IMPULS Lesson Study Immersion Program

2014

Overview Report

April 2015

"IMPULS Lesson Study Immersion Program 2014
Overview Report"
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1 Preface

Project IMPLUS is a newly established project funded by the Ministry of Education, Culture, Sports, Science & Technology of Japan since 2011. The Project is housed in the Mathematics Education Department of Tokyo Gakugei University, Tokyo, Japan. The director of the project is Professor Toshiakira FUJII, and the project members include all the faculty members of the mathematics education department—Professors Koichi NAKAMURA, Shinya OHTA, and Keiichi NISHIMURA. Dr. Akihiko TAKAHASHI of DePaul University joined the project as a specially appointed professor. Ms. Naoko MATSUDA also joined the project as a project staff member. The purpose of the project is two-fold. First, as an international center of Lesson Study in mathematics, Tokyo Gakugei University and its network of laboratory schools will help teacher professionals from throughout the region learn about lesson study and will thereby prepare them to create lesson study systems in their own countries for long-term, independent educational improvement in mathematics teaching. Second, the project will conduct several research projects examining the mechanism of Japanese lesson study in order to maximize its impact on the schools in Japan. Under these main purpose, we are working for ;

1) ***Research*** on Japanese Lesson Study to come up with ideas for establishing innovative teacher education systems for long-term, independent educational improvement in teaching mathematics.
2) ***Professional development*** to disseminate ideas for establishing innovative teacher education systems for long-term, independent educational improvement in mathematics teaching. Workshops and institutes would examine how to implement ideas for Lesson Study and innovative ideas for professional development in various schools with different systems and cultural back ground in order to prepare them to create in their own countries' systems for long-term, independent educational improvement in teaching mathematics.
3) Facilitate opportunities for researchers, administrators, and practicing school professionals throughout the region to ***exchange their ideas*** to improve their education systems for teaching mathematics.

The IMPULS lesson study immersion program was designed to give mathematics education researchers and practitioners from outside Japan an opportunity to examine authentic Japanese Lesson Study in mathematics classrooms. The major purpose of this program is for us to receive feedback on the strengths and weaknesses of Japanese Lesson Study and to discuss how to improve mathematics teacher professional development programs. To accomplish this, we invited leaders of mathematics education to immerse themselves in authentic Japanese lesson study, especially school-based lesson study, and to observe mathematics research lessons in elementary and lower secondary grades.

The program started since 2012 and this year's program was held in Tokyo and Yamanashi in Japan from June 16, 2014 to June 26, 2014. In total 15 mathematics educators (9 form U.S., 4 from U.K and 2 from Australia) including mathematics education professors, principals of school and so on. Two of IMPULS overseas support committee, Dr. Makoto Yoshida (President of GER and Director of Center for Lesson Study in William Paterson University) and Dr. Tad Watanabe (Professor of Mathematics Education at Kennesaw State University) interpret lessons and post lesson discussions observed. All lesson plans were translated by Dr. Makoto Yoshida and Dr. Tad Watanabe and distributed before observation. And one external evaluator, Dr. Nell B. Williams Cobb (Associate Professor of Mathematics Education, College of Education, Depaul University) gave us useful feedback with objective evaluation of program. We would like to take this opportunity to thank all of our overseas support and evaluation committee, cooperative schools which kindly welcomed our visiting and all concerned professionals for their hard work.

2 Contents of Program

This program is designed for deeper understanding of Japanese school-based lesson study and it consist of these contents below.

1) Basic lecture on Japanese mathematics lesson and lesson study (1 day)
2) Observation of research lesson and post lesson discussion (7 lessons)
3) Discussion among participants, Q/A and review session

Detailed schedule is shown as below.

Date	Time	Contents
June 16	AM	Opening Session, Workshop: Mathematics teaching and learning in Japan,Lesson Study in Japan,Teaching through problem solving and Kyouzai-Kenkyu
	PM	Workshop: Japanese mathematics lessons and lesson study
June 17	AM	Preparation for the research lesson observation
	PM	<Research Lesson &PLD1> TGU attached school, Koganei Junior High School (Specially Appointed LS for Fuzoku teachers) (Grade 7, Mr. Sho Shibata)
June 18	AM	Cultural exchange with Grade 3 students at the Matsuzawa Elementary School, Preparation for the research lesson observation
	PM	<Research Lesson &PLD2> Matsuzawa Elementary School(Grade 2, Ms. Haruka Miyamoto)
June 19	AM	< Research Lesson &PLD3> TGU attached school, Koganei Elementary School(Grade 3, Mr. Takeo Takahashi)
	PM	Post Lesson Discussion about the research lesson on the day.
June 20	AM	Preparation for the research lesson observation at Sugekari Elementary School
	PM	< Research Lesson &PLD4> Sugekari Elementary School(Grade 3, Ms. Koko Morita)
June 21	AM	< Research Lesson 5, 6> Tokyo Gakugei University International Secondary School(Grade 7, Ms.Hiroko Uchino)
		Tokyo Gakugei University International Secondary School(Grade 12, Mr. Ren Kobayashi)
	PM	Post lesson discussions
June22		Free
June 23	AM	Showa local educational office and courtesy call, Visit Oshihara Elementary School (observe ordinal classroom)
	PM	< Research Lesson &PLD7> Oshihara Elementary School (School based LS) (Grade 4, Ms. Maki Tsuruta)
June24		Move to Tokyo , Preparation for the research lesson observation on June 25
June 25	AM	< Research Lesson &PLD8> TGU attached school, Koganei Elementary School(Grade 5, Mr. Kishio Kako)
	PM	Discussion to wrap up the Lesson Study Immersion Program
June 26	AM	Discussion to wrap up the Lesson Study Immersion Program
	PM	Closing session

Participants made groups to make observation report for each research lesson.

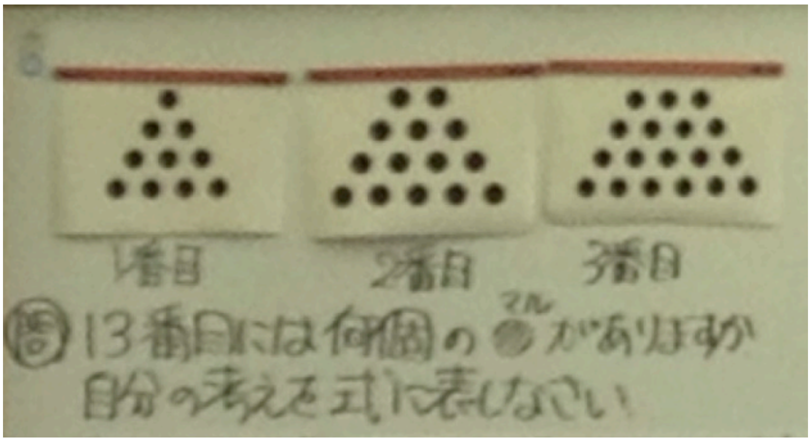
June 17 <Research Lesson &PLD1> TGU attached Koganei Junior High School

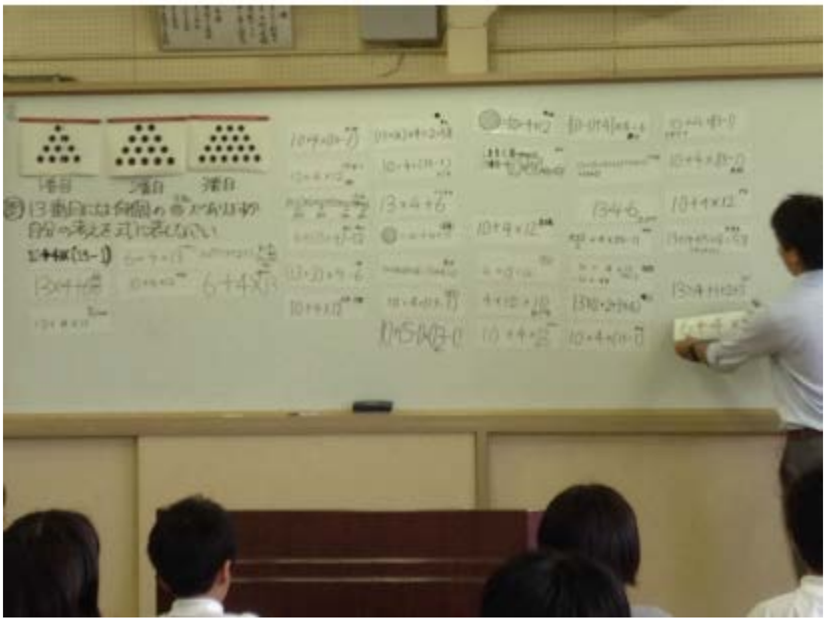
Research Lesson Observation Form (Use photos to document each section)
For Group Facilitation and Report

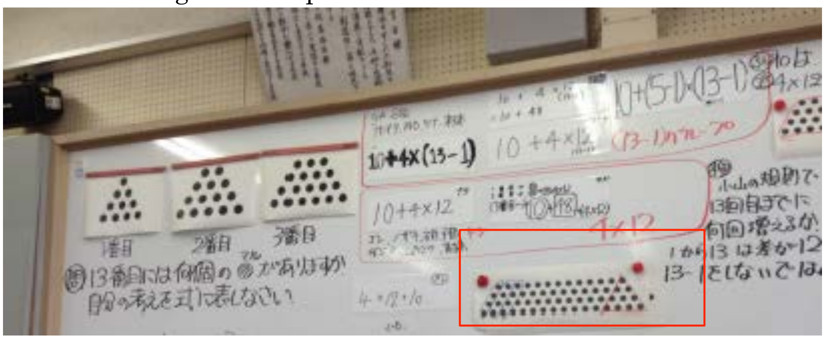
What are the primary lesson goals?

- By interpreting algebraic expressions, students will be able to think about the reasoning and the structure represented by the algebraic expressions.
- Students will think about quantities and relationships/patterns among them, and will try to figure out the total number of dots in the 13th drawing.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
Introduction to the unit; this was the first lesson in the unit of letters and algebraic expressions. Previously at Primary level pupils have used shapes and other quasi variable to represent numbers but this unit is the first time they will have used letters as abstract representations of numbers.

Start &End Time	Lesson Phase	Notes
14.20	**1. Introduction, Posing Task**	**-Strategies to build interest or connect to prior knowledge** -Exact posing of problem, including visuals Used visual display of the first three pictures in the sequence. The teacher wrote the problem onto the board “How many ‘s in the 13th picture?”  He told the class to write their thinking and reasoning in algebraic expressions. Teacher set the pupils off on independent problem solving but quite soon into starting this he reconfirmed the task and that they had to use “algebraic expressions”. He then gave an example on the board of an algebraic expression (not related to the numbers in the task) we assume this was connected to prior learning about what an algebraic expression is. To build interest the teacher gave each student a magnetic strip for them to write their algebraic expression onto then every solution was to be put onto the board.

14.23-14.35	**2. Independent Problem-Solving**	**-Individual, pairs, group, or combination of strategies?** -Experience of diverse learners - Teacher's activities Pupils were working individually, first writing their thoughts into their notebooks then had to write an algebraic expression onto magnetic strips and every pupil put their strip with their expression onto the board. During this time the teacher circulated the room, clarified the question and importance of using algebraic expressions. Many pupils drew out the dots then attempted to see patterns between the numbers. There was a variety of solutions, some saw increasing by four and used that, most starting with 10 as a starting point others starting with 6. Some using 13, others using 12. The mixed responses were all put on the board and the teacher made sure each solution was on the board.
14.35-	**3.Presentation of Students' Thinking, Class Discussion**	**Student Thinking / Visuals / Peer Responses /Teacher Responses** Photos to document chronology (use new box for each new student idea presented)  All pupils' algebraic expressions were put on the board. The teacher then began to organize them based on pupils' feedback. He would choose an expression and ask "is this the same?". Gradually grouping the solutions getting rid of identical solutions if they were similar (e.g. 10+4x12 or 4x12 +10) he would get the class to decide/discuss whether they were the same usually concluding that we cannot say they are the same and that they are different. The final groups were then given names. The teacher named the groups; 13-1 group, 4 x 12 group, +10 group, x13 group, +6 group, -6 group, 1 2 3 group. No pupil input was taken on naming the groups, the teacher named the groups. Teacher started discussion on groups with the most people in them; this was the 4 x 12 group. Teacher asked can you imagine what they did? Teacher used the picture of the 13th pattern for students to explain the thinking.

		**Student 15 (explaining thinking of 4 x 12 group)** 10 is from the first group, 4 x 12 is from 13 dots in the first take one out. She used the diagram and circled rows of 4 dots starting from the left and ended up with the triangle of 10 on the right (see diagram/photos). Triangle is 1 +2 +3 +4. Teacher then asked the rest of the class, do you agree? New student came to explain
		Student 13 10 is the first picture, we are looking for the 13th picture and it follows a pattern. 12 times, which is one less than 13, the same pattern repeats 12 times so the rule is 10 + 4 x 12. Teacher then looked at 4 x 12 + 10 as a comparison.
		Student 37 Used similar diagram to explain that it was 10 + 4 lots of 12 but then said maybe he should have put the triangle on the others side to match the algebraic expressions  This was his demonstration at the bottom right of the photo in the red box.
15.07-15.11	**4.Summary /Consolidation of Knowledge**	Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals. Teacher said he has only had enough time to discuss less than half of the groups. Teacher asked class what is the same and what is difference between these two groups. Student 36 said the difference is 13-1 or 12, 13 is better as looking for 13th pattern. They both are similar in that they look at the same arrangement with 10 as the first pattern. Teacher then summarized that the difference is whether they miss out the step to get to 12 or not. He then highlighted the other solutions, (not yet discussed) which included the number 13. He then asked why are you using 13? Student 33 answered that it is the 13th picture is in the problem, nice to have 13 in the expression. Teacher then probed to see if they could generalize and asked can you find the 50th or 100th patterns using this? Same student answered yes just change the 13 to 50 or 100. This shows initial understanding of substitution for finding the nth term.

What new insights did you gain about mathematics or pedagogy from the **debriefing and group discussion of the lesson**?

Discussion focused on two main themes. One was 13-1 or 12; the other was the starting triangle being on the left or the right. It was argued to be unnatural for the triangle, seen as a starting point, to be on the right as pattern naturally adds to the right so start should be the left but students drew this the other way around. It was also discussed that when solving a problem you should identify what stays constant and what changes. This would have been a good way to introduce the reasoning behind why 6 + 13 x 4 is a better solution than 10 +12 x 4; because it includes the number 13 and can be found by considering what is constant and what is changing. Perhaps this was a missed opportunity. This led to a discussion on quasi variables and their importance. This led us to understand the value of the 13 and that the algebraic expressions demonstrate the thinking behind the pupils answers and which are more profound.

What new insights did you gain about how administrators can support teachers to do lesson study?

We felt that today's lesson demonstrated that whoever is given the role of administrator should be highly knowledgeable about mathematics. They need to have the depth of knowledge to be able to ask insightful questions about the mathematics that occurred in the lesson rather than picking up on superficial details.

How does this lesson contribute to our understanding of high-impact practices?

From seeing this lesson we felt that methods of how to display pupils' solutions need to be carefully considered. In this lesson a lot of time was perhaps wasted organizing the solutions into the different groups. The reason this was done was to ensure all pupils felt involved and that their solution was being considered however we felt that the same advantage could have been reached with a more time efficient way to show solutions.
Overall we felt that depth of mathematical understanding is fundamentally important because when you are making finer distinctions and decisions during the lesson it creates more simplicity for the pupils which encourages understanding of higher level mathematics. The most important thing about doing a good lesson is having the mathematical knowledge which allows you to use pupils' responses in an effective way.

June 18 <Research Lesson &PLD2>   Matsuzawa Elementary School

What are the primary lesson goals?

Using diagrams and words, students are able to think about how to calculate by paying attention to calculations in each place value.

Where is the lesson located in the unit? Lesson # 2

Previously studied topics/Ideas to be studied in the future

- Include topics from lesson 1
- Include topics from lessons 3-13

1. Introduction – Posing Task
    - Introduced problem using projector. Dynamic image showing frogs and tadpoles to build student interest/engagement in problem. Teacher stated there were 45 tadpoles, 27 became frogs, how many tadpoles are left? The problem was clearly presented and it related to a unit of study involving frogs and tadpoles to increase engagement.

The professor thought this was done well. According to him, the goal for this part of the lesson should be for students to develop a sense of curiosity and a desire to learn more.

2. Independent Problem Solving
    - Students worked individually for close to 20 minutesStruggling students used blocks after the teacher invited students who were stuck to come up to the front. Some students use a cherry diagram in their journals. Other students drew place value charts and divided the numbers into tens and ones. At least one student wrote an explanation of his thinking in word form.

3. Presentation of Students' Thinking, Class Discussion:

    - The professor thought she needed to look ahead and think about how students might solve this (anticipated responses). She needed to know more about what she wanted to share.
    - Student #1:
        - Girl: Traded a 10 to the ones place to make 15. She made a mistake and forgot to record 15 ones and sat down.
        - Girl: Took 10 moved it to the ones made 15 and then 15-7 (paused) and then answered 8. 40 becomes 30 then minus 20 = 10. Then 10 +8 =18.
        - Boy: (used magnetic blocks on board) Solved the same way as above.
        - Boy: (used cherry diagram) Solved the same way as above.

        - Our opinion: Limited chance for peers to respond to strategies. And the students shared the same way with different materials.

4. Summary/Consolidation of Knowledge
    - Board Writing: More consideration with board writing plan (banshoo) and more thought about how to share the thinking in a progressive manner, from least sophisticated to most.

- Journals: Students were asked to write a reflection at the end and
- Class Discussion: The discussion wasn't deep enough, didn't allow for comparison of the strategies presented, and the students didn't identify with the strategies presented.

5. What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?
    - The professor reaffirmed the necessity of teaching mathematics through problem solving. Students, as a result, will become better at the procedures themselves, and also become more adept at thinking logically.
6. What new insights did you gain about how administrators can support teachers to do lesson study?
    - The administrators clearly value lesson study as shown by releasing staff for a half day and attending the post discussions. They also supported the teachers by asking for their grade level input that they posted on the board.
7. How does this lesson contribute to our understanding of high-impact practices?
    - Even though there was an attempt to make student thinking visible, it became apparent that there was only one strategy that was highlighted (the standard algorithm).
    - It was clear that thought was put into the sequence of lessons. The prior day involved 2 digit by 2 digit subtraction without regrouping and the day after was another 2 digit by 2 digit subtraction with zero/regrouping.
    - The teacher was effective in introducing the lesson and setting up the students for independent work time (though this was too long and pacing became an issue as she ran out of time), but going forward she can focus on better anticipating student responses and honoring a variety of strategies.

June 19 <Research Lesson &PLD3> TGU Attached Koganei Elementary School

Research Lesson Observation Form (Use photos to document each section)
For Group Facilitation and Report
Written by Katianne Balchak and Richard Cowley

Thursday, June 19, 2014, 3rd period (10:40 – 11:25)
Teacher's Name: Takeo Takahashi, Class: Koganei Elementary School attached to Tokyo Gakugei University, Grade 3, Class No. 2 (35 Students).

What are the primary lesson goals?
There is only one primary goal. "Students will understand that they use division to solve problem situations for finding how many times as much is the given quantity (quantity to be compared) as the base quantity." This is an important aim and if successfully achieved it will mean the students have made connections between division and multiplication.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
This lesson, Calculating for finding "times as much", is the last of nine total lessons in the unit plan. It follows a 5 lesson "Sub-Unit" titled, Quotative division and a 3 lesson "Sub-Unit titled, Partitive division. The students have studied sharing ('partitive'), how many in each group and how many groups ('quotitive'). The students are expected to "expand their understanding of the meaning of division as "number of groups" to "times as much." The goal is that students "notice they are engaged in a solution process similar to quotative division problem situations they learned in previous lessons."

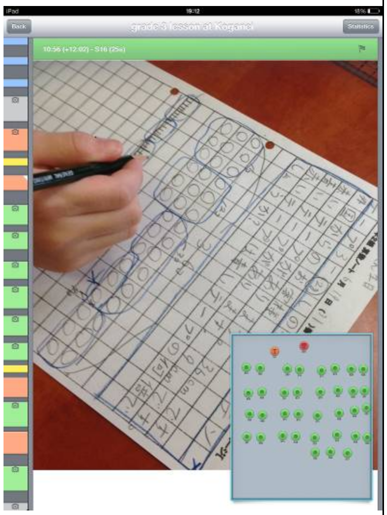
Start &End Time	Lesson Phase	Notes
10:45 10:48	**1. Introduction, Posing Task**	**-Strategies to build interest or connect to prior knowledge** -Exact posing of problem, including visuals 10:45 36cm tape place on board  T: Today we are dealing with tape S: (started shouting out guesses and asking how long is it)? T: It's 36cm 

Start &End Time	Lesson Phase	Notes
10:50		S: Can we draw a picture T: if you like S: how long is that I wonder? T: S: Maybe you are going to ask us how much longer the red tape is than the blue tape… T:Good idea (then writes it on the board – see boardwork The length of a red tape is 36 cm. The length of a blue tape is 9 cm. How many times as long is the red tape as the blue tape? T: (timer on board) Is 6 minutes enough? (Sets timer for 6 minutes). That is a clear statement of the problem.
10:51 10:54	2. Independent Problem-Solving	**-Individual, pairs, group, or combination of strategies?** For six minutes students worked individually as the teacher walked the room with a clip board during post lesson discussion we learned that he wrote notes about what students were writing recorded on a grid (lesson plan). Pictures of students' work follow. **Student 1:**  One possible interpretation of this is that humans involved in any activity beyond their current capacity will, if they feel safe to do so, play at completing the task successfully. We cannot see any mathematical thinking in the approach of this student but socially, we can see participation in a cultural activity; we might call this play. But the participation is mostly in the form of play, not communicating mathematical thinking. This is not necessarily a problem; play might be the first step towards participation in mathematics. -Richard This idea of playing at mathematics is an interesting theory and I have witnessed student behavior to support this idea. Students sometimes "play with numbers" or draw a representation to give at least other students the illusion that they are "working through their

Start &End Time	Lesson Phase	Notes
		mathematical thinking." With a large group of students identified as having learning disabilities this past year, I found that when I asked students to share their ideas with me or with a math partner/classmate they are unable to verbalize their "playing with math" and have begun to say "I wasn't finished with my thinking." -Katie **Student 2:**  This student has drawn a line of 36 circles. Underneath shorthand and calculations show grouping of the circles in nines connected to calculations. The number 1 followed by a small circle in a box followed by an equals sign in a box followed by 9x1 is blocked with the first 9 circles in the line. The number 2 followed by a small circle in a box followed by an equals sign in a box followed by 9x2 is blocked with the first 18 circles in the line. This is consistently continued for 9x3 and 9x4. We interpret this as a notation for, "The first block of circles is 9x1; the second block of circles is 9x2…" and so on. But the first and second and third blocks of circles are increasing in size showing 'times as much' thinking. The student could be thinking, "If we count 1 group of 9, we have 9; if we count 2 groups of 9, we have 18; if we count 3 groups of 9 we have 27…" This looks like 'times as much' thinking. **Student 3:**  This student has squared paper and in one line of squares has divided the square into threes so that 12 squares makes 36 small strips (thirds of a square).Underneath the student is in process of completing 9x1, 9x2, 9x3 (with some other possible Japanese characters we cannot read) blocked against the strips in squares in groups of 9. This does not show 'times as much' thinking as well as the previous example. This is more like ordering the groups of 9 and perhaps this student is thinking, "Here is the 1st group of 9, here is the 2nd group of 9, here is the 3rd group of 9 and so on…"

Start &End Time	Lesson Phase	Notes

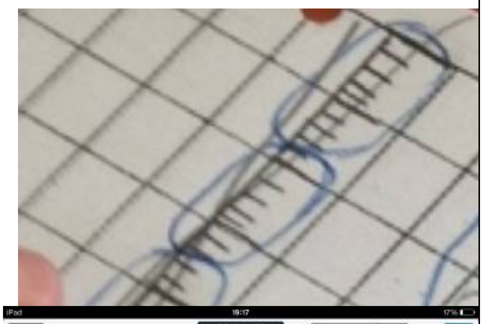
There is very little difference in the appearance of the writing of students 2 and student 3 but the minor difference may reveal a significant difference in thinking. Student 2 is thinking ‘times as much’ but student 3 is thinking ‘groups of 9’.

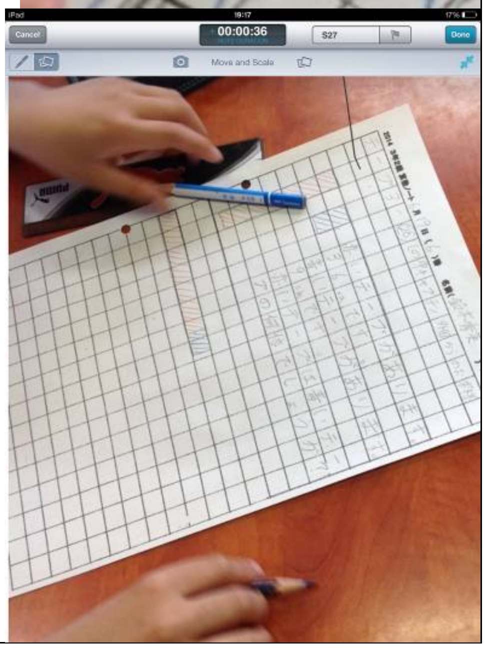


Student 4:

There are two diagrams on the page of student 4. An image of 36 circles is drawn as 2 lines of 15 circles and a line of 6 circles. These circles are blocked into 4 differently shaped groups of 9 circles (two square 3 by 3 arrays and two L-shaped groups, each of 9). This shows an understanding of partitive division and that the representation of 36 circles was probably not planned; this student did not have a sense of 36 being 4x9 before drawing the representation and blocking circles together.

Next, student 4 has drawn out a ruler-style diagram (a scale) and this time blocks lengths of 9 units together. This is the same thinking: blocking in groups and answering the question, “How many groups in?” this is not quite the same as ‘times as much’, which we can see more clearly in the writing of student 2. The students most likely began this representation after the discussion began and sharing of Student A’s(see below) thinking began.





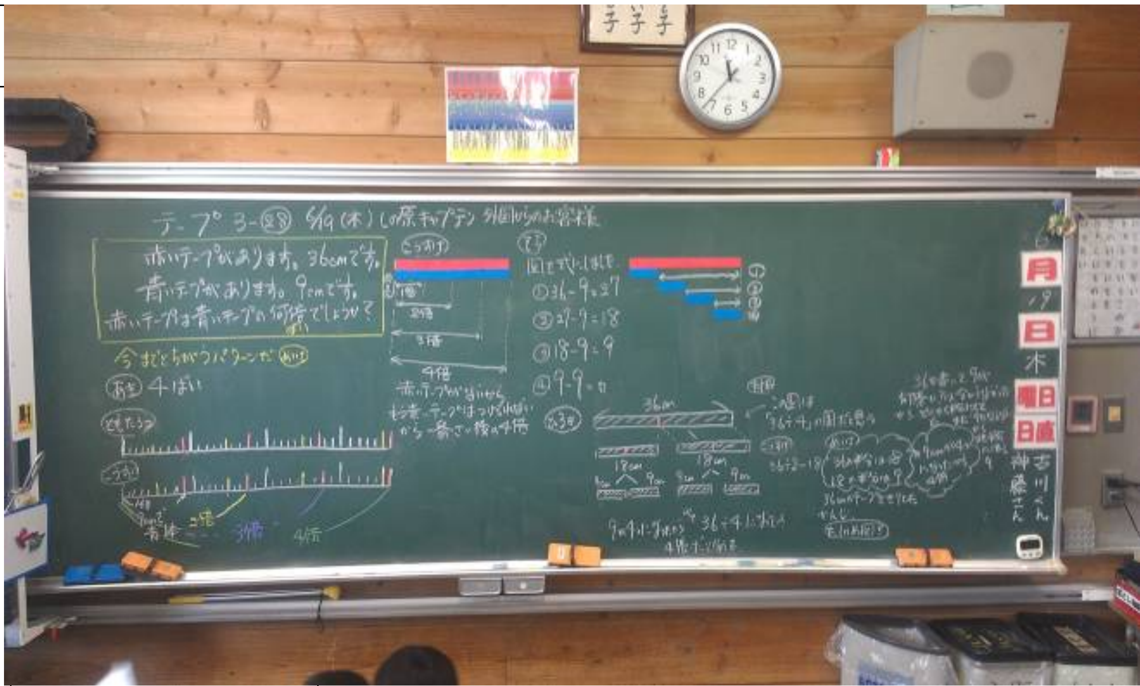
Student 5

Student 5 has some confusion about the scale/proportional relationship between the diagrams of the tape. This is

Start &End Time	Lesson Phase	Notes
		one work sample that we stared at and tried to interpret more than any of the others. It looks like the squares on the page are counted as representing 5 when they are whole but counted as representing 10 when partial. So in the red strip, there are 6 squares to represent 30 (5 each) and then roughly 6 tenths of a square is used to represent 6. The blue strip might be right thinking or wrong; this is not clear.  These five students show various thinking. There are various efforts and points of entry demonstrated as the students attempted to represent the problem situation. During this individual work, the teacher was walking the room using a grid to record a rough idea of what students were doing. I think it would be a very high level of expertise to be able to analyze these student responses at the time so the preparation of predicting student responses is very important.
	3.Presentation of Students' Thinking, Class Discussion	**Student Thinking / Visuals / Peer Responses /Teacher Responses** Photos to document chronology (use new box for each new student idea presented] **Evolution of the board work**
10:58		Timer goes off and teacher transitions to whole class work. **Student A** **T**: (draws line like ruler and some of the students copy this into their books). 

Start &End Time	Lesson Phase	Notes
11:00		There is some discussion of making the scale clear so it is decided colors should be used: white for 10, 20, 30; yellow for 5, 15, 25, 35; red for 9, 18, 27 and 36.  The students applauded this.
11:04		**Student B**    This student has used 4 pieces of blue tape to build up to the length of the red tape. This looks very much like 'times as much' thinking. Some students copy this down. The teacher draws lines to represent 'times as much' thinking and makes a comparison with the ruler representation.
11:10		**Student C**   This comparison of representations allows more discussion of the reasoning behind the representations. The teacher helps to maintain student enthusiasm by listening carefully to them and responding with passion and emotion to their ideas.

Start &End Time	Lesson Phase	Notes
11:14		**Student D** The teacher records on the board the number sentences that Student D has expressed to represent the problem. 
11:16		 Suggestion to use tape to help explain what the calculations mean is taken up by the teacher.
11:19		**Student E** Student E proposes halving and halvng again, which leads to someone calling out, "This looks like sharing." In the post-lesson discussion, we talked about the misconception here. The problem is that this representation is physically sharing into 4 pieces and so suggests the calculation 36÷4 but the calculation we want to get to is 36÷9. 
11:25		

Start &End Time		
11:35		
		So the lesosn comes to an end but the problem is not solved. The answer is to address the misconception directly in th next lesson with the question, "Last lesson we had this problem: The length of a red tape is 36 cm. The length of a blue tape is 9 cm. How many times as long is the red tape as the blue tape? "One student came up with diagrams dividing 36 into 4 pieces of 9cm each and a calculation 36÷4 as a representation of this problem. What do you think of this; good or not?"
	4.Summary /Consolidation of Knowledge	Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals. Because the problem was not solved, the knowledge is not yet consolidated but this goal should not be seen in isolation from other goals. Before the students can consolidate knowledge they must have knowledge that is meaningful and built on their prior learning; that is, their current thinking. This lesson facilitated students to discuss their thinking and compare different representations with respect to this problem. Understanding this concept is difficult so it is worth spending the time on it.

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

The process of unpacking the mathematics and designing the problem so that there is no giveaway of the solution seems important. The thinking is made visible but this is enhanced by comparisons between representations so that students talk about the principles and reasoning they are using. -Richard

Another insight that came from the lesson and group discussion is the importance of purposefully and strategically using number lines in mathematics instruction. During the lesson, a student counted the "zero mark" as a 1 thus resulting in misunderstanding another students' use of the ruler or number line. This misunderstanding of the difference between the number that a "tick mark" represents and the distance between two given "tick marks" is one that seems to be universal. There is much research and debate about how and when number lines should be used in math instruction. What I have gained from this is that the more strategic I am in the planning of using the number line, the more effective, specifically when using it to teach multiplying and dividing fractions. Also, it is worth the time to take the time to address these misconceptions about number lines as they occur. -Katie

Key points from the plan which prompted insights:

"However, students remain somewhat hesitant to share their opinions openly and freely during class discussions." This is of interest to me because I know some strategies that we have had some success with in England to encourage students when they are hesitant like this. For example, asking questions with many possible answers and allowing students to rehearse their answers with each other for a minute before sharing them with the whole class. -Richard

In regards to the hesitation of students to share their math ideas, we are working on some approaches to creating classroom environments in which students feel comfortable and confident with sharing in the United States, as well. One strategy we have been working to implement the past couple of years is to establish norms for sharing and participating in math discussion. We also have tried having students put a finger or two by their chest to indicate they have an idea. Also, when the teacher is walking around as students work independently, in pairs, or team, the teacher may give students a heads up that he/she would like them to share their idea with the class. Again, this gives them the opportunity to mentally or orally rehearse their ideas. I have implemented all of the mentioned strategies in the past two years and I will say it takes a lot of modeling, practicing, and planning to establish these protocols or norms. Ideally, we are moving towards establishing this as part of educational best practices and not just what some "teachers do". -Katie

The phenomenon of reluctance to express ideas in the first instance and the idea of being given the opportunity to rehearse ideas is interesting because it seems to capture a problem and a solution across cultures. I wondered if the idea to 'rehearse ideas' was already in the research and found it in reference to language education for both mother tongue and additional languages. -Richard

I would like to ask the teacher to explain "the difference between diagrams that represent and result from the process of thinking through problem solving and those that are used to explain the result of problem solving." I think this means the diagrams that are drawn/written as the problem is being solved including trials and separately from that diagrams that might be drawn afterwards once the problem is solved to try to explain clearly to others and that these diagrams may be different. -Richard

I have this terminology for division: dividend ÷ divisor = quotient. According to the lesson plan, 'partitive' is the process in answer to a sharing question and calculates the dividend shared by the divisor; and quotitive is the process in answer to a how many times into question and calculates how many times the divisor goes into the dividend. One process shares out the dividend (partitive) and one process multiplies the divisor (quotitive). This lesson plan starts from the processes of working out how many in each group (partitive) and working out how many groups (quotitive) as the two processes. The plan is to deal with the challenge of a conceptual transition from how many groups (counting) to how many times into (times as much). This seems to me to be well thought out and necessary.

-Richard

Prior to this participating in this lesson study, I also had the understanding that dividend ÷ divisor = quotient and that there were two types of division, partitive and quotitive. However, I had not spent much time analyzing what each means and what type of thinking students are using when solve problems of each type. There is a clear difference in the process and need for teachers to understand those differences before teaching. I plan to utilize this deeper understanding of partitive and quotitive in my teaching of whole numbers, fractional numbers, and decimal numbers this coming year. –Katie

I noticed the grade 3 text book uses the phrase 'divide evenly.' Since we have 'even numbers' as a category of integers, to avoid confusion, I would use 'divide equally' as some students may think dividing 'evenly' means dividing into even number. This can result in 18÷2 being answered as 8 and 10 because 9 is an 'odd' number. For quotitive division, the text uses how many people if each person gets so many? And other example like there are 7 flowers in a bunch; how many bunches if there are 21 flowers?

What new insights did you gain about how administrators can support teachers to do lesson study?

Although this was not a formal lesson study, we did get to have a post-lesson discussion with the teacher. I did notice that the principal welcomed us into the school and was present throughout the lesson. This was true of almost every lesson study we observed. These principals were not only present for the lessons but able to knowledgably participate in the post-lesson discussions. This is a piece of lesson study that I would really like to push for in the implementation of lesson study in my school and even district.

How does this lesson contribute to our understanding of high-impact practices?

This question invites a question in response. What are the practices having an impact on? There are pre-lesson planning practices, during lesson teaching practices and post-lesson discussion practices here. The planning practice of predicting student responses captures a teacher's current knowledge, hypotheses and speculations. The teaching practice of problem posing and individual student work tests the predictions so that an attentive teacher might learn something new during teaching. The predictions can be exploited to create a structure for whole class discussions. The discussions might be flexibly adapted depending on the student responses/thinking. The post-lesson discussion might result in the teacher gaining new insights which can be used to predict and structure in the future. This is a high impact on teacher knowledge and capacity to respond to contingency. The individual work that students do is not high impact on its own; you need the whole package; this is what we learn about high impact strategies; they do not stand alone.

During whole class discussion, the teacher makes comparisons between students' representations. This results in reasoning and principles becoming the focus. Because how can we evaluate a representation except by alluding to what it represents? There is something abstract beyond the representation. Having many representations turns the students' attention to the mathematical principles.

June 20 <Research Lesson &PLD4>   Sugekari Elementary School

Research Lesson Observation Form

For Group Facilitation and Report: Kelsey Crowder and Janine Blinko

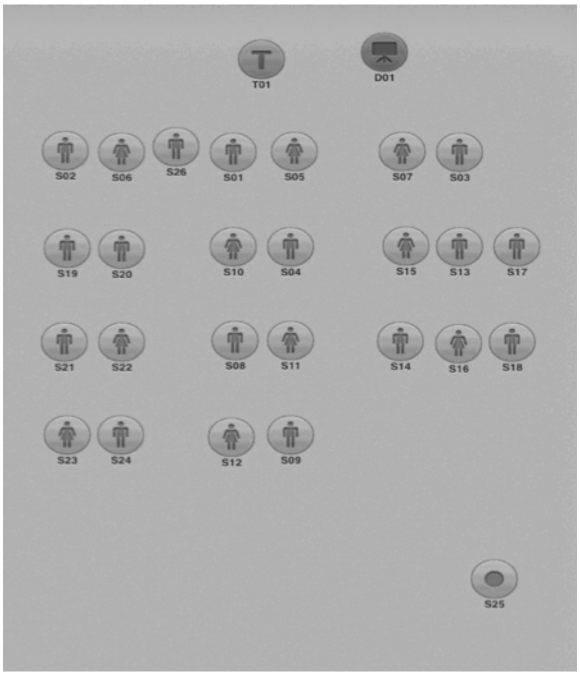
How many packages can we make? How many will be left?
(Division with remainders)
Friday, June 20, 2014 Grade 3 Room 1 (23 students) Teacher: Kohko Morita

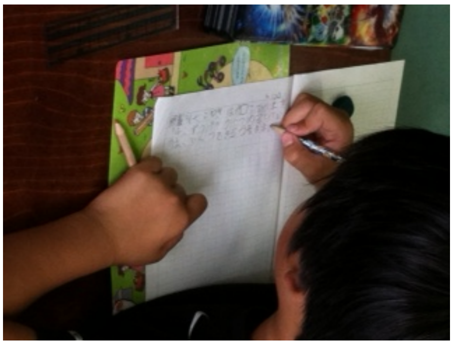
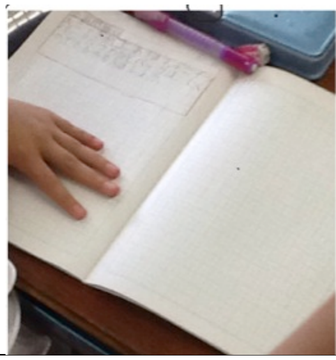
What are the primary lesson goals?

- Students will understand the meaning of division with remainders.
- Students can think about and explain ways to calculate division with remainders using diagrams or by applying the reasoning used while calculating division without remainder.
- Students will examine the size of the remainders and develop a new question about the size relationship of the divisor and the remainder.

