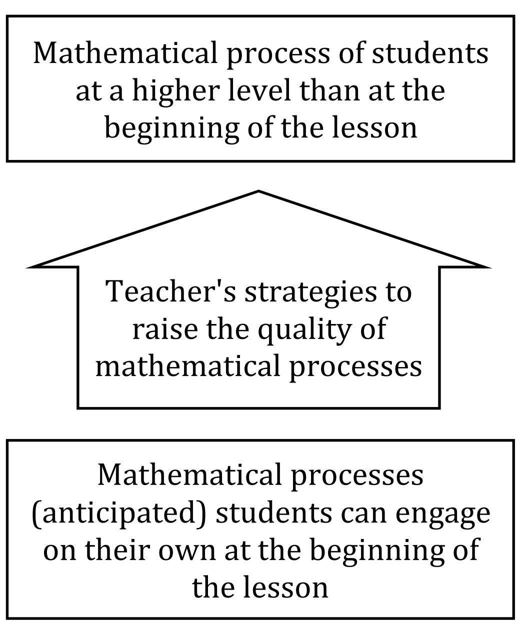


Figure 1 Structure of a lesson that raise the quality of mathematical processes

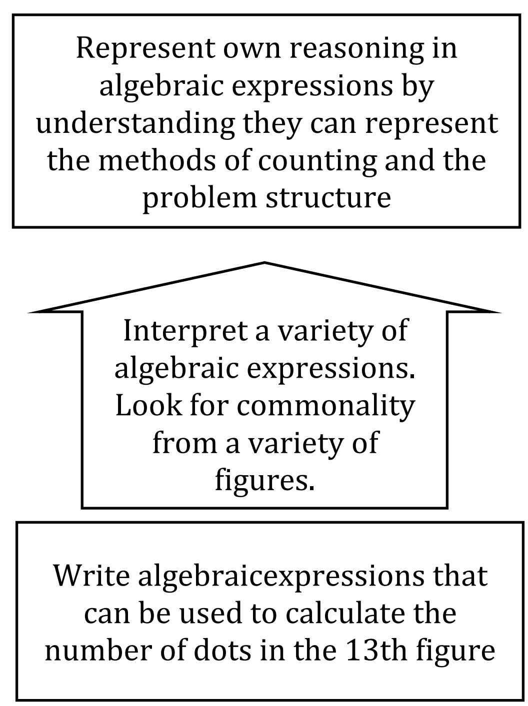


Figure 2 Structure of this lesson

1 Unit: Letters and Algebraic Expressions[1]

2 Goals of the Unit

- Students will become interested in the merits of mathematical reasoning and the enjoyment of mathematical activities by discovering characteristics and properties through the use of letters and algebraic expressions with letters. Students will try to use algebraic expressions with letters in problem solving and mathematical explorations. [Interest, eagerness, and attitude]
- Students will be able to represent relationships and patterns among quantities in various phenomena using algebraic expressions with letters and make generalizations. In addition, students will understand that algebraic expressions with letters can show both the process and the result of calculation through activities such as substituting specific values in the letters and interpreting given algebraic expressions. Students can think about relationships and patterns represented in algebraic expressions. [Ways of observing and reasoning]
- Students will be able to manipulate algebraic expressions with letters such as multiplying and dividing algebraic expressions or adding and subtracting linear expressions. In addition, students can interpret the relationships and patters among quantities represented in algebraic expressions, and also represent relationships and patterns using algebraic expressions. [Representation and manipulation]
- Students will understand the meaning of algebraic expressions with letters and their purposes. [Knowledge and understanding]

3 Assessment standards for the Unit

Interest, eagerness, and attitude toward mathematics	• Students are interested in the necessity and merit of using letters to represent relationships/patterns among quantities generally, and they try to use algebraic expressions with letters to represent relationships/patterns or interpret the given expressions. • Students know how to represent multiplication/division within algebraic expressions with letters and they try to use them to manipulate expressions. • Students try to substitute values in letters to evaluate the value of algebraic expressions.
Mathematical ways of observing and reasoning	• Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. • Students can consider algebraic expressions with letters such as $a + b$ and ab represent both the operations (addition and multiplication, respectively) and the results.

