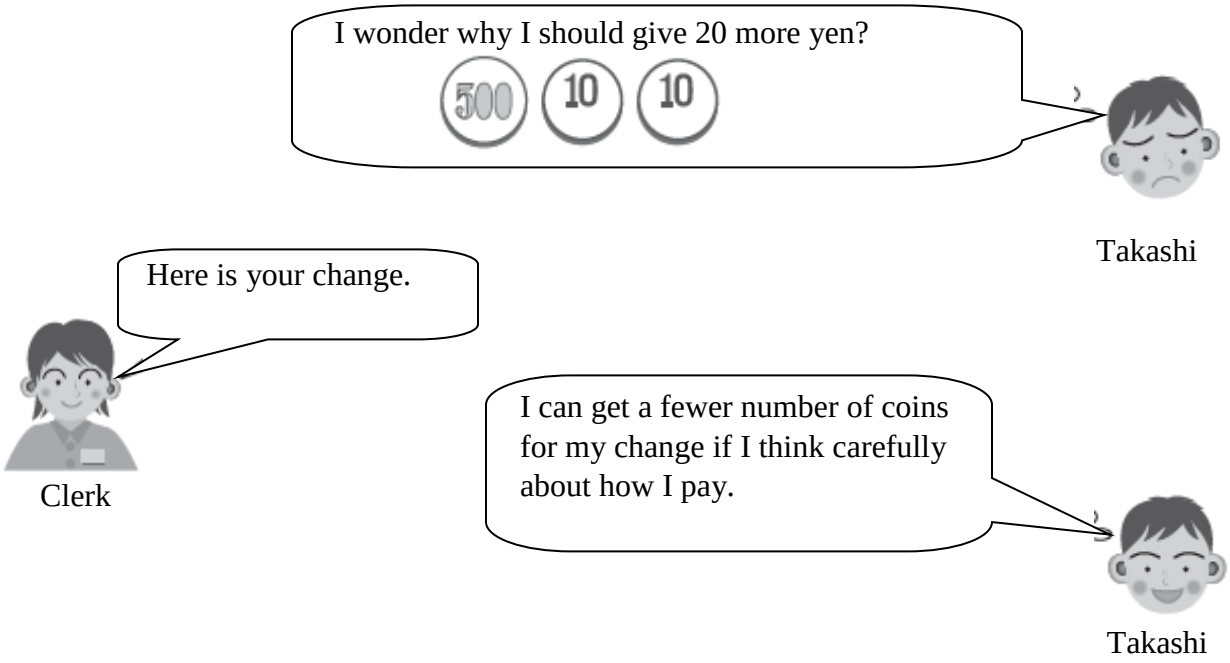
2012 Grade 6 Mathematics Set B

[1] Takashi went shopping.

(1) The cost of an item was 320 yen.

Because Takashi did not have 3 100-yen coins, he decided to pay with a 500-yen coin and get change.

The store clerk asked, “do you have 20 more yen?” So, Takashi gave her 20 yen in addition to the 500-yen coin.



The change Takashi received was in 2 coins of the same type.

Which of the following 6 types of coin did Takashi receive? Write your answer.