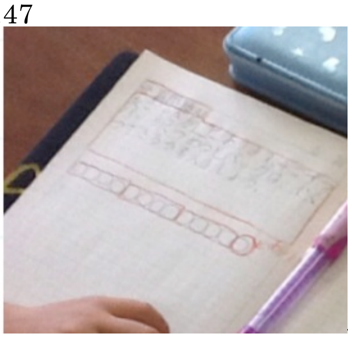
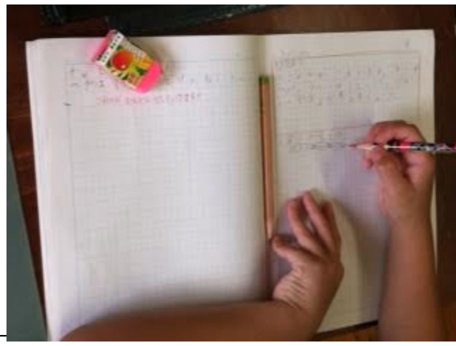
Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

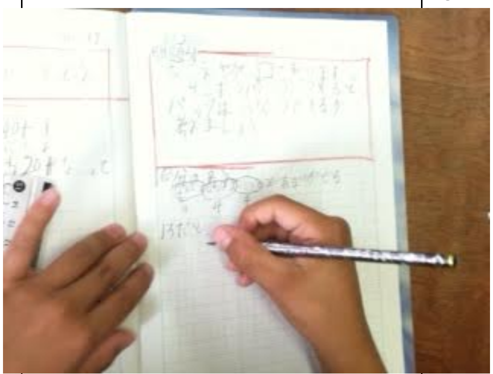
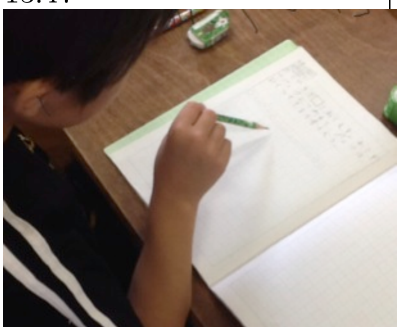
- This is the first lesson in the unit
- Students have previously learned to divide with whole numbers and no remainder using multiplication facts
- Later they will learn the relationships between numbers, the multiplication facts and remainders



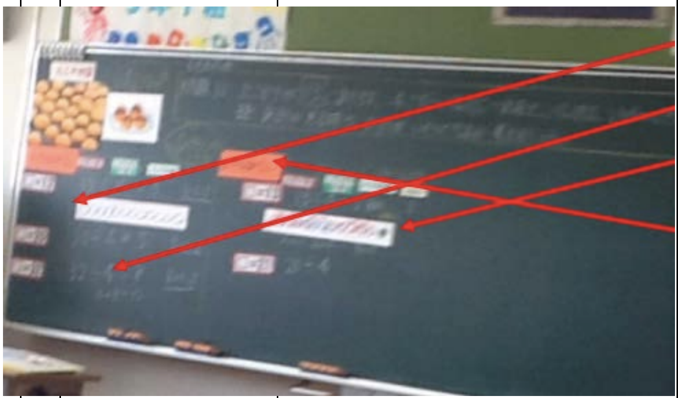
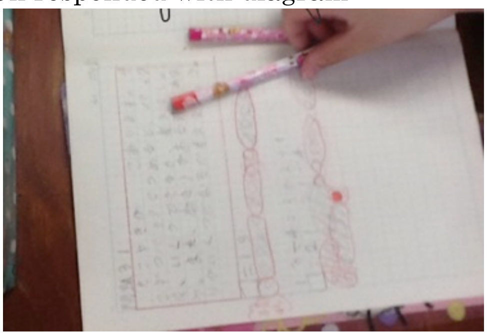
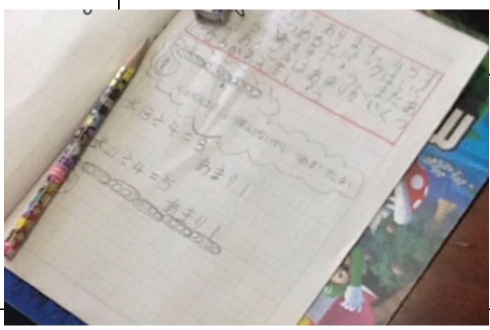
Start &End Time	Lesson Phase	Notes CT- class teacher Ch- students
	1. Introduction, Posing Task	**-Strategies to build interest or connect to prior knowledge** -Exact posing of problem, including visuals • T had learners attention before the lesson began using finger games to engage them while visitors arrived • Learners had all materials ready for the start  • Lesson introduced with discussion about octopus balls using pictures. CT shared problem with students and terminology and enabled the students to interpret the [ ] representation through paired discussion *"Tell your partner what you think it means"* • Wrote on the board for the students to copy "There are [ ] pieces of octopus balls. If we put 4 pieces in a pack, how many packs can we make? • Ch asked to leave a space below the writing and draw the box around the problem   

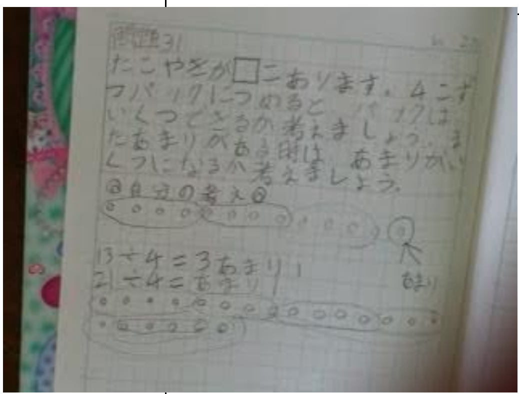
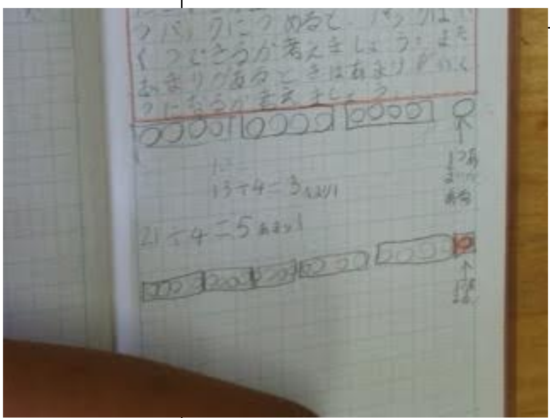
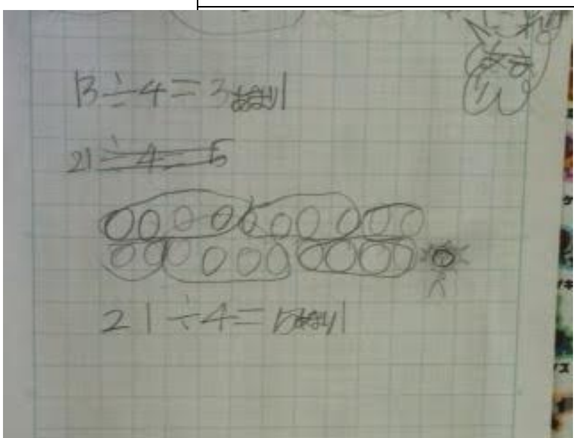
	2. **Independen t Problem-Sol ving**	**-Individual, pairs, group, or combination of strategies?** -Experience of diverse learners - Teacher's activities • (13.35pm) CT explained that they were going to think about what happened when different numbers were put in the [ ]used 'secret' numbers drawn from an envelope. CT then wrote the number in the [ ] and reread the problem CT then modeled recording that ch had met in Gr 2 together with the division sentence 12 ÷4 = 3 (see pic)  • Chidren calling out their answers and then asked to justify them First number 12 Ch ' its 4 because of times 4 Ch ' its dividing' Second number- 20 Ch offer 5 as the answer and asked to justify them CT replaces 12 with 20 in the [ ] but does not draw the diagram • Teacher led discussion to elicit format for division total number ÷ group size and links this to division sentence and records on the board 20 ÷ 4 = 5  • Teacher led discussion to revisit the idea of # of packs vs # in the packs (13.40) Third number- 32 (CT replaces 20 with 32 in the box) Ch offer 8 as the number of packs CT *" what are you doing in your head"* Paired discussion to find out partner view CH " *4 x 8"* CT ' *We use this when we want to do division simply"* (encouraging the use of mult facts to help with division) Students repeat= 32/4=8 CT clarified that students need to say 8 PACKS not just 8 4th Number= 8 CS: Chorus 8/4=2 5th Number 16 CS: Chorus 16/4=4

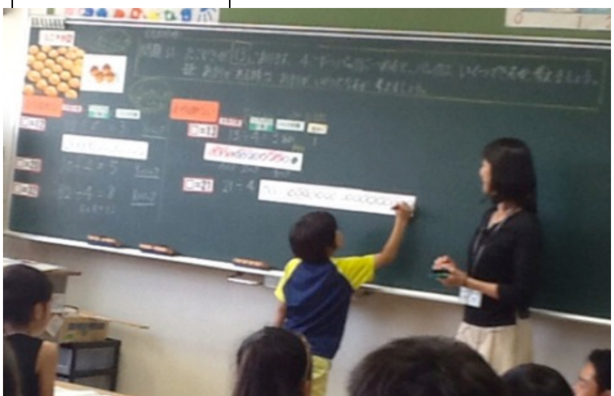
	3.Presentation of Students' Thinking, Class Discussion	**Student Thinking / Visuals / Peer Responses /Teacher Responses** Photos to document chronology (use new box for each new student idea presented]	
		13.44 	6th number out of the envelope- 13 Only 2 students raise hand to answer CT "Whats the operation?" Paired discussion Ch S14 puts up hand to respond CT encouraged response but ch unable to articulate thinking Ch S16 *"It should have been 12* CT "*There will be a remainder* Ch "*Whats that?"*  CT models recording ch met in Gr 2 but extended to remainder asks ch to solve this in their notebook
		13.47 	CH S16 confidently reproduces CT recording leaving a remainder
			Ch Drawing to Solve 

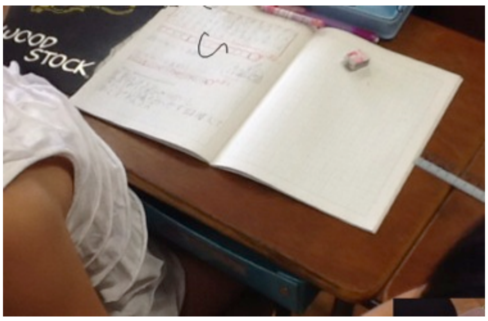
			Ch Drawing to Solve Problem using modeled diagram as a starting point
			Ch Drawing to solve problem
			Ch Drawing to Solve Problem
		13.47 	Ch 214 struggled with solving the problem

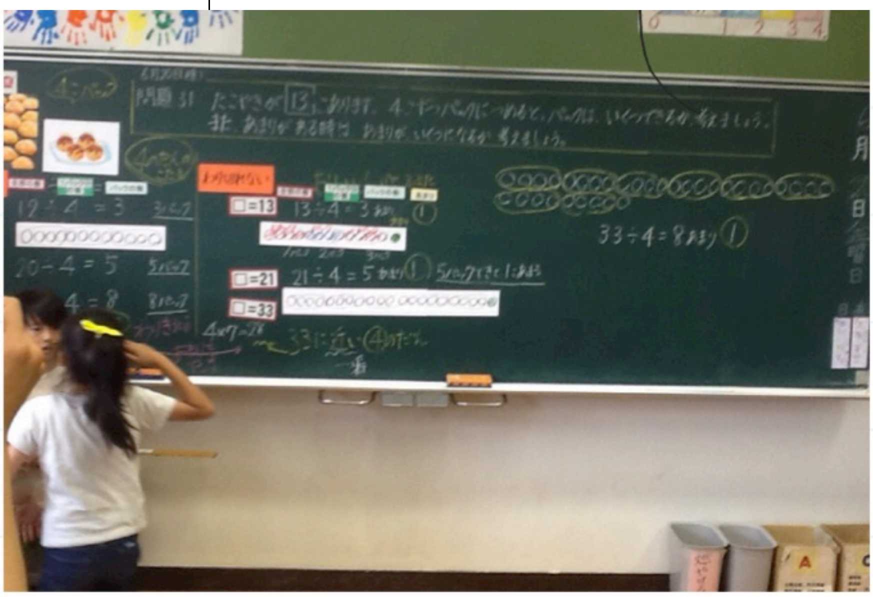
		13.48 	Paired discussion to explain solutions to each other Ch all engaged in discussion- many ch animated in this discussion and using their workbooks to support their discussion
		13.49 	CT "*If you thought your neighbours explanation was really easy to understand please come and share*" Difficult to see on photograph but child S20 draws around circles in groups of 4 and then points to the odd 1 and names it at the remainder
		13.50 	CT "*Does anyone want to add to this idea?* Ch S10 "You have to show the three packs (numbers them) and this one is the remainder CT "Didn't we have 3 packs earlier?
		13.54	Ch S22 *"you have to show that 3 packs makes 12 and then the leftover is the remainder*" CT "*if we didn't have one..?* Ch "*We could eat it*" CT demonstrates how to write the number sentence with the remainder and links it to the diagram 13 ÷ 4 = 3 rem 1 Ct tells ch that we have to add to the word problem, which was what the space was for They add "… and then think about the remainder"

		13.57 	CT led discussion on ‘divisible or not’ Next number – 36 Ch contributed that this number is in the 4s, CT uses the word divisible Ch say “what do you mean” CT explains and introduces the idea of ‘divisible or not’
		14.00 	Early examples Reminder of number sentence 13÷4 = 3 rem 1 example divible/not divisible reminders
		14.01	Next number out of the envelope- 21 Students asked to solve this problem in their workbooks
		14.02	Ch responded with diagram 
			Ch responded with division calc of nearest multiple followed by diagram

		Ch solving Problem using words and symbols
		Ch Solving Problem by grouping in 4's
		Ch Solving Problem.. remainder clearly marked
		Ch Drawing to Solve Problem and recoding using a number sentence

			Ch Solving Problem, each group of 4 octopus balls marked 
		14 	CT invites students to discuss their solutions with their neighbour CH S06 “If you look at the division of 20, you can see that it's the next one, so there is a remainder of 1 CT draws the 21 circles and adds to the board inviting students to come and explain their neighbour’s solution
		14.07	Ch 02 circled groups of 4 as modeled earlier CT “What is he going to say?” Ch explain that 1 is the remainder *CT what is 5?* – 5 packs *What is 1?*- the one piece left over Teacher writes 5 Remainder 1 Discussion establishes that you have to find the nearest multiple below the number Ch invited to show this using the diagram
		14.09 	Next number out of the bag- 33 CT asks ch to solve this in their workbook Ch- I’m pretty tired of drawing circles CT ask ch to stop and draws attention to this comment and asks if the problem can be solved without drawing the circles Paired talk to try to think of a way of solving this without drawing the circles

		14.12	Ch S25 "You have to find the closest 4 to 33" Ch 17 " we have to think about the number of 4s to 33 and use that" Ch S25 " 4 x 7 = 28
		14.14 	Ch S15 says the answer is here and points to the answer to 32÷4 on the board CT "*How can you use this?* Ch S20 " *32 is the closest, if you subtract 28 from 33 you get 5 remainder"* CH: 32/4=8 Close to 33. Answer increased by 1 CH: 32 is divisible. Add 1 and it becomes not divisible Ch "4 x 7 is 28 that's not good CT: Why is 4X7 not good enough? Ch S23 "*There are 5 still left you cant stop here its not good'* *CH: If you subtract 28 from 33 it equals 5. So you would have 7 packs with 5 left over* *(CT draws on board 7 packs with 5 left over)* CT draws 33 circles
		14.18 	Partner talk- "Explain to your partner why this cant be the answer" Ch demonstrates "There are still 5 here, you can make 1 more pack
	4.Summary /Consolidation of Knowledge	Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals.	
		14.20	CT invites students to complete problem and to use a diagram if they wish and to record their findings, learning for the lesson
		14.22 	Ch S16 opts to write arithmetically and with words

14.22 	Ch 14 opts to draw
14.23	Ch invited to share recording and outcomes with neighbour and then with class Ch S07 *Division with remainders can use the fours facts to find the closest and to find the remainder* Ch S01 my discovery is that even without drawing circles we can find remainders using multiplication
	CT summarises learning using key language divisible and not divisible and that some numbers have remainders and some do not Also poses the question about is the remainder always 1? CT "*Next time we will look at the size of remainders*" **Lesson ends**
Post lesson 	Interested students come to the board to talk about what is on there

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- There is a huge advantage in being very intentional about the numbers used in the problem. During the planning process, the teacher changed the numbers to those where the students would be able to use the pattern (4 facts) This enabled the students to begin to develop a deep understanding of the mathematical structure of finding the nearest multiple below the given number, using that to find the remainder
- that the fact that there may be different remainder in different calculations needs to be taught, rather than being an assumed understanding

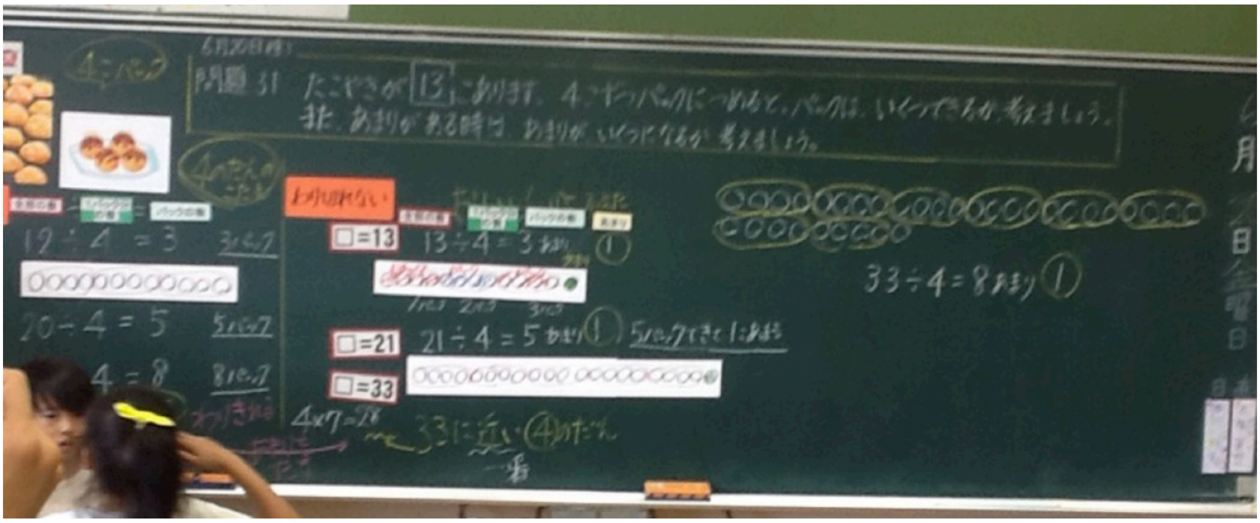
From the final comments...
-It is important to ask students read the question- reading the question is helping students re-check what they wrote
-further engagement could be attained by asking students to have come up to pull the numbers out of the envelope
-there is a query about the use of the equals sign in the equation 13÷4=3 rem 1. Equal sign does show the relationship accurately- maybe we could use an arrow when showing remainders.

What new insights did you gain about how administrators can support teachers to do lesson study?
Interpreting 'adminstrators' to mean leadership staff as well as teaching staff……
-Leaders were very active participants in the whole process, including engagement with the planning process and throughout the discussions. Gives a good sense of collaborative responsibility and cohesion
-Leaders seemed very proud of their lesson study cycles and teachers who were involved
-all staff, whatever their 'rank' held learning and teaching as central to the profession, and took shared responsibility for this

How does this lesson contribute to our understanding of high-impact practices?

- The masterful and planned use of the blackboard which shows developing thinking and progression of ideas contributed to the flow of the lessons. Students who were invited to share their thinking frequently used the board to assist their explanations. Boardwork planning is essential if learners are to use the content for later reference in the lesson (and maybe later lessons)
- Teaching and encouraging students to develop their skills in *meaningful* recording in mathematics enables them to capture their thinking for personal use and for reference, supports them in consolidating their understanding, and gives the teacher a meaningful point of reference for assessing learning and planning next steps.
- Class teacher skilfully framed the lesson so that the students were asking the mathematical questions that needed to be asked, for example "What is the remainder? What does that mean?" "Can we use the 4's to solve this?"
- Written mathematics is used for a range of purposes in the same way as writing is used for a range of purposes, these are skills that children need to be taught early, so that the recording of mathematics is useful, rather than an exercise in neatness.
- Using the end of the lesson to pose a question "Can the remainder be another number than 1?" In this lesson the students recorded their last thoughts in their journals. This will provide a brilliant start to the next lesson. Because the children had recorded their thinking

on this, it would not matter if this ‘next lesson’ was not the next day



June 21 <Research Lesson &PLD5>   TGU International Secondary School

<u>Group Observation - Jacqueline Mann and Kent Steiner</u>
<u>June 21st, 2014-Grade 7</u>

What are the primary lesson goals?

The goals of this lesson, as stated on page ten of the lesson plan, are as follows:

Objective C: Communicating

- Students are able to express the gains using positive and negative numbers, and able to show the gain and loss of Players A and B.
- Students are able to understand the different points of views of the players from the table and are able to read the table effectively.
- Students are able to explain how they think about the mathematical strategy for playing this game in an orderly and logical way.

Objective D: Applying Mathematics in Real-Life Contexts

- Students are able to examine gains in points of each other's decisions or mathematical methods and determine what strategy to use when they need to make a decision about what cards to play to give themselves a mathematical advantage.
- Students are able to think about their own strategy by thinking about and identifying the other players' points of view and outcomes by going back and forth between the two players' points of view.
- Students think about their own strategies using different senses of value, such as "high risk and high return," and "low risk and low return."

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

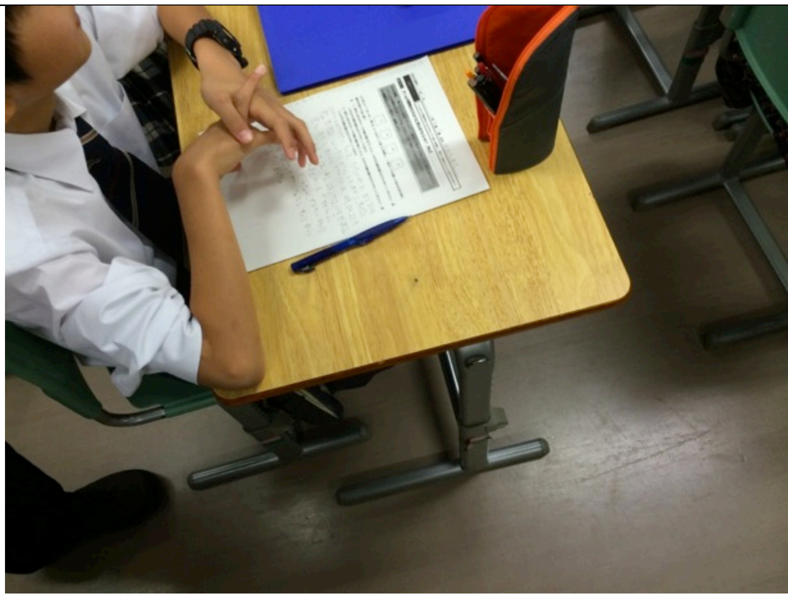
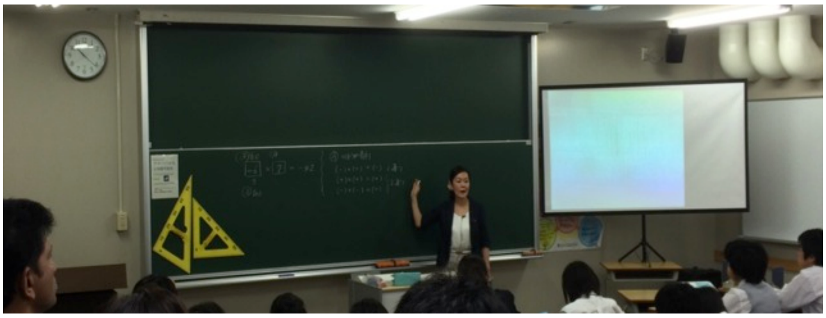
This is lesson 2 of 3 within the unit. In the previous lesson students played the game against their classmates and challenged different students for a total of three rounds. The students recorded their final score for each round and the winner was the student with the highest sum for the three games. In the second (this lesson) students are to think about the different outcomes in terms of gains or losses each player might experience by playing different cards. They are also to use a table of possible outcomes to think about strategies to play the game. In the third lesson they are to compare and contrast strategies.

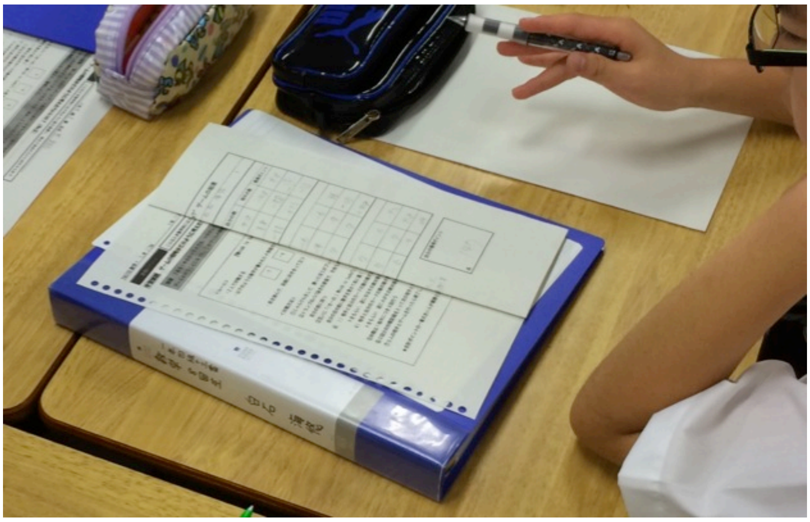
Students will play the zero-sum-like game, provided within the context of a real-life situation; and they will think about strategies to win the game based on their experience playing the game.
Students will think about how to mathematically organize the different cases of gains and examine the methods. Students will also think about the strategies used to play the game using the table created. (This Lesson)
Students think about multiple different ways to play the game and compare and contrast these strategies. The teacher helps students to make connections about what they have learned in these lessons with the real-life phenomena.

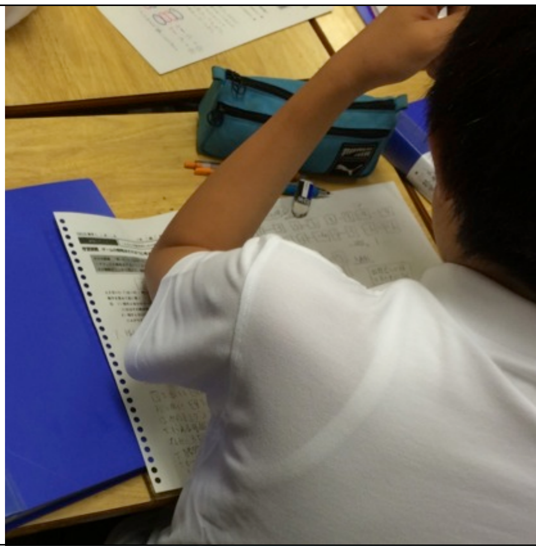
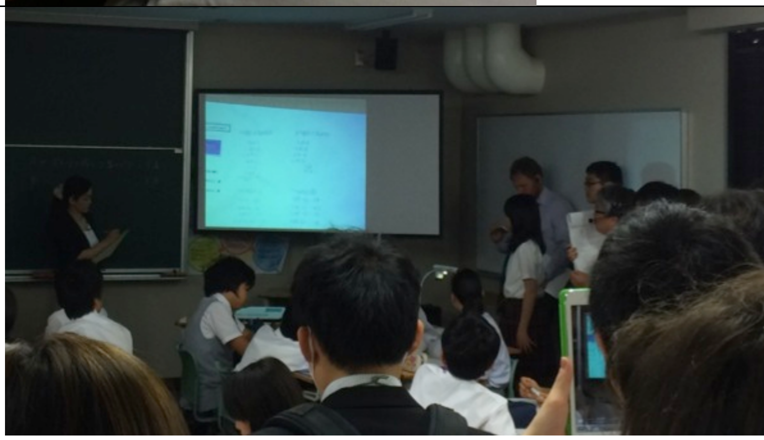
The lesson plan provides the information that the students should be almost finished learning about positive and negative numbers at the end of June. The idea of 'exploration for grasping real-life phenomena mathematically' is one that will continue over the whole of the students' time in secondary education. The lessons were set up to bridge the first two chapters of the textbook.

Start & end time	Lesson Phase	Notes
	Introduction, posing task:	**Strategies to build interest or connect to prior knowledge** **Exact posing of the problem including visuals** As the students waited for the lesson to begin a video of the previous lesson was playing on the projector. Students could be seen engaged in the game and challenging their classmates. This served as a reminder to the students and was useful for the observers in the room. By playing the game the previous lesson students' interest had already been built prior to this lesson. At the beginning of the lesson the teacher announced the winners, and loser, for the previous day. These helped to reengage the students with the previous lesson.  At the beginning of the lesson the teacher expected the students to 'demonstrate little or no conscious effort to utilize an ability to think mathematically'. At the end of the previous lesson the teacher had asked students if they had tried to use a strategy and displayed the results at the start of the lesson. The results supported the teacher's original assessment. In the previous lesson students had played three games against different opponents. Each game had consisted of three different rounds. The students each selected a card and scored points based on their combined choices and whether they were student A or B. When the problem was re-introduced the students were asked to consider 'playing the game one time only and choosing one card only from the four cards, which card should we play?' This was a change from version 1 of the lesson plan. The rewording of the problem posed caused some unanticipated responses from the students. Some students decided they could now ignore the value of the card and simply focus on whether it was positive or negative as the absolute value no longer had any relevance.
	Independent problem-solving	At two points in the lesson the teacher had students work together on the problem. The first time she asked them to think about what card they would choose to put down if they were only playing one card (i.e. a "one shot" game). She asked them to think about their strategy and describe it. Students turned and talked to their neighbours (at this point they were seated in pairs) for a couple of minutes. The second time, after several students had shared their ideas about whether player A or player B would have an advantage and how many combinations of cards existed that would benefit either player A or player B, she had the students get into groups of four or five. She asked them to discuss whether one player

		would have an advantage over the other. Students discussed for two or three minutes. Both formats seemed to be effective in getting students to share their ideas. Students appeared to be deeply engaged and many ideas about strategy and possible outcomes emerged from their discussions. It was difficult, even with simultaneous translation of the lesson, to always pick up the details or nuance of students' responses. Also, from my observation and understanding of the lesson, the goal of the lesson may not have been particularly clear to students. The day before they had played the game with a series of "rounds", rather than "one shot". In today's lesson the teacher changed the focus to what their thinking would be if they could only play one card. However, during the paired discussion time, one student commented, "If I am losing, I'll play smaller cards, like 3 or -4." Another student commented, "I want to be player B and put down the -6 card because I don't think I'll lose as many points." These and other students' comments seem to indicate that they were also still thinking about the game in terms of strategy for playing multiple rounds. Therefore, it is difficult to know whether students were struggling with the thinking, possibly due to their diverse experience levels, or whether they were simply confused or unclear about what they were supposed to be discussing. However, the pairing/grouping and discussion formats seem to offer great value in terms of supporting students with diverse needs. For example, at one point a student shared his or his group's list of all of the possible outcomes they had discovered. Other students commented that there were more possible outcomes because he had forgotten that each card could be paired with it's equal (e.g. 7x7 or -4 x -4). This may seem like a small fact, but much learning happens through the revelation and understanding of many small facts, and in this case the point was central to the discussion of the possible outcomes and what they might mean in terms of advantage or strategy for either player. Other student comments also brought out key ideas related to the game problem, such as the need to look at the decision of which card to play from the perspectives of both players, or that it is important to consider the amount of risk one is taking by playing a given card. Thus, the discussion and sharing likely served to deepen students' understanding of many of the key ideas. I was unable to pay as much attention to the teacher's activities as I did to the students' discussions. I would say that in general, in this lesson compared to most of the others we observed, the teacher in this lesson did not seem to play quite the same role. In other lessons we observed the teacher seemed to be leading the students toward understanding a specific concept and did a good bit of guiding the class toward that understanding through questioning, calling on strategically chosen students, and translating and transcribing their ideas onto the board and thus steering the discussion. In this lesson the teacher simply called on **all** groups to report out after they had discussed in small groups. The groups projected their ideas onto the screen, and very few ideas were

		recorded on the chalkboard. Again, it was unclear to me whether the goal of the lesson was just to generate as much thinking as possible about the "phenomena" resulting from this game situation, or whether it was to have students identify all possible outcomes and use a table to organize it, or identify whether or not a particular player had an advantage or whether there was a possible strategy for winning. The discussion did not seem to have a clear direction, and at the end of the lesson, rather than summarizing some idea (or asking students to summarize what they had learned) the teacher ended the lesson by simply stating "We will continue to investigate your strategies."
	Presentation of students' thinking, class discussion	 During the independent problem solving students wrote down their initial strategies and thoughts. As can be seen here the student has described their thought process using words rather than tables or diagrams. He has listed 8 possibilities, 4 positive and 4 negative outcomes. The student is not yet representing their outcomes as desired by the teacher. I was surprised to see that students wrote in this manner after the elementary school lessons that we have seen. Given their grounding in using diagrams and visual representations I expected far more students to immediately start writing in a more 'mathematical' manner. This could also have been due to the fact that many were educated overseas, as explained in the post lesson discussion.
		 In order to help the students clarify their thoughts the teacher begins a class discussion. Students begin by discussing which player they would prefer to be. The first student discusses how multiplication is

		commutative and that he was thinking about probability and that he considers it is better to be player A. The second student has reduced the problem to positive and negatives and has been able to ignore the value of the card due to the single round. The student demonstrates a misconception when considering the number of ways by failing to recognise that there are two ways to obtain a negative value. The teacher does not address this at this stage but returns to the point later in the lesson.
		 During the group work students discussed and analysed their strategies. Here students can be seen looking back to their work from the previous lesson. Students were discussing and referring to values obtained while playing the game. The video shown at the start of the lesson made it appear that students had just played the game. However here we can see that students did keep a record, although the first table shown here does not appear to be correctly recorded compared to the next two.
		 Here some students can be seen to have progressed in the way that they are recording their results. This student has now represented the data in a tabular format and highlighted the results according to the player that will win. This student was selected to come to the front and explain her method to the class later in the lesson. She stated that she “listed all the combinations. Blue is when A wins and red is that B wins. They have the same chances so there is no difference between those.”

		 However it was unclear how much the students were communicating these thoughts within their groups as a boy on the same table was still trying to compare combinations one at a time. He did not seem to have reached a systematic method to record or process his thinking.
		 Although the students' recorded methods did not seem to progress in the anticipated ways, the students' presentation of ideas did draw out some more anticipated thinking. During the student presentations we heard students considering that if they were A and wanted the greatest number of points they would have to try to score 49 however if they wanted to go for a low risk strategy they should play 3. Students were also still considering that one player may have an advantage. However, many seemed to have reached the conclusion that there was no difference between the two players.
	Summary/ consolidation of knowledge	During this lesson, as in all that we observed, the students used their journals to record their thinking and their work as they attempted to work out their ideas. Also, there was a great deal of class discussion, but, as stated earlier, it did not seem to be as strategically directed by the teacher toward consolidation of knowledge or summary of learning as was the case in most other lessons. The closest the lesson came to arriving at some agreed upon knowledge was that some of the students responded to the teacher's questions about whether player A or player B had an advantage. By the end of the discussion, at least some students had arrived at the conclusion that there were sixteen possible outcomes, half that benefitted player A and half that benefitted player B, although I did not hear the teacher question other students about this to either challenge or confirm it to be true. The teacher did not use the blackboard to consolidate students' ideas, although she did record a few facts relating to the outcomes for different combinations of cards.

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

In the post discussions (including both the one with just IMPULS participants and the one with the Japanese teachers) three different people raised the question of why the teacher decided to have students think about strategies for playing a “one shot” game when there is no true strategy in such a situation. This question highlighted the importance for the teacher of being absolutely clear about the purpose(s) of the lesson and ensuring that the learning activities are aligned to those purposes. For example, of the six lesson objectives bulleted above, the design of this lesson only allowed the teacher and students to address the first and second bullets, because the other four objectives related to determining and explaining the different strategies available to them, which do not apply in a “one shot” game.

What new insights did you gain about how administrators can support teachers to do lesson study?

I was extremely impressed to see a school be able to run a programme such as this on a Saturday. To see the students attending school to take part and the number of teachers who were able and willing to attend shows how valued lesson study is within the education system.

One of the problems I have observed as an administrator is teachers’ reluctance to publicly (i.e. in front of colleagues in a research lesson) tackle particularly “hard-to-teach” standards. It is easier to tackle “safe” standards that students do not struggle as much with and that we already feel comfortable teaching. However, there is not as much to be gained from playing it safe, so school administrators need to ensure that lesson study is a process in which teachers feel safe taking risks. The learning objectives the teacher of this lesson addressed were complex and even a little ambiguously or broadly worded. They are also part of a new set of standards the school will be adopting in full next school year. The teacher should be applauded for tackling them now in preparation for their full implementation next year and for being willing to put herself in the spotlight in the process of learning these new standards. School administrators need to keep in mind that it is not the success or failure of a particular research lesson that is important, but rather what we learn from the process to inform our future instruction.

How does this lesson contribute to our understanding of high-impact practices?

One of the high impact practices is “using blackboard and journals to promote student meta-cognition, reflection, and integration of mathematical ideas.” This was the only lesson we observed in which the teacher did not use the blackboard to fully capture and summarize students’ thinking and to lead them to new understandings. This highlights the value of doing so, because during this lesson some students shared their ideas via the document camera, which projected their writing onto a screen. This was convenient for quick and easy sharing, but resulted in not having a record of students’ thinking at the end of the lesson. This stood in sharp contrast to the other lessons we observed, which all resulted in a generally well-organized presentation of students’ thinking and solutions (skilfully selected, summarized and organized by the teacher) filling the board by the end of the lesson. During the lessons we observed, it was not uncommon to see students referring to what was on the board as they worked and discussed, so the board work served as both a living document/prompt and a summary of the class’s collective thinking. Through its relative absence in this lesson, the importance of strategically using board work became clearer.

June 23 <Research Lesson &PLD7> Oshihara Elementary School

Research Lesson Observation Form (Use photos to document each section)
For Group Facilitation and Report
Grade 4: Let's Make Quadrilaterals Taught by: Masaki Tsuruta

What are the primary lesson goals? Through activity of sorting quadrilaterals from a variety of viewpoints students will attend to parallel asides and understand the properties of trapezoids and parallelograms.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)? Lesson 15 out of 15.

Start &End Time	Lesson Phase	Notes
13:49.17 - 14:00:22 (11 min. 5 sec.)	**1. Introduction, Posing Task**	**-Strategies to build interest or connect to prior knowledge** -Exact posing of problem, including visuals  - Teacher started putting up shapes on the board (a square and a rectangle). He pulled them our slowly and students started to get excited. (High Impact Strategy: encourage students' sense of commitment, interest and capacity to solve challenging mathematical problems.) - Students were saying one was a square and one was a rectangle. - Teacher labeled what students were saying and then put up four more shapes (two parallelograms, 2 trapezoids and one quadrilateral that looked similar to a trapezoid) - Students said quadrilateral when he put up the last four shapes. - Teacher told the students the four shapes he just put up were shapes they made in a previous lesson. (High Impact Strategy: encourage students' sense of commitment, interest and capacity to solve challenging mathematical problems.) - Teacher asked students why they thought the shape was a square? - Student responded that they have two sets of parallel sides. - Teacher asked what about the rectangle. - Student responded the two vertical sides have the same length.

- Teacher asked what other students saw?
- Student responded the opposite sides have equal lengths.
- Teacher asked what is the difference between a rectangle and a square?
- Student replied a square has four equal length sides and a rectangle has two pairs of equal length sides.
- Teacher asked if any of these shapes were surrounding us?
- Students pointed to the white board, a pencil box, posters and objects around the room that were in the shape of a square and a rectangle. Students didn't find as many squares as they did rectangles.

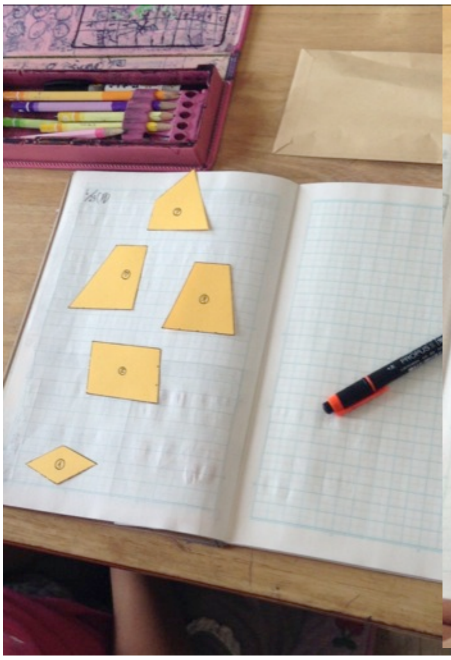
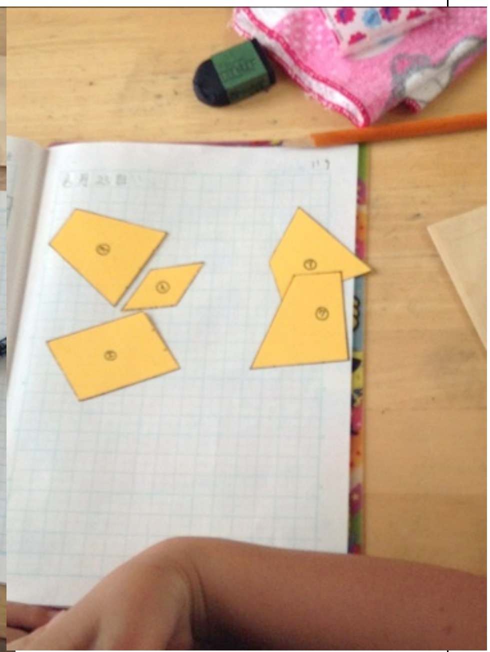
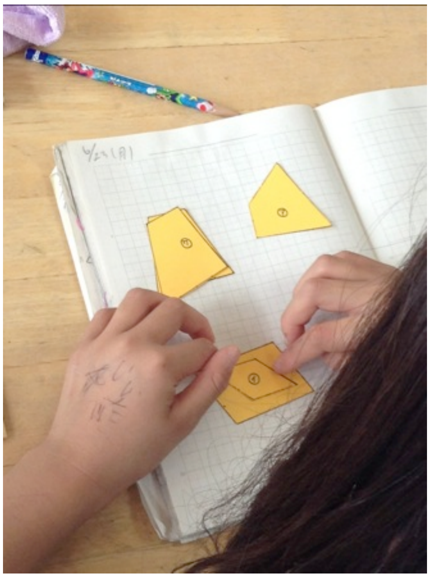
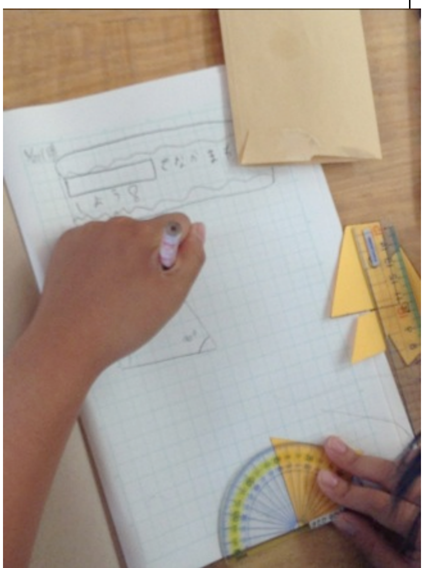
- Teacher asked if students have seen any of the other four shapes?
- Student responded the trapezoid looks like Mount Fuji. Another student said it (the right trapezoid) looks like a slide.
- Teacher stated that we don't see the shapes on the right as much, let's call them quadrilaterals.
- Student said let's give them names.

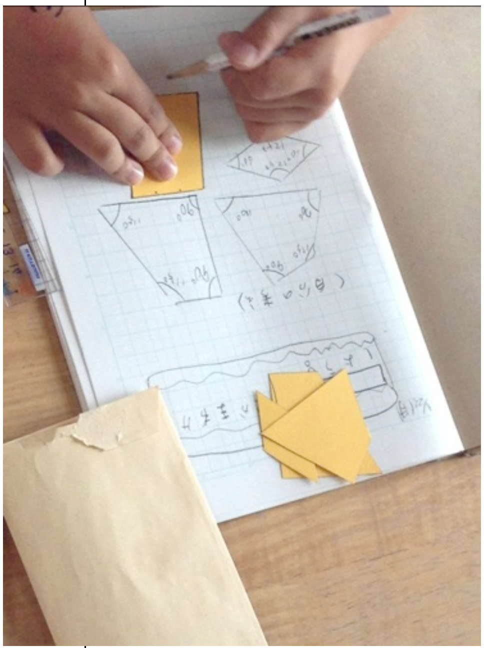
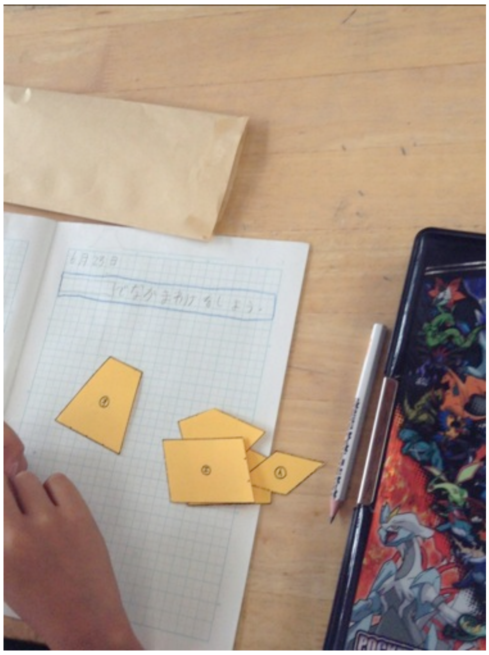
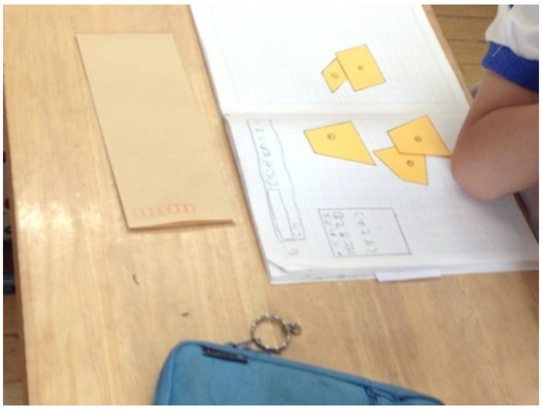
- Teacher asks students what they did in the previous lesson. (High Impact Strategy: Planning for the development of the mathematical concepts over multiple lessons, units and grades.)
- Teacher reminded students that in social studies they sorted trash, if they should burn the trash or not or recycle the trash or not.
- Student said maybe we can sort it by the length of sides.
- Another student said we could break apart the shapes to make squares.
- Teacher responded that he didn't know how that would work.
- Student said you could sort the shapes using parallel and perpendicular. Another student mentioned using angles.