[1] The Japanese word translated as "algebraic expressions," *shiki*, is used to describe both expressions and equations, with or without letters.

	• Students are able to use algebraic expressions with letters to think about concrete phenomena by substituting values in letters. • Students are able to think about ways to calculate algebraic expressions with letters by considering calculations with algebraic expressions as analogous to calculations with numbers.
Mathematical representations and manipulations	• Students are able to represent quantities and relationships/patterns among quantities in phenomena using algebraic expressions with letters, and they can interpret given algebraic expressions. • Students can use algebraic expressions with letters involving multiplication and division appropriately by following the conventions, and they can add and subtract simple linear expressions. • Students are able to evaluate the value of an algebraic expression with letters by substituting values to letters.
Knowledge and skills about numbers, quantities, and geometric figures	• Students understand that by using letters quantities and relationships/quantities among quantities can be represented generally or interpreted from the given algebraic expressions with letters. • Students understand how to represent multiplication and division in algebraic expressions with letters, and they understand how to add and subtract linear expressions by combining like terms. • Students understand the meaning of the value of an algebraic expression.

4 About teaching

In elementary schools, students have used □ and Δ in equations, for example, 5 + □ = 8 and 3 × Δ = 24 to grasp the relationship between addition and subtraction or multiplication and division. In addition, they have studied to represent quantities and their relationships in expressions and equations or interpret the given expressions and equations. In lower secondary school, as the ground work for the study of algebraic expressions with letters, they have learned that letters such as *a* and *x* may be used in place of □ and Δ, as well as expressing direct and inverse proportional relationships using algebraic expressions. Moreover, they have used quasi-variables in, for example, thinking about ways to calculate division of fractions or expressing relationships/patterns of numbers.

In lower secondary school, building on the study in elementary school, students will learn about not only using letters as representations or simply manipulating them but also manipulating and interpreting letters as variables, unknowns, and a representative for a set. Moreover, instead of simply introducing letters and studying calculations involving algebraic expressions with letters, we will introduce letters starting with the examination of quasi-variables, which are numbers that act like variables, and then through activities of interpreting algebraic expressions and their structures and using algebraic expressions as generalization.

By using letters, we can now have numbers in general as the object of study instead of particular numbers such as 1, 3, or 0.7. We can also express various phenomena as relationships in the mathematical world. Furthermore, by transforming the given algebraic expressions or equations, new interpretations may become possible. It is hoped that the study of letters students' mathematical explorations may be deepened and more refined. Such explorations typically take place in trying to prove conjectures or utilize the ideas of equations and functions. However, in this unit, instead of simply positioning the current study as the preparation for those future explorations, the main purpose is for students to experience mathematical manipulations and interpreting their results through activities of representing a relationship in a real-world phenomena as an algebraic expression, transforming it to reflect their own thinking, and interpreting and understanding a new relationship.

5 About students

This year, students have studied positive and negative numbers (integers). In that unit, students learned to consider negative numbers are just like whole numbers. Moreover, by studying calculations with integers, and patterns and properties of operations, they have studied what numbers are. During lessons, I have tried to help students pay attention to own problem solving processes by reminding them to make explicit ideas like "what needs to be considered" and "view point used in reasoning."

In general, students' mathematical achievement levels are high. Some students were able to use, for example, 3 and -2, as quasi-variables as they examined calculations with integers, and most students were able to understand their explanations, indicating most of them understand the notion of quasi-variables. Therefore, it is anticipated that few students will have difficulty generalizing numbers.

Although few students consider mathematics as difficult, there are some students who find it difficult to explain their ideas. However, they have experienced that their mathematical understanding was deepened by clarifying questions other students had. Therefore, I believe that there is a classroom culture where students feel safe to admit something they don't understand openly. Moreover, many students are willing to share their ideas in whole class discussion, and they do not hesitate to share even simple ideas.