Types of coins

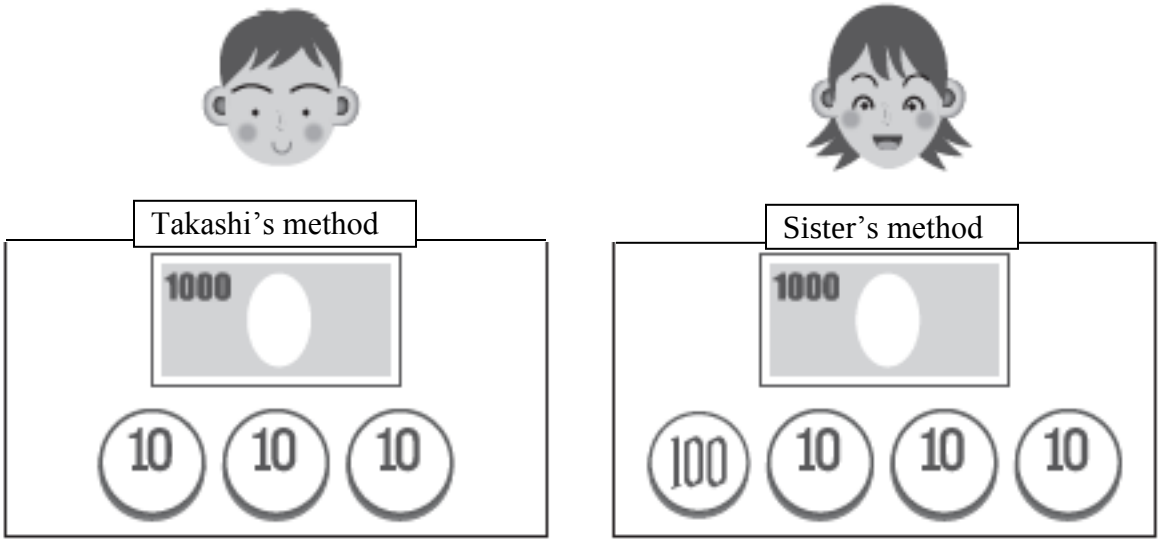
1 yen 5 yen 10 yen 50 yen 100 yen 500 yen

Next day, Takashi went shopping with his older sister.

(2) The cost of the item was 630 yen.

Takashi decided to give 30 yen in addition to a 1000 yen bill in order to reduce the number of coins he would get for his changes.

His sister told him, “If you give 100 yen more, the number of coins will be even fewer.”



The number of coins Takashi will receive as change will be fewer if Takashi uses his sister’s method.

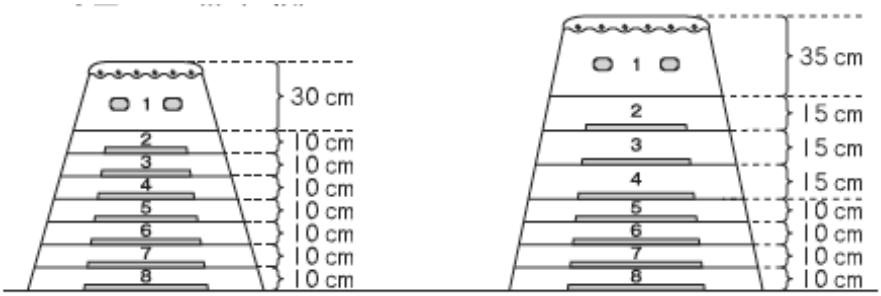
Explain using words and numbers why the number of coins in change will be fewer with Takashi’s sister’s method by comparing the number of coins for each method.

[2] At Yuriko's school, there are two types of vaulting horse, small and medium.

The top layer of the small vaulting horse is 30 cm high, and each layer second through the eighth is 10 cm in height.

For the medium vaulting horse, the top layer is 35 cm high and each layer second through fourth layer is 15 cm in its height. For each layer fifth through eighth, the height is 10 cm.

Small Vaulting Horse (8 layers) **Medium Vaulting Horse (8 layers)**



(1) Which of the following calculations will determine the total height of the medium vaulting horse when all 8 layers are used? Select from (1) through (4) below and write the number.

(1) $35 + 15 \times 8$

(2) $35 + 15 \times 7$

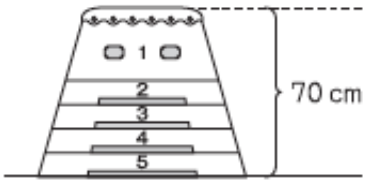
(3) $35 + 15 \times 4 + 10 \times 3$

(4) $35 + 15 \times 3 + 10 \times 4$

(2) Yukari's teacher asked her and her friends to prepare both the small and the medium vaulting horses but make them the same height.

First, they set up the small vaulting horse that they often use to practice with 5 layers. The height was 70 cm.

Small Vaulting Horse (5 layers)



Next, they want to set up the medium vaulting horse so that its height will be 70 cm as well.