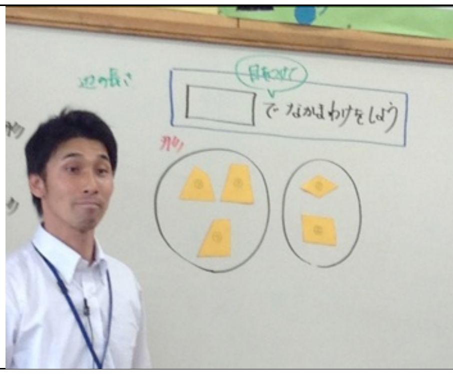
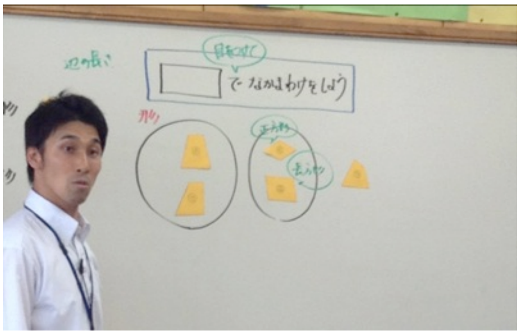
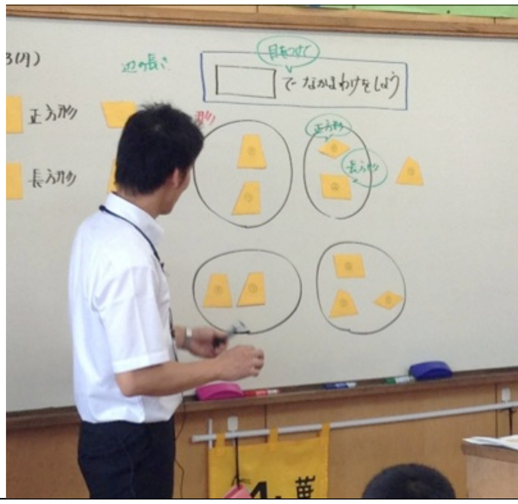
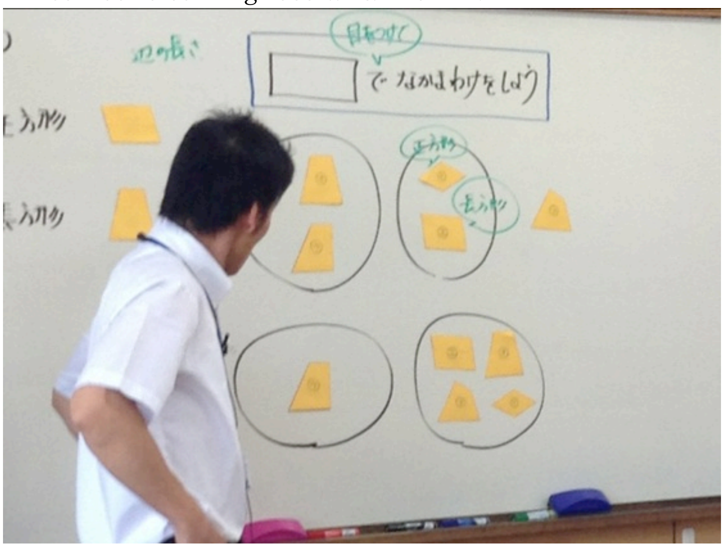
The teacher said there are a lot of things we can use.

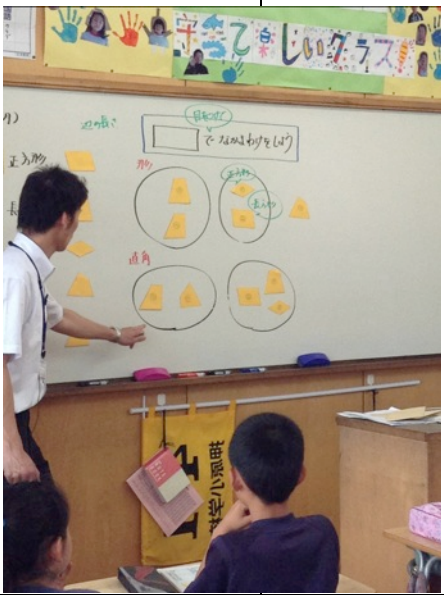
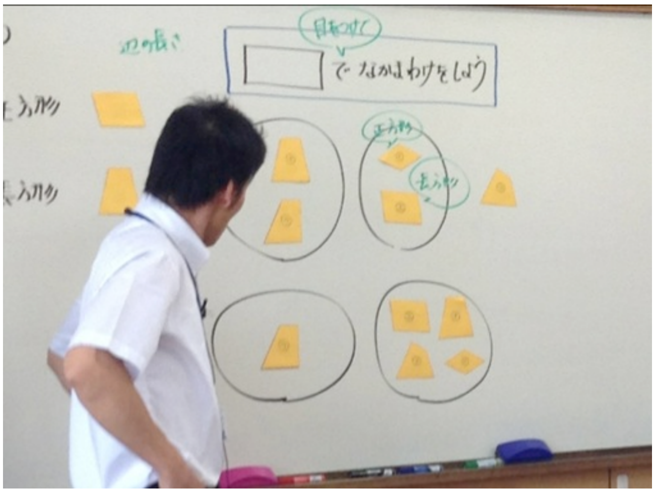
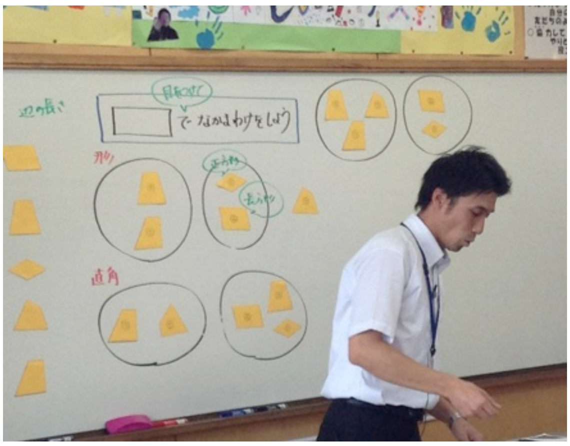
Teacher handed out an envelope to each student. He told them they have the same shapes as him. He then reviewed the task.

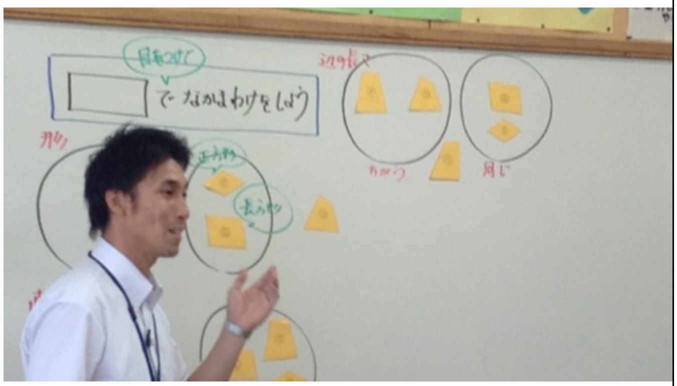
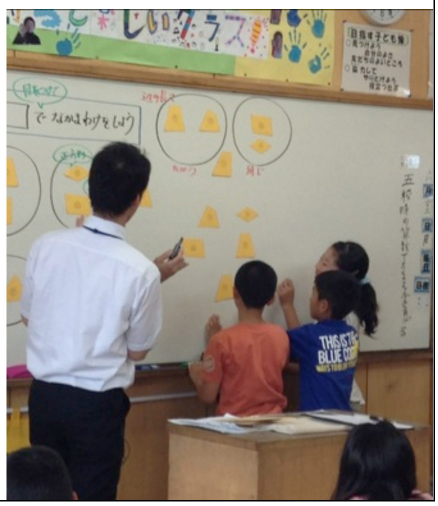
Task: Using _______ let's make groups.

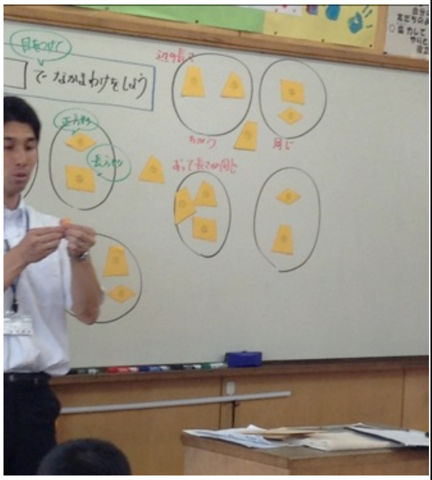
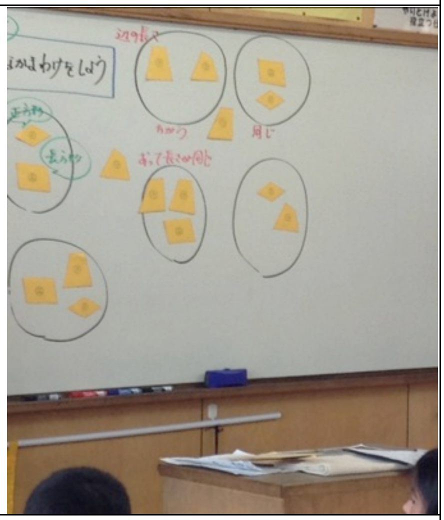
14:00:23 – 14:08:32 (8 min. 9 sec.)	2. **Independent Problem-Solving**	**-Individual, pairs, group, or combination of strategies?** -Experience of diverse learners - Teacher's activities - Students worked individually to manipulate the shapes and sort them into groups based off attributes. Teacher walked around taking notes.   - Student comparing the length of sides. - Student using a protractor to measure the angles.  

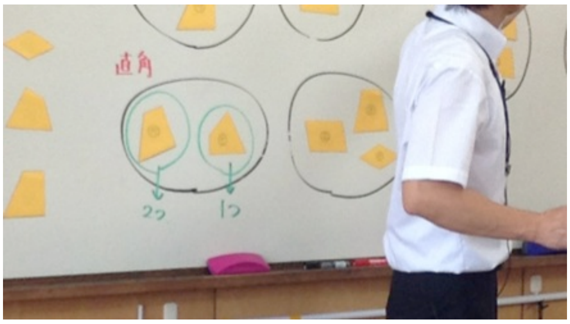
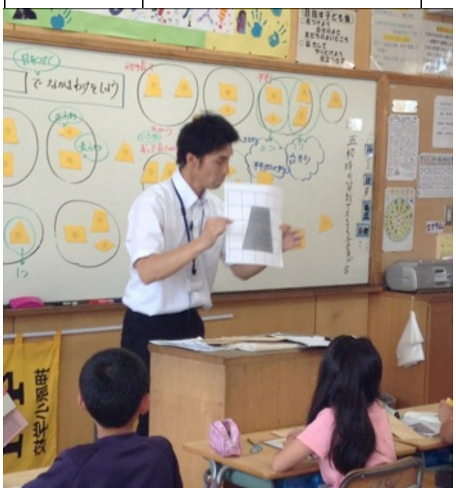
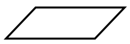
		- Student comparing sides by touching them to each other.  - Student measuring the sides.  - Teacher works with one student individually. He asked how can you use the length of sides? - The student responded that he didn't know.
14:08:33 – 14:37:11 (28 min. 38 sec.)	**3.Presentation of Students' Thinking, Class Discussion**	**Student Thinking / Visuals / Peer Responses /Teacher Responses** Photos to document chronology (use new box for each new student idea presented] - Teachers stopped students and said I want to hear your thinking, it's okay if you aren't done. - A student shared that they sorted them. Teacher asked how they were similar. - A student asked why are there 3 in one group and 2 in another?

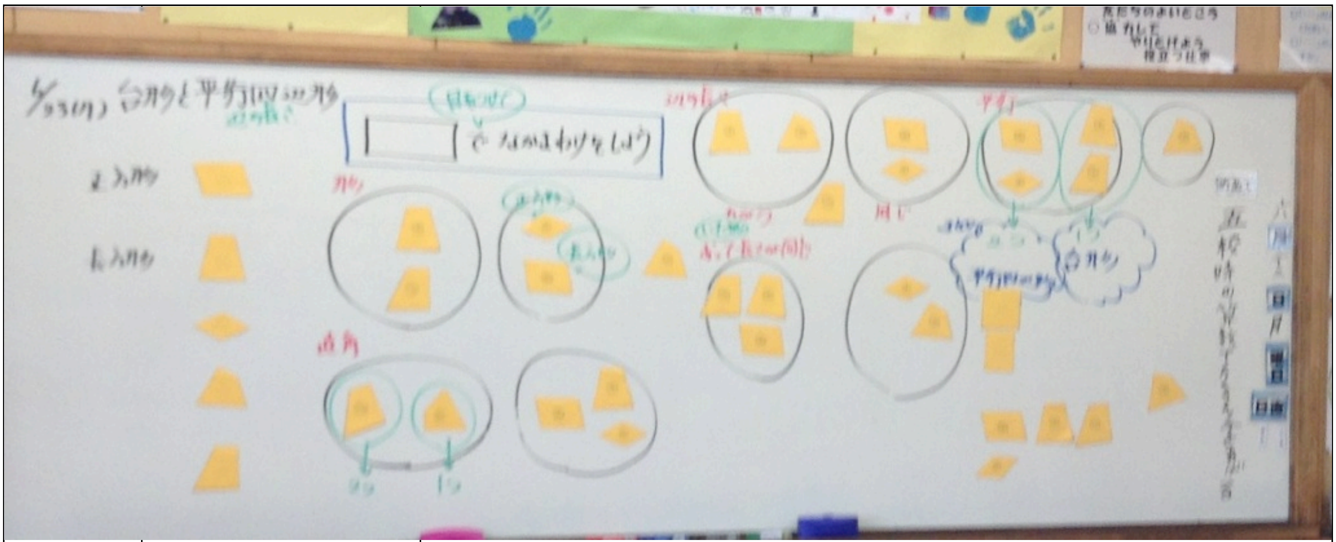
		Teacher asked what the class thought she did. - A student responded if you straighten up the two on the right it makes a square or a rectangle. 
		- A student said he used the same shapes. The teacher said what did he do? - A student responded it's about angels the middle two shapes look alike and one doesn't. 
		- Teacher told the student not to tell how they sorted the shapes, just show us. 
		Teacher asked if someone could guess what he did. 

		- A student came up and switched it. - A student said he looked at parallel sides. Teacher asked where the parallel sides are. - Student responded the top and bottom sides. - Teachers asked the student how he made the groups. The student responded right angles and no right angles. - Teacher asked if there are any other shapes to put in the right angle group. A student came up and changed one. The teacher had to rotate the shape for students to see it was a right angle.
		
		- Teacher asked who sorted it different ?A student put up his thinking and the teacher asked what did he look at? - A student mentioned that it looks like other groups. The teacher stated that two groups may be similar, but the resonaing mught be different. - The student said he looked at the length of sides. The teachers asked him to say more about the length of sides. - The student replied that these two have sides that have the same length (right side), and the others don't. - Another student added that maybe they are parallel and that the opposite sides have the same length. - A student asked, if that was the case what about this one (the right trapezoid). The teacher said maybe it's not necessarily opposite sides that are equal. - Teacher asked students to measure it. 

		- Students measured it and found the adjacent sides were the same length.
		
		- Student rearranged piles. 
		- Teacher asked if anyone tried foldig the paper to sort the groups. Studnets had a moment to try folding paper. 
		- A team of three studnets came to rearrnge the groups. . The teacher asked if they wanted to fold the shapes? - A student said they got two correct. 

		- Student mentioned if you fold the shapes you can se if they have the same length sides. - Teacher asked the student to show the class what she did. She folded it infron of the class and changed the groups. - A student mentioned the three don't match up. 
		- Teacher asked if someone else had a different group. - Student shared and teacher asked what their thought was. 
		- Teacher asked if anyone else had a different way. - Another student said that there are parallel sides. The teachers asked where the parallel sides were. - Student said the one by itself doesn't have any parallel sides. - Another student said they had the same group, but they looked at it differently.
		- Student meentioned she looked at angles. She saw 70 degrees, 90 degrees,… not angels were the same. The other four shapes have atleast a pair of angles with the same size.  - Teacher mentioend that they looked at different ways. The second one doesn't have a right angle.

		- Teacher labeled right angle in a previous sort. 
14:37:12 – 14:39:54 (2 min. 42 sec.) 	**4.Summary /Consolidation of Knowledge**	Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals. - Teacher mentioned that we can give these shapes names. If they have one set of parallel sides they are called a trapezoid and if they have two sets of parallel sides they are called a parallelogram. Teacher labeled it on the board.  Teacher asked where the square would go. Students responded the parallelogram group. Teacher asked where the rectangle would go. Students responded the parallelogram group. Teacher said the square and rectangle both have two pairs of parallel sides. Teacher then held up a picture of another quadrilateral, he tilted the paper, and asked which group this shape would go in? The students responded the trapezoid group. - Teacher then help up a  and asked what group this went in? The students responded the parallelogram group. - The teacher summarized the lesson that trapezoids have one set of parallel sides and parallelograms have two sets of parallel sides. - The teachers asked what the title of this lesson should be. The students responded that it should be called trapezoids and parallelograms.

		
		- The teacher instructed students to write a reflection in their journals about one of their friends ideas.

What new insights did you gain about mathematics or pedagogy from the **debriefing and group discussion of the lesson**?

- First, we gained a new insight that student strategies and solutions should be discussed extensively and incorporated into the research lessons. Teachers should ensure that the discussions are “successful” in advancing student learning. It became apparent during post lesson discussion that a strong connection exist between use of the Japanese problem solving structure (with its associated focus on discussion) and students taking responsibility for their own learning.

- Second, we became aware of new practices for assessing students. These include (i) close observation of student solutions, (ii) recording student solutions and (iii) using observations by other teachers and external commentators. Also, it became evident that we need to augment our reflective practices: ways of recording reflections, discussing the reflections with others, and incorporating them in future planning of mathematics lessons.

- Third, students should be given an opportunity to understand the link between mathematical concepts and daily life. For example, on June 23 at Oshihara Elementary School (Showa Town), the external commentator praised teacher Masaki Tsuruta for having connected the lesson, *Let’s make quadrilaterals,* to daily life.

- Fourth, there is a strong emphasis on structured problem solving lessons across the mathematics curriculum, that is, for teaching through problem solving (rather than teaching problem solving or teaching for problem solving).

What new insights did you gain about how administrators can support teachers to do lesson study?

- First, administrators should take pride in lesson study as a teacher professional development strategy and attend lesson study sessions e.g. lesson teaching and observation, and post-lesson discussion. They may chair the post lesson discussions. Administrators should invite external experts to take part in lesson observation and to sum up the post lesson discussion.

- Second, administrators should facilitate parties after post lesson study for participants to reflect on the lesson study freely. Under the influence of a few bottles of beers, some teachers may bring out significant issues that might not have been raised during the post lesson discussion.

- Third, administrators should facilitate teachers to participate in lesson study in other schools.

How does this lesson contribute to our understanding of high-impact practices?

This lesson study augments our understanding of high impact practices by its emphasis on characteristics like:

- Focus on student learning,
- Focus on content,
- Use of structured problem solving lessons across the mathematics curriculum,
- Linking mathematical concepts to students' daily lives, and
- Embracing new practices for assessing students.

June 25 <Research Lesson &PLD8> TGU Attached Koganei Elementary School

Research Lesson Observation Form—June 25th
Koganei Elementary School—Grade 5—Teacher: Kishio Kako
Comparing with *bai* or "times as much"

Sarah Harris
Leland Dix

What are the primary lesson goals?

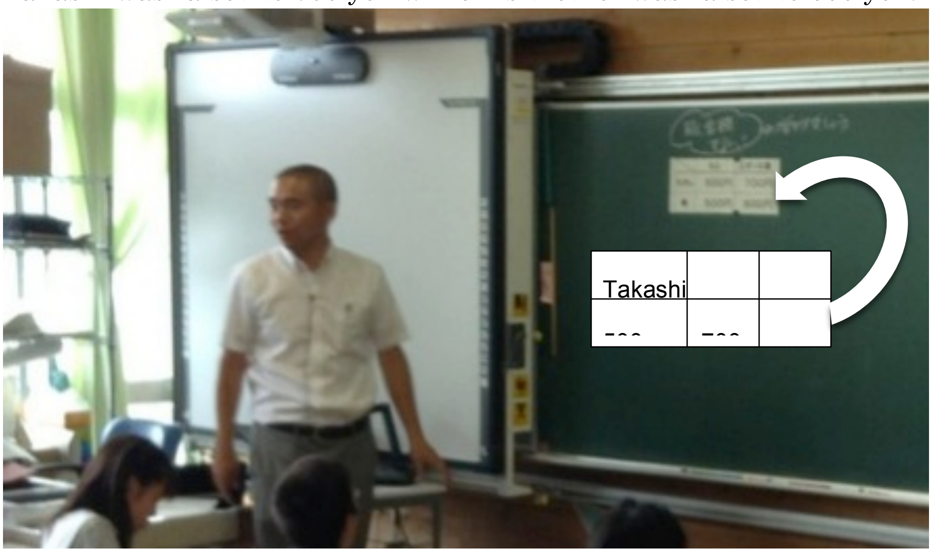
From lesson plan, students will "understand that we can make comparisons using both subtraction and division. By considering the two ways of making comparisons, one based on the difference and another based on bai*, students will understand that the comparison using* bai *is more appropriate when the base quantities are different."*

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

This lesson is the tenth and last lesson in a unit titled, "Division of decimal numbers". Previously, students have calculated a whole number divided by a decimal number, a decimal number divided by a decimal number (more than one and less than one), reasoned about the remainder in these types of problems, and used division of decimal numbers to make multiplicative comparisons.

Based on a task from the Japanese textbook, the teacher "created a new task in which students can more easily consider the two ways of comparison explicitly depending on cases."

In the post lesson discussion, the teacher noted that the concept of warai is difficult for students (and National Assessment test data included in the lesson plan proved this point) and in Japanese textbooks, there are not many lessons on bai before warai so this lesson is for students to experience bai before a formal study of warai.

Start & End Time	Lesson Phase	Notes
10:40-10:45	1. Introduction, Posing Task	**-Strategies to build interest or connect to prior knowledge** Teacher built student interest by bringing up a previous survey about allowance. He asked students how much allowance they receive which got students excited. In a prior draft of the lesson plan, the task was to be about increased prices, or increased bank interest. The lesson's problem is "Takashi's parents raised the monthly allowances for Takashi and his brothers. Whose allowance can we say was raised most?" The problem was posed in two parts (refer to second 3 for the second problem). First, Takashi and his younger brother—both began with 500 yen—and many of the students agreed that was "unfair". Takashi is older and should get more than his younger brother. So the parents agreed to increase their allowance (teacher revealed the new allowance). Takashi was raised to 700 yen while his brother was raised to 600 yen. 
10:45-11:02	2. Whole Class Problem-Solving	Students noticed that now Takashi is receiving more allowance. One student remarked, "it's in the table. The original was the same. Now it's more." Students collectively came to the conclusion that "it's easy to compare if the original amounts are the same". The students took notes along with the teacher and had a collaborate discussion as a whole class. They said "you subtract the original amount from the new amount." 700 – 500 200 ↑ (700-500) 100 ↑ (700-600)

Students also went down the route of subtracting the increases. 200-100= 100 so Takashi got 100 yen more of an increase than his younger brother.

At 10:53, a student said we could use "times as much" (or *bai*). "If we divide 200 by 100, you see it's twice as much!"

- This was a student misconception—the goal of the lesson was to compare original to new amount, finding how many times more the new amount was compared to the original. This student at 10:53 was comparing increases across different students. Technically, speaking in relative terms, Takashi did not receive 'twice as much' of an increase.

The teacher used this as a jumping off point to say "oh, maybe we can use times as much then."

At 10:57, a student in the back came up with the math sentence

700 ÷ 500 = 1.4

Other students explained, "she compared the new amount to the old. She was saying how many times more. Takashi's new amount is 1.4 times his original amount."

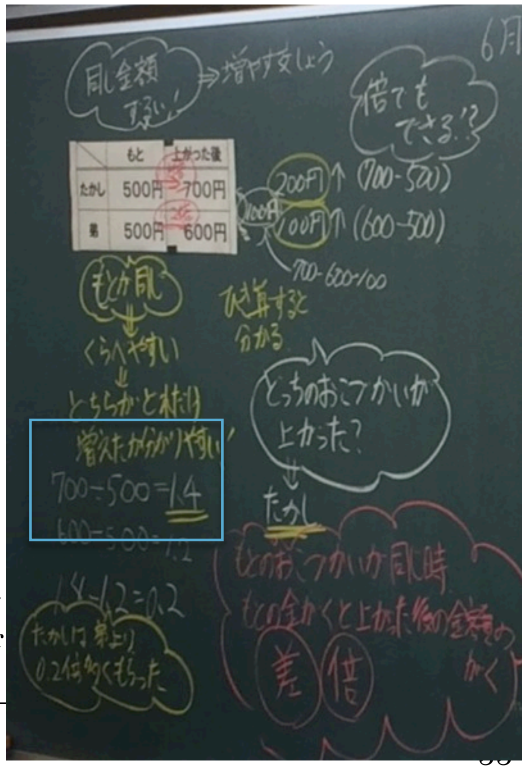
A group of eager students raised their hands to expand on each other's ideas about *bai* and 'times as much'.

At 10:58, an individual student came up to the board to put

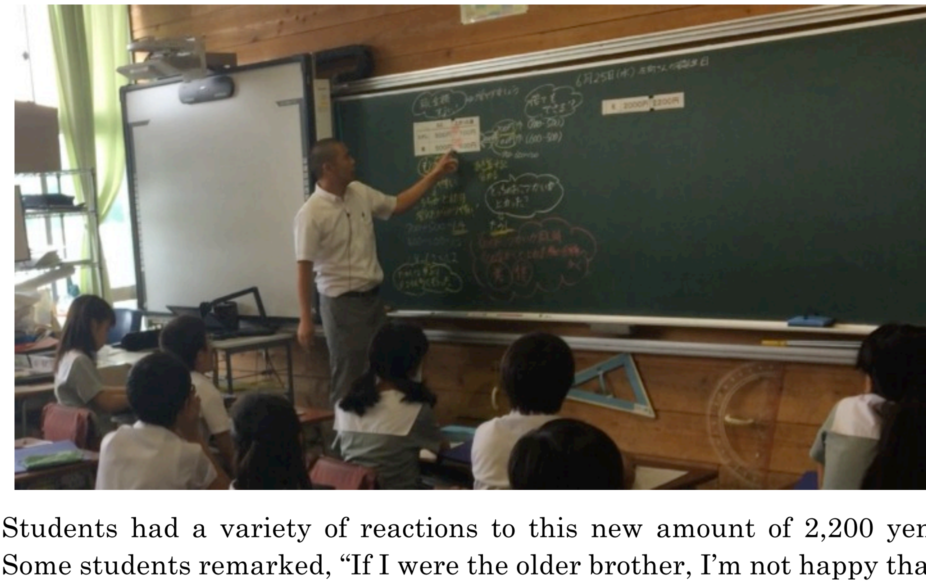
600 ÷ 500 = 1.2

700 ÷ 500 = 1.4

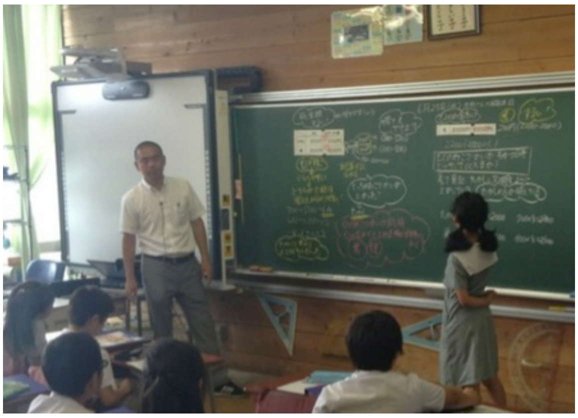
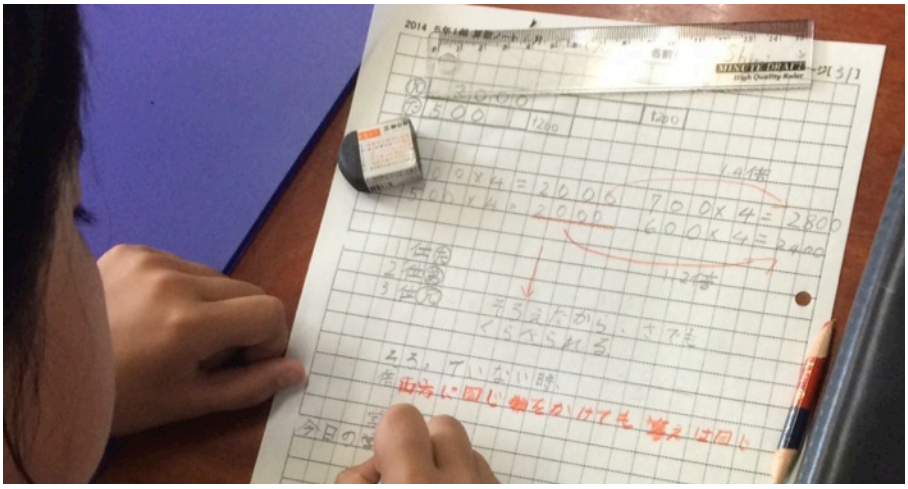
600 ÷ 500 = 1.2

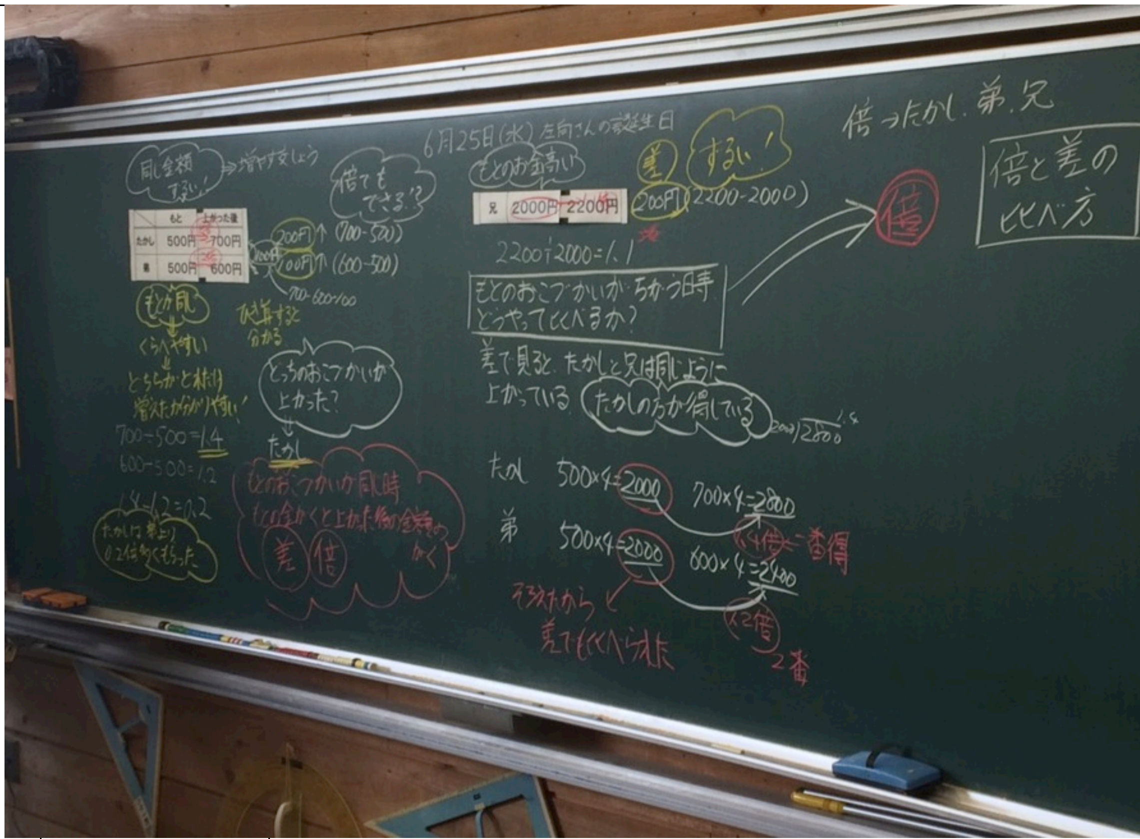


- Then, students turned down another unexpected path. They took the two "*bai*"s of 1.4 and 1.2 and then subtracted them, claiming that Takashi is getting 0.2 times more than his younger brother (which is not true).

		At 11:01, they concluded that "if the original amount is the same, we can use difference or times as much" and circled this conclusion in a pink bubble (see previous picture). Considering the goal of the lesson was "understand that we can make comparisons using both subtraction and division", this conclusion was very aligned with their topic, and made 20 minutes into the lesson.
11:02	3. Teacher posing of task	Secondly, an older brother was introduced. This brother began with 2,000 yen. The teacher had the students think if he should get more, and how much more he should be increased.  Then it was revealed that the other brother's allowance increased to 2,200 yen. Students were asked to compare the allowance increases if the original amount is different.  Students had a variety of reactions to this new amount of 2,200 yen. Some students remarked, "If I were the older brother, I'm not happy that my increase is the same as my younger brother." The task was posed again, "whose allowance can we say was raised the most?"

11:06 – 11:13	2. Whole-Class and Group Problem Solving	The subgroup of 6 or 7 students who have already discussed *bai* with the class were hard at work during this discussion time. Rather than posing this problem and letting students work independently, the conversation flowed and math was revealed quickly. At 11:06, a student from the back said, "the older brother is getting THE LEAST amount of increase. It is only 1.1 times as much!" • This was the major reveal of the lesson, made by an individual student from a subgroup of students already talking about *bai* and *wariai*, during the whole-class lesson. Interestingly enough, this lesson included no independent problem solving. There were two instances where students could turn and talk with the people around them.  These students are now discussing what was revealed at 11:06, that the older brother is getting only 1.1 times as much. Students thought about how to compare with different original amounts.
11:14-11:23	**3.Presentation of Students' Thinking, Class Discussion**	**Student Thinking / Visuals / Peer Responses /Teacher Responses** In response to the question, "what if the original amounts aren't the same?" students had a variety of answers. They realized that you can't just subtract to find the difference, because the increase in both amounts was equal. Takashi 700 – 500 = 200 Older brother 2200 – 2000 = 200 Students decided to use *bai* to divide and see that it was 1.1 x as much. At 11:12 , one student suggests making original amounts the same so we can compare (Takashi and his older brother). Instead of coming up to the board to show *bai*, this student solved the problem of "what if the original amounts aren't the same" by *making* them the same. Using multiplication, this student showed

		Takashi 500 x 4 = 2000 Younger Brother 500 x 4 = 2000 Takashi (new) 700 x 4 = 2800 Younger (new) 600 x 4 = 2400 This demonstrated that “[Takashi] got the largest increase in terms of times as much” because he got 2800 when his older brother only got 2200. Many students write this method of common multiples in their notes.   Then, at 11:16, another student says, "I want to use division" and the students work together again to consider *bai.* At 11:22, a student states, "You still have to use difference with times as much," indicating that when you find how many times as much two amounts are, the *bai* must be compared by the difference.
11:23-11:25	**4. Summary /Consolidation of Knowledge**	The teacher put bubbles on the board to indicate key conclusions reached by students. Students followed along in their student journals.



		At one point in the lesson, a student references what the board says in her thinking/explanation. Students also used their journals to write a final reflection on the day's lesson. They consolidate their learning by writing sentences such as "you can compare new amounts using times as much and the difference."

What new insights did you gain about mathematics or pedagogy from the **debriefing and group discussion of the lesson**?

This lesson was guided by students' thinking and ideas. Teachers often have a hard time being flexible within the lesson, especially if it's not going in the direction that you planned on it going. During the post-lesson discussion, the teacher was asked why he didn't use/show the double number line since it was in his lesson plan as a key tool, and he stated that he did want to use it, but it didn't seem like the students wanted to. He was also asked why there was no independent work in the lesson and he said he felt like it wasn't necessary once the lesson got started and he wanted to follow the natural progression of the lesson and decided to just let it go. The teacher did mention, however, that he might have misjudged stopping for some independent work when a student mentioned using *bai* to compare the allowances. This brings the realization that we, as teachers, have to be flexible, but also make intentional decisions about when to interrupt vs. when to let the lesson flow naturally. This is what makes the process of anticipating student responses such a vital part of the lesson planning process. We also discussed the placement of this lesson in a division unit, and how students walked away with more of a multiplication-based understanding of *bai.* We pondered questions such as "is this the best lesson for today in the unit?" and "what will you do tomorrow/ the future to build off of today?"

What new insights did you gain about how administrators can support teachers to do lesson study?

Administrators can support teachers by allowing for this flexibility in lesson planning. Schools and districts often have specific lessons in a specified order that doesn't leave much room for flexibility. By allowing teachers to plan more flexibly, we are able to meet the specific needs of our students.
Along those same lines, more time for planning would be very beneficial. Planning a beneficial lesson including research and the anticipation of student responses requires much time and effort, something teachers are often in desperate need of.
Administrators and other experienced staff can also help by being an extra pair of eyes and ears—they can script out the *exact* questions (or *hatsumon*) that were posed, keep track of who are answering questions, and even take statistics on talk time.

How does this lesson contribute to our understanding of high-impact practices?

Teachers must make intentional choices of the numbers within our problems. The level of impact a specific problem has on students can be determined by one single number within a problem. For example, in this lesson, the older brother's original allowance amount was 2,000 yen, which allowed students to make the base amount the same, which might have been a bad idea. If the teacher had made the original amount 2,200 yen, students would have had to problem solve about how to compare their allowance increase.

Another important factor is the context of the problem. When deciding whether to compare with *bai* or difference, the context is important to consider. Originally, the problem was a bank situation (deciding which bank had a better return). The teacher decided not to use this context and we discussed whether or not this was a good move. Tad Watanabe brought up an interesting point when he said; "*Mathematically* we can decide when to use bai vs. difference, but *contextually* it might change."

3 Reflection Journals

Adam Bright

From every professional working in the field of education I have discussed Japanese Lesson Study with they have said something along the lines of, 'hands down the best professional learning I have ever been involved in.' It is therefore little wonder that the interest in Japanese Lesson Study in Australia, England and the United States is gaining momentum. The collaboration, increase in teacher content and pedagogical knowledge and the action research teachers experience as being part of a Lesson Study Team make Lesson Study a 'must have' not a 'nice to have'.

Following is my reflection from the 2014 IMPULS Program:

'We will take the best bits out of it and make it our own,' was a view shared by myself and my principal before I embarked on the IMPULS program. Great idea! I thought. Surely we won't need to spend such a long time on this? We are already so busy so how could we possibly justify spending weeks, literally weeks, researching to prepare one lesson? The drawn out process is not a practical way to plan and is potentially a waste of time. The decision was made to steal the best bits of Japanese Lesson Study and make it our own. 'Springside Lesson Study' was the term we coined. Doesn't it sound great? Use the best bits and get rid of the bits that take too long - it will become even better!

So with this idea of creating our own modified version of lesson study at Springside I set of on my journey to Japan to participate in the IMPULS Program 2014. Over the next few days I began thinking about what parts of the Japanese Lesson Study process could be eliminated that would not take away from the success of this professional learning approach?

Should it be the high level of planning for the development of concepts over multiple lessons, units and grades? No, that's pretty important can't get rid of that! Staff analysing professional readings, studying sequences of learning and scrutinising a variety of text books. Seems essential - can't get rid of that.

What about the setting of unit goals? Do we need to spend all that time thinking about what we actually want the students to know, understand and do as a result of the unit. Yeah, that's imperative to better student outcomes, so better keep that too.

O.K then, setting a clear, precise goal for the lesson? Isn't that doubling up? Unit goals and then lesson goals? But you need to have a clear learning intention you want the students to 'get' by the end of that teaching. Wait I'll find something to get rid of soon.

Anticipating student responses, now that must be a waste of time? Why would I work with my colleagues, pretending to be students, trying to think like them? To use it strategically; to plan in advance how to facilitate student discussion and achieve the learning intention of the lesson, my new Japanese friends inform me.

Alright, so what about all the time that's spent with students, explaining their thinking? Why would they spend so much of their time making their thinking visible clarifying their understanding, comparing their thinking to their classmates and linking their thinking with words, pictures, diagrams and mathematical expressions? Ooops I think I answered my own question that all sounds like aspects of a highly effective lesson.

So what about this post lesson discussion part then? We already have enough meetings! Surely we don't need to sit through another one. How wrong was I on this one! Once you sit in an authentic post lesson discussion in Japan and you see the level of professionalism and respect that each teacher displays you are blown away by its effectiveness. The attitude and mind set of Japanese teachers to see this as such a valuable learning opportunity for themselves was amazing. The level of critiquing they shared with their peers was initially confronting for me as I thought about how I would respond to such a challenge of my own teaching practice. Especially when you had worked so hard to put this lesson together in the first place! But not the teachers we observed. They were so eager to learn and hear differing perspectives of the lesson. They took on advice and challenged peers who they did not agree with. The whole time showing the utmost level of professionalism. What a thing to aspire towards in Australia!

Well by now I wasn't having much luck finding any parts of Japanese Lesson Study to get rid of. But what about the knowledgeable other who provides comments at the end of a research lesson? No way! Do not mess with these guys! What a privilege it was to hear the thoughts and opinions of such esteemed mathematicians and educators. The knowledgeable other plays such an important role in helping a lesson study be effective. To have someone with a great deal of experience with lesson study and who can get to the heart of the matter is invaluable. Those moments when the knowledgeable other is sharing their wisdom are precious. You could see in the faces of the Japanese teachers and in how they were scribbling away trying to take note of each word, the importance they placed on the knowledgeable others final commentary.

I had come to the realisation that all elements of Japanese Lesson Study are crucially important. As a whole it works! There is no point tampering with something that has worked for over a hundred years and will continue to be just as effective in the future. When we begin a new lesson study cycle at my school I will keep this in mind and try to instil the importance of each part of the process with my team. I look forward to the challenges that will come with trialling this approach to professional development with my Australian colleagues. And I know that I have developed a great set of buddies to call on, from all corners of the globe, to share these challenges, seek feedback and guidance and celebrate any success.

I hope to do Japanese Lesson Study justice in Australia.

Adam Bright

Gustavo A. Soto

Having taken part in the IMPULS program this summer in Japan I have learned a lot about teaching in Japan and the Lesson Study approach that is implemented in everyday instruction in Japan. One of the aspects that I found to be most refreshing about the Lesson Study approach was the use of a problem solving approach to teaching mathematics. Mainly I found this to be interesting and reaffirming for myself because I've always believed that teaching students to be critical thinkers would lead to students not only gaining a better understanding of mathematical concepts, but that it would also lead to students gaining a better appreciation for mathematics. I was never a proponent for a practice and drill approach to teaching mathematics and this experience has helped to further distance myself from this way of thinking about mathematics instruction. Having observed mathematics instruction in the Japanese classroom where teachers are developing math problems that tap into students' interests and backgrounds I've seen how this approach engages students in the task and how invested they become in solving the problem. I see the importance of developing problems in which all students are given an entry point to have an equal opportunity to succeed in solving a problem.

Another aspect about teaching in the Japanese classroom that I will take away with me is the importance of collaboration in planning research lessons. Too often I've experienced situations where teachers are very closed off from one another and this has lead to inconsistencies on not only the approaches we are utilizing to teach but also inconsistencies in what content we are teaching. More emphasis needs to be put on collaborative planning so that we can all be on the same page and giving our students the same opportunities to succeed. From what I observed in Japan this is one of

the main reasons they are successful in teaching mathematics, content is better aligned and teachers have a better support system than what we see in our everyday teaching. It just makes more sense to have multiple perspectives to not only plan meaningful lessons but to reflect on our teaching practices. This is something I feel we are sorely lacking in our schools and something that I would like to try and improve upon not only as an individual but as a district. Part of why I believe this is successful in Japan is because there is a better attitude among teachers in Japan to open their doors to colleagues and to not take constructive criticism personally. Maybe I am speaking from my personal experiences but I just don't see the same level of camaraderie and enthusiasm as I observed in the Japanese schools that we visited. One of the biggest challenges I foresee for implementing lesson study in our schools is getting teachers to buy into the idea of opening up their classrooms for others to observe their teaching.
Also, making sure that when planning a research lesson that we implement all aspects of lesson study. Something that I've learned from this experience is that in order for lesson study to be most effective taking pieces of it is not the best practice or most effective means for its success.

Another aspect of lesson study that I found interesting and that I will definitely consider more in my everyday teaching practices is the Japanese teachers use of board work. I learned the importance and impact that being more selective about the work being displayed on the board can have on student understanding of concepts. This also ties into the importance of anticipating student responses when lesson planning. When we think about the responses that our students will come up with it makes it easier to scaffold instruction during a lesson so that all of our students can have the best opportunity to be successful with the material. I will also take away from lesson study the importance of being more strategic when having students share out their work. I observed some great situations where students seemed to be leading the lesson with their discussions and ideas. Students were able to build on each others ideas and strategies to build a deeper understanding of the concepts being taught. It will be a goal of mine this school year to be more conscious about giving students more opportunities to share out their thinking and to build a better community of learners that are willing to take risks by being comfortable sharing their ideas. Student journal writing is another aspect of lesson study that I will put more emphasis on this school year in my everyday teaching. I found it to be as important instructional tool that if used correctly can serve various purposes. In my observations I saw journal writing used as an assessment tool where students were able to show their understanding of the concepts being taught through their problem solving strategies. The journals also served as a means for all students to participate in the lesson whether they shared out their ideas or not.
Lastly, the journals were used by the students and teachers to be reflective on the lesson and could be used as a tool to look back on and build on previous content learned. I've used journal writing before but I will have students share out more what they have written, especially when the lesson is done to sum up their thinking and gage their understanding of the concepts.