6 About mathematics

I consider the most valuable aspect of the task used in this lesson is that there are a variety of strategies to determine the total number of dots. The algebraic expressions representing those strategies also vary. From each algebraic expression, it is possible to interpret the reasoning of the student who wrote it as well as the mathematical structure behind his/her reasoning. By using this particular task as the introduction of the unit on letters, I believe students can not only understand that letters serve as variables, unknowns, and a generalized number but also realize that different algebraic expressions represent different ways of reasoning. Moreover, they can realize that transformed algebraic expressions can lead to new interpretations of the original phenomena or their graphical representations. As a result, I believe students will not only gain the knowledge of letters as variables, unknowns, and generalized numbers but also make it possible for them to utilize algebraic expressions creatively and with sophistication as they engage in future mathematical study and explorations.

7 Scope and sequence in lower secondary school

	A. Numbers and Algebraic Expressions	C. Functions
Gr. 7	[Letters and Algebraic Expressions] --> [Equations]	[Direct and Inverse Proportions]
Gr. 8	[Calculations of Algebraic Expressions] --> [Systems of Equations]	[Linear Functions]
Gr. 9	[Polynomials] --> [Quadratic Equations]	[Functions in the form of $y = ax^2$

8 Unit plan

	Content	Main Evaluation Points
I	Section 1: Algebraic expressions with letters • Merits of using letters in place of numbers • Representing various quantities using letters • How to write algebraic expressions with letters • Interpreting algebraic expressions with letters • Substituting values in letters of algebraic expressions and the meaning of the value of an algebraic expression	• Students are interested in the necessity and merit of using letters to represent relationships/patterns among quantities generally, and they try to use algebraic expressions with letters to represent relationships/patterns or interpret the given expressions. [Interest, eagerness, and attitude] • Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning]
II	Section 2: Calculations of algebraic expressions • Relationships between terms and coefficients • Combining like terms • Addition and subtraction of linear expressions • Multiplying and dividing linear expressions	• Students know how to represent multiplication/division within algebraic expressions with letters and they try to use them to manipulate expressions. [Interest, eagerness, and attitude] • Students are able to think about ways to calculate algebraic expressions with letters by considering calculations with algebraic expressions as analogous to calculations with numbers.

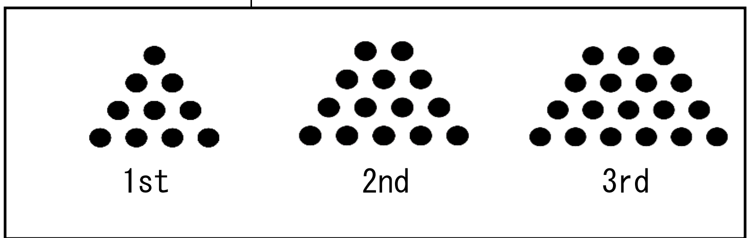
		[Mathematical ways of observing and reasoning] • Students can use algebraic expressions with letters involving multiplication and division appropriately by following the conventions, and they can add and subtract simple linear expressions. [Mathematical representations and manipulations]
III	Section 3 Applications of algebraic expressions with letters • Quantities represented by algebraic expressions • Algebraic expressions that represent relationships	• Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning] • Students are able to represent quantities and relationships/patterns among quantities in phenomena using algebraic expressions with letters, and they can interpret given algebraic expressions. [Mathematical representations and manipulations] • Students understand that by using letters quantities and relationships/quantities among quantities can be represented generally or interpreted from the given algebraic expressions with letters. [Knowledge and skills about numbers, quantities, and geometric figures]

9 Today's lesson

(1) Goals

- By interpreting algebraic expressions, students will be able to think about the reasoning and the structure represented by the algebraic expressions. [Mathematical ways of observing and reasoning]
- Students will think about quantities and relationships/patterns among them, and they will try to figure out the total number of dots in the 13th drawing. [Interest, eagerness, and attitude]

(2) Flow of the lesson

min	Learning Activity	Anticipated responses	❖ Instructional consideration ○ Evaluation
5	[Opening] Post the figures below.		
			