Is it possible to set up the medium vaulting horse with the height of 70 cm?

Select the correct answer from 1 and 2 below and write the number. Also, explain why you chose that answer using words and numbers.

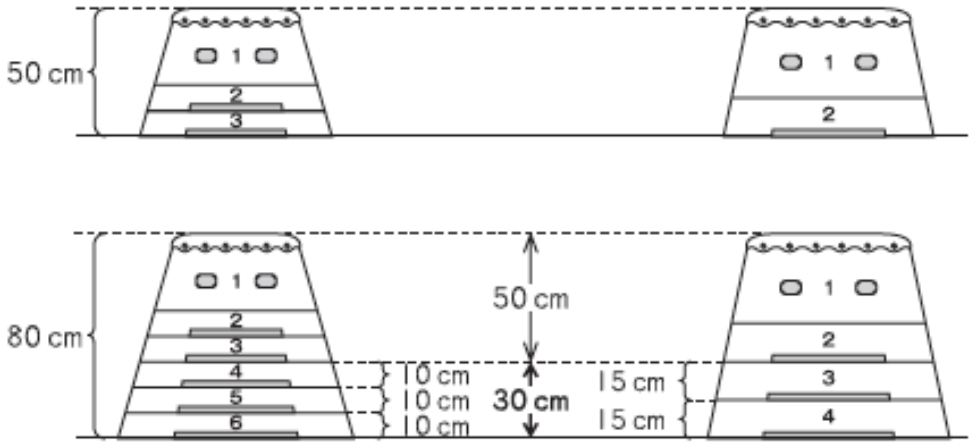
1 It is possible to set up the medium vaulting horse with the height of 70 cm.

2 It is not possible to set up the medium vaulting horse with the height of 70 cm.

(3) Yukari and her friends decided to investigate when the two types of vaulting horse will have the same height.

They found that the heights are the same when there are 3 layers for the small vaulting horse and 2 layers for the medium vaulting horse. The heights were also the same when there are 6 layers in the small vaulting horse and 4 layers in the medium vaulting horse.

Small Vaulting Horse **Medium Vaulting Horse**



The heights when the two types of vaulting horse are the same are 50 cm and 80 cm. Thus, we know that if we increase the height of each vaulting horse by 30 cm from 50 cm, the height will be the same.

Why does the height of the two types of vaulting horse stay the same when the height is increased by 30 cm? Select the reason from 1 through 4 below and write the number.

1 It is because “30” of 30 cm is the greatest common factor of 10 and 15.

2 It is because “30” of 30 cm is the greatest common factor of 15 and 30

.

3 It is because “30” of 30 cm is the least common multiple of 10 and 15.

4 It is because “30” of 30 cm is the least common multiple of 15 and 30.

[3] Atsuko is investigating the area of a quadrilateral drawn inside a rectangle.

(1) She is going to determine the area of the rhombus drawn inside the rectangle whose length is 6 cm and width is 10 cm, as shown in **Figure A**.

If the diagonals of the rhombus are drawn, the rectangle will be divided into 8 right triangles.

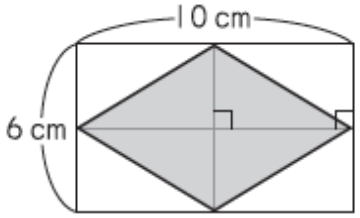


Figure A

If right triangles of the equal area are marked with ○, the result will be as shown below in **Figure B**.

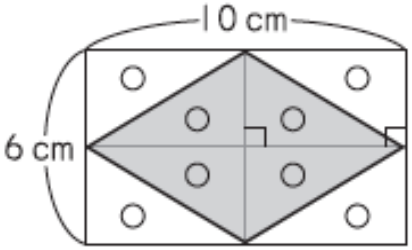


Figure B

If we determine the area of the rhombus based on **Figure B**, it will be as follows.