There is so much that I have taken away from this experience and I hope to be able to implement what I have learned going forward with my educational career. The pre and post conferences that we were a part of were very enlightening and helped to better understand that lesson study model. However, one thing that became very apparent to me was that I need to become better familiarized with not only the content taught in my grade level but in all grade levels. If I want to truly be able to teach to the best of my abilities I have to put forth more effort in reaching out to other grade levels to learn their content as well. As educators we can't be complacent and must continue to learn and adapt to new teaching approaches. I feel so much eagerness to put into practice what I have observed and learned from the IMPULS Program and am thankful for the opportunity to have taken part in the program.

Jackie Mann

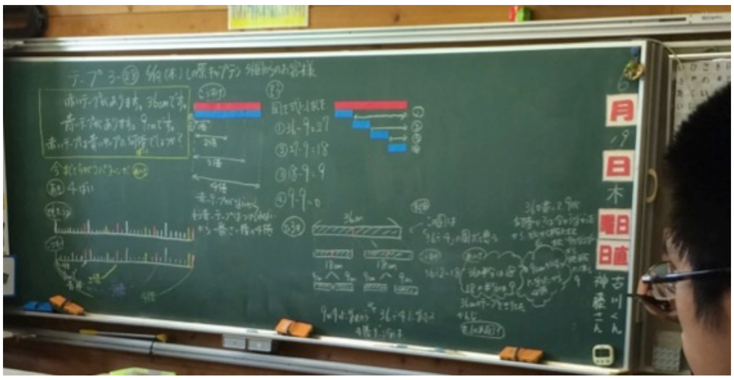
Mathematics Teaching and Learning in Japan

I was privileged to be invited to take part in the IMPULS Immersion Program 2014 due to my involvement in the Bowland Maths Lesson Study Project in the UK. Having had the pleasure of being involved in the rich professional development that is a result of lesson study I was looking forward to experiencing the authentic model. I had heard much about how Japanese students studied Mathematics using problem solving and was excited that I was going to see this first hand.

The program included 8 different lessons across 6 different schools from grade 2 through to grade 12. Each lesson that I observed began with a problem to consider and solve. I was amazed to see that a whole lesson could be spent on a seemingly simple problem. During the grade 2 lesson the class spent the entire time discussing and exploring the methods for solving 45-27. All the problems chosen were accessible, yet had the potential for deepening conceptual understanding. Students understood that the point was not to reach a solution, but to explore the different methods by which a solution could be reached. The first 10-15 minutes of the lesson would be spent on individual problem solving before the class shared their work. Students were not afraid to try solving the problems presented. They seemed confident that they had the tools needed to find a solution and knew that they could use any method that they wished to. This is a complete contrast to the UK where problem solving is considered to involve lengthy multi-step problems that can take considerable time to solve. Due to length of time required to solve a problem there is little time to compare and discuss solutions. In Japan the emphasis was on understanding the thinking of others and, in many of the lessons observed, students were asked to explain the method that another student had used.

It became clear early on that boardwork and diagrams are an integral part of teaching and learning. Teachers carefully plan how they are going to lay out their board for the lesson. The board captures the entire lesson in one place and serves as a progression of ideas. The teacher aims that the board will flow from the simple methods into the more sophisticated methods. By circulating the room during the individual problem solving the teacher can identify the methods used and the students they wish to call upon.

The first methods usually make a greater use of diagrams, starting from simpler representations. This was seen during the lesson 'Times as much'. The problem was stated as 'The length of the red tape is 36 cm. The length of the blue tape is 9 cm. How many times as long is the red tape as the blue tape?'



The first method demonstrated by a student employed a number line and counting up in multiples of 9. Later methods may still employ diagrams but require a deeper conceptual understanding. In the lesson seen students used the tape to employ a repeated addition or subtraction model; as can be seen in the centre of the board. A final method may dispense with the use of diagrams and use a purely written approach. The progression of solutions ensures that all students will have an accessible method for solving the problem and students can also see the next step to improvement. It can also serve to show the links between diagrammatic representations and the equivalent numerical representations. When the students write in their journals the lesson is captured as a whole in front of them making it easier for them to record the flow of the lesson.

Diagrams were used in most of the lessons observed. The use of diagrams is introduced from grade 1. Studying the textbooks you can see that the progression for the use of diagrams is well thought out.

For example, a diagram that will be needed in grade 6 is introduced in a simplified manner in grade 2. The correct use and context of diagrams is considered and students know where to use each diagram correctly. I was impressed by students correctly using diagrams and their precision in labelling them. Students become intuitive in their use of diagrams thorough repeated and frequent use. This ensures that Japanese students are able to grasp problems that their Western counterparts would struggle to solve through their inability to visualise the problem correctly. Students are also far more advanced in their use of correct terminology which introduces phrases such as partitive and quotitive division at an early stage.

Mathematical reasoning was a key aspect in most of the lessons and research themes. Even at grade 2 it was argued in a post lesson discussion that students should have been looking more deeply at the reasons behind their methods and not just the methods themselves. Japanese students are well versed in having to explain not just their own thoughts but why another student may have employed the method that they did. It was wonderful to witness this culture of sharing being built during a grade 3 lesson. When a student became shy the teacher gently approached the student, she crouched down in front and lowered her voice to help the student feel secure in participating. In almost all of the lessons observed the learning was a collaboration of ideas and reasoning that moved the class forward together rather than a result of teacher instruction.

On several occasions during the program misconceptions as learning points were discussed. Misconceptions are considered when discussing which numbers should be used in problems in order that they can be drawn out and discussed. They can also be deliberately introduced in order to give an opening for the following lesson in a unit (division with remainders). In the 'Times as much' lesson a significant amount of time was spent on one students misconception with the teacher skilfully questioning until the student realised his own error. Misconceptions are not seen as a problem that needs to be fixed. Instead they are viewed as an opportunity to deepen understanding.

During the lessons and post lesson discussions that I observed, it became clear that the three levels of teaching are at the heart of the Japanese education system. Everything I saw stems from a desire to achieve that third level not just for the teacher but for the benefit of all students and education as a whole.

<u>Japanese Lesson study</u>

A research lesson is a desire to learn in greater depth about how to teach and learn a particular concept. The results of the research are shared in the form of an observed lesson within a community, whether school-based, district or wider. The lesson is then examined and critiqued in a post lesson discussion. A research lesson is not intended to be a demonstration lesson.

The lesson study process begins with research using the available textbooks and current teaching strategies from various sources. In the UK reliance on textbooks can be looked down upon, partly due to lack of faith in textbooks that on occasion containing conceptual errors themselves. However the Japanese textbooks have been developed through lesson study and a great deal of time and thought has been put into the problems and, in particular, the numbers used. By using textbooks as resources, teachers can not only see how other educators feel a topic should be taught, but also see how to draw out common misconceptions through the numbers that are used.

Teachers can then begin writing a collaborative lesson plan. 'Lesson plan' however does not accurately reflect the document that is actually produced. The document will usually contain background to the research theme, background of the students' prior learning, a unit plan and then the individual lesson plan for that lesson.

Every school will have a research theme for that year i.e. 'Designing lessons to raise the quality of mathematical processes' and 'Instruction that helps students eagerly grapple with mathematics-aiming to improve students' expressive abilities'. This will guide teachers on the initial design of a lesson. The document will usually explain the background and/or context of the research theme. It will also explain the students' prior learning, attainment and attitude towards mathematics. This along with the unit plan for the topic allows observers to set the lesson in context. It is also one of the indicators that each research lesson is planned for that individual class at that moment. The result of a research lesson is to have a deeper understanding on how to teach that concept not to have a perfected lesson that can be taught again.

The final part of the document is the actual lesson plan. The lesson plans shared followed a similar format. The lesson would usually introduce a new problem and students would be asked to consider this problem individually. Each of the lesson plans contained anticipated student responses which were then used to plan the strategies the teachers would employ. This is a clear defining point of the differences between Japanese and Western teaching. The use of anticipated students' responses and the comparison of these led to deeper conceptual understanding. Students' confidence in problem solving is increased as it is an integral part of lessons not a stand-alone subject to be taught in isolation of learning new concepts.

At the start of every post lesson discussion the lead teacher had an opportunity to share their thoughts on the lesson and, where appropriate, to emphasise the key research questions. Before passing over to the chairperson and the group as a whole the teacher would then sincerely ask for feedback and suggestions for improvement. Although this is partly due to the Japanese culture it also struck me as due to a genuine desire to learn, not only from the lesson itself but from the wealth of experience and knowledge present at the post lesson discussion. Rather than teachers being judged on what they don't know they are able to admit that they have conducted the research lesson because they want to improve. The teacher will have chosen what they know to be a weaker area of their teaching and are therefore unafraid to ask for help.

This culture of respect does not exist purely between teachers and educators but also between teacher and students. Names are written on the board next to students' ideas giving recognition to the students. On two occasions during post lesson discussions I heard teachers voice regret that they had not been able to give validation to all of the students ideas. Students were also very respectful of each other and the teachers. At times the classrooms were noisy and energetic with students enthusiastically calling out, however as soon as an individual was sharing an idea the class was quiet and listening attentively.

<u>Conclusion</u>

It is impossible to ever fully express how much I have learnt from this program, particularly in such a short space. I have so much to take back to the UK, for my students, department and school as well as the wider network I am building through my involvement in projects such as this. I have learnt the true definition of problem solving and cannot wait to incorporate this into my teaching in the next academic year. In Japan it is not just school students that are learning. Through lesson study teachers are constantly developing and improving. It is ingrained that learning is through experience and as a group of participants we experienced this in our discussions. As frustrating as we found it at times, we were not given direct answers to our questions, instead we were encouraged to discuss them as a group. The desire for the quick answer can be hard to let go of. I have also learnt that it is important to have colleagues that will challenge me openly and honestly if I wish to improve as a teacher.

The immersion program was not just about the lessons but also being part of a like minded community. Most of us could not stop talking from morning to night about what we were seeing, learning and experiencing. The excitement generated was amazing to be a part of. Despite the variety of backgrounds and experiences we were drawn together by a shared aspiration and the knowledge of the difference lesson study can make.

My lasting impressions are of the children excited by their mathematics lessons, the wonderful Japanese culture, the kindness of the people I met, the openness of every school we were privileged

to be invited to and the desire of everyone involved to improve mathematics teaching and learning for all students. I witnessed lessons that did not just teach but instead allowed students to truly experience mathematics. This should be an aim for teachers in every part of the world.
Please allow me to once more thank everyone who made this experience possible. It was a privilege and honour to take part.

Jana Morse

Mathematics Teaching and Learning in Japan

A 45-minute math lesson? That's all? To introduce and make sense of a problem, allow for individual student work time, facilitate a group discussion to deepen understanding for all, and allow for the writing of a reflection by every student? It's not possible. Or is it?
I was able to observe 8 lessons as a result of my participation in the Japanese Immersion Program, Project Impuls, and though there were a few teachers who stated they "ran out of time" during the post-lesson discussions, they were still able to accomplish more in 45 minutes than many American teachers, including myself, during a lesson lasting an hour or more. Why?
A Japanese teacher approaches a lesson with *intention*. The teacher's *intention* – to pose a challenging problem and incorporate anticipated student responses as part of his boardwork plan with the goal of leading a group discussion to deepen understanding for all, requires the following to be in place; the teacher must understand and be able to identify the students' prior learning or knowledge, be familiar and aware of the concept progression beyond just his grade level, understand the math content he is teaching, be familiar with common misconceptions, anticipate student responses, and be able to effectively ask questions to help students identify their "soft spots" and deepen their understanding. A teacher who teaches with such intention seeks to understand his students' thinking and can still be flexible in the moment. In fact, though lessons seem scripted, they allow for flexibility and/or contingencies based on the needs of the student (vs. a typical American lesson that is followed regardless of the student's state of learning).
In a Japanese classroom the focus is not on getting the answer but on understanding the concept. According to the New York Times article, "Why Americans Stink at Math," "Most American math classes follow the same pattern, a ritualistic series of steps so ingrained that one researcher termed it a cultural script. Some teachers call the pattern 'I, We, You.'" The "I" refers to the teacher demonstrating a new procedure, the "We" refers to the teacher leading the students to try out a sample problem together, and the "You" refers to the students as they work through or practice similar problems on their own, usually in the form of a worksheet. "The answer-getting strategies may serve them well for a class period of practice problems, but after a week, they forget. And students often can't figure out how to apply the strategy for a particular problem to new problems."
Contrast this to a Japanese classroom where the structure is more similar to a "You, Y'all, We." One problem is presented to the class for the entire lesson, and the students are given time to work and make sense of it on their own (You). Afterwards, they may discuss the problem first with their peers (Y'all), and then as a class (We). This structure promotes sense-making as opposed to the answer getting most typically seen in American classrooms and makes it much more likely for students to be able to apply their understanding to new problem situations.
To illustrate this point, while observing Mr. Takahashi's 3rd grade class in Japan, the teacher interrupted the students after less than 10 minutes of working individually on a problem, and when students exclaimed they needed more time, the teacher (working with intention and following his script or plan) said they didn't need it. He asked for the answer to which many students replied "4 times," and then spent the rest of the lesson (at least 20 minutes) recording the different strategies, some he had anticipated and some he had not. His decision to ask for the answer before the class

discussion showed that he valued working together as a class for the purpose of building understanding as a group (we).

Another interesting point of fact is that the Japanese math classroom is heterogeneous, meaning the students are not tracked. This is deliberate because the Japanese believe that the class discussions (that take up roughly 85% of the lesson), are made much richer by the diversified thinking. Furthermore, the Japanese don't differentiate in the classroom. They choose problems or tasks that have "low floors and high ceilings," meaning that all students can access the problem, and those who need more challenge can go deeper and become more challenged by the same problem. The purpose of differentiation, according to the Japanese, is to allow different pathways with the expectation of achieving the same goal. If not, the gap will become greater and greater over time. The Japanese provide different entry points for all students but expect all students to reach the same goal.

Finally, the Japanese classroom, unlike the American classroom, has access to and uses textbooks, that are "focused, coherent, rigorous, and problem-solving oriented." The cohesive curriculum found in Japan "builds new ideas upon prior knowledge and organizes its contents in a purposeful sequence." This is a distinct advantage for Japanese teachers because the focused and coherent curricula "help students acquire fundamental mathematics knowledge each year with sophisticated topics built upon previously mastered knowledge through careful sequencing." And this careful sequence, complete with a plan for the number of days each unit should be taught, can also build capacity in teachers as they study the curriculum through *Kyozai Kenkyu* and learn about the content and its progression.

Japanese Lesson Study

So how is all of this possible? Lesson Study is the main source of professional development of math in Japan. It is the process by which a team of teachers will work together to carefully plan and implement a "research lesson." The planners and observers then discuss how to improve the lesson and note the impact on student learning.

Grade levels only get to participate in one lesson study a year and teachers only get to teach one, maybe two research lessons during their 6 years at a school. This makes implementing lesson study more achievable. Official planning begins one month before the research lesson is taught and the group will meet once a week during that time, though they are also expected to do research on their own, *Kyozai Kenkyu,* where they analyze materials on the topic that will be taught.

There are many facets of lesson study that I found fascinating and enlightening, but the one aspect that struck me most was the professionalism exhibited during the conversations that took place during the post-lesson discussions.

The post-lesson discussion features a moderator, the teacher who taught the lesson, and the final commentator, usually a university professor. Also present are the lesson study team members, as well as the school principal and other school colleagues or those in education who observed the lesson. After the teacher shares her thoughts and reflections on the lesson, the members of her lesson study team have an opportunity to speak and answer questions. This discussion can last up to an hour, after which the university professor or final commentator will have the last words. The final commentator's role is an important one for several reasons, not the least of which is the professionalism that it brings to the process. His critique (not criticism) is important in developing this professionalism that I witnessed throughout my two weeks in Japan. The final commentator can help to establish a new perspective, deepen the content knowledge, and bring theory and practice together. And though he is purposely not part of the team, he may be asked to give advice during the planning.

As an outsider observing the post-lesson discussions, I was decidedly uncomfortable in the beginning

because I worried for the teacher who taught the lesson, especially if it hadn't gone as planned. But after observing several more of the post-lesson discussions, I began to realize that the goal of everyone, including the teacher, was to gain new insights into teaching and learning. It wasn't personal! All involved were interested in improving the craft of teaching, in understanding how students learn best, and the post-lesson discussion allowed them to do this in the most professional of ways.
This fall, I am fortunate enough to be taking part in a lesson study at my school. I am excited to apply what I learned from my time in Japan and to share it with my team with the goal of making our research lesson as strong as possible. I remain frustrated by the textbooks we have available to us at this time and wish we had a coherent and rigorous series like the Japanese, but am grateful to have access to the Tokyo Shoseki books thanks to Project Impuls.

Janine Blinko

I write this about 3 weeks after returning to the UK, following a fascinating experience in Tokyo on the lesson Study Immersion Programme. I am still processing much of the experience, but have noticed that in conversation with colleagues and friends many of my sentences are beginning with "In Japan……" . So, the impact of the experience is already evident for me.

This was a great opportunity to see how the education system (or part of it) functions in Japan and it would be impossible to reflect on it without drawing comparisons with the way the parallel system functions 'back home'. As the programme progressed it became increasingly clear from discussion with colleagues in the programme, that the systems in place for education in Australia, the USA and the UK were all grappling with more or less the same challenges as one another. Despite a great deal of discussion about problem solving, and innovation in mathematics learning, what actually happens in many of our classrooms in terms of teaching and learning mathematics remains very didactic and closed. Speculating why this is the case, would take this conversation into a broad range of topics which would no doubt include politics, consumerism and history, and end up running round the same track as many discussions have before. However, I would like to pick up on some overriding themes that keep recurring in my post-Impuls pondering to date. They are cohesion, clarity, commitment and confidence…

Cohesion

Throughout the programme, including all the visits to schools, research lessons and discussions I felt an over-riding sense of cohesion within the education system. I suspect there are some tensions which come from outside the profession, but from the inside it seems as though there is a belief in the structures that are in place…. rightly so, compared to much of the rest of the world, they are working. It was exciting to see how the university worked in harmony with the local schools and teachers, particularly in the schools that are formally linked to Tokyo Gakugei University. I found myself making 'notes to self', with the names of colleagues from a range establishments next to questions and ideas that I wanted to share, such as on-site schools to enable the building of strong links between training teachers, graduate students and practicing teachers.

This cohesion was also evident within each school. The lesson study process really supports the professional development of *all* professionals in the school and their associates, with everyone who is part of the discussion taking away something that is applicable to their own context. The rare occasion when this feeling of collective learning was disrupted was very noticeable. The challenge in the UK is to reverse the current proportions, where few professional development contexts build this cohesive agenda, where *everyone* learns *something*. In the research lessons that I have been fortunate to be part of since returning from Japan, this expectation that we are all learning something needs to be made explicit, rather than implicit as it is in the Japanese context. In Japan, everyone present is familiar with the process and knows that everyone is taking part in the research lesson and follow-up discussion with the collective aim of improving lessons and learning.

Cohesion is also clearly seen in the classrooms, where there is a strong sense of collective learning, extremely high engagement and purposeful discussion. Because there is no sense of learners being labeled as 'top ', 'middle' or 'lower', there is a sense that everyone is moving forward. The carefully chosen and planned problem solving tasks and discussion, which are accessible to all students, enable 'high flyers' to extend their thinking by being expected to articulate and model solutions. Students who are finding solutions less forthcoming learn from peers and are often expected to re-explain and question explanations and diagrams that are modeled to them.

There was a palpable sense of how this cohesion provides a really strong core to the education system, and it shows itself in all aspects of that system.

Clarity

I have not seen text books or schemes used well in the UK, and I suspect the same is true in North America and Australia. At the extreme, there is a tendency for dependence on the published material, so the teaching of mathematics becomes an issue between the materials and the students, and the teacher is missed out of the equation. The text books in Japan appeared to be used differently. They are used as a dependable track upon which the teaching and learning of mathematics could run.

This gives a clarity of purpose, of content and progression.
In both planning and in post research lesson discussions, reference is made to where the research lesson sits, not only in the current unit of work, but also in the long term plan for student learning over a period of years. In many of the lessons seen, it was very clear that children were building on previous learning (not just experiences, the children had really learned what had come before). For example, the same terminology 'times as much' was used by the students in a year 3 lesson where the goal of the lesson was :

"Students will understand that they use division to solve problem situations for finding how many times as much is the given quantity (quantity to be compared) as the base quantity"
taken from lesson plan for Gr3 at: Koganei Elementary School Thursday, June 19

In a Grade 5 lesson, the students were using this skill with the following goal:

"By considering the two ways of making comparisons, one based on the difference and another based on bai, students will understand that the comparison using bai is more appropriate when the base quantities are different."
taken from lesson plan at Koganei Elementary School Gr 5 Wednesday, June 25,
In the discussions with students in the second lesson, it was clear that, at least for those children who contributed to the discussion, 'times as much' was embedded learning, that they were able to draw upon and apply to this new situation. The clarity of progression was clear, and the purpose of the earlier learning was to be able to use it and build upon it in later learning.

There is also clarity in the use of the chosen models and images. Throughout the texts books, which inform the progression of learning in schools, models are used consistently. These have been chosen specifically to enable students to not only get to grips with new mathematical ideas, but also to enable them to build their understanding of bigger mathematical ideas.

The approach to learning is also very clear. In the UK, as early as 1931, the Hadow report recommended that:
"work should encourage children to solve problems and make discoveries for themselves"

Later, the Cockroft Report (1982) recommended:
243 Mathematics teaching at all levels should include opportunities for

- *exposition by the teacher;*
- *discussion between teacher and pupils and between pupils themselves;*
- *appropriate practical work;*
- *consolidation and practice of fundamental skills and routines;*
- *problem solving, including the application of mathematics to everyday situations;*
- *investigational work.*

In Japan, the lessons I saw were 'living the dream'. In every lesson, there was discussion between students and between teacher and students. Where there was practical work it was always appropriate, and contributed to the learning and the development of mathematical ideas. The mathematical steps students were taking, were very clear and built out of earlier learning. All the choices made in lesson planning bear the purpose and the students prior learning in mind.

So, not only is there clarity of purpose and content, but also a clarity of methodology, the 'how' of teaching, exemplified in the way lessons are conducted and in the way that learning is expected to happen.

Commitment

There is an extraordinary sense of commitment from all stakeholders.
Teachers, leadership, university staff, masters students and young learners are all committed to enabling the development of mathematicians, who behave as mathematicians do.... As such, the students in all the schools visited take responsibility for getting their heads around the mathematical ideas being taught. This is exemplified in the way they ask questions of both the teacher and their peers if they do not quite follow what is being discussed. So rather than sit passively and become increasingly unclear, there are many interjections and requests for clarification. Students who do not 'chip in' remain engaged in the task. Very occasionally, I saw children appear not to be engaged. However, within a very short space of time, minutes, they were re-engaged with the problem in hand again without any intervention from the teacher.

In the same vein, the nature of lesson study means that there is a strong sense of all members of the teaching profession moving forwards in their own learning. The heart of the lesson study is reflecting on practice and questioning decisions, with a view to continual improvement. The depth of the conversations, and the strength of the overview given by the *koshi* ensure that the effectiveness of teaching practices is constantly in a state of review and improvement through adjustments and modifications to teaching strategies, and in turn, through further review. So the model of ownership for learning that is engendered in the students is also apparent in the way the adults in the profession work together as well.

This commitment appears also to be extended to the text book companies, who, if there is any conflict between them at all, it is based on a deep seated belief about mathematics, and the best way to enable learners access to it.

Confidence

There is a quiet confidence amongst education professionals in Japan which appears to stem from a trust in the system. This emerges in a number of ways.

Firstly there is a confidence that learning mathematics through problems solving is by far and above the most effective way to do so. There is an unarguable body of research evidence that supports this. So the processes in place in Japanese schools have firm roots in educational research, which, so far, has not been contaminated by league tables or the need to evidence every bit of learning.

The teachers have confidence in the text books. There is a trust that they have been designed to support learning through clear pathways to secure mathematical understanding. They appear to use them as the backbone to their planning, and this ensures that progression is clear, models and images are consistent, and mathematical language is accurately and consistently used.

The lesson study process builds a confidence in the adults involved, in that all questions are valued and considered. This mutual respect acknowledges that all 'teachers', in the broadest sense of the word, come with different experience and different points of view. Collectively, they bring a sense of a powerful learning community within which developing professionals can feel safe to develop their craft.

This sense of being part of a learning community is echoed in the classroom, where all contributions from students are listened to and respected. These contributions are skillfully managed by the teachers, to enable all members of the community of students in the classroom to contribute to the learning, and to have a sense that they are part of something that they can influence. The students are happy to make suggestions and all suggestions are respected. Many of their ideas are used to move the learning on for all students. It was extraordinarily exciting to see so many confident and engaged learners in the classroom. In one classroom, there was one student who was new to the school, the class teacher was quick to encourage him and make him feel safe in the classroom and to show him that he was safe to make suggestions in his new world.

Next steps

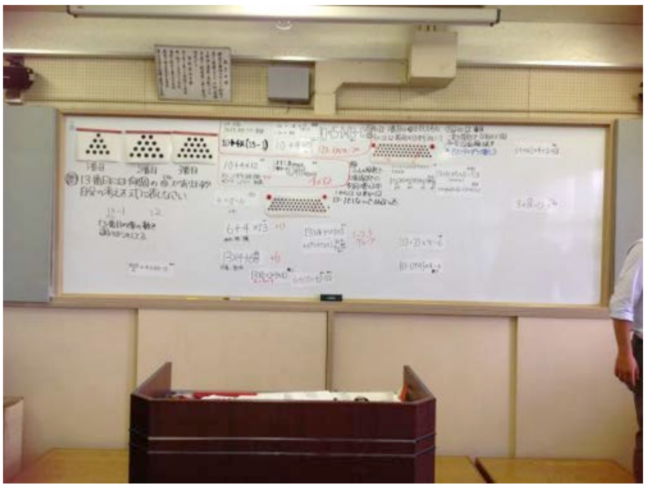
In the UK we are in the throes of establishing a 'new' national curriculum. Problem has a high priority in this curriculum, but it is certainly not central to it, in the same way as it is in Japan, or even in the way the common core curriculum in the USA is attempting to do so. Many teachers have only trained and worked with the regime of the National Strategy, which was most commonly interpreted as being very prescriptive. I work as a freelance consultant for mathematics, mostly in primary schools. Being part of the Impuls lesson Study Immersion Programme has really helped me to crystallise my thinking about strategies for supporting UK teachers as they regain the reins for teaching and learning mathematics. It has been a really exciting milestone for me, so thank you not only to the Implus Team, but also to my fellow professionals in the group for such engaging and thought provoking experiences and discussions.

Katianne Balchak

In reflecting on my participation with Project IMPULS in Japan, I am overwhelmed by the true immersion into Japanese culture and mathematics that I experienced, both in the classroom and as a part of Lesson Study. Throughout the ten day experience, I recorded many personal insights, new understandings, and ideas for implementation in my classroom, school, and district. As I review my notes and pictures and recall the observations and rich discussions; some key insights and ideas resonate with me.

On the first day, one of the first aspects of teaching in Japan that was introduced is the idea that there are Three Levels of Mathematics Teachers. Although I have sat through many mathematics professional development sessions and, along with every other teacher in the state, receive a yearly teacher evaluation, these levels seem to clearly define the expectations of teachers, yet allow for math teachers to continue growing. A level 3 teacher "can provide students opportunities to understand basic ideas, and support their learning so that the students become independent learners." I think the key phrase that distinguishes a level 3 teacher from the others is *opportunities to understandÄi0. From what we observed in the classroom observations and post-lesson discussions, it is clear that in order to provide these quality opportunities for students, teachers must be willing to go beyond just posing a question or task and having students share their thinking. Teachers must collaborate with other teachers, purposefully select tasks and numbers that allow students to develop basic math ideas, and be willing to examine the math and delve into a discussion about anticipated student responses. This level of teaching does require a lot of time, effort, and a willingness to keep informed about the newest research on students' understanding of math.*

Another element of mathematics teaching in Japan that was consistent in all of the observed lessons was the use of board to show the flow of the lesson. The lesson began with the teacher writing the given problem or task on the board (with a box around it in elementary grades). Students then recorded it in their math journals. Some teachers had students participate in an open class discussion then work independently. Few had students "pair share" at a specific point in the lesson. Few had students write their equations and/or representations on paper and stick them to the board and explain. Few had other students interpret the representations of thinking of their classmates on the board. While some teaching styles were different than others, the boards consistently showed the order in which strategies and student ideas were shared, had equations and representations clearly labeled in relation to the context, and showed the relationship between the equations and representations as part of the problem solving process. Some boards, if time permitted, had a prompt for students to reflect on or a generalizing statement. From reading the lessons, it is clear that the teachers put much thought and planning into what the "final board" would look like based on anticipated student responses. Some boards showed that students took a slightly different problem solving path than the teacher anticipated, but still show the flow of the mathematical ideas in relationship to the task.



Another new understanding I gained about mathematics teaching is the use of the double number line. The plan to use the double number was included in the lesson on division of decimal numerals in a fifth grade class. Although we did not see any students use this strategy, we were able to discuss it during our post-lesson discussion. Seeing how the use of the double line progresses through the grade levels or standards, [from representing *How much is 14 tens? in 2nd grade (Japanese student textbook 2A, pg. 39) to Multiplying by a Decimal Number in 5th grade (Japanese students textbook 5A, pg. 26),] will help me in providing representations that allow students to understand basic math concepts and continue to build upon them.*

I plan to apply some of my new understandings and insights in my mathematics classroom by:
· _Continuing to keep challenging myself to be a "level 3" teacher by providing ample quality opportunities for students to access mathematics and guide them in becoming independent learners
· _Strategically choosing problems and numbers that best fit the goal of a given lesson
· _Keep abreast of the most current research about students' understanding of math and implementing new practices when applicable

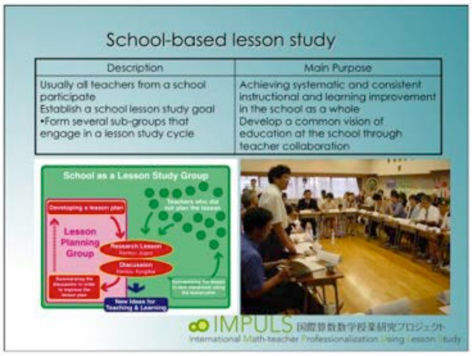
· _Include a "board writing plan" as part of some of my math lessons to guide me and my students in creating a board that shows the flow of the given lesson
· _Utilize the double number line representations to provide students access to understanding, specifically when teaching multiplication and division of decimal numbers, relationship between operations, and multiplication and division of fractional numbers

After observing the first lesson study, including the observation of lesson and post-lesson discussion I was impressed with the amount of effort, attention to detail, collaboration, and commitment to student learning that was demonstrated by all of the participants. As I continued to observe the lesson study process, I noticed that the post-lesson discussions all followed the same format:
1. An introduction (usually by the principal) – including the breakdown of time to be spent on the discussion, introduction of moderator (someone who was not a part of the lesson planning process), introduction of guest speaker
2. Teacher explains a bit about the planning of the lesson and provides some reflection of the lesson
3. Teachers and other participants discuss their observations, critiques, and questions about the lesson as guided by the questions and statements they recorded and posted during the 15-20 minutes between the end of lesson and beginning of discussion
4. Guest Speaker (usually a professor) speaks on theory related to the lesson
5. Conclusion

In all of the post-lesson discussions it was clear that the participants had a respect for each other and this format. When thinking about how to implement a true math lesson-study in my school district, I began to think about all of the logistics of organizing and planning. However, in reflecting on the Japanese lesson-studies I observed, I've concluded that it truly is a team effort with everyone committed to making it a meaningful professional development experience.

Another insight, shared by several participants, that was discussed on our last day is the idea that there is a difference between criticism and critiquing. It is my opinion, based on what I have experienced during my career thus far, that educators often mistake constructive critiques as criticism and get defensive. I am not sure exactly why this need to defend ones teaching methods and choices is such an automatic instinct of educators. I could delve into political changes in education throughout the state and country which have strongly impacted the culture of education but, that be taking a step backwards. Instead, I would like to see a shift occur in the mindset of teachers. By definition, the word critique means, "to evaluate (a theory or practice) in a detailed and analytical way." We educators analyze student data and work in order to evaluate their understanding to facilitate their learning so they continue to grow as learners. There is no ceiling for what we, as educators and humans, can learn. In order to make lesson-study productive and meaningful a willingness to make the shift from interpreting critiques as criticisms to using critiques to learn and grow as a professional needs to occur.

Another crucial aspect of School-based Lesson Study is that a "lesson planning subgroup" develops a plan as group. Although one teacher conducts the lesson which is observed, the entire subgroup collaborates to write the lesson, participates in post-lesson discussion (along with teachers who were not in planning subgroup), then summarizes the discussion in order to improve the lesson plan. From observing the lesson studies it is evident that collaboration of this subgroup of teachers to write the lesson does help to deepen the common vision of the school.



Slide from IMPULS power point, presented by Dr. Takahashi, used to summarize and interpret school-based lesson study process

I plan to apply some of my new understandings and insights about Japanese Lesson-Study in my school and district by:

· _Co-presenting the data and facts about Japanese Lesson-Study to teachers, administrators, and professors at a "World-View Conference" in August, 2014

· _Slowly immersing other teachers in Lesson-Study by inviting them to attend and view –the following quote is from our discussion on the final day: "Show others what we know and they can extrapolate."

· _As a lead member of my school's math team, I plan to initiate a shift from having one person write the lesson to having a subgroup collaborate to write the lesson for Math Studio (a smaller scale lesson study) and invite all teachers from the school to attend

· _Looking into more research about using critiquing to improve morale, build a sense of community, and most importantly, help teachers grow as professionals (this idea is of special interest to me)

· _Conducting some "dry rehearsals" before some lessons (even if they just include my grade level team, curriculum coach, AIG Specialist, and others who would like to attend; it would be a start to some great discussions)

On the last day of the Lesson Study Immersion Program, Dr. Takahashi stated, “This is one chance in one life; these ten days are not replaceable.” I am grateful for the opportunity to be a part of this program and the cultural and professional events I got to experience. It truly was the professional and personal experience of a lifetime. As I prepare for another school year, this experience leaves me feeling a renewed sense of enthusiasm and passion for teaching and learning.

Kelsey Crowder

The Lesson Study Immersion Program was an eye opening and amazing experience. I have learned so much from this experience and I can’t wait to embed my new understandings within my teaching practice and with my fellow colleagues. Before participating in this program I was fortunate to be a part of a school wide lesson study where I planned and taught one lesson and was able to watch three others. Due to this experience I had some background knowledge about Japanese Lesson Study and I was so excited when I got chosen for this opportunity to learn even more about how this type of professional development. The three key takeaways that I want to focus on are: planning, board work, and the post lesson discussion.

Planning

While reading the lesson plans for all of the research lessons one aspect that really stood out to me was how precise and meticulous the planning was. I was impressed with how intentional the teachers were in planning for all the components within that particular lesson. The teachers were very aware of their student’s prior knowledge and where they were on the learning progression for that particular concept. I noticed that this helped the teacher be prepared for the anticipated responses.

Another aspect of planning that the teachers really honed in on was student engagement. I noticed in some of the lesson plans the goals of the lesson had to do with student engagement and enjoyment. The research theme (How many packages can we make? How many will be left) on June 20 was “I did it! I got it!” Designing mathematical lessons students will be engrossed: Teaching strategies that value students’ questions and help students enjoy reasoning and expressing themselves. This particular lesson also considered student’s interest, eagerness, and attitude (IEA) when assessing the whole unit. Before participating in this program I had always heard educators state how important it was to have high student engagement, but after observing lessons in Japan I truly understand why it is so important to have high engagement and buy in. Teachers don’t plan for high engagement lessons only to excite students for the first 10 minutes of the lesson, but rather to impact the level of discussion and work produced by the students. I realized that the level of student engagement directly correlates to the quality of the student led discussion. If students are interested in their topic they will be excited and eager to solve the problem at hand. Students will be more engaged in their thinking and their peers if there is a high level of engagement.

When planning teachers were also very intentional in the particular problem that they were posing to their students. Teachers considered how that particular problem would lend itself to the particular concept and their student’s needs. In the post lesson discussion the teachers would talk about the particular reasons why they chose to use that problem and how they had solved the actual problem with all the various ways that the students would. The teachers would include all of the anticipated responses within their lesson plan along with possible redirects Teachers also presented the problem in a manner that engaged their particular group of students. One teacher polled his class and intertwined the results within his problem. Another teacher discussed how he modified the particular problem from his textbook to meet the needs of his students. The Japanese teachers didn't only consider which number/problem was the best to present to their students but also how they would present it to their students.

Board Work

One component of the Japanese teaching that is not as prevalent in the United States is board work. I now realize how crucial board work is to one's lesson. Not only does board work help guide and frame discussion but it also is a tool that students can use to take notes independently. The Japanese teachers pre-planned for their board work in their lesson. The Japanese teachers knew exactly what they wanted on the board, where it was to be placed, and in what order. I also noticed how interactive the board work was with the Japanese students. The students and the teacher were constantly referring to the possible solutions on the board.

Post Lesson Discussion

This component of the Japanese Lesson Study had the biggest impact on me. The post lesson discussions that we were able to observe were done with such high levels of rigor and teacher involvement that allowed for meaningful discourse. Each school structured their post lesson discussions a little differently (one particular school had teachers observe specific students, others discussed as grade level bands first), but they were still produced a high level of discussion. I think that the post lesson discussions were meaningful because all the teachers came well prepared and willing to participate. The teachers had a good understanding of the lesson plan and were prepared to discuss the questions that the lead teacher had posed. The teachers used these key questions when they were observing and these observations helped guide the post lesson discussions. The teachers weren't afraid to offer critical feedback and the lead teacher welcomed their comments and ideas. The lead teacher was also very reflective in their practice (which is a key component in lesson study) I believe the teachers were very willing to participate because not only where they strengthening their knowledge, but their collective knowledge as a school as well.

As I am getting ready to embark on my second year of teaching and second year of Lesson Study I am excited to be able to take my experiences from this program and integrate them in my teaching practice as well as my colleagues. My personal goals are to focus on being more precise in my planning while setting up daily math journaling and board work in my classroom. As a school I want to help elevate our post lesson discussions to a higher level of rigor than before. I would like to extend my deepest gratitude to the Project IMPULS team for this wonderful professional development opportunity. Thank you!

Kent Steiner

Reflections on Japanese Math Education and Lesson Study

If school improvement were as simple as identifying what works, we would have much better schools. However, as we learned from presenters at the IMPULS program, researchers in the U.S. have been advocating for decades for changes that have been shown to improve mathematics instruction, but by and large those changes have not been implemented in the U.S. There are many reasons why those changes have not been implemented, from factors relating to education policy, to the culture of teaching and education in the U.S., to a basic lack of knowledge of how the suggested processes or programs work. Some of those factors are beyond the control of local school leaders and teachers. However, many of them are not. In fact, I believe that none of the factors outside of our direct control (e.g. national, state or district education policies) absolutely preclude the implementation of change, such as implementing lesson study (although they can make it very difficult). Lesson study is something we need.

There are aspects of lesson study that apply mostly to teachers in the classroom and there are others that are more relevant to school administrators. Therefore, during the course of our visit to Japan I sometimes looked at what we were seeing and learning through the lens of a teacher who is learning to engage in lesson study and teaching through problem solving. Other times I looked at it through the lens of a school administrator who has to figure out how to implement lesson study at his school. I will share my reflections through both of those lenses, focusing on just a few topics I found to be the most revealing and relevant for me.

The Lens of a Teacher

Re-teaching

One of the most interesting topics we touched upon in our discussions was the question of "re-teaching" a lesson. Before coming to Japan I was under the impression that one of the goals of lesson study was to identify better lessons, in part for the purpose of sharing them with other teachers who could then take them and use them. I didn't believe that there would ever be such a think as a perfect lesson, but I did think that lessons might be taken, almost whole cloth, to be used by different teachers. After being in Japan and discussing with Japanese educators I am clearer that this is not likely to be the case. While it is true that teachers in Japan learn from the research lessons by observing them and participating in post-discussions, as well as from sharing written copies of research lessons, it is apparent that Japanese educators do not believe that they can simply take someone else's lesson and teach it to their class. The differences between any two groups of students is likely to be too great to allow that. Professor Fuji explained that a lesson is not like a car, where are all the parts are standardized and always work the same way and always work well together. A lesson should be custom made for a particular group of students based on their prior learning and their needs, levels of development, etc. However, while this is true, there is also a sense that certain parts of lessons (e.g. certain math problems) have been proven to work best to elicit the desired solutions from students, as, for example, when certain numbers are used in the given problem. One set of numbers may lead students to one strategy or solution, but a slightly different set of numbers may lead them to other strategies that might be more desirable.

On the other hand, there is a natural desire among teachers to not have to "reinvent the wheel". Even given the many possible differences between different groups of students, there are many aspects of a given lesson that would probably work well for many classes. This seems to apply to Japanese teachers as well, as we learned that Japanese lesson study teams sometimes teach the research lesson to another teacher's class to try it out before teaching it to their own class as a public lesson. Also, teachers in Japan can get published copies of past research lessons. So, in Japan there seems to be a nuanced understanding about the value of the research lesson; there is a place for sharing ideas, methods, math problems, even whole lessons, but with an understanding that each class is different and lessons cannot be treated as "one size fits all". Seeing and hearing about how this works in Japan was helpful for developing my understanding of this nuance.

The Lens of an Administrator

Lesson Study: A Slow and Steady Process of Improvement

This is something equally important for policymakers to think about, but one of the most striking things about lesson study in Japan is that it has been going on, in one form or another, for many decades. I believe we learned that it is about 100 years old. The process and methods we observed have taken decades to be developed, and the changes brought about in Japanese education have been, by definition, slow and steady. While policymakers will need to understand this for lesson study to be implemented on a wide scale, it is also something for administrators who hope to implement it now to understand. This is because, in the best-case scenario, it will take years to implement lesson study and teaching through problem solving at any one school. These are not "quick fixes" to low student achievement. There are many aspects of teaching practice and culture and organizational structure and culture that will have to change for lesson study to fully take root and for teaching to improve. Given the constant pressure school administrators are under to quickly raise standardized test scores, administrators will have to resist the temptation to turn lesson study into something it is not or to drop it altogether if it does not result in immediate improvements in student achievement.

The Value of the Research Lesson to Observers

Another aspect of lesson study that was helpful to learn more about was the value of the research lesson for those teachers who are not on the planning team. The value of a research lesson is multiplied many times when it is school-wide, district-wide or cross-district. Generally speaking, the more teachers who observe a research lesson, the more students who benefit. Likewise, the more research lessons each teacher gets to observe, the more her students will benefit. However, I believe it may be difficult to fully realize this value when a school and a group of teachers is just learning about and beginning to do lesson study. This is because teachers are probably not used to observing/critiquing other teachers or being observed/critiqued by other teachers. Teachers may not be comfortable inviting anyone other than the planning team or, at most, teachers from their own school to observe the research lesson. I believe teachers should be given this option, at least in the beginning when they are so new to the process and the methods. This is a cultural norm that will take time to change. However, as a school and a faculty get more comfortable with lesson study and it becomes a part of their culture this should not be much of a problem.

At my school we are only one year into learning about lesson study, so my first concern has been for the experience of the teachers on the planning team, and in particular for the teacher who teaches the lesson. By this I mean that I wanted them to have as positive an experience as possible, which for some of them meant a lot of support and a lot of safety. We needed to create buy-in, and word of mouth is probably the most valuable tool we have for that. Last year we asked for volunteer teams of teachers to plan research lessons. We needed them to have a positive experience so that they could convey the power and value of lesson study to their colleagues. To that end we allowed the planning team to decide how many teachers to invite to observe the research lesson. This year we will probably do the same, since this will still be most teachers' first experience on a research team. However, I hope some of our research lessons this year will at least be school-wide, if not open to teachers from other schools, and down the road I would like to make school-wide research lessons the norm.