	"How many dots are there in each drawing?"	• There are 10 dots in the first one. • There are 14 dots in the second. • There are 18 in the third. • Is it increasing by 4?	❖ In order to avoid restricting students' reasoning, we will not discuss "how the numbers of dots are increasing."
20	[Development]		
	Main *hatsumon*: In the 13th drawing, how many ● will there be? Represent your reasoning in an algebraic expression.		
	• Independent problem solving • Students share their algebraic expressions • Sort the shared algebraic expressions	1. 13 + 14 + 15 + 16 2. 4 × 13 + 6 (2)' 6 + 4 × 13 3. 4 × 12 + 10 (3)' 10 + 4 × 12 4. (13 + 16) × 2 5. (13 + 16) + (14 + 15) 6. (13 + 16) × 4 ÷ 2 7. 13 × 4 + 6 8. 12 × 4 + 10 9. 16 × 4 - 6 10. 10 × 10	❖ Distribute mini-white boards and markers for students to use during the sharing time. ○ Students will think about quantities and relationships/patterns among them, and they will try to figure out the total number of dots in the 13th drawing. [Interest, eagerness, and attitude]
20	[*Neriage*] • Interpreting algebraic expressions		
	Main *hatsumon* for *neriage*: Let's try to explain the counting strategy each algebraic expressions represent.		

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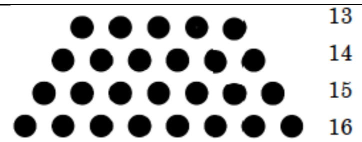
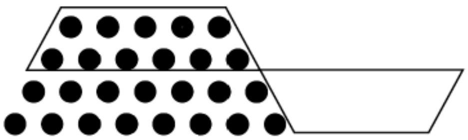
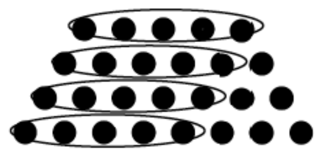


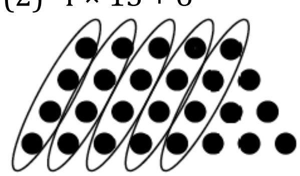
(1)' Although the student wrote 13 + 14 + 15 + 16, their reasoning is more accurately represented by (13 + 16) + (14 + 15) = 29 × 2.



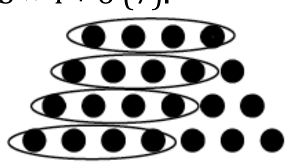
(1)'' Using 13 as the starting point, think about other rows as "one more than 13," "two more than 13," etc.. It can be represented more accurately as 13 + (13 + 1) + (13 + 2) + (13 + 3).



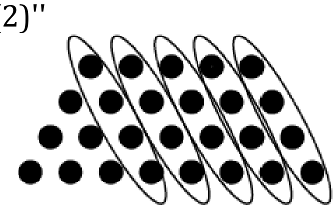
(2) 4 × 13 + 6


6 dots

(2)' If we distinguish the multiplier and the multiplicand, the algebraic expression will be 13 × 4 + 6 (7).



(2)''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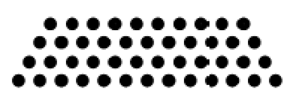
○ By interpreting algebraic expressions, students will be able to think about the reasoning and the structure represented by the algebraic expressions. [Mathematical ways of observing and reasoning]

❖ If a student says something like, "My idea was similar to ___'s" or "My idea was different," ask him/her what is similar/different and have students think about the correspondence between algebraic expressions and diagrams.

❖ Have students use the sheet with the 13th drawing as they explain.