How to find the area

The area of the rhombus is the total of 4 of the triangles marked with ○. The un-shaded part of the figure is also the total of 4 of the triangles marked with ○. Since the area of the rectangle is the sum of the area of the rhombus and the area of un-shaded part, it is the total of 8 triangles marked with ○. Therefore, the area of the rhombus is a half of the area of the rectangle. The calculation to determine the area of this rhombus will be [(1)], and the answer will be [(2)] cm^2.

In "**How to find the area**" above, write the appropriate calculation for [(1)] and the answer in [(2)].

(2) Next, Atsuko is going to investigate if the area of quadrilaterals drawn inside a rectangle will always have an area that is a half of the area of the rectangle.

As shown in **Figures 1** through **3**, right triangles are drawn inside the quadrilateral constructed within a rectangle. The right triangles with equal area are marked with symbols like ○ and △.

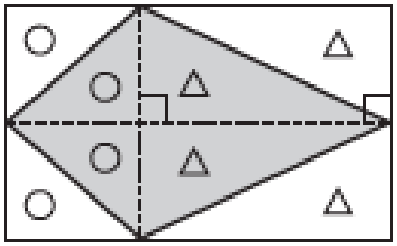


Figure 1

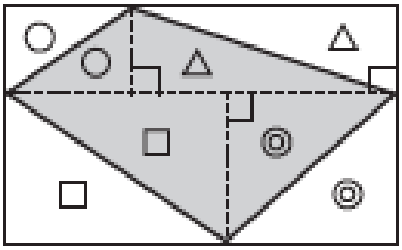


Figure 2

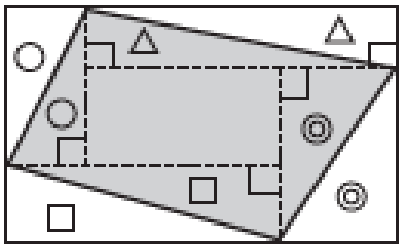


Figure 3

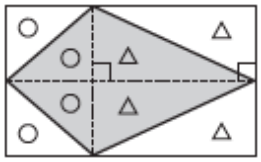
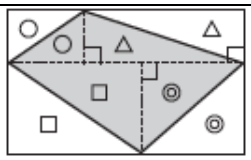
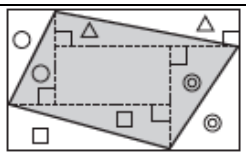
Inside the quadrilateral in **Figure 3**, there is a part with no symbol ( ).

Atsuko

If you examine **Figures 1** through **3**, you will notice that the area of the rectangle is the sum of the area of the quadrilateral inside the rectangle and the area of white (un-shaded) parts.

We are going to investigate the relationship between the area of the rectangle and the area of the quadrilateral by comparing the area of the quadrilateral and the area of the white (un-shaded) parts.

The table below summarizes this comparison.

	 **Figure 1**	 **Figure 2**	 **Figure 3**
Area of quadrilateral	The sum of 2 parts with ○ and 2 parts with △.	A	Thus sum of the areas of the parts marked with ○, △, □, ◎and the area of  .
Area of white parts	The sum of 2 parts with ○ and 2 parts with △.	B	Thus sum of the areas of the parts marked with ○, △, □, and ◎.
Relationship between the areas of the rectangle and the quadrilateral	The area of the quadrilateral is a half the area of the rectangle.	C	The area of the quadrilateral is not half the area of the rectangle.

Sometimes the area of the quadrilateral is half the area of the rectangle and sometimes it is not half.

Atsuko

Fill in the spaces A, B, and C, using words and symbols.

[4] Hiroshi's class will be making rice and potato salad for a home economics lesson.

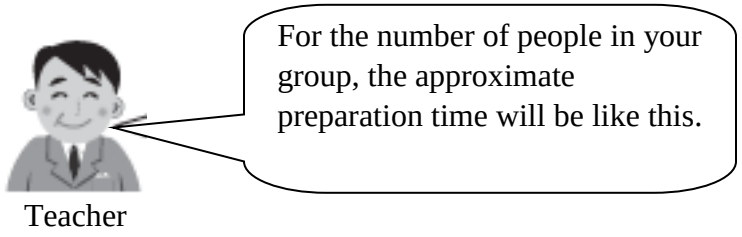
(1) The class is planning how to prepare rice.
Rice will be cooked 40 minutes after it is put on a stove.
In order to have rice ready by 11:30, what is the latest time it should be put on a stove? Write that time.