Related to this, it was helpful in Japan to get a clearer idea of how many research lessons a Japanese teacher typically participates in each year, either planning or observing. If, as we learned, each teacher typically serves on a planning team one time per year and actually teaches a research lesson once every four years, then, to me, this highlights again the importance of having teachers attend more research lessons as observers. This seems true not only for individual teachers, but for fostering an ongoing dialogue among teachers that crosses grade levels and subjects. In my experience, professional development that happens once a year is an "event", something that is not followed up on and is more of a show than a learning experience or a process for improving instruction that is part of the school's culture.

Laura Burrell

IMPULSE, The International Math-teacher Professionalization Using Lesson Study, facilitated a life changing professional development opportunity within the Lesson Study Immersion Program. This program immersed a group of educators across the world in authentic mathematics Lesson Study in Tokyo and Yamanashi, Japan. We had the opportunity to observe eight mathematics research lessons developed through Lesson Study in elementary and secondary schools, participate in post lesson discussions, and have deep conversations with other professionals about content and pedagogy.

In Japan, Lesson Study encompasses all components an educator needs for professional development. There are three types of Lesson Study are school based, district based, and cross district based. We were fortunate enough to be a part of the three types. This consistently implemented process provided to all teachers creates the highest level of educator, a level three teacher, who can provide students opportunities to understand basic ideas so students become independent learners. A level one teacher is a teacher who can tell students important basic ideas of mathematics and a level two teachers is a person who can explain the meaning and reasoning of important basic ideas to understand them. Japanese math educators focus more on content than pedagogy, but feel strongly that knowing the content isn't sufficient to be a level three teacher.

Overall the Lesson Study Cycle revolves around a school based theme created by the faculty and administration. With this theme a team comes together to create a research lesson plan where one teacher will teach this lesson to their students while the other educators observe with a previously given purpose. After the lesson there is a post lesson discussion where the team discusses how the lesson went. This cycle will continue within the school with another teacher and grade level throughout the year. The mindset of the teachers is that you are not just teaching your class, you are teaching the school. It is a collaborative effort for all students to learn. The teachers share their ideas a lot, they are not competitive in a negative way, and their work is open to the public. Teachers also have the opportunity to come in contact with many different educators in the span of their career because teachers are moved usually every six years. That gives a continuous new prospective to a faculty.

The lesson plan research proposal is the first part of the Lesson Study cycle. It is created by a team of teachers who collaborate and research the content embedded in Japanese textbooks, analyzes progressions across grade levels, and keeps in mind the ten high impact strategies to create a powerful lesson, this is called kyozai kenkyu. The team comes up with one real life problem solving problem that creates a need for students to want to solve it. Problem solving means that students engage in a task for which the solution methods is not known in advance, which is a powerful approach for developing math concepts and skills. Within this problem or task the teachers analyze what appropriate numbers would best be suited in order to provide students with an opportunity to solve the problem using a variety of strategies. Students are expected to build on prior knowledge and apply that to a different context where they have to investigate on their own. The research plan takes about five weeks to create. During the planning the teachers look at the scope and sequence, what knowledge students were previously taught and what they will be taught in later grades, and plan out anticipated student responses. Other parts of the research lesson plan are; the goals of the unit, how this lesson is related to the schools theme, the lessons within the unit, a detailed flow of the lesson to explain what observers should expect to see. With this knowledge the teachers can plan the bancho, or board writing. This is the blackboard that the teacher will use to pose the problem and have students share their strategies. By the end of the lesson the bancho tells a story about the lesson.

The next part of the Lesson Study cycle is for the teacher to teach the lesson. All educators have read through the lesson prior to observing, and they are given a focus when they observe. The lesson structure starts with a short five to ten minute introduction to the lesson.

This is where the teacher poses a problem and engages students. After the problem is posed students have about ten to fifteen minutes to explore the problem independently in their journal. While the students are solving the problem the teacher circulates recording student's strategies. At times when a student is struggling the teacher will conference with them and question them to clear up any misconceptions. During the independent time the participants who are observing have the chance to walk between the desks, kikan jyunshi, and take pictures of student work or record students ideas. The teacher will then regroup the class together, for around 20 to 30 minutes, so students can share their strategies. It was stressed by majority of teachers we observed that it was okay if the student had not yet completed the task. Throughout this student discussion time the teacher became the facilitator and asked questions like: did anyone do this differently, do you have the same work but you thought about it differently, how are these thoughts similar or different, and can someone tell me what you think this student did. The teacher utilizes their conference chart to strategically take the insights from the students to share in an order where the ideas and concept of the lesson builds upon each other's thoughts in order to build the concept to meet the predetermined goal of the lesson. The students would have conversations between each other stating multiple solutions and comparing and contrasting their strategies. When the student discussion has accomplished the teachers intended goals the teacher will wrap up the lesson with a summary. At this time the teacher takes a couple minutes to consolidate the information that was shared and at times write the name of the lesson above the task. The students then have a chance to reflect in their journal.

Prior to the post lesson discussion the teacher will assess the students based off informal observations from conversations and their journal entry. This gives teachers data to support if the lesson went well or what their next steps may be. When looking at a journal there are four levels of entries. The first level is when a student explained if the task was easy or difficult, the second level is explaining if the task was easy or difficult and WHY, the third level is when a student compares their strategy to a friend's strategy, and the final level is when a child goes beyond the task. After the teacher has time to assess their students they meet with the people who observed the lesson. During the post lesson discussion the teacher will not only gain insight form other lenses such as an administrator and an expert in the area of mathematics, but will also have a chance to reflect on the lesson themselves. The post lesson discussion starts off with a moderator, who introduces the discussion, then the teacher reflects on the lesson stating what they learned from the data and what they felt went well with the lesson and how they might improve the lesson or what next steps they will take. Teachers across grade levels then have a chance to ask questions and make comments. The perk to having teachers in different grades is that they all see the lesson through a different lens and all have different things to contirubte. Through all of this the teachers were always very processional and open to hear what suggestions people had to offer. The commentator, a specialist in mathematics, is always invited to do the closing comments. This closing statement is full of content knowledge, high level suggestions, and next steps for teachers to think about.

I am honored to have been a part of this program and to have grown as an educator to the extent I did. I feel as though I have so much to bring home to the United States and share with my colleagues. Being able to have a hand on professional development regarding Lesson Study where we were a part of every component was vital to our learning and will help us transfer it in our county. This way of work is so beneficial and grows teachers into level three teachers. A couple of the most important things that stand out to me are the effort that is put into developing the lesson, the high level conversations students were having on their own comparing and contrasting their ideas, and the openness and cohesiveness of the post lesson discussion. I look forward to implementing all of the components I learned this summer in my district this year.

Leland Dix

My first reaction to the news of being accepted for the grant was to 'hoot and holler' on the subway line at 125th street in Harlem, NY. My biggest reaction after the project was finished was to question, "How can I go back?" Being involved with Project IMPULS was single-handedly the best professional development I've been involved with. I have learned many insights throughout the process, including ways to improve my mathematics instruction, but for this reflection I'll focus in on lesson study and how to implement it.

How can we adapt this process to work abroad?

If you're looking for the best experience, in short answer, you can't. From the first day of the program, we heard about people who try to take *'the greatest hits'* of lesson study, but that is *not* the most effective ways to develop your teachers. Let me be clear, lesson study *can, will,* and *has been* effective in the United States, but not when you only take it in pieces. Lesson study is a culture of support and critique and should not be watered-down.

In order to effectively use lesson study abroad, it should have all of these components.

1. A Shared Theme or Research Focus
    - This broad goal is often established at the beginning of a year, before the lessons, and could look subject-specific like "attack mathematics problems eagerly", or across subjects like "improve the quality of independent journal writing," or "attentively listen and respond to peer responses."
2. Targeted Content
    - Lesson study is not 'perfecting a lesson' and one could choose a subject other than mathematics, like language arts, science, social studies or the arts. Regardless of the subject, the content chosen should be a concept that teachers find difficult to teach, a gap or weakness in student learning, and improve upon these areas. There are differences between a research lesson and a demonstration lesson. Many times in America, we can feel like we're being observed so we want to pick our favorite lesson—but that would be *demonstrating*, not researching a targeted content.
3. Planning and Kyozai-Kenkyu
    - When preparing for the research lesson, there should be a planning team assembled. This is often the grade team but could also include other teachers, or even leadership as well. The teachers meet at least one month in advance, and take part in weekly meetings including *kyozai-kenkyu,* meaning investigating instructional materials. The planning team studies the subject content with scope and sequence (multiple textbooks, teacher's manuals, and standards), considering manipulatives and other instructional tools, and think about targeted misconceptions in regard to the content and research focus. During this planning process, the team intentionally selects the task *and* the specific numbers that will be used (if a mathematics lesson). I learned that the planning team might have 10 potential tasks or options, but then they choose the one that is best.
    - These weekly meetings typically end with "homework" as each member has something to accomplish before their next meeting. The planning team is not simply useful for planning and multiple perspectives but also is important in the post-lesson discussion as well (see #5).
    - Before the live observation stage, the teachers of the school must have access to the research lesson plan *and* must have studied it carefully. For schools or participants new to lesson study, there should be a pre-lesson discussion, where all observers have read the lesson plan, discuss it and maybe even try the mathematics for themselves. I

found that I was the best observer I could be when I walked into the live observation with a question or curiosity about how students would [*x,y,z*].

4. A Live Observation of Students / Lesson
   - At my school in New York, we've done several research lessons, but most are observed via videotaped sessions. I fear that this is a common adaptation that cuts some corners. You can't fully assess students' understandings and experiences if you aren't in the room, in the moment. By viewing a video of the lesson, emphasis is often put more on the teacher, when the whole reason for doing the research lesson should be your theme or research focus—which is based on the students. During a live observation, teachers can be assigned roles: such as studying a typically low, average, or high performer, taking notes on students' responses, scripting out the exact questions or *hatsumon* (posing of a problem) that teachers use in the lesson, taking pictures of student work, or documenting the *bansho* (blackboard writing) and printing it out before the discussion. Administrators and schools wanting to use lesson study must think carefully of a way to coordinate live observations—after school, before school, on planned professional development days etc.
   - Videotaping the lesson should still be done, but more used for documentation or potentially to confirm a finding from the discussion, and not for people to get a first observation of the lesson.
5. Post Lesson Discussion
   - For the discussion, the participating organization should have a panel assembled. Members, and their frequency of occurrence, can include:
     i. A moderator, who did not take part in the planning process, commonly an assistant-principal (always)
     ii. The teacher of the research lesson (always)
     iii. Another representative from the planning team (often)
     iv. Leaders of the school (often)
     v. Final Commentator or "Knowledgeable Other" (always) who did not take part in the planning
   - After the moderator welcomes and finishes the introduction, the teacher of the lesson has a chance to reflect upon the lesson, and usually includes some curiosities or questions that he/she invites the audience to answer or ponder as well. The members of the panel bring questions, and the teacher isn't always one that has to answer. This is the importance of having a planning team. Often times, people hesitant to use lesson study think that they are afraid of being "too rough" on the teacher, and in America I've found that observers spend more time complimenting and praising the teacher before coming to any critiques. The questions regarding planning of the lesson can be fielded by members of the planning team, and thus, does not create a situation where the teacher has to feel defensive in any manner.
   - In Japan, I learned that there is a large difference between **critiquing** and **criticizing**. In lesson study, we share our observations, question, and offer productive **critiques** of what we saw. There is no "criticizing."
   - After the panel asks questions, the other audience members / teachers are invited to participate in the discussion. This next item is one of the BIGGEST differences I've found between lesson study at my school and the research lessons in Japan. <u>Typically, a school carves out 1 ½ hours for this discussion.</u> It could be shorter, but they hold this time as sacred time, and with such a large window, there is never a feeling of wanting to hurry up and finish, or wishing colleagues would hold questions so we could get on to the next professional development item. Lesson study *IS* the major professional

development; there is no scheduling conflict or over-booking of the afternoon. After the discussion runs dry, which could potentially be 90 minutes later, the final commentator (or "knowledgeable other") is invited to lecture. The knowledgeable other is typically a reputable professor from a local university and this "lecture" is often what the audience has been waiting for, akin to a headlining act of a rock concert. Just short of high-pitch screams and fainting, all the audience members in this venue get out a fresh page of paper are rejuvenated with the fresh-take from the professor. The knowledgeable other *must* be familiar with the lesson study process—they could potentially derail the whole process if they overly-criticize and pick apart a lesson. The scintillating observations of an effective knowledgeable other typically last for about 30 minutes.

6. Informal Post Lesson Celebration
    - Hopefully, not too many Japanese teachers get word that I'm telling you the true secret of lesson study. After the lesson (that evening of the next), all observers and panel participants meet to celebrate the lesson. Typically they meet at a local restaurant, sharing food and choice beverages with each other. Everyone splits the bill, except for the teacher and knowledgeable other whose costs are observed by the other individuals (no school funding is put toward this event). At this informal celebration, there are toasts, smiles and many cheers for the teacher. This helps create a positive, memorable culture of revering the hard work of the planning team and teacher, and also creates a perfect avenue for further discussion. Much of the talk around the tables is about the lesson from that day, and everyone is able to approach the teacher with gratitude and other curiosities/questions that they feel more comfortable asking in this setting.

My Next Steps

Now that I know the effective, complete process, there are ways that I can make them all work at my school network. This year I'd like to be a part of several planning teams to help move the process forward. Our elementary schools at Harlem Village Academies are currently K-2. A reasonable expectation could be doing 6-9 school-based research lessons this year, depending on our availability of Wednesday afternoons. Our school has early dismissal on Wednesdays, and if we could block off one Wednesday a month, then we would have 10 potential slots.

If every grade (K, 1st, and 2nd) simply did two-a-year that would be 6 lessons, and then we could have an opt-in from other teachers such as art, music, P.E. or science. In regards to the workload of all the teams, they would only need to be in 2 planning teams a year (a 3rd if they opted into helping art, music, P.E. or science) and a teacher would actually *teach* a research lesson once every 2-3 *years.* If there is pushback from school about Wednesday afternoons, we potentially *could* do less live-observations, but as I stated earlier, that severely decreases the effectiveness of the process.

There are different types of lesson study: school-based, district-wide, cross-district, and public. The difference, I'd assume, is that the first three are only for teachers, and a public research lesson can invite parents or other people/ media in an "open-house" fashion. I'd love for HVA to have a 'district-wide' lesson study day where we invite teachers from the four other schools in our network, and also other invited guests to observe some lessons at our school. This could be on a Saturday or a network-wide professional development day, and simply have 1-2 lessons with post lesson discussions.

In the future, I'd hope to open it up to the public, teachers from other schools in NYC could come in the morning for an introduction to lesson study, preview the lesson plan, try it out and observe a live lesson and post lesson panel discussion. I'd also love to have fellow participants in the Project IMPULS program come to HVA and take part in research lesson as well. I still have some concerns about local outsider Knowledgeable Others, and I'd feel no greater honor than to have Dr. Toshiakira Fujii and Dr. Akihiko Takahashi to come back to Harlem Village Academies as Final Commentators.

My final step is right now to propose a new role in the Project IMPULS program. I could be one of the first members of this role. Instead of being a Project IMPULS *participant* that receives a grant, I could perhaps be a Project IMPULS *observer* who funds his or her own way. I will sit in on the lessons and discussions but promise I won't take away from others' thoughts or experience! I could be silent the entire program, simply taking in the information. My only request for future programs is to have it happen in the beginning of July—I missed two weeks of instruction. If it costs me $4,000, it will be worth every penny— I must go back to Japan.

Lorna McCance

Before I embarked on the adventure that was the IMPULS project in Japan I had already had several experiences with Lesson Study. I would like to outline these first before I continue. In my first year of teaching as an NQT I was part of the Bowland Lesson Study project in the UK which involved working with other schools in the Midlands and being part of research lessons in various contexts. Until this project I had not heard of Lesson Study nor knew anything at all about teaching and learning in Japan. During this year long project I was part of the planning team for three research lesson including one that I taught myself. I also visited other schools on several occasions to observe research lessons. The first research lesson I was part of was at my school and I was a member of the planning team. Due to the nature of the project we had to use Bowland Maths resources. The first job was to find a task that we thought was appropriate for the class we had decided to use. Once this had been decided we had several planning meetings to discuss and decide what to do during the lesson. None of us knew what to expect from the process of lesson study or what was really expected of us as project participants. An advantage right from the beginning was working collaboratively with colleagues and discussing teaching and learning ideas. I felt that as a new teacher I learnt more than I gave to others but that was in a way expected. We felt the first research lesson went well however I found the post-lesson discussion, especially looking back on it now, to be rather unfocussed and as a result not as productive as it potentially could have been. Although we learnt from experienced others it seemed many of us were lesson study novices and were still stuck on the British, or Western, idea that we shouldn't be over critical of the teacher.

As the year progressed I feel I had many unparalleled learning experiences through the lesson study project. Our department was discussing teaching and learning much more as a result. We were lucky enough to have Japanese visitors to our school twice throughout the year, once to see a lesson my skilled mentor was teaching, this was discussed at great length in the post-lesson discussion and the Japanese colleagues were very impressed with how the teacher had planned and executed the lesson; in particular they noted how well he had anticipated the pupils responses and that was like Japanese teaching. The second time we were graced with the presence of Japanese experts was to watch a research lesson I was teaching. I did my best to produce a detailed lesson plan with many anticipated responses. I remember being nervous before the lesson but as soon as it started I forgot about everyone else watching and just focussed on my pupils and their responses. It was an incredible experience and so many points were raised in the post-lesson discussion that I still think of when planning a lesson today. The whole project was a real highlight of my first year as a teacher, I couldn't imagine better professional development than having these opportunities to plan in great depth, then analyse the outcomes in the presence of some of the most influential experts in the field. I felt privileged to have had the opportunity to learn from so many esteemed individuals.

Following this I moved schools and became part of a new lesson study project in my new school, organised by similar people through the University of Nottingham and chosen due to my links with the project from the previous year. Now I was seen as more of a lesson study expert as I had done it before so therefore could help others understand the nature of it. It was these experiences that led me to being selected to come to Japan and I was so thankful for them and incredibly excited to see and learn more.

Our first morning in Gakugei Univeristy already highlighted several big misconceptions I had about Lesson Study. I felt if I learn this much every day here I really will be able to make an impact back at my own school and beyond. The main things I took away from that first day was that back at home we weren't putting enough emphasis on the planning phase of a research lesson and that some things were being done in the wrong order. Planning should start with a research question and then develop with the individual class at the heart of consideration, including where they are in their knowledge and what sequence of learning should come before and after the lesson. In my experiences so far we were firstly finding a problem that we wanted to do and then developing this, the class it was done with was almost irrelevant. I almost felt embarrassed inside about this lack of consideration for individual pupils. The second misconception compounded this embarrassment; this was that a research lesson should not be retaught. At home we almost felt that this was part of the "lesson study cycle" as it had been sold to us. That you teach a lesson, revise the plan, then teach the new revised plan to a different class. This again seemed abhorrent to the Japanese professionals as it disregards the individualism of your class. Every lesson should be planned with the class in your thoughts, re-teaching a research lesson does not allow for this.

Even after the first day I knew things I wanted to take home. This leaning continued to grow throughout my time in Japan. The focus on planning continued to strike me. Planning is the core to good teaching in Japan, the teacher doesn't just demonstrate to pupils he/she creates the opportunities for pupils to learn from **the lesson** through whole class discussion, not from the teacher's examples. This happens through meticulous planning. I think there is a barrier to this style of teaching in the UK and that is that pupils (and their parents) have an attitude of expectance; you are the teacher, teach me. This idea continued to trouble me but if pupils can get the enjoyment from the lessons then surely they would be on board and willing. Necessary for good planning is strong maths content knowledge. This again does not seem to be central in the UK, possibly for more practical reasons, we are short of Maths teachers therefore accept people who are not always highly qualified Mathematicians, this is especially a problem in Primary schools; people want to teach Primary age children and a proportion of them admit to hating maths and not wanting to teach it, perhaps coming from their own educational experiences where lessons weren't exciting and Maths was a series of facts that had to be learnt. This creates a vicious cycle where their negative attitude to Maths breeds a hate of maths in their pupils. This idea was deeply concerning, by the time pupils get to me at Secondary school many pupils already hate maths so are bound to be uncooperative; this needs to change. Through working with Primary schools we need to foster a love of Maths from day one and let pupils discover the Maths for themselves. Deeper understanding will then follow from this as pupils are not just being given a series of facts to learn (which they will no doubt forget) they are building a knowledge network of connected processes that they have reasoned for themselves.

Another part of Japanese teaching and learning that struck me whilst in Japan was the incredibly large proportion of a lesson spent doing "Neriage". All of the lessons we saw consisted of at least 85% whole class discussion time. This is vastly different to the UK; the "plenary" or summary section of the lesson is usually no more than 15 minutes (25% of the lesson), although encouraged to summarise using "mini plenaries" throughout the lesson the idea that the majority of the lesson should be whole class discussion is almost an alien concept in the UK. From seeing it I thought it cultivated a motivation from the whole class to get involved and allowed the pupils to build understanding together for themselves. However again there are barriers for this strategy in the UK. The first coming from the perspective of participation; how can you ensure that every pupil is participating in a whole class discussion where the louder more confident pupils may dominate? Another question may be raised about the hot topic of "differentiation"; how can you be sure every pupil is working at their appropriate level of challenge during whole class discussion. The class will only make progress at the pace of the slowest learning in this circumstance and this is not fair to the more able pupils in the class. These are difficult issues to overcome and from my point of view if the culture of Japanese classrooms can be developed then every pupil will be participating and everyone will be challenging themselves however this is hard to measure and hard to evidence and in an OFSTED culture where everything a teacher is doing needs to be evidenced unfortunately these issues present challenges.

My understanding of what neriage should consist of also developed hugely during my time in Japan. The phrase "beyond show and tell" really hit home for me. I have always been passionate about problem solving in Mathematics and always try and include it in my lessons wherever possible however I felt I was perhaps guilty of "show and tell" when pupils were sharing solutions. I now know it needs to be so much more than that; solutions need to be evaluated, compared and developed. This is made more productive if solutions are anticipated before the lesson in great depth and the sequence of progression within solutions should be planned. The importance of planned is highlighted once again as if solutions are not anticipated effectively the teachers job during the lesson to collate and sequence each solution would be much more difficult.

As aforementioned I have always been passionate about problem solving however even the nature of problem solving is so different in Japan to in the UK. Even the basic definition of problem solving that we were presented with on the first day opened up my understanding; "engaging in a task for which the solution method is not known in advance". This definition allows for a more flexible concept of problem solving which can be incorporated into every lesson. In Japan all lessons are taught **through** problem solving, every concept is built starting with pupils' response to a problem. In the UK problem solving is almost seen as a separate thing and is often taught in isolation to content lessons. The basic definition of problem solving helped me to incorporate problem solving into every lesson when I returned to the UK and this is something I want to do more and more.

Finally I want to discuss some cultural differences I noticed in Japan. Before coming to Japan I thought the pupils would be impeccably behaved and this would be seen in silent work during lessons. This was shown to be a complete fallacy. Although mostly well behaved lessons certainly weren't quiet, pupils were constantly shouting out their ideas really wanting to be heard and participate in the lesson. Something that is different is the culture of responsibility that is built throughout pupils lives. In grade 1 pupils are given a plant plot to nurture, they are responsible for this and even over their holidays they must look after their plant. Pupils serve their own school lunches and clean their school; this shows responsibility for others and their surroundings. During lessons pupils build their knowledge for themselves; they are trusted with this responsibility. Pupils take responsibility for their journals, they write whatever they feel is relevant from the lesson. Responsibility is all encompassing; I want to build this into the lives of my pupils within my classroom and the wider context of my school.

Richard Cowley

Current London Context

I am engaged in lesson study in London, England, UK. I act as knowledgeable other for a lesson study project in Barking and Dagenham, London. As part of my role at the Institute of Education, London University I am working to set up lesson study in our partnership schools. We have partnerships with over 600 schools in London with varying levels of involvement from initial teacher education to joint professional development, consultancy and research. The English context for mathematics teaching is particularly interesting currently (2014) due to a number of policies and initiatives coming into play simultaneously. There is a new National Curriculum, which is statutory from September 2014. The British government has set up 30 'maths hubs' across the country; these are led by schools and will coordinate all professional development for mathematics teachers across England. The maths hubs initiative is one of a number of ideas taken up by politicians and adapted for England from processes identified in high performing jurisdictions (based on international tests).

Reflections

It was interesting to hear about the history of lesson study from Akihiko Takahashi; how it originated in the United States of America and was adopted by Japan over a hundred years ago. Lesson study forms the core of professional development for teachers in Japan and is well established. Other forms of professional development such as training courses and expert consultancy are also used but often these are motivated by questions arising from lesson study. This creates an interesting dynamic between those involved in research, consultancy and directly in school teaching.

Focussing these ideas, I perceive some cultural expectations of lesson study in Japan that result in and are a result of accepted formal relationships between the people involved. And I have some intuitions about the general principles of lesson study and how it relates to other forms of teacher learning activity. As a teacher educator I have reflected on my perceptions, intuitions and ideas in relation to:

- cultural expectations of lesson study in Japan
- formal relationships between the people involved in lesson study in Japan
- general principles of lesson study
- lesson study in relation to other forms of teacher learning activity

Cultural expectations of lesson study in Japan

When we engage in an activity we call 'lesson study' what are we expecting to get out of it? As a teacher educator, I consider this question in terms of my understanding of how teacher development can be enhanced in professional processes. There are lots of things we want teachers to know and ways we want them to act for better teaching. The most common approach to teaching teachers has been to tell them what to do and to regulate and inspect their work. Unfortunately, this has repeatedly failed as a sustainable approach in that it does not seem to permanently change teaching. There is sometimes a mismatch between the way we theorise learning for children in schools and the way we theorise learning for teachers. For a long time there has been broad agreement that transmission of knowledge does not work. If we assume that students in school need to construct their own knowledge or co-construct it with their teachers we should also assume that teachers need to construct their own knowledge or co-construct it with teacher educators. Teaching through problem solving has been proposed as a methodology for teaching mathematics that will facilitate deeper learning and avoid limiting students to procedural learning. Compare this to the transmission method often adopted in teacher education where teachers are told how to teach as if teaching is not problematic or complex, as if we think learning to teach mathematics is easier than learning to do mathematics. My question is how we can learn to teach by considering it to be a problem solving activity. *"Our problem today is how to teach mathematics."* What would it look like if teacher educators and teachers co-constructed their solutions to this complex problem? I think lesson study is one possible form of answering this question. In lesson study we have the opportunity to see mathematics teaching as problematic and to regard teaching mathematic s as a problem solving activity.

Formal relationships between the people involved in lesson study in Japan

The notion that we might co-construct knowledge about mathematics teaching rather than transfer it acknowledges two aspects of reality. Firstly that teachers and teacher educators have a shared enterprise and some common knowledge but that they also bring specialist knowledge and know different things from each other (Jaworski, 2008). And secondly that our ideas about teaching need to be referable back to a lesson we have all been present for. Referring to a generalised account of school teaching leads to overgeneralising and a mistaken reduction of the complexity and contingency of classroom activity. I reflect further on these realities in the next two paragraphs.

Participating in the IMPULS lesson study immersion programme provided many examples of teachers and teacher educators having common knowledge, bringing specialist knowledge and co-constructing an understanding of the research lessons. In our post-lesson discussions we would discuss the mathematics, where it fitted in to a learning sequence, the forms of teaching activity, and the things that students said and did and wrote during the lesson and how the ideas being expressed by students developed during the lesson. This discussion was always in relation to a lesson we had all been present for. As a result, it was always possible to ask, "What did you see that you are interpreting in that way?" This ability to ask for a description and to compare what we had noticed facilitated a sense of joint enterprise in the present and immediate moment and in which we could explore each other's ideas. This is different from a general joint enterprise in which idealised lesson episodes are discussed and in which general solutions are offered. Here we discuss the activity of particular students and the nature of particular mathematical problems. However, I do not think this is inevitable or certain to happen. This process could easily be distorted by judgmental and evaluative commentary made by people with disciplinary authority in the setting.

It is tempting to claim the lesson study approach I observed during the IMPULS immersion programme inevitably keeps our ideas about teaching continually referenced to a lesson we had all been present for. However, I think this is a fragile aspect of lesson study and we need to be vigilant to keep it so. A simple example of the dangers of failing to refer back to practice can be seen in how teachers and teacher educators think about 'problem solving'. The specific definition of 'problem solving' given by the IMPULS team during the IMPULS immersion programme follows Pólya (1945) and matches closely that of the NCTM that problem solving means engaging in a task for which the solution method is not known in advance. But some have the habit of referring to exercises of familiar or previously solved problems as 'problem solving'. Once students know how to solve problems of the type posed, such problems become practice questions and exercises. So whether a particular question will result in 'problem solving' depends on the knowledge of the students when the question is asked. So it is possible to talk about 'problem solving' without reference to a lesson we have all been present for and to be talking about two different types of activity. Here we have a difficulty revealed and potentially addressed by lesson study. It is the observation and discussion of a lesson we have all been present for that results in a legitimate and deliberate questioning of terminology. If or when differences of meaning of terminology are revealed it is the reference to a specific episode in a lesson we have all been present for that facilitates our realisation of the difficulty with our terminology and possibly opens the door to developing a specialised vocabulary that could be seen as an essential element of transforming professionalism in mathematics teaching.

General principles of lesson study

As Akihiko Takahashi explained during the IMPULS lesson study immersion project, regarding the impact of lesson study, there are statistics showing grade improvement in Chicago. But there are many reports of lesson study not having any impact on teacher or student achievement. The IMPULS team are working with the assumption that the cultural phenomenon of Japanese lesson study has to be done a certain way but it is not clear what the essential elements or principles are. This is the research needed in the future. There is not much research into the process of lesson study. On reflection, I have some intuitions about what might be general principles of lesson study.

Formality

Elements of lesson study are formally planned and participants know how they are expected to act at each stage. It is therefore possible to keep other non-developmental processes out of lesson study. In the English context this is important. Lesson study must not be about evaluating teachers, appraisal or grading lessons. Formally separating lesson study from accountability is important.

Variety of expertise

Lesson study must bring together people who know different things. This avoids parochialism, creates challenge and requires courage and humility. In such circumstances, teacher educators have to make their ideas relate directly to events all participants have witnessed and they have to show their expertise is applicable. This is a challenge for teacher educators and it is an act of courage to step away from the comfort of the university lecture theatre and pre-packaged educational materials. Teachers invite experts to observe public lessons and taking this is risk requires courage. Both teachers and teacher educators will need humility in these circumstances.

All participants observe the lesson

Participants pay attention to the relationship between what has been seen (what can be described in matter of fact terms) and how it has been interpreted (which is open to dispute). One test we might apply is to ask the theoretical question, "Could a person who did not observe the lesson participate in this discussion?" If the answer to this question is 'yes' then the discussion is too general and not related to events and observations closely enough. The first work we do in a post-lesson discussion is to compare what we have noticed and we assume everyone has noticed something. A non-observer would not have noticed anything and would only be able to comment in general terms on what others had noticed. We should recognise the importance of a simple affirmation such as, "Yes, I noticed that too."

Lesson study in relation to other forms of teacher learning activity

There are other forms of professional development activity that seem to hold on to these principles of lesson study. Teacher inquiry is a similar process that can involve different expertise; for example, I have cited Barbara Jaworski's work on Mathematics Teacher Inquiry. There are other forms of action research that seek to facilitate a relationship between teachers, consultants and researchers in a joint enterprise. When we formalise teacher development activity we should ensure that the expertise of all participants is recognised so that everyone expects to learn; that all participants are witnesses to actual teaching and learning episodes being discussed; and that all participants understand they have something to learn.

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Sarah Harris

Japanese Mathematics Instruction

Several key ideas stick out in my memory from my experience in Japan. First, the intentional difference between problem solving and exercises. In Japanese instruction, exercises are not part of the every day math class; they are saved for the last few days in a multi-lesson unit for students to practice what they've learned. Usually, in my classroom (and most that I know of) teachers may present a problem, then give exercises to practice what they learned. In every Japanese mathematics classroom I observed, problem solving was the entire lesson.

Also, a big push in my district is the use of "I can" statements. We are instructed to post them before each and every lesson in every subject. I find it interesting that in the problem-solving classroom in Japan, this is considered a bad practice because it tells the kids how to solve the problem and there's no need for them to figure this out on their own. When distinguishing between which is more important (figuring out which calculation to use vs. the calculation itself), the Japanese consider the problem-solving aspect more valuable, because you can always use a calculator to solve in the real world!

While in Japan, the topic of differentiation came up several times. Many schools, from what I understand, have leveled classrooms and students with special needs may not be in the regular education classroom. This is very different from U.S. classrooms. However, I did learn that in the lesson planning process, anticipating student responses is a form of planning for what may happen at different ability levels, which is planning for a range of learners. Additionally, it is important to differentiate entry points rather than differentiating the task itself to avoid the risk of differentiating the expectation, which is not best practice.

After this trip, I am convinced that anticipating student responses is one of the most valuable things I can do as a teacher before a lesson. Dr. Fuji mentioned that teachers are somewhat evaluated on their math content knowledge by the quality/validity of the anticipated responses within the lesson plan. I feel that one of the biggest problems we have in America is many teachers teaching without solid content knowledge, especially in mathematics.

An idea that one of the final commentators brought up is to encourage students to think about the meaning of mathematical concepts so that they become better at mathematical procedures. I think, as a fourth grade teacher, when I have students that have difficulty adding, for example, my response is to give them more practice, when I should really be asking myself if they missed the meaning of addition in previous years and that is getting in the way of their ability to calculate.

Finally, the role of journals and board writing in the Japanese mathematics classroom is something I want to implement more fully in the future. I tried these components this past year as part of the Teaching Through Problem-Solving project through Mills College but did not make them consistent, daily procedures for myself or the students. This year, it is my goal to implement these consistently as a regular part of my mathematics classroom.

Japanese Lesson Study

Lesson study is the majority of professional development among Japanese educators. The fact that the administration is so involved was very surprising. One thing I especially love about Japanese Lesson study is the role of the final commentator. This was the part of the post-lesson discussion that I looked forward to the most and I am sure many of the Japanese teachers felt the same way. It is also a valuable piece because of the sense of community it brings to the education system. The idea of school-wide and district-wide lesson studies with a final commentator, often from the university level, showcases the idea that the education of a community/city's children is not solely the responsibility of one person. This is a powerful idea that is often missing in U.S. schools. In Japan, there is no competition among schools and districts. All educators are responsible for all Japanese children.

One thing that surprised me was the teacher analysis in the post-lesson discussion. In lesson studies in which I have participated in America, most of the post-lesson discussion is based on the progression of student idea and discourse. In the first post-discussion we observed in Japan, I almost felt sorry for the teacher. However, after several more lesson studies, I realized the teachers did not see themselves as being attacked. Instead, they saw value in their colleagues and superiors critiquing them. We are often too sensitive when we get critiqued because we see it as criticism.

The amount of time and effort that goes into a lesson for the lesson study process really surprised me. Kyozai-Kenkyu, the process of studying a variety of teaching materials, is intriguing to me. Dr. Fuji compared this process to food when he said, "If you don't cook, you can't eat!". I find myself writing lessons based solely on our state's standards. For lesson study, Japanese teachers study a variety of teaching and learning materials, methods, the process of student learning (including misunderstandings and mistakes), and research related to the mathematical content. Dr. Fuji might say that a Japanese lesson study research lesson is a gourmet meal as opposed to a microwaveable frozen meal that we too often serve our students!

The idea of a pre-lesson discussion was brought up as a potential new component of Japanese lesson study. In my district's classroom studios (which is similar to lesson study), we always have a pre-lesson discussion. During this time, we do the math together which is an important piece, in my opinion. However, it seems that Japanese teachers do this weeks before the lesson as part of the lesson planning process, so maybe it wouldn't be as necessary. I am interested to see if this component becomes part of the Japanese lesson study model and what would take place during this pre-lesson discussion time.

Final Thoughts

Finally, after studying my notes and reflecting about my experience upon my return from Japan, I made a list of specific things I am taking away from this project that I will use as an educator in addition to the entire lesson study process as a whole:

- Cherry diagram (I look forward to using this tool with addition and subtraction work.)
- Double number line (I was fascinated by the presentation Dr. Takahashi gave at the end of our project on the progression of the number line throughout Japanese textbooks. I want to study this further, especially the use of it during work with multiplicative comparison.)
- Journals (specifically the structure I saw in Japanese classrooms --> problem and reflection on the left and solving/calculations on the right)
- Board plan (After the TTP project in Chicago in July 2013, I was not sold on this component; however, after seeing it in use in authentic classrooms, I am anxious to use this in my own lessons.)
- Post-discussion comments (Using a different colored card for positive and negative comments is something I want to implement in my school's lesson study process.)
- Division algorithm (One final commentator showed a division algorithm that is similar to, but not exactly like one we use in the U.S. I want to integrate this into my division unit somehow.)

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  1

- Order of division work - quotative/partitive (In Japan, the two are taught at the same time. In America, usually "fair share" (partitive) is taught first, which I feel is a mistake. Working through different types of division makes it easier to move to fraction division and ratio division later on. Also, I would like to intentionally distinguish between the two types in my division unit. The discussion we had and the lesson we saw on remainders was so powerful.)
- Role of different individuals participating in lesson study, in addition to classroom teachers (Principal, Moderator, upper/lower grade teachers, special guest/final commentator --> I am not sure how much "power" I have in my district to make this happen, but it is definitely something I will investigate. Perhaps an invitation is all someone needs to be willing to participate!)
- High Impact Strategies for Teaching Mathematics (I will share these with my colleagues and superiors to better mathematics instruction in my school/district.)

Sibeso Likando

From 16 to 26 June 2014, I was one of the sixteen mathematics educators and practitioners from Australia, UK and USA who participated in the IMPULS (acronym for International Math-teacher Professionalization Using Lesson Study) Lesson Study Immersion Programme in Tokyo, Japan. The purpose of the lesson study programme was twofold: (i) to give mathematics educators and practitioners from outside Japan an opportunity to examine authentic Japanese Lesson Study in mathematics classrooms in elementary and lower secondary grades, and (ii) to receive feedback on the strengths and weaknesses of Japanese Lesson Study and to discuss how to improve mathematics teacher professional development programmes. This report gives the reflections of my participation in the programme.

1. The participants

A total of sixteen participants was drawn from Australia, UK and USA. Two participants came from Australia, four from UK and 10 from USA. The team was full of energy as can be seen from Photo 1.

Photo 1: Participants with programme facilitators

2. Programme organisation

My first impression when I considered the items on the itinerary was that the program would be too busy. However, the programme was well-organised, and all activities for each day were as planned. There is a strict culture in Japan on keeping time. After observing a lesson study, participants had a social gathering in the nearby bars to reflect on the day's work.

The communication between participants and programme organisers was effective. Prior to my participation in the programme, for example, I received updates through emails. Also, during the programme, participants were provided with the most efficient logistical assistance. We were thoroughly briefed on the programme, the venue and facilities, and we had name badges to be easily identified.

3. The new insights I gained from the programme

Assessment practices and reflections

I augmented my knowledge on new assessment practices. The practices include close observation of student solutions; recording student solutions; asking students to write about their strategies/learning during the lesson, and using observations by others.

Photo 2: Teaching and observing a research lesson

Also, it became evident that educators need to augment their reflective practices: ways of recording reflections, discussing the reflections with others, and incorporating them in future planning of mathematics lessons. In my research on the "Implementation of lesson study in Mathematics, a case of Zambia," I shall investigate, among other issues, the changes in the reflective practices of teachers engaged in lesson study in terms of the ways of recording reflections and discussing with other teachers.

Focus on problem-solving

What I read and heard about Japanese structured problem solving and its focus on one main problem, with students sharing their strategies and solutions, was witnessed during the programme. In my opinion the lessons I observed (based on problem-solving) were very "successful" in advancing student learning. Of course, there were challenges that were uncovered, such as failure by a teacher to present the problem clearly to students. I see that such structured problem-solving lessons should be mainstreamed across the mathematics curriculum in other countries. In my research on the "Implementation of lesson study in Mathematics, a case of Zambia," I shall consider studying the changes (if any) teachers have made or intend to make regarding the use of problem-solving in their mathematics lessons, as a result of their participation in lesson study. I shall also consider investigating additional support teachers think they need to achieve their goals relating to problem-solving.

The role of discussion in mathematics lessons

I was intrigued by the extent to which extensive discussion of student strategies and solutions were considered during post-lesson discussion. I gained a new insight that student strategies and solutions should be discussed extensively and incorporated into the research lessons. Teachers should ensure that discussions are "successful" in advancing student learning. It became apparent during post-lesson discussion that a strong connection exists between the use of Japanese problem-solving structure (with its associated focus on discussion) and students taking responsibility for their own learning.

I now consider discussions as necessary conditions for advancing student learning. In my research on the "Implementation of lesson study in Mathematics, a case of Zambia," I shall investigate the place of discussions like ones I experienced in Japan in the mathematics lessons in Zambia. I shall find out how (if at all) the teachers' views about discussion in mathematics lessons have changed as a result of participation in lesson study. I will also investigate the connections (if any) teachers see between the use of the Japanese problem solving structure (with its associated focus on discussion) and students taking responsibility for their own learning.

Post lesson-discussion

The post lesson-discussion began by head teacher, who chaired the meeting, outlined the agenda for the discussion and invites the teacher who taught the lesson to comment on his or her reactions to the lesson. Photo 3 shows one of the post-lesson discussions.

Photo 3: Post-lesson discussion

Thereafter, questions and observations were invited. After all questions and observations are exhausted, the external expert was invited to comment on the lesson (what went well and any difficulties) and make recommendations. Personally, all the research lesson and post-lesson discussion shown in Table 1 were professionally informative to me.

However, the most informative post-discussion for me was at Oshihara Elementary School. The external expert emphasized that students should be given an opportunity to understand the link between mathematical concepts and daily life. The external commentator praised teacher Masaki Tsuruta for having connected the lesson, *Let's make quadrilaterals*, to daily life.

Table 1: Lesson study I immersed in during the programme

DATE	TOPIC	SCHOOL	GRADE
June 17	Algebraic Expressions	Koganei Lower Secondary School attached to Tokyo Gakugei University	7
June 18	Subtraction Algorithm	Matsuzawa Elementary School	2
June 19	Calculations for Finding "Times as Much"	Koganei Elementary School attached to Tokyo Gakugei University	3
June 20	Division with remainders	Sugekari Elementary School	3
June 21	Research Lesson 1- "Utilizing Mathematics" activities in which students express and think about phenomena mathematically	Tokyo Gakugei University International Secondary School	7
June 21	Research Lesson 2- Generating differential equations as mathematical models	Tokyo Gakugei University International Secondary School	12
June 23	Trapezoids and Parallelograms	Oshihara Elementary School	4
June 25	Division of decimal numbers	Koganei Elementary School	5

Role of school administrators

It became apparent from the experience that administrators should pride in lesson study as a teacher professional development strategy and attend lesson study sessions, especially lesson teaching and observation, and post-lesson discussion. They may chair the post-lesson discussions. Administrators should invite external experts to take part in lesson observation and to sum up the post-lesson discussion.

Furthermore, administrators should facilitate parties after lesson study for participants to reflect on the lesson study freely. Under the influence of a few bottles of beers, some teachers may bring out significant issues that might not have been raised during the post-lesson discussion. Also, administrators should facilitate teachers to participate in lesson study in other schools.

4. Conclusion

From the preceding discussion, my examination of authentic Japanese Lesson Study in mathematics classrooms in elementary and lower secondary grades, brings to the fore the strengths of Japanese Lesson Study which include a strong focus on student learning and content; use of structured problem solving lessons across the mathematics curriculum; linking mathematical concepts to students' daily lives; and embracing new practices for assessing students.

Simon Terrell

First Impressions

When I first arrived in Japan, I was very worried because I don't speak Japanese. However, I was given good directions to find the proper train station. From that time forward, we were taken care of by Ishiharasan and the graduate students. This made it very easy to relax and concentrate on the lesson study that we would be learning about. I think my experience would have been very different if we had not had the help of so many people.