❖ Distinguish the multiplier and the multiplicand.
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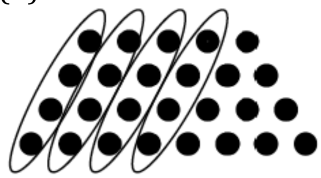
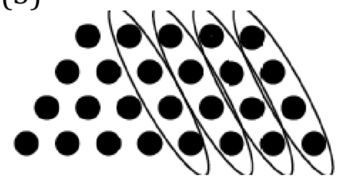
(3) 4 × 12 + 10



(3)'

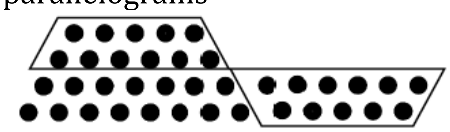


(3)''

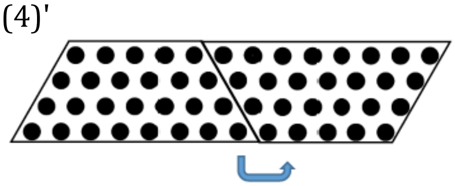
	1	2	3	4	…	12	13
	10	14	20	24		54	58

First term + 12 increases
10 + 4 × 12

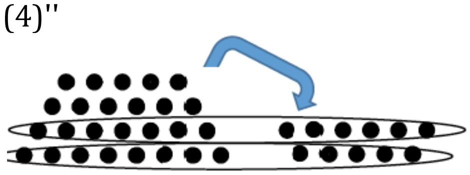
(4) (13 + 16) × 12
Using the formula for the area of parallelograms



(4)'



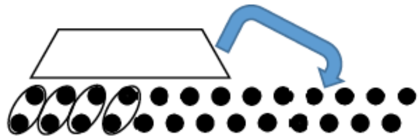
(4)''



There are two sets of 13 + 16.

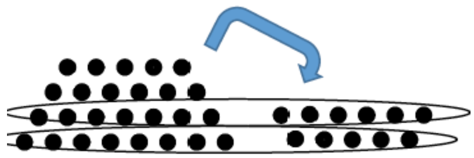
❖ If students notice that different counting strategies can be interpreted from the same algebraic expressions, discuss that idea.

(4)''' There are (13 + 16) groups of 2 dots.

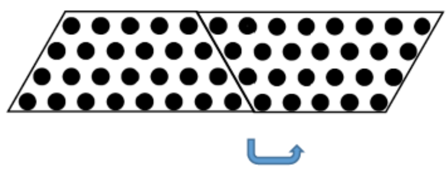


If we distinguish the multiplier and the multiplicand, it should be written as 2 × (13 + 16).

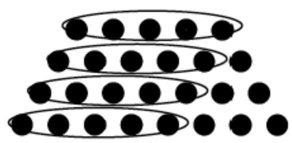
(5) (13 + 16) + (14 + 15)



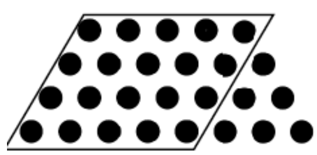
(6) (13 + 16) × 4 ÷ 2



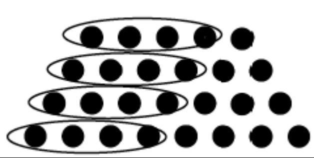
(7) 13 × 4 + 6

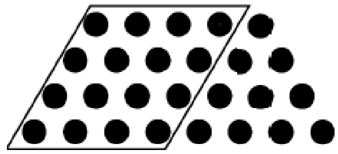
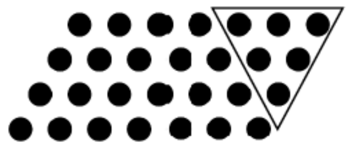
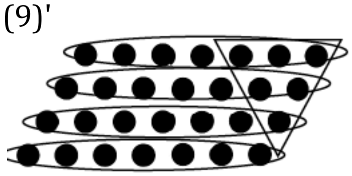