(2) Hiroshi's group is wondering if it is possible to prepare the potato salad during the 40 minutes rice is cooking.

They asked the teacher about the preparation time for potato salad, and she gave them the following information.

Approximate time needed to prepare potato salad

Step 1 (weigh → wash → cut)	10 ~ 15 minutes
Step 2 (boil → season → plate)	20 ~ 25 minutes



Teacher

After looking at this information, Hiroshi's group decided that they could prepare potato salad during the 40 minutes that the rice is cooking.

If you reason as shown below, we can see that it is possible to prepare potato salad in 40 minutes or less.

> Even if it takes [(1)] minutes for Step 1 and [(2)] minutes for Step 2, we can prepare potato salad in 40 minutes or less.

Write the appropriate numbers in (1) and (2) above.

(3) Now it is time for the cooking lesson. The ingredients and the amount necessary to prepare rice are as follows.

Ingredients and amount per person

Rice ••• 80 g Water ••• 120 g (The weight of water is 1.5 times of the weight of rice.)

Hiroshi is going to measure the amount of rice the group will need.

First, he placed the empty container on a scale, and the weight was as shown in **Figure A**. After he placed rice in the container, the weight was as shown in **Figure B**.

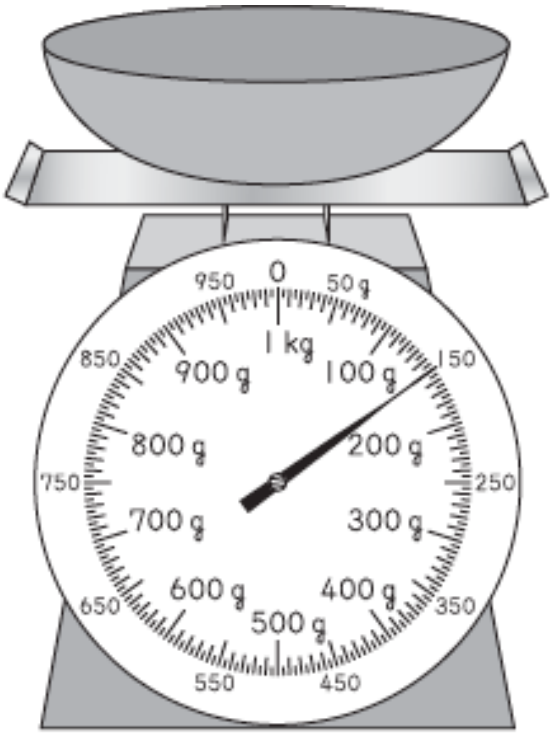


Figure A

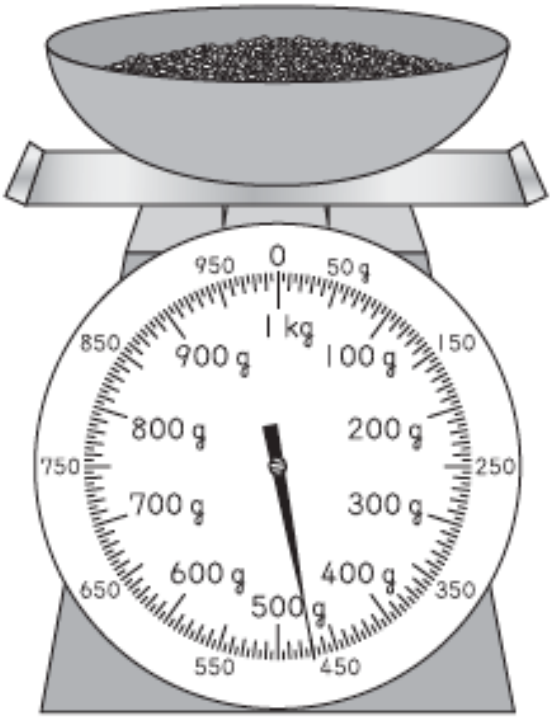


Figure B

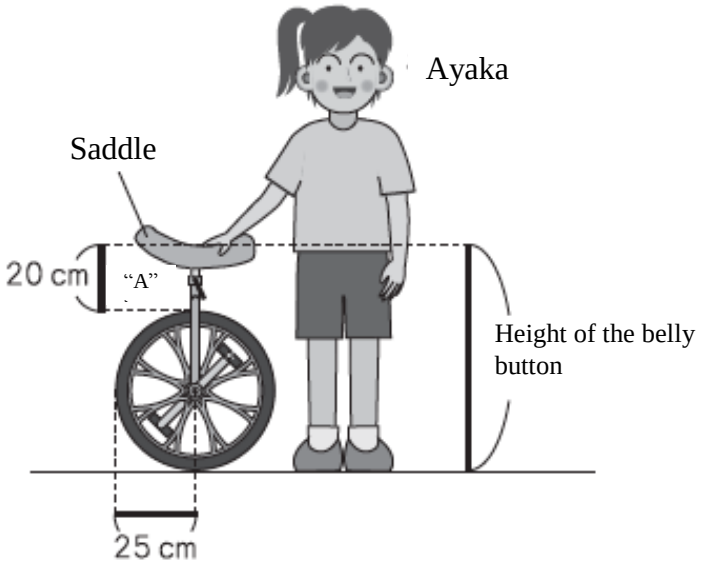
How many g of water will Hiroshi's group need? Write how we can figure out the amount of water needed using mathematical expressions and words. Also, write your answer.

[5] In Ayaka's class, many students enjoy riding unicycles.

(1) Ayaka is adjusting the height of a unicycle.

The most appropriate height of a unicycle is said to be the height of a person's belly button off the ground. After adjusting the height of the saddle, the length of the section shown as "A" in the figure below became 20 cm.

The radius of the tire on the unicycle is 25 cm.



How many cm is the height of Ayaka's belly button from the ground? Write your answer.