Our first lesson was at the university and I was struck by how simple the classroom was. It was not completely empty but, at the same time, there was a small amount of things up on the walls comparative to American classrooms. On top of that, there was only a chalkboard and no noticeable technology, neither computers nor interactive whiteboards or video projectors. Although there are many schools in the US without these things as well, there seems to be a belief that having classroom technology is very important.

Classroom Environment

Before I left for Japan, I mentioned to many of my colleagues that I would be going to learn about lesson study in Japanese classrooms. Almost universally, their reaction was that the Japanese classroom and the American classroom couldn't be compared because of the difference between the two cultures. My colleagues felt that students would be much more respectful comparative to our students and that because of that they would do much better in their lessons. Because of this, I wanted to see if I could find students who were bored, disengaged, or distracting others during the lesson.

In most American elementary school classrooms, teachers have a behavior management chart where students can either move progressively down the chart for misbehaving or move up the chart for being good. I didn't notice until about a week after leaving Japan that I hadn't seen any sort of device like this in any of the classrooms that I visited. I wasn't able to ask any teachers about this, so it will be something that I will continue to ponder.

All in all, I felt that the students were very respectful, and though the classrooms were actually relatively noisy, the chatter seemed to be about the work that they were doing and not so much off task behavior.

After reading lesson plans and hearing the discussions after lessons, it was very clear that Japanese teachers put a lot of thought into student engagement. I often heard students reacting positively to problems that teachers were posing. In each lesson, the problem was posed in some interesting way. For example, a 3rd grade teacher used a story of packing Octopus Balls into groups of 4 in order to do division and many students said, “Those look tasty”, or, “ I would look to eat those”. In another classroom, a teacher had done a survey ahead of time in order to get a feeling for students' understanding and interest in weekly allowance. The effect was that in every case, almost all students were interested in solving the problems (a much higher rate than I have experienced in my own classroom).

Although there are definite cultural differences between our two countries, I can't attribute that as the only thing that separates our systems. I did see students who were not fully engaged and did see some lessons that didn't work for all students, but the level of preparation and thinking about student interest was visible and I think that this is something that can exist anywhere.

Japanese Text Book

We had several conversations about the Japanese textbook. We were told about the thought and research that goes into choosing each lesson. The numbers used in an addition lesson are chosen based on which problem solving methods are desired for the particular phase of student learning. This was all geared toward using problem solving on an almost daily basis. I was struck by the simplicity of the book but also at the depth that was hidden in each lesson.

After using the text at times during the last school year, I noticed that my understanding of the mathematics needed and the deeper mathematics in something seemingly simple as addition of two digit numbers got much better. I feel that by using the lessons from the Japanese math book, an elementary teacher who does not have much confidence in their own mathematical understanding could improve on this very quickly.

Post-Lesson Discussions

The post lesson discussion is a very important and useful part of the process. The environment of the room was very collegial and safe. I got no sense that anyone was there to criticize the teacher of the lesson. The teacher's colleagues asked questions about things that had noticed during the lesson. The discussion gives all the teachers in a school the opportunity to reflect on a particular lesson and see the embodiment of the goals that they had chosen at the beginning of the lesson study cycle. The discussion also gives everyone a chance to look at and talk over the mathematics involved in the lesson and how those mathematics appear at each grade level.

I really appreciated the opportunity to ask questions and hear the questions of others. I came away from the discussion with a deeper understanding about students, mathematics, and teacher decisions during the flow of a lesson and how each decision had a different effect.

The final speaker plays the important role of giving deeper insight to a subject of their choice. I heard a lot about the direction that the teacher needed to go in next, philosophy (John Dewey was cited by one speaker), or a critique of the lesson itself that served to show all in attendance work that needed to be done by the school.

I'd say the one aspect about the post lesson discussion that I enjoyed the most was when the school went out to celebrate afterwards. First of all, the teacher of the lesson was celebrated for the hard work that they had done. I really appreciated this because it sends the message that the lesson study is not a test or an opportunity to put the teacher down, it is an opportunity to uncover strengths and weaknesses so that the whole school can improve. I got the chance to speak with Japanese teachers in a relaxed setting and really enjoyed the positive atmosphere and togetherness. It really helped to bring the participants closer together.

Challenges for US schools

One of the things that struck me deeply was when Dr. Takahashi told us that all parts of lesson study are equally important. I am paraphrasing here, but he said something like, “We don't know which part of lesson study is most important so doing each part is necessary” (I'm apologize if I completely misremembered his words). Because of the importance of doing each part of lesson study, there are a few hurdles that would need to be overcome in order to do lesson study properly in our school.

There are at least two hurdles, the timing of the lesson itself, and finding a final speaker who understands the purpose and limitations of their role.

In a Japanese school, all the teachers in the school are able to watch the lesson and participate in the post-lesson discussion. This is necessary in order to make whole-school progress towards a set of shared goals. Unfortunately, not having this culture firmly established in the US, we would either have to limit the number of teachers who could observe that lesson due to the cost and availability of substitutes, or teach the lesson outside of normal school hours. Being that we are a rural school, a large portion of the students take the bus to and from school. If we taught a lesson outside of regular school hours, the likelihood of having a complete classroom set of students would be very small. Having a normal classroom and not a mixture of students is very important to seeing the students and teacher in as organic an environment as possible.

The final speaker is a very important and understood role in Japanese lesson study. The final speaker is very experienced and knows that their job is not to talk down to the teachers of a school or prove their superior knowledge, but to enrich the discussion by imparting some wisdom or a perspective that serves to leave the school and participants better off. In the US, it is less likely to find someone from the university who understands this role well and we risk possibly having someone who is overly critical and sours the participating teachers on the intended goals of a lesson study.

Last Thoughts

In finishing, I want to say how much I appreciate the opportunity I have had to attend this immersion program. I feel so fortunate to have met so many people who care so deeply about mathematics education and who think about it at such a level. It was life changing for me and I can't thank the IMPULS program and Dr. Takahashi, Tad, Makoto, Professor Fuji, Ishiharasan, and the graduate students from the University. I was really honored to be able to see the level of joy in the classroom and was inspired to try to bring this back to my own school. Once again, thank you to all who helped this be a life changing experience.

4 External Evaluation of the program

2014 Immersion Program Evaluation Report
Nell Cobb, DePaul University, Chicago, IL, USA
December 5, 2014
"We come from different places but we all have a shared experience and common language"
(Field notes)

Background
The comment above by a participant is an appropriate description of the 2014 collective. From June 16th – 27th, The International Math-Teacher Professionalization Using Lesson Study, (Project IMPULS) of Tokyo Gakugei University, organized a ten-day lesson study immersion program designed to familiarize an international group of educators with authentic Japanese lesson study and Japanese mathematics education. As part of the program, participants experienced seven school visits in Tokyo and Yamanashi regions—touring school facilities, interacting with students and teachers, and observing research lessons and post-lesson discussions. Many participants attended the Chicago Lesson Study Conference the summer prior to this program. This provided them with a shared experience and common language about lesson study.

Fifteen educators from three countries (Australia, Great Britain, and the United States) and five U.S. states (Florida, California, Illinois, North Carolina and New York) took part in the 2014 lesson study immersion program. This is one less participant from 2013 and 25 less than 2012. The three countries are consistent with the 2 former years, 2012 also had at least one participant from Singapore. There were a variety of educators including elementary teachers K-7, math coaches, college math educators, one literacy coach and one Ph.D. candidate. All participants had prior experience with mathematics lesson study (60% of the participants had up to a year, 27% had up to 2 years, and 13% had 3 years or more experience with lesson study). In contrast to last year's participants, 80% of this year's group actual planned, taught, and participated in the post discussion for a research lesson. There was one participant who served as an external expert or knowledgeable other in their district based lesson study.

This report is a mixed data study, which includes quantitative as well as qualitative measures. Data collection consisted of a pre-post survey based on the last two years, daily reflections, interviews, program summaries, and group reports. An analysis of the pre-post test was conducted and discussed in the report. There is a descriptive comparison of the last two years. A summary of results and recommendations for improvement are offered.

Executive Summary of Findings

"Since the trip I have given more importance to mathematical content and anticipating student responses because these are essentials for planning a successful lesson. I believe before the trip, I certainly thought these things were useful but perhaps didn't think they were the number one priority. However, now I feel if these are considered then other things will fall into place."

"I think learning about how lesson study is conducted in another country and the organization of the post lesson study discussion were at the top of my want to know list and those expectations were met. I didn't expect to learn so much about strategies for making student thinking visible since I can't read or understand Japanese but between the translations and inferring I gained quite a bit of understanding about that component."

These were two responses to a question in the post survey about any changes made to participants' initial responses of the 25 features of Lesson study after the trip. For those who remembered what they choose, they reported that they were more focused on "making student thinking visible," "anticipating student responses," and "the study of materials and resources (kyozai-kenkyu) as a result of their immersion experience." However, the mean learning about program elements (Posttest Rating) indicated that "How lesson study is conducted in another country", "organizing a successful post-lesson debriefing session," "how to build students' problem solving," and "supporting participants to have powerful and effective lesson study experiences were the areas chosen by this year's participants" (See Figure 1 showing mean participant ratings of learning of 25 program elements). This was consistent with last year's participants who also reported they learned the most about "how lesson study is conducted in another country." The 2014 participants seemed to learn slightly more about topics like "organizing a successful post-lesson debriefing session" and "math content." Both groups reported that "differentiating/offering support for struggling learners" was the least significant topic they learned compared to other lesson study program elements.

A dependent t-test was performed on the thirteen statements related to Mathematics Attitudes of the 15 participants to ascertain if the post-test scores were significantly higher than the pre-test scores. The post-test scores ($M = 35.14$, $SD = 2.18$) were significantly higher than the pre-test scores ($M = 32.57$, $SD = 3.99$), $t(14) = 2.06$, $p = .06$, $d = .74$. The effect size between the pre- and post-test scores approached a large effect size (Cohen, 1988, 1992).

This was the third year that IMPULS has funded and organized this immersion experience, and this year's program benefited from recommendations that emerged from the 2013 program:
1.) More time to process observations. The scheduling of some of the post discussions, prior to school visits, did not allow for thorough discussion of the observations as well as discussion of lesson to be observed on that day.

This year's schedule provided ample time for participants to discuss the lesson before and after observations. In particular, after each lesson there was at least one post-lesson discussion immediately following the classroom observation. For at least two school visits, there were two post-lesson discussions. In each of those cases, there was a private post lesson discussion for the IMPULS group.

2.) Small group discussions as opposed to whole group interactions after each lesson and post-lesson discussion. It was suggested that using daily reflections to guide discussion after lessons might also be useful.

The small group discussions came in the form of group reports, informal discussion during breaks and group dinners, and talks while traveling. For each lesson there were at least two people who discussed the lesson and wrote a group report. Many of the ideas, from these reports as well as the daily reflections, surfaced during the post-lesson debrief. As suggested last year, participants could be informed that an initial draft daily reflection form should be completed prior to the post-lesson discussion. This will also eliminate any last minute completion of these reports. The group reports should be better integrated during these discussions as well.

3.) Other suggestions from participants included opportunities to a) plan a research lesson, b) observe a planning meeting, c) analyze student work, and d) select lessons for observation by grade level, though it was acknowledged that these would be difficult to fit into the agenda.

This suggestion was not realized in its entirety this year. As stated, it is difficult to fit these experiences into the agenda. However, there were many discussions about planning meetings, analyzing student notebooks and work, and selecting lessons for observations during the post-lesson discussions with Japanese teachers.
There were two main categories that were captured by participants during the experience, reported in the surveys and discussed in the summative journals: 1) Intentional Problem Solving and 2) Teacher Content Knowledge.

1. Intentional Problem Solving

In the Japanese classroom, the focus is on conceptual understanding. In the recent New York Times article, "Why Americans Stink at Math." Dr. Takahashi told his story about being transformed by his mentor professor and inspired "to encourage passionate discussions among children so they would come to uncover math's procedures, properties and proofs for themselves". With this goal, he later embarked on his journey to the US, the place where the National Council of Teachers of Mathematics first proposed this approach in the 1980s and later in the early 1990s. There was a shift from prescriptive instruction to intentional problem solving. However, as Dr. Takahashi reported in this article, as he readily states in many discussions, he was disappointed to see that many classrooms in the US were not engaging students in intentional problem solving. As one IMPULS participant stated in a review of this article,

> "Most American math classes follow the same pattern, a ritualistic series of steps so ingrained that one researcher termed it a cultural script. Some teachers call the pattern "I, We, You." The "I" refers to the teacher demonstrating a new procedure, the "We" refers to the teacher leading the student to try out a sample problem together, and the "You" refers to the students as they work through or practice similar problems on their own…"

Another participant talked about the role of problem solving in her district compared to what happens in Japanese classrooms.

> "Usually in my classroom (and most that I know of) teachers may present a problem, then give exercises to practice what they learned. In every Japanese mathematics classroom I observed, problem solving was the entire lesson."

Some participants discussed time spent on student problem solving and the various problem entry points.

> "I was amazed to see that a whole lesson could be spent on a seemingly simple problem. During the grade 2 lesson the class spent the entire time discussing and exploring the methods of solving 45 – 2. …Students understood that the point was not to reach a solution but to explore the different methods by which a solution could be reached."

> "I see the importance of developing problems in which all students are given an entry point to have an equal opportunity to succeed in solving a problem.

One participant discussed the co-construction of knowledge in teacher preparation programs.

> "My question is how can we learn to teach by considering it to be a problem solving activity? What would it look like if teacher educators and teachers coconstructed their solutions to the problem of how to teach mathematics."

The New York Times article provides a good reference for how to teach mathematics effectively. The authors also state that additional professional development for teachers is warranted. In addition, Ball, Hill, Bass (2005) discuss the kinds of knowledge that successful teachers must have. Teacher content knowledge is among those knowledge bases.

2. Teacher Content Knowledge

Through out the experience, teachers were engaged in problem solving and discussion about mathematics content. There was also strong interest in the use of a double number line. In many cases teachers were investigating the meanings of the mathematics concepts written in the lessons and observed in the classrooms. So the teachers talked about gaining content knowledge in three subcategories: extended learning from lesson plan and observation, learning from textbook, and learning from knowledgeable others during the post lesson discussion.

a) Extended Learning from lesson observation

In the June 25th lesson for the 5th grade class, the teacher in his lesson proposal stated that he noticed that the success rate for problems involving wariai (used when comparing the quotient) is rather low. In the lesson, the teacher provided a rationale that "students will understand the idea of wariai more deeply if they have opportunities to compare situations using the idea of bai (the quotient)." He anticipated that the students would need to decide what they are going to use as the base amount. There was a rich discussion about using these concepts. The essential question was "When should students use the differences to compare and when should students use bai." The participants also saw the double number line, as an anticipated response, in the lesson. This is when many participants requested to learn more about the double number line and when to use it. Dr. Takahashi explained that in the second grade the topics involve measurement situations. The use of the tape in the lessons would lead to the number line. In grade three, there is an exploration of division with the questions: “How many groups?” and “How many in each group?” Dr. Takahashi also introduced the participants to the double number line. This was a new tool for some of the participants. All participants were engaged and interested in learning more about this problem solving tool/strategy.

Much of the teacher content knowledge also came during the textbook discussion and in the post lesson by the knowledgeable other.

b) Learning from textbooks and other materials -(kyozai-kenkyu)

> “In the UK reliance on textbooks can be looked down upon, partly due to lack of faith in textbooks that on occasion contain conceptual errors themselves. However, the Japanese textbooks have been developed through lesson study and a great deal of time and thought has been put into the problems and, in particular, the numbers used.”
>
> “Kyozai-Kenkyu, the process of studying a variety of teaching materials, is intriguing to me. Dr. Fujii compared this process to food when he said, "If you don't cook, you can't eat!" I find myself writing lessons based solely on our state's standards. For lesson study, Japanese teachers study a variety of teaching and learning materials, methods, the process of student learning (including misunderstandings and mistakes), and research related to the mathematical content. Dr. Fujii might say that a Japanese lesson study research lesson is a gourmet meal as opposed to a microwaveable frozen meal that we too often serve our students!”

c) Learning from the Knowledgeable Other

Dr. Takahashi has written an article, The Role of the Knowledgeable Other in lesson study: Examining the final comments of experienced lesson study practitioners. In this article he examined the actual comments and interviews of three highly respected knowledgeable others to determine the characteristics of effective comments in post lesson discussions. He also states that the knowledgeable others determined that they are responsible for:

(1) bringing new knowledge from research and the standards;
(2) showing the connection between the theory and the practice; and
(3) helping others learn how to reflect on teaching and learning (Takahashi 2013, p. 9).

One participant started this trip to Japan by asking what are the best bits of lesson study. It was his thinking once he identified the best bits, he bring those back to his district. Here are his views on the knowledgeable other:

> “well by now I wasn’t having much luck finding any parts of Japanese Lesson Study to get rid of. But what about the knowledgeable other who provides comments at the end of the research lesson? No way! Do not mess with these guys! What a privilege it was to hear the thoughts and opinions of such esteemed mathematicians and educators.”

Recommendations for Program Improvement

This section provides considerations for the program for future years. There is a discussion of the most professionally informative lesson and post lesson discussion, the least professionally informative lesson and post lesson, the number of lessons observed, additional comments about itinerary, information about hotel accommodations and general suggestions.

a) The most professional informative lesson and post lesson discussion

The lesson and post lesson discussion that participants choose as the most professionally informative was the June 19th lesson on Division- Calculation for Finding "Times as Much." This 3rd grade lesson was identified by 53% of the participants as the most informative. The fact that this lesson was not a research lesson was appreciated by the participants. In addition, participants were able to gather multiple ways to engage students in independent and collective problem solving.

> "First of all I appreciate that the lesson we observed was part of an "everyday" lesson and not a formal [research] lesson because it showed me how lesson study can impact and shape everyday teaching. I felt that Mr. Takahashi's classroom culture was conducive to problem solving and that his questions were effective in addressing misconceptions…I also appreciate how he built student engagement from the moment the lesson began by labeling it "Tape."

> "This was an 'every day' lesson. I really like how Mr. Takahashi was able to guide the student and have them do most of the work. Then they together put it together and started making sense of each other's methods. The post- lesson discussion was great since he had wonderful points about turn-and-talks and how the students and teacher build the lesson together. I can't wait to share these points with my colleagues."

b) The least professional informative lesson and post lesson discussion

Seventy–nine percent of the participants selected June 21, Lesson 2 Generating differential equations as mathematical models, to be the least informative. In some cases, most participants did not understand the mathematical content.

> "I was so lost with the mathematical content!"

> "The truth is I choose this because it was the content I knew less about so I struggled to follow along and comprehend the content. Also students seemed like they were having a hard time and the teacher never quite got around that issue."

> "This was beyond my level of understanding in mathematics as I haven't studied this for a LONG time. It was therefore difficult for me to understand the lesson and the post lesson discussion."

For those who did understand the content, they felt the structure of the lesson, group discussion, and integration of technology was least effective.

"I felt that the neriage stage of this lesson was underdeveloped and that some important points were missing by the teacher. I also found it difficult to follow the solutions that were being displayed… The post lesson took to get the crux of the issues…"

"The lesson seems to introduce forms of activity the students were not very familiar with including the computer software and group discussion. This made progress more difficult for them and detracted from the focus of the lesson on problem solving. It would have been better to keep as much constant as possible."

c) The number of lessons observed

The participants were generally pleased with the number of lessons observed during the program. There were 87% that indicated the number of lessons observed were just right. Eighty-Seven percent of the participants disagreed with the statement, "There were too many items on the itinerary, as a result, the program felt too busy." Of those 87% indicating the lesson observed were just right, 27 % strongly disagreed. This is an indication that the itinerary for this experience was appropriate.

"I think this program was great and I learned so much from it. I hope to learn even more as I go forward with lesson study."

"It was a very rigorous program, however, I wouldn't have wanted any less. I feel that the professors sensed when we were burnt out and have us some early dismissal dates…Perhaps having a planned 10 minutes for silent reflection would have been helpful for jotting down these notes [quick write] while they are were very fresh in our mind."

"Although it was an intense experience I wouldn't have wanted anything less on the program as wanted to make the most of the time we did have. I felt the sessions given before research lessons where we had the opportunity to discuss questions about the lesson plan and attempt the problem were particularly useful in preparing us for productive observations."

There were some discussions about decreasing the number of lessons:

"The reason I said 'too many' lessons is because after our trip to Yamanshi, it felt redundant to see more lessons. We weren't really seeing anything new at that point. Our brains were all full, and perhaps some time to process it all would have been helpful. Also, some discussion among the group of 'next steps' when we get back home would have been nice.

"I know that I was exhausted after the two public open house lessons on Saturday, but other than that the number of lessons we watched were appropriate."

"Time devoted to work on our group observations would have been useful at the end of the trip. I loved the fact that we had such a full and rich experience however this meant little time to get together to work as a group."

d) Comments about itinerary

The changes to the trip itinerary included more discussion about homework and assessment; more time to discuss the math content, more post-lesson discussion within our group; a better balance of grades, equal number of lesson distributed between primary and secondary lessons, and extended time observing one teacher.

> "I think I would have liked to learn more about the roles that homework and summative assessment play in Japanese system.'

> "I would have liked a bit more time to discuss the math that students were doing that day (like we did on the 1st observation day when 7th graders found how many dots were in the 13th image) and discuss the anticipated responses as a group before observing the lesson."

> "I would have perhaps liked to see more of a balance of grades. Although we did see lessons from a wide range the majority were primary level which was very interesting but perhaps as an ideal I would have liked to see an equal dispersion for primary and secondary lessons."

> "I would have liked to observe 2-3 lessons in the same classroom of one of the stronger teachers (from Koganei or Sugekari) because we could have learned more about making adjustments from the first lesson through analysis of student work/journals to inform the next day's instruction. It would have been helpful to look at student work, discuss next steps, watch the second lesson, and then reflect on the process and outcome."

e) Accommodations

For the other aspects of the trip, the participants were satisfied with: accommodations (Hotel Mets – M=4.67); Meals (Hotel Mets – M =4.20); accommodations (Hotel Fuji – M=4.07); Meals (Hotel Fuji – M= 4.20); Communication with program staff prior to arrival – M=4.40; and Communication with program staff during the program, 4.60.

f) Program staff

> "Many thanks to Ishikara sensei, we were very well taken care of by her. Also to the graduate students, Dr. Takahashi, Dr. Watanabe, and Dr. Yoshida, their hard work and preparation was very helpful. I enjoyed every aspect of this trip and hope to return in the future as well as do lesson study in my school and county."

g) Hotel Fuji

> "Hotel Fuji was quite an experience! But I will say I am glad it was only 1 night of floor sleeping! But I am so thankful I got to experience the traditional room and amazing hot springs! After all of the work we did each day, it was nice to comeback to the peace and quiet of our own room. Thank you for that."

> "Although I was personally not happy with the meal at Hotel Fuji this is because I do not like fish very much. The meal itself was beautifully presented and it was just personal preference that prevented me enjoying it."

h) Other Comments and suggestions

"The IMPULS program was excellent. The activities were well organized and informative and there was a good balance between learning, socializing and culture/sightseeing."

"I am thankful for having the opportunity to participate in the program. I hope to continue to learn about lesson study and mathematics teaching practices to not only improve myself as an educator but to help others understand the importance of learning using the lesson study approach."

"For future groups, it may be helpful to let them know just how much walking they will be doing. It was fine and felt good after a few days but knowing I would be walking to the university in the muggy heat in professional attire may have influenced my clothing options I brought. Also, the bringing of small gifts was a bit confusing. I would have appreciated some more clarification about how many adults, children we may need to bring gifts for and some good suggestion especially with luggage. I know it said it was optional but I definitely felt like a fool not having anything to give a class the welcomed me for lunch."

"Can we have a STAGE 2 Project Impulse Immersion Program for teachers who want to continue learning?"

i) Feedback on Survey

"To compare in #7, tell us during the first survey to write them down."
"Took an hour to complete"
"it took me 90 minutes to complete"

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Daily Learning Highlights

This section describes what was planned, actually happened and provides participant feedback from daily reflections and other documents.

Day 1, June 15

Planned Engagement: Opening Session, Lesson Planning and Kyouzai-Kenkyu,
workshop on Japanese mathematics lessons, and welcome reception

What actually happened:

The first day was hosted at Tokyo Gakugei University. The staff, consisted of 9 master students, 2 elementary teachers, 1 secondary teacher and a coordinator. Ms. Kiyoko Ishihara provided the opening address going through the agenda for the day. Dr. Toshiakira Fujii gave the welcoming address emphasizing that the International Math Teacher Professionalization Using Lesson study (IMPULS) was a 6-year project funded by the Minister of Education for which this is the third year. He encouraged participants to experience the authentic Japanese experience and expressed hope that they enjoy their experience.

Dr. Fujii informed the participants about other educators who will also participate. He also informed the group about Ms. Matoda who is on maternity leave, Dr. Fujii stated that she controls everything in this project. This was followed by the introduction of members of Project IMPULS and participants and staff.

Dr. Fujii explained that the teachers took one year off to study at the university to become master teachers and they will return to the classroom. He stated that the Japanese government is supporting the graduate student from Romania to study Japanese and mathematics for one and a half years.

Dr. Takahashi welcomed the group and indicted that some participants have been practicing lesson study already. The purpose of this immersion program is for participants to see what happens in Japan with lesson study and how Japanese teachers design and teach lessons. Dr. Takahashi encouraged the group to compare what they are doing to what is the ideal way and consider what they see over the 10 days. The participants were encouraged to bring those things back to their country to compare ideas with ways to improve teaching and learning.

Dr. Takahashi went through the packet of materials found in the bag. He explained that the Tokyo Gakugei University was 100 years old and thus the oldest teacher training institution in Japan. The university has about 6,000 students. He also explained how popular the multicolor pen is and described how the pen is used by Japanese Teachers in lesson study to describe what teacher and student says. Dr. Takahashi also indicated that the Suica transportation card has enough money for travel all 10 days but if the participants do extra travel, they would need to add additional money. The card can also be used for purchases at stores. The participants were most appreciative to have these resources as indicated by multiple, "thank yous."

Dr. Takahashi explained that they have lesson plans and a few more will come after they are translated in to English. He encouraged participants to take notes on the right side of the plans. Participants were informed again that they could find these on Base Camp to download. Dr. Takahashi provided an overview and very welcoming explanation of the agenda and experiences for the participants. He explained that the first public lesson will be June 18th at Matsuzawa in a 3rd grade classroom. School lunch will be served and participants will eat in the classroom with the students. Dr. Takahashi also informed the group that on the 19th, this will be everyday teaching not a research lesson. Here the IMPULS Project wanted to provide participants an opportunity to see variety in planning and teaching. Dr. Takahashi stated that on the 21st, two lessons will be observed at the Tokyo Gakugei University International Secondary School: lesson 1 grade 7 and lesson 2 grade 12. It is important to note that the IMPULS group will have their own private post lesson discussion with the teachers as suggested by previous year participants.

Dr. Takahashi explained that on the 23rd participants would take a 2-hour bus ride to the Oshihara Elementary School. Participants will check out of the hotel on Monday and move to Yamanshi by bus from Kokubunji. In this school, participants will be able to take pictures of student work, board writing, and student notes.

In his discussion, Dr. Takahashi explained that Japanese teachers are doing problemsolving daily. Once you write a script you do not have to study. The Japanese Teachers design a lesson so students can learn from it. He talked about the lesson study cycle and the 3 levels of teachers. Usually, it is a lesson study theme. Many US teachers try to select their favorite lesson, a lesson that worked in past experiences. Japanese teachers do not select their favorite lesson; they select topics that are a challenge to design a lesson around it. For Japanese teachers, they do not choose their best lesson. Teachers do research and design a lesson that has not been taught. The fundamental difference is not to try to demonstrate accomplishments, Japanese teachers research to study and become better. The planning session is important. The lesson plan is a research proposal. Dr. Takahashi elaborated that Japanese teachers really want students to accomplish what is planned.

Lesson study was first implemented in US in the late 1990s. Dr. Takahashi said that he had an opportunity to review the lesson study research articles. In these articles he found comments like, "We tried lesson study but could not do authentic lesson study so we changed something," "We might not have strong results because we could not do lesson study as designed," and "They provide a tool kit for teachers doing lesson study."

Dr. Takahashi stated that in a recent randomized controlled trial, it was demonstrated that when the researchers measured teacher and student growth, out of 643 studies in mathematics only 2 have a strong impact and one of them was lesson study. If you do lesson study right, that means a lot.

Dr. Takahashi also discussed how Chicago uses NWEA for decisions like promotion of students to the next grade level. One participant, a Chicago administrator, talked about how the test is used for teacher and school ratings.

Dr. Takahashi stated that this immersion program is important for you as participants and also for the Japanese educators. He said, "we learn a lot from your reports and reflections."

Dr. Takahashi also stated that Japan has been doing lesson study for more than 100 years. The original idea came from the US. The Teaching Gap was published in 1999, not many participants had read the book.

Dr. Takahashi introduced participants to Lesson Note. Many of the participants borrowed iPads from the university. He stated that it is important to keep the time so the length of the learning cycle can be determined.

Day 2, June 17

Planned Engagement: Preparation for the research observation; School Visit #1,
Tokyo Gakugei University – Koganei Lower Secondary School, and post lesson
discussion

What actually happened:
Dr. Watanabe reviewed the lesson for the problem, "in the 13th drawing, how many dots are there in each drawing? Dr. Watanabe indicated that the Japanese word for this representation is shiki. This is a Japanese word translated as "algebraic expressions"

which is a stronger indicator used to describe both expressions and equations with and without letters. He explained that all translations will be American and apologized to the Australian and UK participants. The teachers themselves actually solved the problem. He asked the participants to think about the problem and solve the problem individually. Dr. Watanabe stated, "Your task is to figure out the number of dots and come up with an algebraic expressions." Participants spent approximately five to ten minutes trying to solve the problem. Dr. Watanabe had participants talk to each other about the different strategies used. Dr. Watanabe modeled monitoring and selection of participants' work to use in the reports outs. One question posed by Dr. Watanabe that had participants thinking and pondering was: "Which expression is correct? 4 groups of 13 + 6 or 13 groups of 4 + 6." The Common Core writers would interpret this problem as, 'number of groups' followed by 'group size'.

After much discussion and manipulations, Dr. Watanabe was able to demonstrate that it is logical to first show how much in each group or the group size. So the expression should show: 'group size times # of groups'. This is the Japanese convention.

Dr. Takahashi strongly encouraged participants to read through the problems. He stated that Dr. Watanabe did a good presentation of the problem, you will see a similar problem during the lesson. Dr. Takahashi also encouraged participants as they observe the lesson to consider:

1. Is every thing coming from the students?
2. Is it true that students come up with the solutions anticipated by the teacher?

Dr. Takahashi informed participants that the post lesson discussion would be amongst our group. Dr. Takahashi also talked about the preparation of lesson as Kyozai-Kenkyu: Kyo (Teaching); Zai (Material): KenKyu (Study). So Kyozai-Kenkyu is the study of teachin materials. He also introduced Banshu, board writing. Dr. Takahashi talked about the 7 textbook companies that publish Japanese textbooks. The Ministry of Education approves all textbooks.
Dr. Takahashi discussed problems from the fifth, first, and third grade textbooks. The problems focused on comparative diagrams. Dr. Takahashi said that participants should ask what is/are the reason(s) for teaching this idea at this point in the curriculum?

Some reflections from participants:

(High-Impact Strategies for Teaching Mathematics)

"The problem was posed to students without any set up to build their interest. However, the open ended nature of the task and the sharing of all student solutions served to increase students' commitment and interest, and the problem was just challenging enough to build capacity to solve it."

"The student's mathematical thinking was first made visible to themselves individually through the act of writing and through completing the act of writing their shiki on their mini whiteboards."

(Mathematic Practices in the U.S. Common Core State Standards)

"Students were given a problem to make sense of their won. They demonstrated reasoning when writing expressions to represent their thinking and made viable arguments when explaining the expression of another student. They were able to model with mathematics by using the picture of the 13th pattern to represent an expression, and the problem was all about looking for/making structure and repeated reasoning."

Day 3, June 18

Planned engagement: Cultural exchange at Matsuzawa Elementary, School visit in
Matsuzawa Grade 2, and Post Lesson Discussion.

What actually happened:
Participants were taken to the gymnasium to engage with the 3rd grade students around various activities ranging from origami, Ohajiki (the stone game), Ayatori (String Game), Yo-Yo, Kendama (ball on a string connect to cups), Fukuwana (pin the facial parts on a picture of a face), Ostedoma (juggling bags), and other games. The students seem to enjoy having the participants actively engage in these games. The students practice their English and were enthusiastic about showing the games. A small group of students would come to the participant and say "Hello" then escort them to an area in the gym where they would play a game. The IMPULS participants enjoyed the exchange as evidenced by their laughter, great energy while playing and their willingness to rotate from station to station.

The participants then went to the classrooms to have lunch in the classroom with the elementary students. This is when some participants gave gifts to the children and teacher. After lunch the group observed Ms. Haruka Miyamoto teach the 2nd grade lesson.

Prior to the lesson, Dr. Yoshida informed IMPULS participants that the school curriculum was organized around problem solving. Students will review what they learned in the 1st lesson of the topic. Today is the second part of the lesson. They learned to subtract 2 digit numbers from 2 digit numbers without regrouping. Now the student shave started to use the subtraction algorithm. These students have not learned the vertical algorithm for subtraction. They will use their knowledge of addition to solve the problems.

Dr. Yoshida explained that regrouping is learned in the 1st grade. Textbook authors think about the connection between subtraction and addition across what students learn and need to know. Through problem solving, the students will make the connections.

Dr. Fujii stated that 45 – 27, read 45 subtract 27, is not a good problem because of the use of manipulatives. Dr. Fujii has written about designing tasks in the Japanese lesson study. He has focused on the role of the quasi-variable. Dr. Yoshida stated that,
"Japanese teachers will talk about appropriate numbers for a problem. Dr. Takahashi stated, "coming up solutions is not the goal. We are teaching mathematics. If you only want to teach one method, that should not be a goal… so textbooks use the most appropriate number."

Some reflections from participants

(High-Impact Strategies for Teaching Mathematics)
"The classroom teacher began lesson by writing algorithm to match the context, also discussed the common error with the children…reversing the digits (i.e. calculating 47 – 25 instead of 45-27 just because it is easier!)…before the children made the error…

"The teacher wanted block method and the cherry method, so she chose students who used different methods – but the answers were really the same. 45 = 30 + 15 these were different representations but not really different methods.

(Mathematic Practices in the U.S. Common Core State Standards)
"[Use appropriate tools strategically] Base ten blocks and cherry diagrams. Students used these (which ever they were comfortable with) to problem solve and to show the action of "regrouping." I tried to see students use these tools on their own and choose which made sense to them. I am sure the teacher did some important work prior to setting up this environment."

Day 4, June 19

Planned engagement: Preparation for research lesson observation, School Visit #3 –
Koganei Elementary School – Grade 3, Mr. Takeo Takahashi, 2 Post Lesson Discussions

What actually happened:
Dr. Yoshida in the preparation for the research lesson explained the term 'times as much'. He provided an example of two times as much. Dr. Yoshida then provided an example of 12 cookies divided by 3 people. How many cookies does each person get? Dr. Yoshida also talked about the quotative meaning of division, which is repeated subtraction. In 5th grade, the emphasis is on the per unit quantity. How many per person in each area of the room. Dr. Yoshida stated, "the partitive meaning of division is extended to talking about the rate."

The multiplication can be interpreted as # of groups 'times' # of each group = Total

Dr. Yoshida, stated that today we are discussing, the concept of 'how many times as much.' Participants were directed to the problems in the textbook. On page 29 of the 3A textbook: the problem reads: 20 cm of ribbon if you cut it into 5cm pieces, how many pieces can you make?
In the 3B book, page 22, it states that to find 'how many times', you can use division.

Dr. Takahashi discussed the concept of :
# of groups 'times' # in each group = Total
So, the Total ÷ # in each group = # of groups
This is the quotative interpretation

# of groups 'times' ? = Total
Total ÷ # of groups = # in each group

This is the 'fair share' interpretation

Dr. Yoshida stated that, "Japanese teachers spend time on both interpretations. Most students have more experience with fair share interpretations".

Post Lesson (1)

Mr. Takahashi (through Dr. Watanabe's interpretation), the 3rd grade teacher, explained that he would use different numbers. Maybe 42 and 6. He stated that he collects student notes and reflective journal and use that to plan the next lesson.

Mr. Takahashi further explained that he maintains students' focus by not allowing students to see the topic of the lesson until it starts. Then students:

- Share the answers
- Give their reasoning
- Constantly ask if there is anything they want to add
- Appreciate seeing students' names on the board next to their ideas

He believes that the lesson is something that the teacher and student cooperatively create.

IMPULS participants commented that there was excellent interaction between the teacher and the children. Problems involve different numbers.

Post Lesson (2)

Participants continued the discussion without Mr. Takahashi. The topics discussed were; the teachers' choice of student work; questions about the use of the ruler diagram; students confusing the number as opposed to the number of jumps.

Dr. Takahashi asked a series of questions between the discussions:

What should Mr. Takahashi do for the next lesson?

What would you do for the next lesson?

What journal entry would you use to start the next lesson?

These questions were discussed extensively. Dr. Takahashi stated that our discussion is not for reteaching. It is for what are we going to do the next day. Japanese teachers do not reteach their lessons.

Dr. Fujii provided information about the textbook. 3A, page 29 problem #2 and 3B page 22.

Some reflections from participants:

(High-Impact Strategies for Teaching Mathematics)

"The teacher planned and organized the first 3 examples on the board very well and it was easy for kids to see the progression of thinking."

"…The teacher used the strategy of taking ideas from students and asking if any one did not understand what they said. I think the strategy of asking for alternative explanations of what has been said is better because it is more open. Asking for students to say they do not understand is less likely to bring out ideas. Overall the range of thinking was revealed from 'difference', 'times as much' and 'bai'."

(Mathematic Practices in the U.S. Common Core State Standards)

"[Look for and express regularity in repeated reasoning] subtracting 9 four times as a division strategy. I always saw this as a negative strategy (since it's not as efficient with larger numbers) but after today's lesson I realize that it models division well and could be used effectively with smaller numbers and be valuable for student understanding."

Day 5, June 20

Planned engagement: Preparation for the research lesson observation; School visit
#4, Sugekari Elementary School, Grade 3, Ms. Koko Morita, Post Lesson Discussion

What actually happened:
Dr. Takahashi was the translator for this session. The principal stated that the school shifted their style of research. After last year they usually discussed in their grade level teams. The teacher did mock up lessons before the final plan. Teachers pretended to be children. By doing this, despite their knowledge, some things might be possible. So the new lesson is what they learned from the mock-up lesson. Teachers discuss again. The principal also join the discussions. The teachers in this school gave themselves one month to finalize the lesson. The teachers review the lesson three times and give feedback within one month. Then based on how the lesson goes focus on what to do for next lesson. Each teacher has a content focus. This teacher, Ms. Kohko Morita, focuses on math. Last year she was in 6th grade, this year she is a 3rd grade teacher. Japanese teachers usually change grades periodically. The teachers can teach any grade.
Dr. Watanabe facilitated the lesson discussion. The context of the lesson was octopus balls, which is a common Japanese snack food. There are [ ] pieces of octopus balls. Dr. Watanabe asked why the teacher use [ ]. The discussion focused around having the [ ] was intentional. The number is divisible by 4. The teacher wanted students to generate their own questions. Then they solve the problems they develop. The question for the students would be, can we have other number remainders?
Dr. Watanabe explained that Japanese teachers think about, what should be told if children can figure it out, they should do this. He also explained:
13÷ 4 = 3 rem 1 and using the Transitivity idea, a=b, b=c so a=c
13 ÷ 4 = 3 rem 1
16 ÷ 5 = 3 rem 1
13 ÷ 4 = 3 .
Dr. Watanabe finds that expressing the remainder as a fraction is 'weird'.
Japanese teachers think about teaching and learning about the 4 basic operations. They work with students on how to express the meaning of the operation and how to do the calculation.

Some reflections from participants

(High-Impact Strategies for Teaching Mathematics)

"The teacher didn't anticipate different responses, as in other plans we've seen, as much as anticipate the flow of the lesson and key points were she would need toask questions to guide students' thinking or let students discuss to draw out their thinking. This planning was very effective."

"Students seem encouraged to solve the problem, however because of the nature of the content there were not different representations of the solutions."

"As mentioned, only one representation was used and linked clearly to the mathematical sentences and structure of the sentences. Connections were also made between multiplication and division math sentences so links were made in other ways rather than using different representations."

(Mathematic Practices in the U.S. Common Core State Standards)
[Reason abstractly and quantitatively] "Word problems and division -- attaching context to each value. Division is one of the most difficult concepts for my students to understand. Even students who do well with multiplication seem to hit a wall sometimes with division. I set up story contexts, but students still want to think of the numbers abstractly, which I think is more difficult. I want to take some ideas from this lesson in my teaching of division this year."

Day 6, June 21

Planned engagement: School visit #5 Tokyo Gakugei University International Secondary School; Research lesson 1, Grade 7, Ms. Hiroko Uchino; Research
Lesson 2, Grade 12, Mr. Ren Kobayashi; two Post lesson discussions.

What actually happened:
Dr. Takahashi explained that each teacher would talk about the lesson. For grade 7, the teacher, Hiroko Uchino, stated that she changed the introduction to the lesson. The goal of the lesson is to raise the level of students examining the products. She decided not to use the table for this lesson but to have students discuss and analyze the strategies.
The 12th grade teacher, Mr. Ren Kobayashi, explained that his students were considering differentiated equations for everyday situations. He wanted students to express the changes involved in mathematical ways. Mr. Kobayashi also explained that students learned about sequences using a recursive formula. He further explained that students ran the simulation and saw the changes but did not see the need to express the changes. Students did not understand the question because the question was not clear. Therefore, students had no reason for seeing or interpreting the changes. This was a perfect opportunity, since this public open house involved a large number of teachers from different schools, to have the IMPULS group participate in the regular post lesson discussion after the group's private post lesson discussion.

Some reflections from participants (1)
(High-Impact Strategies for Teaching Mathematics)
"Student responses were anticipated and the lesson was taught to another class of the same level as a result of this, the lesson plan was changed as pupil responses were not good. The lesson was re-planned very carefully to elicit deep student thought. Numbers were chosen so that the situation was fair, this was intended so that students could not tell easily which strategy to use so were forced to investigate more deeply to be sure."

"Students did not use diagrams or outcome space very much. Most of the work was verbal ad this made formulating an argument difficult for the students.
As a result, students were debating what to consider."

(Mathematic Practices in the U.S. Common Core State Standards)
"Class spent a long time discussing the strategies and persevering through reasoning about the game. No one gave up!"

"Students shared strategies and responded to each others' ideas and whether they agreed or disagreed and why."

"Both the Mathematical Practices described above require much time to establish as learning expectations. I still wonder how much of these practices are influenced by culture?"

Some reflections from participants (2)
(High-Impact Strategies for Teaching Mathematics)
"The teacher anticipated many possible students' responses however due to the high challenge of the problem and reticence on the part of the students the class did not make the progress that was expected."

"The real life context of the problem was the main strategy to encourage commitment and interest. It also promotes flexibility of thought."

(Mathematic Practices in the U.S. Common Core State Standards)
[Construct Viable Arguments and Critique the Reasoning of others]
"Towards the end of the lesson, the discussion led to students thinking what is most efficient? Students suggested looking at number of people infected per week vs. number infected per day, which led to the discussion about efficiency. (This was the way I interpreted what was happening during the lesson based on translation of lesson and post-lesson debriefing discussion as this content was a bit above my mathematical knowledge base)."

Day 7, June 23

Planned engagement: Move to Yamanashi; Courtesy visit to Showa Local Educational Office; Visit Oshihara Elementary School, observe ordinal classroom,
School visit #6, Oshihara Elementary School, School Based Lesson study, Grade 4,
Ms. Maki Tsuruta, Post Lesson Discussion, Move to Hotel Fuji; Japanese style
dinner with Japanese teachers.

What actually happened
Drs. Takahashi and Fuji invited the math education professors, one from England and the other from the U.S. to participate in the visit to Showa Local Education office. The IMPULS subgroup was warmly welcomed and they were given tokens from the area as souvenirs. Once participants were in the school, they were given a tour of the school building, which was made of green material. Participants had lunch with various classrooms and the observed the 4th grade lesson and participated in the post lesson discussion.