(7)' Using the formula for the area of parallelograms

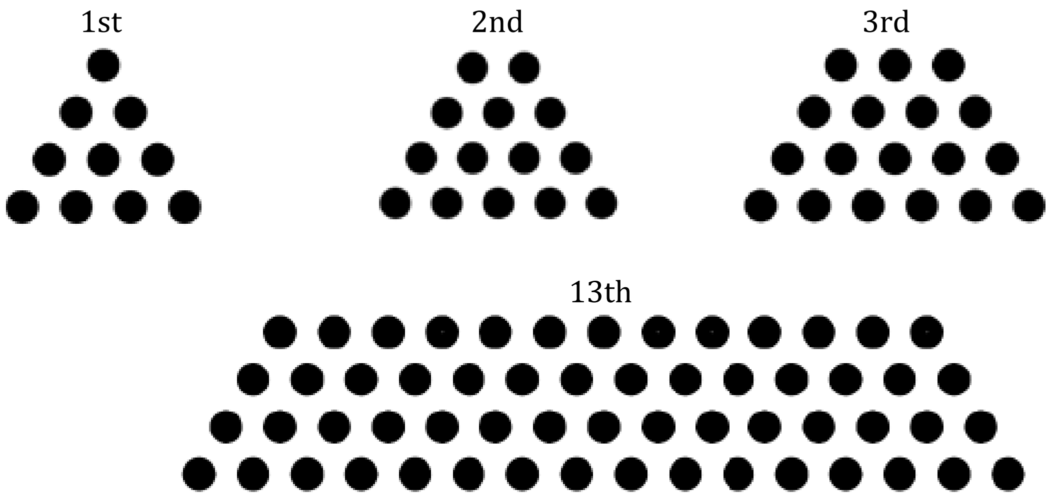


(8) 12 × 4 + 10



		(8)' Using the formula for the area of parallelograms  (9) 16 × 4 − 6  Add 6 more dots to make an arrangement in the form of a parallelogram (9)'  Add 6 more dots to make 4 groups of 16. (10) Since the first drawing has 10 dots, if we multiply 10 by 13, we can determine the number of dots in the 13th drawing. (incorrect answer)	
5	[*matome*] • Creating new algebraic expressions "Which counting strategy was similar to yours? Which one had the same algebraic expression as yours but the counting strategy was different from yours?"	• "I wrote 13 + 14 + 15 + 16, but I noticed some of my classmates were using the algebraic expression, 13 + (13 + 1) + (13 + 2) + (13 + 3). They were trying to use the number 13 from the "13th" drawing." • "I made a large parallelogram by doubling and used the algebraic expression, (13 + 16) × 4 ÷ 2. But, there	○ By interpreting a variety of algebraic expressions, students understand that algebraic expressions represent not only the steps of calculations but also their results and the structure behind the problem. [Knowledge and skills

	"What was the difference in your strategies?"	were people who added 6 more dots to make a parallelogram."	about numbers, quantities, and geometric figures]



10 Points for observing the lesson

As discussed above, this lesson focuses on the idea of "raising the quality of mathematical processes." The hypothesis proposed in today's lesson is "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer." Finally, the research objective for this lesson is "To demonstrate the viability of the hypothesis, "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer" through a lesson that introduces algebraic expressions."

Therefore, as we observe the lesson, we would like to focus on "the mathematical processes students can engage in on their own," "higher quality mathematical processes students engaged in as a result of the lesson," and "teacher's strategies." In particular, we would like to consider the following questions.

1. During the independent problem solving time, were the students able to represent their own reasoning processes using numbers such as 13? (the mathematical processes students can engage in on their own)
2. Through the activity of interpreting and comparing algebraic expressions, did the students come to understand that algebraic expressions represent not only the steps of calculations but also the results of the calculation and the structures of and changes in the diagrams that represent the problem situation? (higher quality mathematical processes students engaged in as a result of the lesson, and teacher's strategies."

By focusing on these two questions, we hope to address the proposed hypothesis and the research objective of the lesson.