(2) Ayaka wanted to know how far the unicycle moves forward when its tire makes one full rotation. So, she collected data on the distance the unicycle traveled and the number of rotations its tire made.

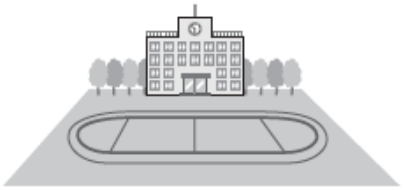
Distance and the number of rotations

Number of rotations (turns)	1	2	3	4	
Distance traveled (cm)	157	314	471	628	

After examining this table, Ayaka noticed that the distance traveled is proportional to the number of rotations made by the tire. She decided to use this fact to determine the length around the track in the school play ground.

When she circled around the track once, the tire made exactly 120 rotations.

Select the calculation needed to determine the length of the track from (1) through (4) below and write the number.



1 157×120

2 314×120

3 120×3.14

4 157×3.14

(3) Ayaka wondered who can ride a unicycle more, boys or girls in her class. So, she asked her classmates and summarized the results as shown in the table below.

People who can ride a unicycle

(people)

	Can ride a unicycle	Cannot ride a unicycle	Total
Boys	9	6	15
Girls	12	8	20

After examining the table, Ayaka said the following.

Ayaka

The number of people who can ride a unicycle is 9 for boys and 12 for girls. So, I wonder if we can say more girls can ride a unicycle than boys.

After hearing Ayaka's comment, Taro said the following.

But, the total numbers of boys and girls are different. Why don't we compare using the proportions instead of comparing the actual numbers of people?

Taro

For boys and girls, we are going to compare the ratios based on the total number of students in each group. Which group has the greater proportion, boys or girls? Select your answer from 1 through 3 below. Also, explain why you chose that answer using words and mathematical expressions.

1 The proportion of people who can ride a unicycle is greater for boys.

2 The proportion of people who can ride a unicycle is greater for girls.

3 The proportion of people who can ride a unicycle is the same for boys and girls.