Reflections from participants

(High-Impact Strategies for Teaching Mathematics)

"Students displayed categories of shapes on the board without saying how they grouped the shapes. The remaining students then had to explain how these students had group the shapes and discuss if they had grouped shapes differently, they could then add their thinking."

"[The teacher] asked children to do a second sort at the end of the lesson which provided a good set up for the next step in learning. The children showed how they were engaged in clarifying their thinking through discussion. The use of the board enabled children's thinking to be transparent to other learners. For example, children were linking rectangles and squares to parallelograms and rhombi. The children were beginning to think about classifying shapes in different ways and that they have more than one property."

(Mathematic Practices in the U.S. Common Core State Standards)

"[Use Appropriate Tools Strategically] Many students pulled out rulers and protractors to precisely measure length of sides and/or measure of angles to use as justification for their sort. Overall, students seemed to be using these tools in a purposeful way."

Day 8, June 24

Planned engagement: Sightseeing at Takeda shrine; Lunch at lake Kawaguchi; sightseeing at Oshino-Hakkai; Preparation for the research lesson observation on June 25.

Day 9, June 25

Planned engagement: School Visit #7, Koganei Elementary School, Grade 5, Mr. Kishio Kako; Post Lesson Discussion; Discussion to wrap up Lesson study
Immersion Program.

What actually happened

Dr. Watanabe talked about the "per unit quantity." Students were to compare by differences or ratio idea. Dr. Watanabe stated that "times as much" is not always used. He stated that people must decide what you are using as the base amount. 'Compared' is 'Scale Factor' times as much. The concept of crowdedness per unit amount involves the number of students and size of the room. The teacher anticipated that students would use this approach.

Reflections from participants

(High-Impact Strategies for Teaching Mathematics)

"The teacher specifically chose a context and numbers that would be contextually relevant to these students, depositing Yen in the bank and the interest gained or changes between "raises in allowances" as compared among siblings. "

"The teacher planned to use a double number line as a link between the verbal explanation of students' thinking and the mathematical representation, bai that were being compared. I was surprised that he did not use the double number line during the lesson. However, he did use board work to link words to mathematical expressions."

(Mathematic Practices in the U.S. Common Core State Standards)
"Look for and Make use of Structure – Students were noticing the relationship between finding the difference between numbers (700 – 500 and 700 – 600) and using division to express it as well (700 ÷ 500 = 1.4). Students also recognized they could use multiplication to multiply 500 x = 2000 to get a base rate."

Day 10, June 26

Planned engagement: Discussion to warp up Lesson study Immersion Program and Farewell Party at Umenohana.

Teaching and learning math in Japan:
What would you like to take back to your schools:
There were five themes that emerged from the discussion:

Problem Solving
- Greater emphasis on discussion methods to solve problems and deepen understanding
- Distinguish between exercise and problem solving
- Intentional choice of numbers in problems

Planning
- Anticipated responses, well thought out problems
- To focus teachers on the mathematics. To become more content driven.
- Using kyozai-kenkyu to vertically study curriculum above and below my grade level

Note Taking and Journaling
- Use of journals with students

Textbook usage
- I am going to take back the progressions from Japanese textbooks

Board Work
- Investigate planned board work

Questions and concerns: There were two themes that emerged in the category.
Assessment
- What is traditional assessment in Japan?
- How do teachers assess student journals?
- How are students assessed on a daily basis?

Students
- I am going to take back the progressions from the Japanese textbooks.
- When students shout "HAI" and raise hands, it seems that speed is emphasized and others lose out on think time.
- How does differentiation look like in a Japanese classroom?

Lesson study in Japan
What would you like to take back to your schools? Five themes emerged in the category.

General Lesson study
・The children feel 'safe' to offer contributions and to make their thinking transparent.
・You need to do all the parts of Lesson study for it to be most effective. (Not just the "best bits")
・Teachers articulating their professional knowledge in public.
School based
・Don't 'reteach' the same research lesson.
・How many people work so well as a team and everyone is life long learner.
・There is a collective sense of responsibility for improving teaching and learning that drives lesson study
Planning
・The depth of planning and research that goes into a lesson plan.
・It's not a lesson that has worked well previously or been re-taught
・Emphasize the importance of 'Kyozai-Kenkyu' and a research lesson properly.
Post lesson discussion
・How invested/ involved all teachers are during post lesson
・The more formalized format of the post lesson discussion, (everything really!)
Final comments
・The idea of having a guest speaker for final overview/comments of the lesson.
・I enjoyed and looked forward to the final commentator- will look into resources at home for this component

Questions and concerns: The same five themes emerged in this category
General Lesson study
・Which forms of activity are essential elements of lesson study?
・Which forms of activity are essential elements of lesson study
・How have/do schools use lesson study in order subjects?
School based
・How to get administration and local university buy-in?
・Where should I start to implement lesson study in my school?
・There are many facets to implementing Lesson study in a culture where it is not embedded… Do those facets have hierarchy?
Planning
・What are some resources that teachers use when researching?
・In the U.S. differentiation of instruction is included in most lessons/consideration for lessons. How is/is differentiation incorporated into lesson study
・How is time made for teachers to meet/discuss and plan?
Post lesson discussion
・How do you ensure that the main focus of the post lesson discussion is the mathematics?
・How do you get the post lesson discussion to contain such rigor and have so many people to contribute?
Final comments
・How is the final speaker for Post-lesson discussion chosen? In the U.S., who would we chose?
・What are all the steps/ how can (I) become a knowledgeable other like Dr. Fujii?

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Figure 1: Reported Mean Learning about Program Elements (Posttest Rating)

Annex;

(1) List of participants

Name		国	School/ Department
Mr	Gustavo A. Soto	U.S.	3rd grade bilingual/ESL teacher Daniel Boone elementary School, Chicago, IL.
Ms	Jana Morse	U.S	School Based Elementary Math Coach at Daves Avenue Elementary School, Los Gatos Union School District
Ms	Katianne Balchak	U.S.	5th Grade Teacher W.D. Williams elementary School Buncombe county Schools, NC
Ms	Kelsey Crowder	U.S.	Kindergarten teacher, O'Keeffe School of Excellence Chicago, IL
Mr	Kent Steiner	U.S.	Assistant Principal , South Shore Fine Arts Academy Chicago, IL
Ms	Laura Burrell	U.S.	School Based elementary Math Coach At Holiday Hill School Duval county Public Schools
Mr	Leland Dix	U.S	Kindergarten Teacher, Harlem Village Academies-Leadership Elementary, New York, NY
Ms	Sarah Harris	U.S	4th grade teacher, Emma Elementary School, Asheville, NC
Mr	Simon Terrell	U.S.	6th-8th grade teacher, main mathematics coach for the 3-5 grades Santa Rosa, CA
Ms	Sibeso Likando	Aus	Ph.D. student, Deakin University, Australia
Mr	Adam Bright	Aus	Primary Literacy Coach, Springside College, Caroline Springs Victoria
Mr	Richard Cowley	UK	Lecturer of Math Education Institute of Education (IOE)
Ms	Janine Blinko	U.K.	Primary Maths Consultant
Ms	Jackie Mann	U.K	Secondary teacher Robert Clack School
Ms	Lorna McCance	U.K	Secondary teacher George Spence School

Grade 7 Mathematics Lesson Plan

Tuesday, June 17, 2014
Period 6 (14:20 ~ 15:10)
Grade 7 Classroom B
(20 boys and 20 girls)

Teacher's Name: Shou Shibata
Koganei Lower Secondary School
attached to Tokyo Gakugei University

Location: Educational Technology Room (3rd Floor)

0. Research theme and its intent

Designing lessons to raise the quality of mathematical processes

(1) Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University

This lesson was developed as a part of the activities of the Mathematics Research Group of Secondary Schools Attached to Tokyo Gakugei University. The purpose and rationale of the research group are as follows. In the Japanese mathematics education, we have emphasized not only the mathematical content (results of explorations) but also the processes of exploring mathematical problems and the development of skills and ways of reasoning utilized in the process of explorations. However, in spite of this emphasis, we are concerned that mathematics teaching overwhelmingly focuses on the mathematical content. We are not suggesting that we should treat teaching of the mathematical content less seriously, nor are we suggesting that processes and contents should be treated separately. However, we wonder if Japanese mathematics lessons are indeed emphasizing "mathematical ways of observing and reasoning" or "mathematical activities" even though we have been discussing their importance for a long time.

To emphasize mathematical processes means to emphasize the processes of creating and applying mathematics. As an activity, we can consider those processes as "mathematical activities," and the ways of observing and reasoning utilized in those processes can be considered "mathematical ways of observing and reasoning." Therefore, in our research group, we call the totality of the processes involved in creating and applying mathematics as "mathematical processes," and the purpose of our group is to continuously examine the nature of mathematics lessons that raise the quality of "mathematical process." Therefore, our research theme has been established as "designing lessons to raise the quality of mathematical processes."

(2) Proposal in today's lesson

As I planned this lesson, I have chosen the following as the working definition of "raising the quality of mathematical processes." Students are engaged in a higher quality mathematical process when they re-engage in a mathematical activity with a new perspectives or purposes gained internally or externally after the initial engagement with the mathematical activity. In a mathematics lesson, this rise in quality of mathematical processes should take place when students re-engage in the problem for which they had previously found a solution independently with a new idea or purpose gained internally through reflecting on own problem solving processes or externally through their peers or the classroom teacher's comments. Therefore, as we plan a lesson, it is necessary to clearly articulate mathematical processes that could surface in the selected mathematical activity and to describe strategies to raise the quality of mathematical processes. This idea is shown visually in Figure 1. Figure 2 shows the rise in the quality of mathematical processes and strategies employed in today's lesson.

The hypothesis being proposed in today's lesson is "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer."

Based on the above, the research objective for today's lesson has been established as follows.

> Research objective
> To demonstrate the viability of the hypothesis, "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer" through a lesson that introduces algebraic expressions.

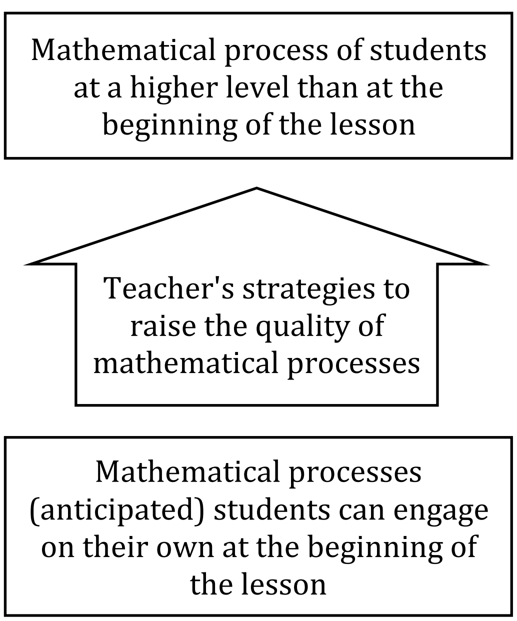


Figure 1 Structure of a lesson that raise the quality of mathematical processes

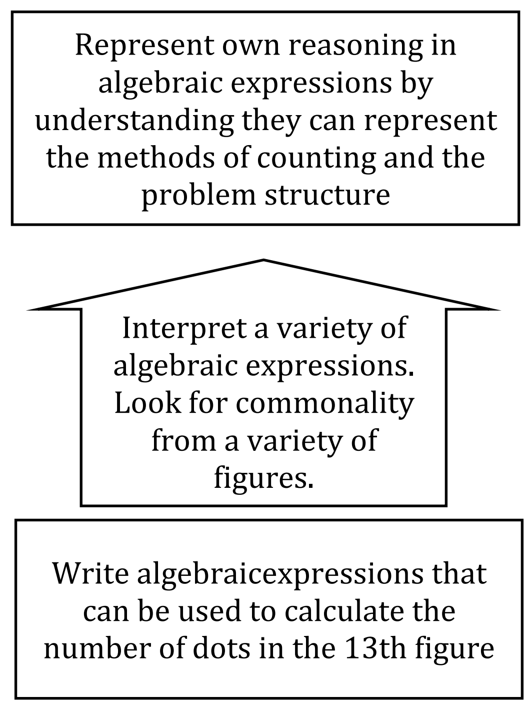


Figure 2 Structure of this lesson

1 Unit: Letters and Algebraic Expressions[1]

2 Goals of the Unit

- Students will become interested in the merits of mathematical reasoning and the enjoyment of mathematical activities by discovering characteristics and properties through the use of letters and algebraic expressions with letters. Students will try to use algebraic expressions with letters in problem solving and mathematical explorations. [Interest, eagerness, and attitude]
- Students will be able to represent relationships and patterns among quantities in various phenomena using algebraic expressions with letters and make generalizations. In addition, students will understand that algebraic expressions with letters can show both the process and the result of calculation through activities such as substituting specific values in the letters and interpreting given algebraic expressions. Students can think about relationships and patterns represented in algebraic expressions. [Ways of observing and reasoning]
- Students will be able to manipulate algebraic expressions with letters such as multiplying and dividing algebraic expressions or adding and subtracting linear expressions. In addition, students can interpret the relationships and patters among quantities represented in algebraic expressions, and also represent relationships and patterns using algebraic expressions. [Representation and manipulation]
- Students will understand the meaning of algebraic expressions with letters and their purposes. [Knowledge and understanding]

3 Assessment standards for the Unit

Interest, eagerness, and attitude toward mathematics	• Students are interested in the necessity and merit of using letters to represent relationships/patterns among quantities generally, and they try to use algebraic expressions with letters to represent relationships/patterns or interpret the given expressions. • Students know how to represent multiplication/division within algebraic expressions with letters and they try to use them to manipulate expressions. • Students try to substitute values in letters to evaluate the value of algebraic expressions.
Mathematical ways of observing and reasoning	• Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. • Students can consider algebraic expressions with letters such as $a + b$ and ab represent both the operations (addition and multiplication, respectively) and the results.

[1] The Japanese word translated as "algebraic expressions," *shiki*, is used to describe both expressions and equations, with or without letters.

	• Students are able to use algebraic expressions with letters to think about concrete phenomena by substituting values in letters. • Students are able to think about ways to calculate algebraic expressions with letters by considering calculations with algebraic expressions as analogous to calculations with numbers.
Mathematical representations and manipulations	• Students are able to represent quantities and relationships/patterns among quantities in phenomena using algebraic expressions with letters, and they can interpret given algebraic expressions. • Students can use algebraic expressions with letters involving multiplication and division appropriately by following the conventions, and they can add and subtract simple linear expressions. • Students are able to evaluate the value of an algebraic expression with letters by substituting values to letters.
Knowledge and skills about numbers, quantities, and geometric figures	• Students understand that by using letters quantities and relationships/quantities among quantities can be represented generally or interpreted from the given algebraic expressions with letters. • Students understand how to represent multiplication and division in algebraic expressions with letters, and they understand how to add and subtract linear expressions by combining like terms. • Students understand the meaning of the value of an algebraic expression.

4 About teaching

In elementary schools, students have used □ and Δ in equations, for example, 5 + □ = 8 and 3 × Δ = 24 to grasp the relationship between addition and subtraction or multiplication and division. In addition, they have studied to represent quantities and their relationships in expressions and equations or interpret the given expressions and equations. In lower secondary school, as the ground work for the study of algebraic expressions with letters, they have learned that letters such as *a* and *x* may be used in place of □ and Δ, as well as expressing direct and inverse proportional relationships using algebraic expressions. Moreover, they have used quasi-variables in, for example, thinking about ways to calculate division of fractions or expressing relationships/patterns of numbers.

In lower secondary school, building on the study in elementary school, students will learn about not only using letters as representations or simply manipulating them but also manipulating and interpreting letters as variables, unknowns, and a representative for a set. Moreover, instead of simply introducing letters and studying calculations involving algebraic expressions with letters, we will introduce letters starting with the examination of quasi-variables, which are numbers that act like variables, and then through activities of interpreting algebraic expressions and their structures and using algebraic expressions as generalization.

By using letters, we can now have numbers in general as the object of study instead of particular numbers such as 1, 3, or 0.7. We can also express various phenomena as relationships in the mathematical world. Furthermore, by transforming the given algebraic expressions or equations, new interpretations may become possible. It is hoped that the study of letters students' mathematical explorations may be deepened and more refined. Such explorations typically take place in trying to prove conjectures or utilize the ideas of equations and functions. However, in this unit, instead of simply positioning the current study as the preparation for those future explorations, the main purpose is for students to experience mathematical manipulations and interpreting their results through activities of representing a relationship in a real-world phenomena as an algebraic expression, transforming it to reflect their own thinking, and interpreting and understanding a new relationship.

5 About students

This year, students have studied positive and negative numbers (integers). In that unit, students learned to consider negative numbers are just like whole numbers. Moreover, by studying calculations with integers, and patterns and properties of operations, they have studied what numbers are. During lessons, I have tried to help students pay attention to own problem solving processes by reminding them to make explicit ideas like "what needs to be considered" and "view point used in reasoning."

In general, students' mathematical achievement levels are high. Some students were able to use, for example, 3 and -2, as quasi-variables as they examined calculations with integers, and most students were able to understand their explanations, indicating most of them understand the notion of quasi-variables. Therefore, it is anticipated that few students will have difficulty generalizing numbers.

Although few students consider mathematics as difficult, there are some students who find it difficult to explain their ideas. However, they have experienced that their mathematical understanding was deepened by clarifying questions other students had. Therefore, I believe that there is a classroom culture where students feel safe to admit something they don't understand openly. Moreover, many students are willing to share their ideas in whole class discussion, and they do not hesitate to share even simple ideas.

6 About mathematics

I consider the most valuable aspect of the task used in this lesson is that there are a variety of strategies to determine the total number of dots. The algebraic expressions representing those strategies also vary. From each algebraic expression, it is possible to interpret the reasoning of the student who wrote it as well as the mathematical structure behind his/her reasoning. By using this particular task as the introduction of the unit on letters, I believe students can not only understand that letters serve as variables, unknowns, and a generalized number but also realize that different algebraic expressions represent different ways of reasoning. Moreover, they can realize that transformed algebraic expressions can lead to new interpretations of the original phenomena or their graphical representations. As a result, I believe students will not only gain the knowledge of letters as variables, unknowns, and generalized numbers but also make it possible for them to utilize algebraic expressions creatively and with sophistication as they engage in future mathematical study and explorations.

7 Scope and sequence in lower secondary school

	A. Numbers and Algebraic Expressions	C. Functions
Gr. 7	[Letters and Algebraic Expressions] --> [Equations]	[Direct and Inverse Proportions]
Gr. 8	[Calculations of Algebraic Expressions] --> [Systems of Equations]	[Linear Functions]
Gr. 9	[Polynomials] --> [Quadratic Equations]	[Functions in the form of $y = ax^2$

8 Unit plan

	Content	Main Evaluation Points
I	Section 1: Algebraic expressions with letters • Merits of using letters in place of numbers • Representing various quantities using letters • How to write algebraic expressions with letters • Interpreting algebraic expressions with letters • Substituting values in letters of algebraic expressions and the meaning of the value of an algebraic expression	• Students are interested in the necessity and merit of using letters to represent relationships/patterns among quantities generally, and they try to use algebraic expressions with letters to represent relationships/patterns or interpret the given expressions. [Interest, eagerness, and attitude] • Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning]
II	Section 2: Calculations of algebraic expressions • Relationships between terms and coefficients • Combining like terms • Addition and subtraction of linear expressions • Multiplying and dividing linear expressions	• Students know how to represent multiplication/division within algebraic expressions with letters and they try to use them to manipulate expressions. [Interest, eagerness, and attitude] • Students are able to think about ways to calculate algebraic expressions with letters by considering calculations with algebraic expressions as analogous to calculations with numbers.

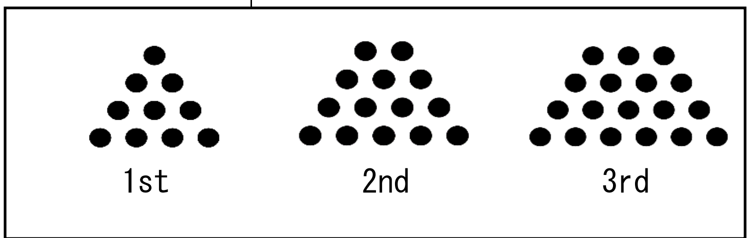
		[Mathematical ways of observing and reasoning] • Students can use algebraic expressions with letters involving multiplication and division appropriately by following the conventions, and they can add and subtract simple linear expressions. [Mathematical representations and manipulations]
III	Section 3 Applications of algebraic expressions with letters • Quantities represented by algebraic expressions • Algebraic expressions that represent relationships	• Students can represent and think about quantities and relationships/patterns among quantities in phenomena generally by using letters. [Mathematical ways of observing and reasoning] • Students are able to represent quantities and relationships/patterns among quantities in phenomena using algebraic expressions with letters, and they can interpret given algebraic expressions. [Mathematical representations and manipulations] • Students understand that by using letters quantities and relationships/quantities among quantities can be represented generally or interpreted from the given algebraic expressions with letters. [Knowledge and skills about numbers, quantities, and geometric figures]

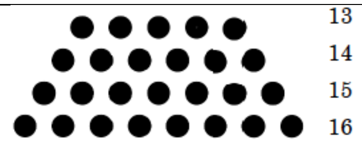
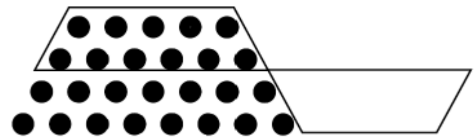
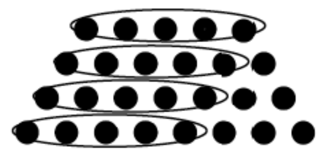
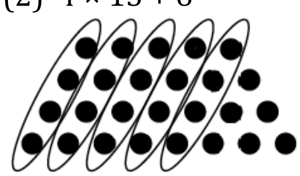
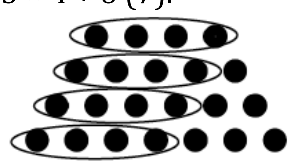
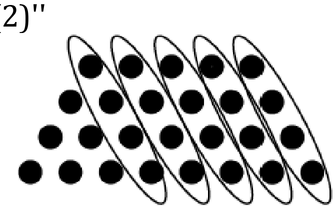
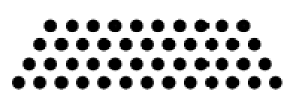
9 Today's lesson

(1) Goals

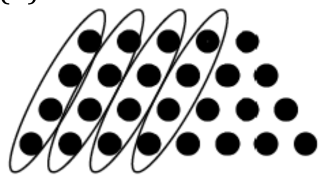
- By interpreting algebraic expressions, students will be able to think about the reasoning and the structure represented by the algebraic expressions. [Mathematical ways of observing and reasoning]
- Students will think about quantities and relationships/patterns among them, and they will try to figure out the total number of dots in the 13th drawing. [Interest, eagerness, and attitude]

(2) Flow of the lesson

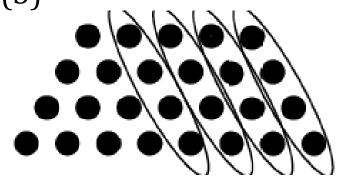
min	Learning Activity	Anticipated responses	❖ Instructional consideration ○ Evaluation
5	[Opening] Post the figures below.		
	 		
	"How many dots are there in each drawing?"	• There are 10 dots in the first one. • There are 14 dots in the second. • There are 18 in the third. • Is it increasing by 4?	❖ In order to avoid restricting students' reasoning, we will not discuss "how the numbers of dots are increasing."
20	[Development]		
	Main *hatsumon*: In the 13th drawing, how many ● will there be? Represent your reasoning in an algebraic expression.		
	• Independent problem solving • Students share their algebraic expressions • Sort the shared algebraic expressions	1. 13 + 14 + 15 + 16 2. 4 × 13 + 6 (2)' 6 + 4 × 13 3. 4 × 12 + 10 (3)' 10 + 4 × 12 4. (13 + 16) × 2 5. (13 + 16) + (14 + 15) 6. (13 + 16) × 4 ÷ 2 7. 13 × 4 + 6 8. 12 × 4 + 10 9. 16 × 4 - 6 10. 10 × 10	❖ Distribute mini-white boards and markers for students to use during the sharing time. ○ Students will think about quantities and relationships/patterns among them, and they will try to figure out the total number of dots in the 13th drawing. [Interest, eagerness, and attitude]
20	[*Neriage*] • Interpreting algebraic expressions		
	Main *hatsumon* for *neriage*: Let's try to explain the counting strategy each algebraic expressions represent.		

		 (1)' Although the student wrote 13 + 14 + 15 + 16, their reasoning is more accurately represented by (13 + 16) + (14 + 15) = 29 × 2.  (1)'' Using 13 as the starting point, think about other rows as "one more than 13," "two more than 13," etc.. It can be represented more accurately as 13 + (13 + 1) + (13 + 2) + (13 + 3).  (2) 4 × 13 + 6  6 dots (2)' If we distinguish the multiplier and the multiplicand, the algebraic expression will be 13 × 4 + 6 (7).  (2)'' 	❍ By interpreting algebraic expressions, students will be able to think about the reasoning and the structure represented by the algebraic expressions. [Mathematical ways of observing and reasoning] ❖ If a student says something like, "My idea was similar to ___'s" or "My idea was different," ask him/her what is similar/different and have students think about the correspondence between algebraic expressions and diagrams. ❖ Have students use the sheet with the 13th drawing as they explain.  ❖ Distinguish the multiplier and the multiplicand.

(3) 4 × 12 + 10



(3)'

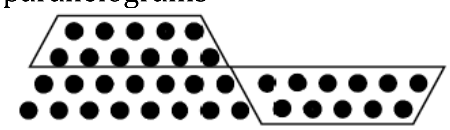


(3)''

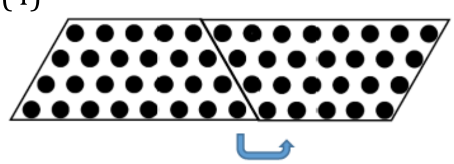
	1	2	3	4	…	12	13
	10	14	20	24		54	58

First term + 12 increases
10 + 4 × 12

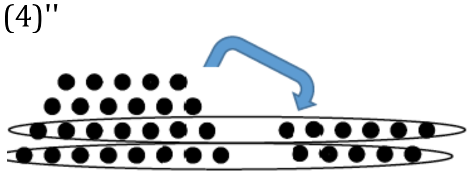
(4) (13 + 16) × 12
Using the formula for the area of parallelograms



(4)'

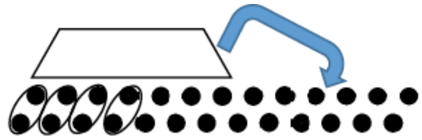
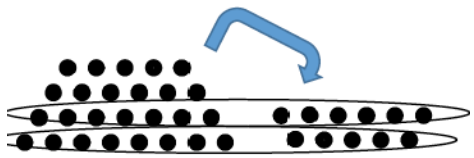
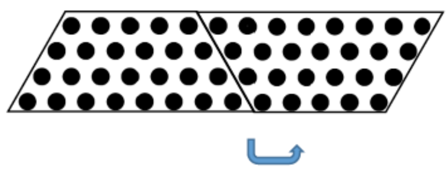
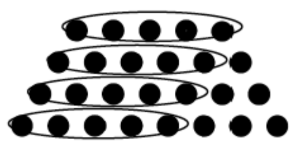
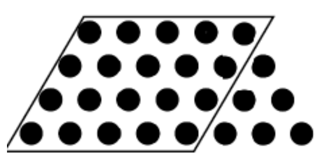
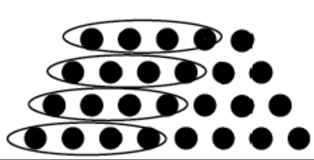


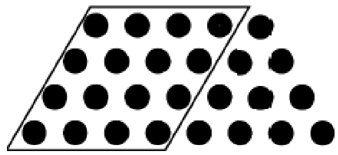
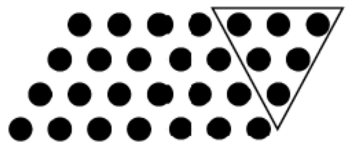
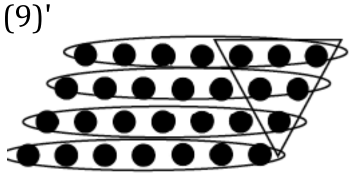
(4)''



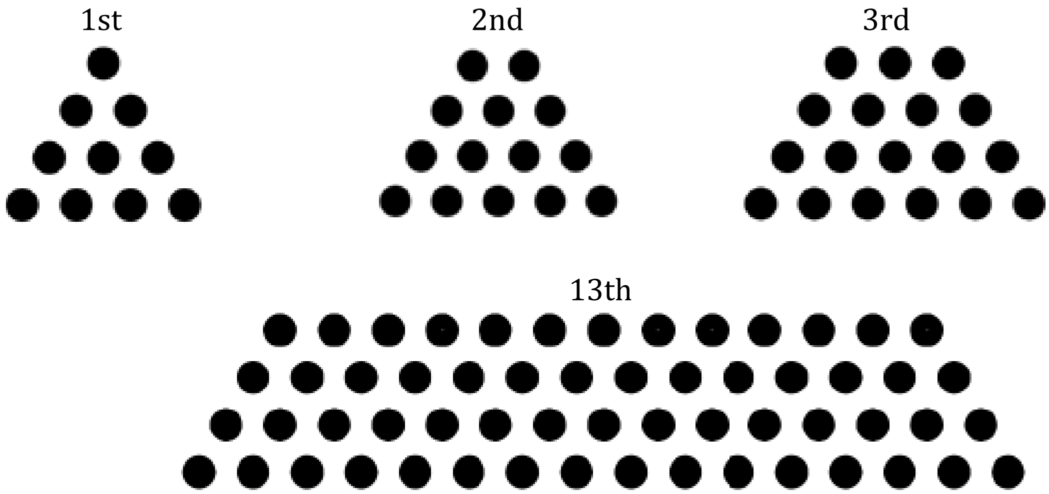
There are two sets of 13 + 16.

❖ If students notice that different counting strategies can be interpreted from the same algebraic expressions, discuss that idea.

		(4)''' There are (13 + 16) groups of 2 dots.  If we distinguish the multiplier and the multiplicand, it should be written as 2 × (13 + 16). (5) (13 + 16) + (14 + 15)  (6) (13 + 16) × 4 ÷ 2  (7) 13 × 4 + 6  (7)' Using the formula for the area of parallelograms  (8) 12 × 4 + 10 	

		(8)' Using the formula for the area of parallelograms  (9) 16 × 4 − 6  Add 6 more dots to make an arrangement in the form of a parallelogram (9)'  Add 6 more dots to make 4 groups of 16. (10) Since the first drawing has 10 dots, if we multiply 10 by 13, we can determine the number of dots in the 13th drawing. (incorrect answer)	
5	[*matome*] • Creating new algebraic expressions "Which counting strategy was similar to yours? Which one had the same algebraic expression as yours but the counting strategy was different from yours?"	• "I wrote 13 + 14 + 15 + 16, but I noticed some of my classmates were using the algebraic expression, 13 + (13 + 1) + (13 + 2) + (13 + 3). They were trying to use the number 13 from the "13th" drawing." • "I made a large parallelogram by doubling and used the algebraic expression, (13 + 16) × 4 ÷ 2. But, there	○ By interpreting a variety of algebraic expressions, students understand that algebraic expressions represent not only the steps of calculations but also their results and the structure behind the problem. [Knowledge and skills

	"What was the difference in your strategies?"	were people who added 6 more dots to make a parallelogram."	about numbers, quantities, and geometric figures]



10 Points for observing the lesson

As discussed above, this lesson focuses on the idea of "raising the quality of mathematical processes." The hypothesis proposed in today's lesson is "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer." Finally, the research objective for this lesson is "To demonstrate the viability of the hypothesis, "if we clearly articulate the mathematical processes students can engage in on their own and higher quality mathematical processes that can result from the lesson, strategies the teacher should employ will become clearer" through a lesson that introduces algebraic expressions."

Therefore, as we observe the lesson, we would like to focus on "the mathematical processes students can engage in on their own," "higher quality mathematical processes students engaged in as a result of the lesson," and "teacher's strategies." In particular, we would like to consider the following questions.

1. During the independent problem solving time, were the students able to represent their own reasoning processes using numbers such as 13? (the mathematical processes students can engage in on their own)
2. Through the activity of interpreting and comparing algebraic expressions, did the students come to understand that algebraic expressions represent not only the steps of calculations but also the results of the calculation and the structures of and changes in the diagrams that represent the problem situation? (higher quality mathematical processes students engaged in as a result of the lesson, and teacher's strategies."

By focusing on these two questions, we hope to address the proposed hypothesis and the research objective of the lesson.

Lower Grade Level Group

<< Theme of the Research >> Instruction that Helps Students Eagerly Grapple with Mathematics! -- Aiming to Improve Students' Expressive Abilities --

1. **Students' State of Learning:**
   ○ A number of students want to solve problems on their own using and enhancing the knowledge they learned previously.
   ○ Students are developing note-taking skills gradually.
   Δ Students are still not accustomed to writing their thinking/ideas.
   Δ Even when students write their mathematical thinking/ideas, their explanations are often insufficient.

2. **Goal for An Image of Students:**

○ Students willingly and enthusiastically write their own mathematical thinking/ideas in their notebooks and explain their thinking/ideas to others. ○ Students use what they have learned previously in math class and describe and explain their thinking/ideas to others.

3. **Instructional Ideas for Improvement:**

Ideas for Helping Students' Eagerly Grapple with Mathematics
• Provide problem situations that are familiar to the students • Use instructional materials that help students become interested in visual representations, such as using real objects, illustrations, photos and/or PowerPoint presentations. Support students as they show an increasing willingness for problem solving, expressing their interest by making statement, such as, "I want to find a solution to this problem/task!"
Ideas for Improving Students' Expressive Ability
• Expand student's thinking and reasoning via problem solving situations where students engage in making transition from concrete materials and illustrations to diagrams, and from diagrams to mathematical expressions and words (explanations). • Teach students how to carry out mathematical conversations and discussions in math classrooms. Expand students' explanation skills in pairs, small groups, and eventually whole class.

4. **Grade 2, Name of the Unit:** Subtraction with an Algorithm

5. **Focal points of this lesson**; i.e., points that we want to discuss during the post-lesson discussion.
   Focal Point 1: Was the setting and launch effective in generating and making evident students' eagerness to engage in problem solving?
   Focal Point 2: Was asking students to use the block or cherry diagrams an effective way to draw out students' thinking about regrouping?

Grade 2, Mathematics Lesson Plan

Date & Time: 5th period, Wednesday, June 18, 2014
Class: Grade 2, Class No. 3 (31 Students)
Instructor: Haruka Miyamoto

1. **Name of the Unit:** Subtraction Algorithm

2. **Goals of the Unit:**
   ○ Help students deepen their understanding of subtraction and develop their ability to utilize subtraction.
   ○ Help students understand the bidirectional (inverse) relationship between addition and subtraction and foster their ability to explain this relationship using math expressions.

3. **Evaluation Standards:**

Interest, Eagerness, and Attitude [IEA]	Mathematical Way of Thinking [MT]	Mathematical Skills [MS] on Quantities and Geometric Figures	Knowledge and Understanding [KU] on Quantities and Geometric Figures
• Students are eager to think about how to calculate subtraction of 2-digit numbers. • Students are eager to utilize subtraction of 2-digit numbers in their lives and for learning.	• Students think about how to calculate subtraction of 2-digit numbers. • Students are investigating the property of subtraction, thinking about how to calculate using it, and utilizing the property as a method to check calculated answers.	• Students are able to confidently calculate subtraction of 2-digit numbers, by understanding the procedure of algorithm calculation.	• Students understand that subtraction calculations of 2-digit numbers consists of basic calculations of 1-digt numbers, by utilizing the place value of digits in numbers. • Students understand how to calculate subtraction of 2-digit numbers.

4. **Three Pillars of Instructional Material:**
   (1) Our view of the instructional material:

The goals of the unit are for the students to think about the process of subtraction calculations using the algorithm and applying these calculation skills to solve problems. In addition, students will be able to grasp the relationship between addition and subtraction and be able to express the meaning of addition and subtraction using tape diagrams.

The students learned in the previous unit how to use the algorithm to add 2-digit numbers with regrouping by aligning each place value. In this unit students learn the more complicated calculation process of 2-digit subtraction with regrouping, i.e., 2-digit minus 2-digit or minus 1-digit numbers that involve regrouping.

We would like to foster students' disposition for solving problems on their own and thinking about better ways to explain their ideas clearly to others, through discussing how they manipulate blocks and utilize cherry diagrams. Through such discussions, we want to help students understand the meaning of calculation involving regrouping, when a 10 is decomposed before finishing the calculation process. Lastly, we would like to help students be able to do the calculations confidently by practicing after they deeply understand the process and meaning of subtraction calculations with the algorithm.

(2) Students' expressive ability that we would like to foster in this unit:

◎ What and how students express their thinking:

- Students utilize the knowledge they learned previously

  Student express their process of thinking by drawing the block or cherry diagrams and making connections with what they learned in previous lessons, such as regrouping calculations in Grade 1 and 2-digit addition regrouping calculations in a previous Grade 2 unit. For students needing more support, encourage them to use blocks to express and represent the process of their thinking and reasoning.

◎ Mathematics students need to express

- Students express the process of algorithm calculation.

  Students understand that the process of regrouping in subtraction, "break a group of 10 into 10 single ones," is the same regardless of whether they use the block or the cherry diagrams to express the process. By doing so, students realize the merit of the base ten system and understand that even as the number of digits/places increases, the calculations used in each place is as simple as the calculations they learned in Grade 1, such as: 1-digit minus 1-digit numbers without regrouping and 2-digit (teen numbers) minus 1-digit numbers with regrouping calculations.

(3) Evaluation during the lessons:

★ Evaluation during the time students solve a problem on their own:

Are the students trying to come up with ideas for how to calculate subtraction with the algorithm and apply the calculation process?

Level A: Students are trying to regroup one (1) from the tens place to make 10 ones by using block and/or cherry diagrams and writing the explanation of their thinking process in words.

Level B: Students are trying to solve the calculation by using block and/or cherry diagrams, and writing the explanation of the process in words.

★ Evaluation during the time students engage in whole class discussion:

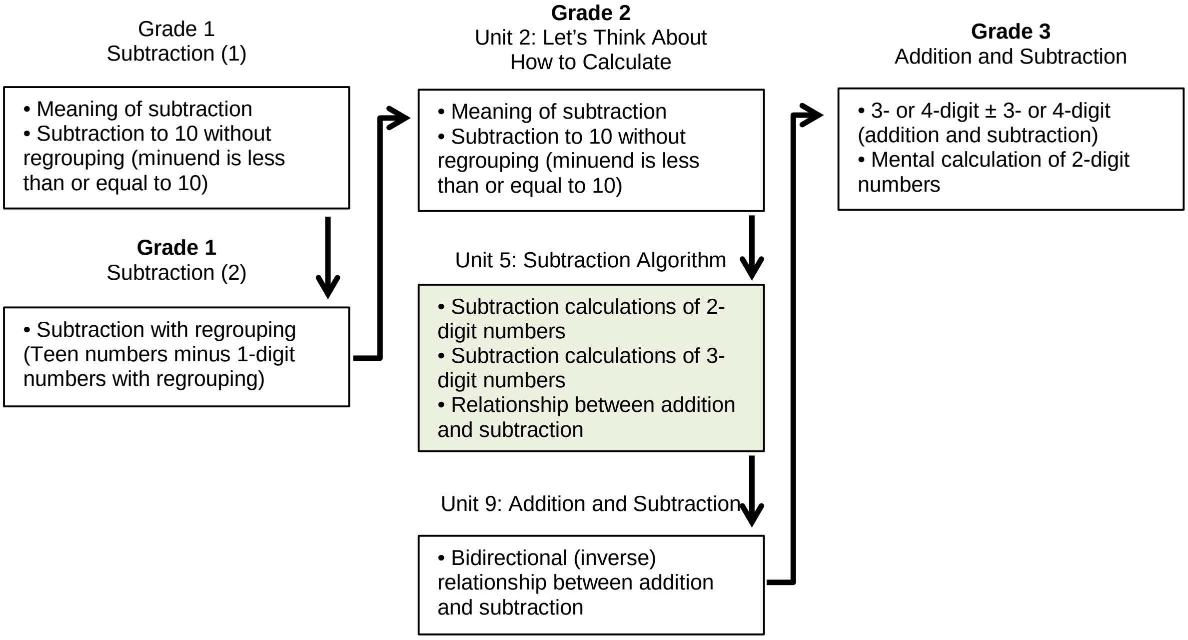
Level A: Based on their diagrams (e.g., block and cherry diagrams), students are able to present their ideas for regrouping one (1) from the tens place to make 10 ones and explain what they've written in their notebooks. Furthermore, they grasp the similarities and differences between their own ideas and their classmates' ideas.

Level B: Students are able to present their ideas for regrouping one (1) from the tens place to make 10 ones, based on diagrams such as block or cherry diagrams. They explain what they have written and diagrammed in their notebooks.

★ Student's reflection about their learning from the calculation work recorded in their notebooks:

Level A: Students understand the meaning of regrouping from the tens place by comparing their own ideas and their friends' ideas and by grasping the similarities and differences among these ideas.
Level B: Students are able to understand the meaning of regrouping.

5. Scope and Sequence of Subtraction



6. About Instruction:

Devise the plan of the lesson

Utilize concrete materials corresponding to individual needs

○ Instructional Plan:

The suggested plan of this lesson is for students to think about how to calculate 45 minus 27 (45-27) and discuss the algorithm calculation. However, we want to stress that when students are expressing their own thinking, we decided not to include discussion of the algorithm calculation. Instead, we would like to provide more opportunities for students to present their own ideas in front of the class. In regard to the subsequent lesson, we plan first for students to summarize the learning from this lesson, and then engage in discussing the process of algorithm calculation.

○ Instructional that corresponds to individual student needs:

For students who struggle to write their own thinking easily, we will provide blocks and ask them to manipulate the blocks to represent the story problem. When the students need to regroup one (1) ten in the tens place into 10 ones in the ones place, we plan to help them understand how a ten-group of blocks in the tens place can be broken/decomposed into 10 individual ones in the ones place. Furthermore, during the whole class discussion, students will be asked to use blocks and to manipulate them to show the regrouping process. This process will be repeated many times to help students understand the process deeply. Through this process of discussion, I hope that the students notice and conclude that "if we start calculating from the tens place, we might make a calculation mistake or run into difficulty. "

7. Students' State of Learning:

In my class, there are students who think they are not good at mathematics or capable of being successful as strong mathematics learners. Moreover, there is a large gap in the speed with which my students solve problems and write down their thinking in their math notebooks. There are some students who can explain their own thinking clearly to others, but generally there are more students who do not have confidence in their ability to explain their thinking to others. They think and judge that they are not "good at it" (math).

For the addition algorithm calculations that students learned in the previous unit, some of the students could recall what they learned in the last year and utilize it to reason through the calculation process in this unit. Many of them understood calculation could be done by splitting tens and ones and then conducting calculations separately on each respective place value. There are some students who can compare the presented ideas and find the similarities and differences among them. However, there are some students who cannot describe the calculation process and are not yet able to express their own ideas in words or diagrams.

8. Unit and Evaluation Plan:

Sub-Units	Lesson	Content	Interest, Engagement, and Attitude	Mathematical Way of Thinking	Mathematical Skills	Knowledge and Understanding
Subtraction with 2-digit numbers	1	• Understand problem situations that are applicable to subtraction and establish math expressions • Think about how to calculate 2-digit – 2-digit without regrouping using the algorithm based on the knowledge gained from addition algorithm calculations. • Think about the process of algorithm calculation by corresponding to how blocks are manipulated. • Think about how to calculate 2-digit numbers minus tens and 2-digit minus 1-digit numbers • Practice how to write and calculate using the algorithm.		• Through analogical thinking, consider how the format of the subtraction algorithm can be constructed in a similar way to the addition algorithm.		• Students understand the subtraction problem situation, meaning of subtraction, and how to calculate subtraction.
	2*	• Find the difference between algorithm calculations with and without regrouping. Think about the calculation process by understanding the meaning of regrouping and by utilizing the block diagram and cherry diagram.	• Students are eager to think about the process of calculation by drawing block or cherry diagrams and writing explanations in words.	• Students are thinking in an organized way about the process of regrouping by corresponding regrouping to the block and cherry diagrams.		

	3	• Understand how to calculate subtraction algorithm calculations with regrouping. • Practice subtraction algorithm calculations with regrouping. • Think about how to deal with zero (0) and a vacant place by applying algorithm calculations, such as 70 – 23 and 34 – 26.			• Be able to do the algorithm calculations such as tens minus 2-digit numbers and the difference of 2-digit minus 2-digit numbers that become a 1-digit number.	
	4	• Think about how to calculate 2-digit minus 1-digit algorithm calculations with regrouping.			• Be able to do subtraction algorithm calculations of 2-digit – 1-digit	
Practice		• Deepen the understanding of the content learned in this unit.			• Be able to do subtraction algorithm calculations with regrouping	
Subtraction that involves a minuend of more than 100	5	• Think about the calculation process that splits the minuend in order to calculate easily and utilize calculation that is based unit of tens. • Think about how to manipulate blocks to calculate the algorithm calculation involving regrouping from the hundreds place to the tens place.		• Students notice the regrouping calculation process from the hundreds place to the tens place is similar to the regrouping process of tens place to the ones place and are able to think and express the process using diagrams and math expressions.		
	6	• Think about calculation process of algorithm calculation involving two regrouping (the hundreds place to the tens place, and the tens place to the ones place) by using block and Cherry diagrams.		• Students are thinking the process of calculation orderly by first regrouping the tens to the ones then regrouping the hundreds to the tens.		
	7	• Think about the process of algorithm calculation when the tens place of the minuend is a vacant place.		• Students are thinking about the calculation process by regrouping from the hundreds place and do use the new numbers in the tens place to do the regrouping calculation of the ones place when the tens place of the minuend is a vacant place.		

Subtraction with 3-digit numbers (minuends)	8	• Think about calculation for hundreds minus hundreds. • Think about calculation for 1000 minus hundreds.				• Students understand that they can calculate easily by a hundred as a unit. Also they understand how to subtract hundreds from 1000.
	9	• Think about how to calculate 3-digit minus 1- or 2-digit algorithm calculations and pay attention to the regrouping that is necessary during the calculations.			• Students are able to express 3-digit minus 1- or 2-digit calculations using the algorithm and carry out the calculation correctly. .	
Relationship between Addition and Subtraction	10	• Investigate the relationship among minuend, subtrahend, and difference using the tape diagram and the word math sentence, and think about the relationship between addition and subtraction. • Use addition to check the answers to subtraction problems.				• Students understand that subtraction is an inverse calculation (operation) of addition and addition can be used to check the answer of subtraction.
Which calculation should we use?	11	• Grasp the relationships of quantities such as part + part = whole and whole - part = part using tape diagrams. Use mathematical expressions to represent these relationships			• Students are able to read the tape diagram and present it with it corresponding math sentence.	• Students understand the relationship between addition and subtraction as an inverse relationship.
Practice	12	• Deepen understanding of the content studied in this unit			• Students are able to do calculations with the algorithm for 2-digit minus 2-digit and 3-digit minus 1- or 2-digit with or without regrouping.	
Power Builder	13	• Review and check the content studied in this unit.				
		• Understand the meaning of *Mushikuizan* (arithmetical restorations). • Think about how to solve the problem by utilizing the mechanics of the algorithm learned previously. Students explain the solution process.	• Students show interest and motivation for calculations through problem solving with *Mushikuizan* problems.			

*** This Lesson**

Goals of This Lesson:

Using diagrams and words, students are able to think about how to calculate by paying attention to calculations in each place value.

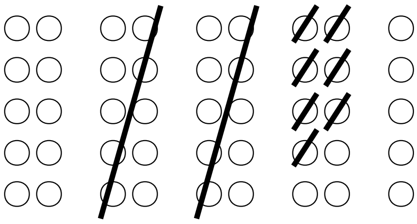
9. Flow of the Lesson (Lesson 2/13)

	■ Learning Activities *Hatsumon* (T), Students' Anticipated Responses (C)	○ Points to Remember ◆ Evaluation ★ Support
Understand the Problem and Make a Plan for Solving the Problem	**■ Understanding the Problem and Make a Plan for Solving the Problem** T1: There were 45 tadpoles. 27 of them became frogs. How many tadpoles are left? T2: What is different about this problem from the one we worked on yesterday? C1: The numbers used in the problem are different. T3: Up until yesterday, how have we been calculating? C2: We calculate numbers keeping the tens and ones separate. T4: Okay, let's calculate this one by splitting up the tens and ones. Let's write the algorithm (vertical notation) on the board. For the tens place the calculation becomes 4 minus 2 (4 – 2) ... For the ones place the calculation becomes, 5 minus 7 (5 – 7)… C3: We can't calculate this. C4: We can't subtract 7 from 5. This is different from the problems we worked on before. T5: What should we do? Let's think about how to calculate 45 – 27. T6: Let's think about how to calculate this problem.	○ Show the slides of tadpoles and frogs and help students understand the problem. ★ Setting up a familiar topic as the problem situation (i.e., tadpoles in the biotope), brings out students' interest in the problem. ○ Writing the algorithm on the board helps students understand visually how the numbers in the ones place cannot be subtracted easily.

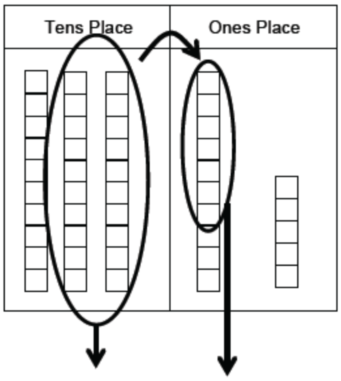
Student Solving the Problem on Their Own

■ **Student Think about How to Calculate**

② Draw circles with groups of 10s and subtract 27 from there.



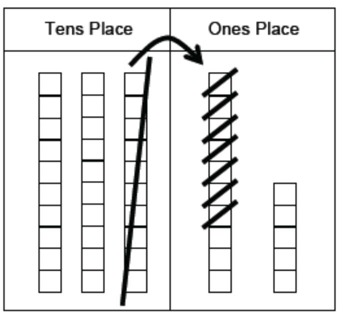
③ Think about the calculation using blocks. (A)



30 – 20 = 10　15 – 7 = 8

10 + 8 = 18

③ Think about the calculation using blocks. (B)

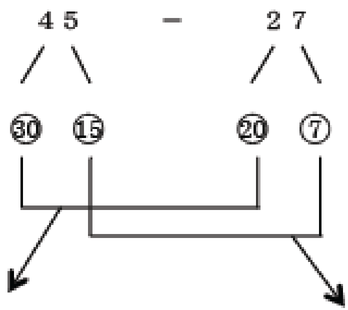


30 – 20 = 10

15 – 7 = 8

10 + 8 = 18

④ Think about the calculation using the Cherry Diagram



３０－２０＝１０　　１５－７＝８

１０＋　８＝１８

Answer: 18 tadpoles

★ For students having difficulty coming up with their own ideas, provide blocks they can manipulate to represent the story problem as they work to understand the problem.

○ Help students to think about what they can do when they can't subtract numbers in the ones place. Help them notice and pay attention to how they can use the number in the tens place to solve not having enough ones (to subtract 7).

○ If students are drawing circles, suggest that they draw the circles organized into groups of tens.

★ [IEA] Students try to solve the problem by drawing block or cherry diagrams.

Presenting and Discussing (Whole Class)	■ **Presenting Own Ideas** T7: Let's present your own ideas. ■ **Finding Commonality of Ideas** T8: When you compare the methods, do you see any commonality? C5: The calculations were done at each place separately. C6: Carry out subtraction in each place separately and added what was left in each place. C7: Moving 1 from the tens place and made 10 in the ones place. C8: Moving 1 from the tens place and calculated 15 -7. C9: Split 45 into 30 and 15 and carry out the calculation 15 – 7, so it is also similar to other ways.	○ Ask students to draw their diagrams on paper and post them on the board. ○ Put blocks on the board and ask students to manipulate them. ○ Record student ideas and keywords on the board. ★ [MT] Students are able to present the process of calculation, breaking one (1 ten) in the tens place and showing the 10 in the ones place. Students do this by using the block and cherry diagrams they recorded in their math notebooks.
Summarizing	T9: This calculation process, moving 1 from the tens place and changing the 1 ten to 10 in the ones place, is called "regrouping." When we "regroup" we can used the calculation methods that we learned in Grade 1, can't we? ■ **Check the Ideas for the Calculation by Looking at the Slides of Tadpoles and Frogs** T10: In the next lesson, let's represent the calculation process we discussed today using the algorithm. ■ **Writing Reflections**	○ Ask students to check the ideas for the calculation by showing the slides of tadpoles and frogs. ★ [NU] Students are able to understand the meaning of the regrouping calculation process (moving a ten from the tens place to the ones place).

10. Evaluation:

- Students are able to think about, represent, and explain how to calculate using diagrams, such as block cherry diagrams.

11. Board Plan:

Wednesday, June 18

Problem

There were 45 tadpoles. 27 of them became frogs. How many tadpoles are left?

Math Sentence:
45 – 27 = 18

45
- 27

Answer: 18 tadpoles

Objective Let's think about how to calculate 45 – 27!

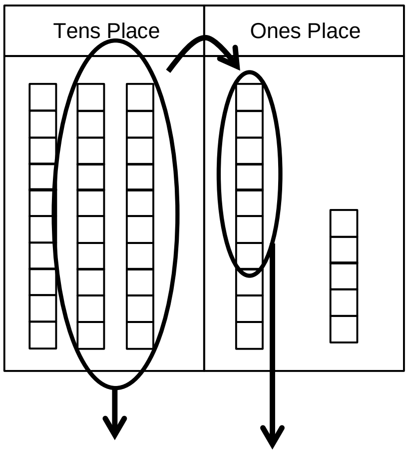
My Idea | **Friends' Ideas**

Method ①
Draw 45 circles and subtract 27 circles.

Method ②
Draw 45 circles using groups of 10's. Take away circles from the 10's

Method ③
Thinking about the calculation using a block diagram.

Method ④
Thinking about the calculation using a cherry diagram.



30 – 20 = 10   15 – 7 = 8

10 + 8 = 18

- Calculations were done at each place value separately.
- Results of calculations are added together.
- Moving 1 from the tens place to make 10 in the ones place to calculate.
- Splitting 45 into 30 and 15 then carrying out the calculation 15 – 7 = 8.

Grade 3 Mathematics Lesson Plan

Thursday, June 19, 2014
3rd period (10:40 – 11:25)
Teacher's Name: Takeo Takahashi
Class: Koganei Elementary School
attached to Tokyo Gakugei University, Grade 3,
Class No. 2 (35 Students)

1. Name of Unit: *Division – Calculations for Finding "Times As Much"*

2. Goals of the Unit:
   ○ Students eagerly try to understand the meaning of division and the calculation process by manipulating concrete materials and making connections to multiplication. (Interest, Eagerness, and Attitude)
   ○ Students understand partitive and quotitive division as one operational meaning of division and represent the division process of calculating with concrete materials, diagrams, and mathematical expressions. (Mathematical Way of Thinking)
   ○ Students learn and are able to consistently and accurately carry out division calculations. (Mathematical Skills)
   ○ Students understand problem situations that involve division, the relationship between division and multiplication, and the meanings of division. (Knowledge and Understanding)

3. About the Lesson:
   In general, students in my class are active and are starting to feel the joy of exchanging ideas among classmates during lessons. My students are developing the skills needed to solve problems, such as explaining their ideas to others in words, drawing diagrams, writing mathematical expressions, and anticipating or speculating about their classmates' ideas. However, students remain somewhat hesitant to share their opinions openly and freely during class discussions.
   When solving problems such as those involving quotitive division, time, and elapsed time, my students are accustomed to using diagrams, including tape diagram models and array models and using grid lines in their journals. However, most have not developed an understanding of the difference between diagrams that represent and result from the process of thinking through problem solving and those that are used to explain the result of problem solving.
   Falling within the "Division" unit, this lesson is the first lesson of the sub-unit "Calculation for Finding Times as Much." Prior to this sub-unit, students learned the two meanings of division: partitive and quotitive division. If the meaning of multiplication is identified as [number of objects in each group] x [number of groups] = [total number of objects], two meanings of division are distinguished as follows: (1) when division is used to find the number of objects in each group, it is called "partitive" division (dividing an amount into a given number of groups, to find the number in each of the

equal-sized groups/parts) and (2) when division is used to find the number of groups it is called "quotitive" division (dividing an amount into a given number in each group to find the number of those equally-sized groups).

In this lesson, the meaning of division as "number of groups," will be extended to the meaning "times as much." As students expand their understanding about the meaning of division, I would like them to discover that in the case of finding how many "times as much," they can use the same division process for finding the "number of groups" (quotitive division). To do this, we will discuss solution ideas that students might use for finding the answer, namely solutions which are associated with repeated addition and repeated subtraction. I expect to see them utilizing and manipulating diagrams (e.g., tape diagrams) and hear them discussing as well as using mathematical expressions. In addition, by making connections to the multiplication expression [length of 1 tape] x ☐ = [total length of the rope], I want students to notice that they are engaged in a solution process similar to quotitive division problem situations they learned in previous lessons.

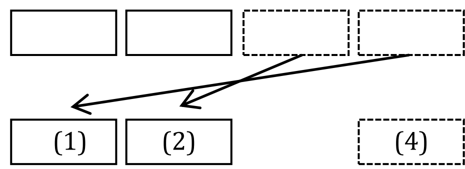
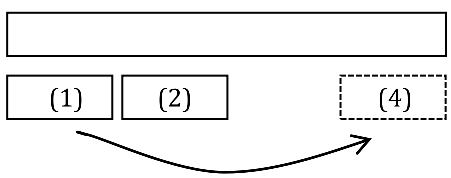
4. Plan of the Unit (Total: 9 lessons):
   - Sub-Unit 1: Quotitive division (5 lessons)
   - Sub-Unit 2: Partitive division (3 lessons)
   - Sub-Unit 3: Calculation for finding "times as much" (1 lesson, described below)

5. Instruction

   (1) Goals of the Lesson:
   - Students will understand that they use division to solve problem situations for finding how many times as much is the given quantity (quantity to be compared) as the base quantity.

   (2) Flow of the Lesson:

Process	Activities and Students' Anticipated Responses	○ Instructional Points to Remember ★Evaluation Points and Methods
Grasping	**1. Grasping the problem situation** The length of a red tape is 36 cm. The length of a blue tape is 9 cm. How many times as long is the red tape as the blue tape? T1: Write your solution methods and the reasons why the methods work clearly in your notebook, so your friends can understand your thinking.	○ Provide scissors, rulers, 36 cm strips of red tape, and 9 cm strips of blue tape (for students). ○ Ask students to show their own solution methods using diagrams.
Investigating and Confirming	**2. Solving the problem on their own** < Student Anticipated Solutions > C1: (Repeated subtraction)	○ Ask students to manipulate the blue and red strips of tape on the board. Ask them to express how they manipulate the tape and make

	 Just as I show my thinking in the diagram, I tried to find out how many 9 cm strips of tape I can take from the longer (36 cm) tape. I took them one by one and lined them up next to the bottom 36 cm strip. After all the 9 cm strips of tapes are aligned to the bottom, I counted the number of strips of tape I moved. There are 4 strips of tapes so I think it is 4 times as much. The math sentences for this is: 36 – 9 – 9 – 9 – 9 = 0. So it is 4 times as much. C2: (Repeated addition)  As you can see, I placed 9 cm tape one by one next to the long 36 cm tape. When you do that you place the 9 cm tape 4 times. So it is 4 times as much. The math sentence is: 9 + 9 + 9 + 9 = 36. (*When a student says something like "I placed 9 cm tape…,," ask the student why s/he decided to place 9 cm strips of tape several times (to clarify he difference between C1's method and C2's method). Through this discussion, I want students to grasp the idea of measurement, such as "how many times a 9 cm tape can be fit into or taken away from the 36 cm tape.) C3: 9 x ☐ = 36 C4: 36 ÷ 9 = 4	connections among the manipulation process, their words and math sentence(s). By doing so, help students see and make the connection with the manipulation process of quotitive division that they learned previously. ○ Go over each solution method with students though discussion and clarify the similarity and differences of the solutions. ○ The order of the students' presentation should be: Repeated subtraction method → repeated addition method → multiplication method → division method (with math sentence). When the division method is presented, ask the students if and why they can use division although the problem asks them to find out "how many times as much (as long)." ○ Ask students to be sure they put headings in their notebook that indicate which solution method is their own and which solution methods are their friends'. ★Are the students eager to solve the problem on their own? ○ If there are students who used the division symbol (÷) to solve the problem, make sure the students know this problem situation is different from the division situations they studied previously. Press the students to think about how and why they can use division in this problem situation.
Presenting Summarizing	**3. Presenting and Summarizing** C5: I tried to find out how many 9 cm strips of tape I can take from the 36 cm tape. I took the 9 cm strips of tape one-by-one and aligned them to the bottom 36 cm strip. After all the 9 cm strips are moved to the bottom, I counted	★Do the students understand that division is used when problem situations ask how many times as much a given quantity is as a base quantity?

	the number of tapes. There are 4 strips of tape, so I think it is 4 times as much. (Same as anticipated response C1 above) T2: What do you think? C6: I did this the same way. T3: Do you have something to add to C5's explanation? C7: I don't have anything to add to C5, but I did it a similar way. T4: Please explain your way. C8: As you can see, I placed 9 cm strips of tape one-by-one next to the 36 cm long tape. When you do that you can place the 9 cm tape 4 times. So it is 4 times as long. (Same as the C2 anticipated response above) T5: I see, do you have anything you want to add or do you have a a similar way? C9: I used multiplication. C10: The diagram that C2 used shows addition of 4 tapes that are 9 cm. So, just like we studied before, the math sentence is 9 x [4] = 36. C11: C1's math sentence could be multiplication, because there are 4 tapes of 9cm ... but it is shown as subtraction. T6: Do you have something to add to that? T7: It looks as if each one of the methods includes the math sentence 9 x [4] = 36 and the manipulation process of division that we learned previously. So we have found that in the case of problem situations for finding "times as much" using calculation, we can use division also. This is the end of the lesson, please write your reflection in your notebooks.	○ If students are not actively responding after *hatsumon* question T5, ask "Can you write math sentences for methods C1 and C2 using a math sentence structure we learned before today's lesson?" Refer students to the solution methods on the board. If the *hatsumon* helps bring out the multiplication sentence, 9 x [4] = 36, or the division sentence, 36 ÷ 9 = 4, ask students to think about why they can use the division sentence in this situation. After the discussion, help students to understand the relationship between the division sentence and multiplication sentence. ○ When students respond similarly to the response of C9, be sure to ask the students the reason why they are using the idea of multiplication, which is related to what they learned previously,. If the reasons are something related to the manipulation process for quotitive division, help students understand that division can be used. Although the problem situation is different from those learned previously, it is similar in its process of manipulation and diagramming used in quotitive division.

June 20
Ver.2

Grade 3 Mathematics Lesson Plan

How many packages can we make? How many will be left? (Division with remainders)

Friday, June 20, 2014
Grade 3 Room 1 (23 students)
Teacher: Kohko Morita

Research Theme: "I did it! I got it!" Designing mathematics lessons students will be engrossed: Teaching strategies that value students' questions and help students enjoy reasoning and expressing themselves.

1. Name of the Unit: Division with remainders

2. Goals of the Unit

   Students will understand the meaning of division and be able to use it.
   a. Students will learn about division with remainders and think about ways of calculating the answers.
   b. Students will understand the relationship between division and multiplication and subtraction.
   c. Students will be able to calculate accurately division where the divisors and the quotients are both 1-digit numbers.

3. Assessment Standards for the Unit

Interest, Eagerness, and Attitude (IEA)	Students realize that division can be used even when there will be remainders, and they try to use division in various situations.
Mathematical Way of Thinking (MT)	Students can think about the meaning and the ways of calculating division with remainders using manipulatives and drawings while making connections to concrete situations.
Mathematical Skills (MS)	Students can calculate division with remainders, and they can deal with remainders appropriately.
Knowledge and Understanding (KU)	Students understand the meaning and appropriate use of remainders, and they understand how to calculate division with remainders using the division algorithm.

4. About the Unit

(1) Goals

In the previous unit, Division, students learned about the meaning of division and the way to determine the quotients (without remainders) using the basic single-digit multiplication facts. In this unit, the goal is to deepen students' understanding of division by examining division with remainders. Students will think about the meaning of the remainder and think about ways to calculate division with remainders. In addition, they will explore the size relationship between the divisor and the remainder.

(2) About students

Students in this class are generally enthusiastic about mathematics. They are willing to share their ideas freely, and they seem to enjoy tackling problems arising from the tasks given during mathematics lessons.

On the other hand, there are significant individual differences in students' mathematical knowledge, skill mastery, the ability to apply their knowledge, and the ability to express their ideas. During the previous unit, Division, there were students who could represent problem situations using diagrams on their own and explain their ideas logically, while others had difficulty making sense of problem situations and had to work closely with the teacher and with the aid of manipulatives.

(3) Mathematics in the Unit

In this unit, students will think about the meaning of remainders while examining both division without remainders and division with remainders. In addition, students will explore the size relationship between the divisors and the remainders by varying the dividend while keeping the divisor constant. By engaging in those explorations, it is hoped that students can expand the range of numbers in which division can be used.

Moreover, another goal of the unit is to nurture students' ability to think logically and express their ideas clearly. To do so, an emphasis will be placed on activities in which students will devise ways to calculate division with remainders and explain their ideas to other students, utilizing what they have learned up to this point such as multiplication, division, and various diagrams.

5. Scope and Sequence

【Grade 2】 Multiplication

・ Meaning of multiplication
・Basic single-digit multiplication facts

【Grade 3】 Multiplication

・ Multiplication with 0
・ Multiplying multiples of 10 and 100 by 1-digit multiplier
・Multiplication algorithm; mental calculation

【Grade 3】

・Meaning of division
・Division (without remainder) using the basic multiplication facts; dividing 1 and 0

【Grade 3】 Division with

・Division with remainders using the basic single-digit multiplication facts.

【Grade 3】 Division with

・(2- and 3-digit number) × (1-digit number)

【Grade 4】 Division: 1-digit

・Division algorithm
・Dividend = Divisor × Quotient + Remainder
・ (1- ~ 3-digit number) ÷ (1-digit number)

【Grade 4】 Division: 2-digit

・ Division with 2-digit divisors
・ Properties of multiplication and division

6. Unit Plan (Total of 5 lessons)

Sub-Unit	No.	Learning Activity	Assessment			
			IEA	MT	MS	KU
Division with remainders	1	• Explain the meaning of division with remainders using words and diagrams. • Think about and explain ways to calculate division with remainders using diagrams or by applying the reasoning used while calculating division without remainder. • Examine the size of the remainders and develop a new question about the size relationship of the divisor and the remainder.	○	◎		○
	2	• Verify that the remainder is less than the divisor. • Learn the way to check the result of division calculation.		○	○	◎
Problems	3	• Solve word problems involving partitive and quotitive division problems (with remainders). • Write word problems involving division with remainders from a given picture and sample problems.	○		◎	
Mastery	4	• Deepen the understanding of the unit content.			◎	◎
	5	• Consolidate the understanding of the unit content. • Think about how to evenly split juice in two different containers.		◎	◎	

7. Today's Lesson

(1) Goals of the lesson

- Students will understand the meaning of division with remainders.
- Students can think about and explain ways to calculate division with remainders using diagrams or by applying the reasoning used while calculating division without remainder.
- Students will examine the size of the remainders and develop a new question about the size relationship of the divisor and the remainder.

(2) Proposals in the lesson

Up to this point, students have come to understand the meaning of division. They are also able to calculate division without remainders using the basic 1-digit multiplication facts.

Today's lesson is about division with remainders. For the students, this is the first time they encounter division with remainders. Through today's lesson, they will expand the range of numbers with which they can use division. I have devised some strategies to generate questions like "What does it mean to have a remainder?" or "Can we use division for this situation, too?" as students are presented with the problem situation where there will be left overs when items are distributed. Then, by having students explain the remainder using diagrams, I want to clarify the meaning of the remainder, as well as the meaning of division with remainders.

In addition, through the activity of judging "if there is a remainder" as the motivation, I want students to think about ways of calculating division with remainders on their own and explain their ideas to others. From these experiences, I want students to realize that we can use the basic 1-digit multiplication facts to calculate division with remainders just as we did with division without remainders. Moreover, I want to nurture students' ability to express their own ideas logically by incorporating the activity to explain their ideas using not only words and equations but also diagrams.

Then, at the end of the lesson, I want students to generate the question, "What is the relationship between the divisor and the remainder?" by having students think about the size relationship between the divisor and the remainder.

(3) Specific strategies to address the research theme

① Engage students with a problem and draw out questions. (Grasp)

In the introduction, problems that involve division with remainders will be mixed in with those that involve division without remainders that students have already learned. By doing so, I want to draw out comments and questions like "We can't evenly share these" or "There will be left over. What can we do?" I will then inform them that they can still use division in those situations and help students clearly understand the meaning of division equations using diagrams. Furthermore, by having students think about whether or not 21 ÷ 4 and 33 ÷ 4 can be divided evenly, help students have ideas for how to calculate these division problems and make connection to the independent problem solving time.

② Help students experience the joy of solving problems on their own as they solve their questions. (Explore)

During the independent problem solving time, students will think about how to calculate 21 ÷ 4 and 33 ÷ 4 using mathematical expressions, diagrams, and words. For those students who are stuck, I will provide small group mini-lesson to help students get to "I got it!" and "I did it!"

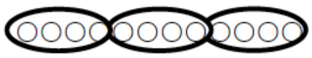
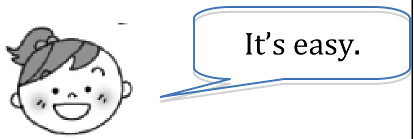
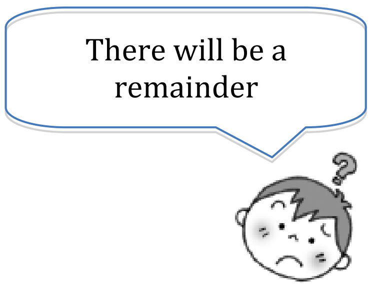
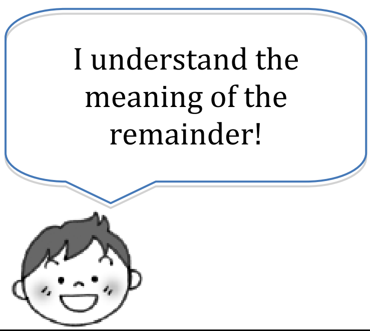
③ Answer their questions and deepen their understanding through the activity to interpret and verify other students' ideas. (Deepen/Heighten)

In this phase, I will have students explain why 21 ÷ 4 = 5 rem. 1 and 33 ÷ 4 = 8 rem. 1 using diagrams. As they do so, instead of simply accepting their words, I will ask "Is it really so?" or "Are you absolutely sure?" to enhance students' ability to explain their ideas logically. Then, I will call on a student who did not use any diagram while we are thinking about ways to calculate 33 ÷ 4 and ask, "How did you figure out 33 ÷ 4 = 8 rem. 1?" Through discussion in pairs and also as a whole class, we will examine ways to calculate division with remainders, and verify the reasoning using diagrams.

④ Draw out new questions through the activity of summarizing and extending. (Summary/Extension)

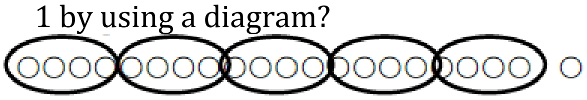
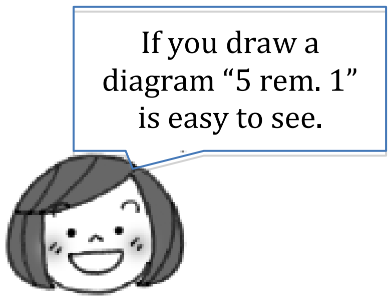
In the summary step of the lesson, we will reflect on the lesson according to students' thinking shared during the lesson. Then, since all division problems discussed in the lesson had the remainder of 1, I want to generate the question, "Is the remainder always 1?" Students should be able to realize that "the remainder can be 2 or 3, too," and I will have them explain their ideas using mathematical expressions and diagrams. Finally, I want to draw out the new question, "Can the remainder be 4 or 5, too?" as the motivation for the next lesson, "I want to investigate the size relationship between the divisor and the remainder."

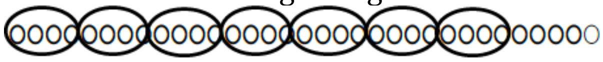
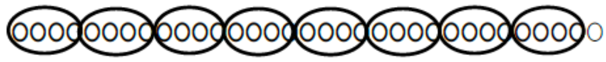
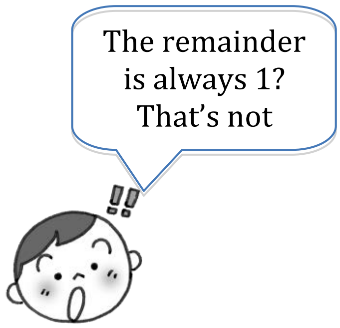
8. Flow of the lesson (Lesson 1 of 5)

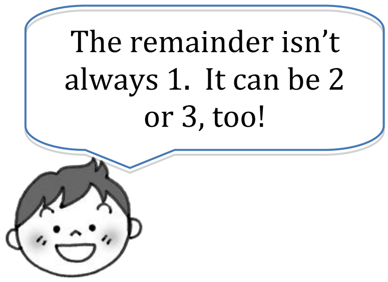
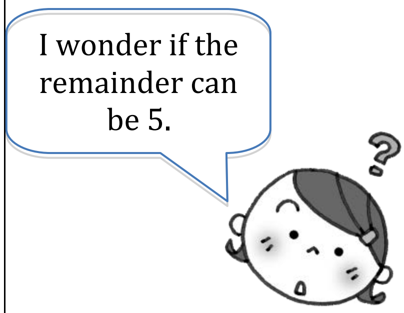
	Learning activities (Main *hatsumon* and anticipated responses)	□ Strategies to address research theme ○ Support and instructional considerations ⦿Assessment
G R A S P	1. Understand the task. There are [ ] pieces of octopus balls. If we put 4 pieces in a pack, how many packs can we make? (If there is any remainder, think about how many will be the remainder.) ← To be written later. T ① What if the number in the [ ] is 12? C We can make 3 packs. C Because 12 ÷ 4 = 3. T If we draw a diagram, it will look like this, right?  T ② What if the number in the [ ] is 20? C Because 20 ÷ 4 = 5, we can make 5 packs. T ③ What if the number in the [ ] is 32? C Because 32 ÷ 4 = 8, we can make 8 packs. T ④ What if the number in the [ ] is 13? C Whoa? Something is wrong. T Why do you say "whoa?" C We can't make 13 exactly. C There will be remainder[1]. T Remainder? What do you mean? Can you draw a picture? C Yes, I can. T Please draw a diagram in your notebook. C I have it.  **3 packs** **remainder** C If there are 13 pieces, we can make 3 groups of 4, and there will be 1 remainder. T I see. If we have 13 pieces, we can make 3 packs, and there will be 1 piece remainder. Even when there is a remainder, like this	□ By discussing division without remainders first, naturally generate the question, "What can we do when we cannot divide evenly?" when the division with remainder is posed as the 4th question. ○ Have a diagram prepared.   □ Have students draw diagrams in their notebooks so that it will be easy to understand. 

[1] It is probably more accurate to translate this statement as "There will be left overs." However, the same Japanese word, *amari*, is used for both "left over" and "remainder." Therefore, the term "remainder" in this lesson plan does not have the significance of formal mathematical term as is the case in English.

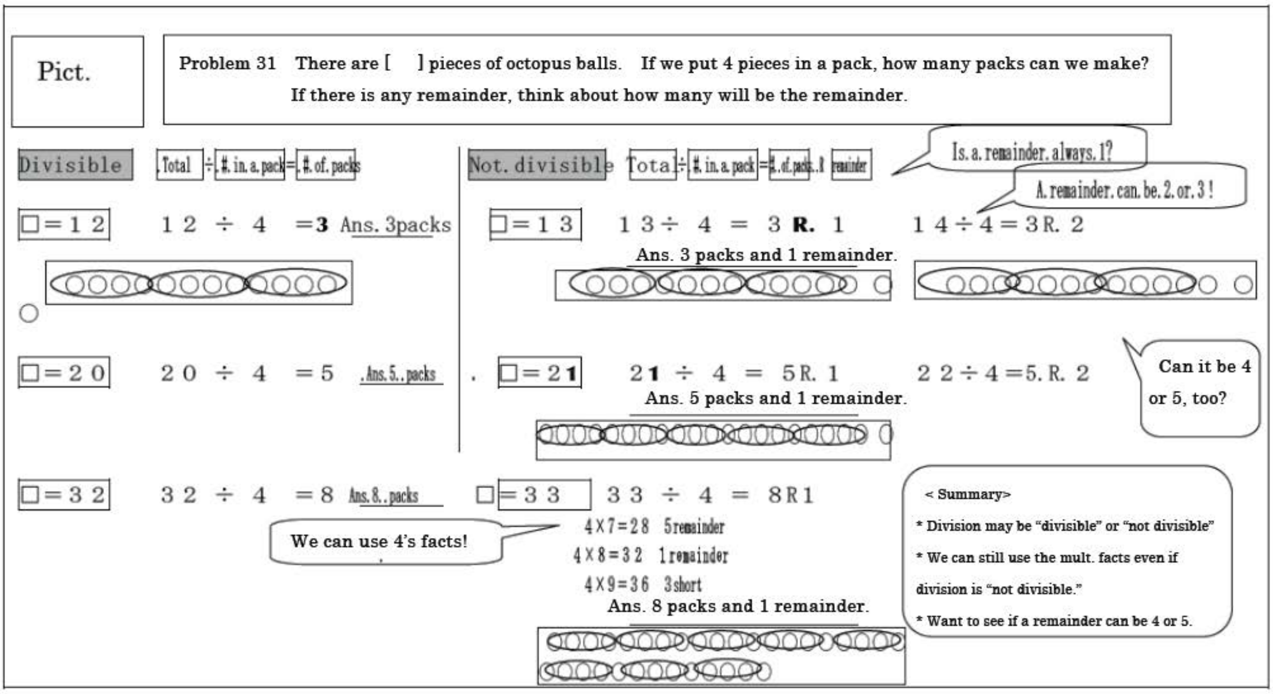
	case, we can still use division. If we represent this situation using a division equation, it will be written as 13 ÷ 4 = 3 rem. 1. Please write it in your notebook. C – Write the division equation below the diagram – T Division we have looked at so far did not have any remainder, and we can always divide equally, can't we? Division like 20 ÷ 4, when there is no remainder, we say "divide evenly." Division like 13 ÷ 4, when there is a remainder, we say "does not divide evenly." Today, let's think about division with remainders. I am going to add something to our problem. Please write, "If there is any remainder, think about how many will be the remainder." T Oh, I have more problems. What if there are 21 pieces of octopus balls? What if there are 33 pieces? How many packs can we make? If we had 21 pieces, do you think there will be a remainder? C Yes. T Do you think there will be a remainder if we have 21 pieces? C Yes. T Are you sure? C Yes, absolutely. T OK, let's find out if there will be a remainder. Please solve 9 ÷ 4 and 21 ÷ 4 using mathematical expressions and diagrams. You can start with either one.	○ Tell students that division can be used even when there is a remainder and show hot it is written. ○ Have students write the additional statement in today's problem. ⊙ Students have their own question and try to solve the problem eagerly. [Interest, Eagerness, and Attitude]
E X P L O R E	2. Represent division with remainders in expressions and think about ways to calculate using diagrams. When there are21 pieces. 21 ÷ 4 = 5 rem. 1  When there are 33 pieces. C: It will be really tedious to draw a diagram.	□ Have students draw diagrams in their notebooks so that it will be easy to understand. □ For students who are stuck, conduct a small group mini-lesson. Hint: 1. What is 20 ÷ 4? 2. Can you draw a diagram to show 20 ÷ 4?

	C: I think we can do it without drawing a diagram. **4× 8 = 32 Since we use 32 octopus balls of 33, there will be 8 packs and 1 remainder.**	3. So, what happens if you have 21 ÷ 4? ⦿ Based on the way to calculate division without remainder they have learned previously, students think about ways to calculate division with remainders using diagrams. [Mathematical Way of Thinking]
D E E P E N	3 Discuss how to calculate division with remainders. T Let's start with the case where there are 21 pieces of octopus balls. Please explain your idea to your neighbor. Please share your equation and answer. 21 ÷ 4 = 5 rem. 1 T How many packs can we make and how many is the remainder? C We can make 5 packs, and there is 1 remainder. T Can someone explain why 21 ÷ 4 = 5 rem. 1 by using a diagram?  C If you look at the diagram, you see there are 5 packs and 1 remainder. So, the equation is 21 ÷ 4 = 5 rem. 1. T Can you see "5 rem. 1" in this diagram? C This part. T I see. I think we can conclude that 21 ÷ 4 = 5 rem. 1. T OK, the other problem with 33 pieces of octopus balls. What is the equation? What do you think is the calculation? C 33 ÷ 4 = 8 rem. 1. So, you can make 8 packs and there will be 1 remainder. T _(name of a student)_ didn't draw a diagram in her notebook. How do you think _____ thought about this calculation? Discuss it with your neighbor.	By incorporating pair-sharing time, give each student to explain his/her idea logically.  By incorporating pair-sharing time, give each student to explain his/her idea logically.

	C I think we can use the 4's facts. T Even when we cannot divide evenly, can we still use the 4's facts? C 4 × 7 = 28, and there will be 5 more pieces. 4 × 8 = 32, and there is 1 remainder. 4 × 9 = 36, and we don't have enough. So, 33 ÷ 4 = 8 rem. 1. T Let's check it using a diagram.  4 × 7 = 28 there are 5 remainders  4 × 8 = 32 there is 1 remainder T We can find the answer for division with remainders using the multiplication facts just as we did with division without remainder, can't we?	
	4 Summarize the lesson and generate a new question about the size of the remainder. T We discussed division without remainder through yesterday. Today, we studied that division sometimes has the remainder. We were able to write equations and diagrams, and we understand the meaning of the remainder. Also, we learned the way to calculate division with remainders. Now we know that we can either divide evenly or we will have the remainder of 1. C No, that's not true. The remainder can be 2. C The remainder can be 3, too. T What? The remainder can be 2 or 3? But look at what we did today. The remainder is always 1. C Yes, but if we change the total amount, we can get the remainder of 2. T OK, please discuss with your neighbor how we need to change the total number so that the remainder will be 2. T If your neighbor's explanation was easy to understand, please raise your hand. OK, __________, could you explain for when we will have a remainder of 2?	• By intentionally making a false claim, generate a new question in students and have them express their ideas eagerly.  • By incorporating pair-sharing time, give each student to explain his/her idea logically.

	C If you draw a diagram for the case when there are 14 octopus balls.  T OK, so what number sentence should we write? C It will be 14 ÷ 4 = 3 rem. 2. C Because you get 14 when you add 2 to 4 × 3 = 12, the remainder will be 2. T I see. We can have a reaminder of 2, too. C It can be 3, too. T What? The remainder can be 3, too? C If we add 1 more, the remainder will be 3. T I see. The remainder isn't always 1. It can be 2 or 3, too. I wonder if there is any division where the remainder will be 4 or 5? C I don't think that's possible … T The remainder cannot be 4 or 5? C Absolutely not. T OK, then let's think about if there is any division where the remainder is 4 or 5 tomorrow.	○ Students can explain their own ideas or their friends' ideas. [Mathematical Way of Thinking]  
	T Let's write the summary of the lesson. C With division, we sometimes we have the remainder and other times there is no remainder. But, the way to calculate is the same for both cases. C The remainder is not always 1. It seems like a remainder can be 2 or 3. I wonder if it can be 5.	○ Students have a new question and eagerly trying to solve it. [Interest, Eagerness, and Attitude]

9. Board writing plan



10. Observation points for the lesson

1. In order to have students generate their own questions, division without remainders and division with remainders were posed together in the beginning of the lesson. Was it effective?
2. Instead of simply accepting students' responses, the teacher posed follow-up questions and had students discuss their ideas in pairs. Did that strategy lead to the activity of students explaining and expressing their ideas logically?
3. Was the activity of having students draw diagrams in their own notebooks a useful strategy to help students independently organize their own thinking and express their ideas?
4. Other

Mathematics Public Research Lesson 1

"Utilizing Mathematics" Activities that Students' Express and Think about Phenomena Mathematically

Mathematics Lesson Plan

Date & Time: 10:00 a.m. to 10:50 a.m., Saturday, June 21, 2014
Students: Tokyo Gakugei University International Secondary School, Grade 7, Class No. 4 (26 students: 11 boys and 15 girls)
Teacher's Name: Hiroko Uchino

The International Baccalaureate Middle Years Programme (MYP) published the *Next Chapter* in May of this year with a newly revised curriculum and evaluation. Because of this change, beginning in the next school year, our school will enforce the *Next Chapter* fully. To make this transition, I will incorporate the *Next Chapter* in my plan of instruction and classroom practice as a forward-thinking research project.

1. Unit Planning considering "Backward Design"

1.) MYP Statement of Inquiry:

Organizing patterns, properties, trends, and relationships of phenomena with numbers, mathematical expressions, tables, and graphs, and using these to grasp the phenomena mathematically, helps us to plan, implement, problem solve, make projections and conscious decisions related to the phenomena.

2.) **Name of the Unit in Our Curriculum:** TGUISS Mathematics 1: Chapter 2, "How to Look at Phenomena"

The educational goals of our school include fostering Mathematical Literacy; that is, students think about how to make a real-life phenomena better, problem solve, and expand ideas using mathematics. Based on these goals, our textbook was created by incorporating real-life phenomena as exploratory topics that include important mathematics content. Students think about these topics mathematically; they problem solve and practice with selected problems to be sure they utilize their learning within a real world situation context. This structure and flow of learning is represented in the diagram below, "Teaching and Learning in the IB," described in MYP's *Next Chapter*. (See Figure 1.) Our school's vision and plans for instruction follow MYP's vision of education.

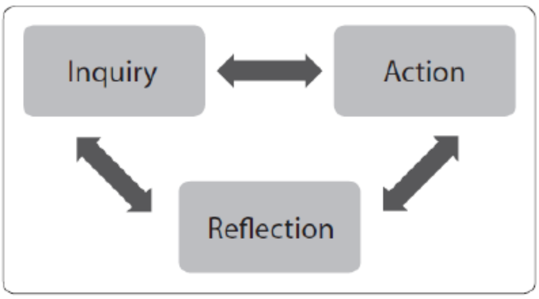


Figure 1

In Chapter 2 of our Grade 7 curriculum, we have included a unique chapter called "How to Look at Phenomena." In the regular curriculum under the guidelines of the Japanese Course of Study, Grade 7 students are expected to study the concept of functions developed through and including proportional relationships, algebraic expressions using letters to denote variables or constants, and equations. However, our curriculum takes a more comprehensive view of this content and considers the mathematical concepts students learn as very effective and clever methods for organizing, examining, and grasping real-life phenomena. For example, in regard to functions, the learning is focused on fostering students' ability to express the relationships between two quantities associated with real-life phenomena using tables and graphs. The learning stops at the point where students are using mathematical expressions to represent the reflexive relation of two discrete quantities. Actual learning about proportional relationships and function expressions is explored in Grade 8. In addition, the focus of studying algebraic expression using letters and equations aims at grasping the mathematical relationships of quantities in the real-life phenomena studied. Thus, Chapter 2 of our curriculum is a unique chapter that reflects our school's vision of mathematics education, fostering mathematical literacy. In other words, Chapter 2 in the first trimester of the first year at our secondary school sets up student's exploration for grasping real-life phenomena mathematically over six years of secondary school mathematics learning. Grade 7 students examine numbers from many different points of view and apply the concepts they learn in real contexts in Chapter 1. For example, students have learned the property of prime numbers, merits of prime factorization, relationships between prime factorization and the greatest common factor or the least common multiple, merits of Euclidean Algorithm, residue class (classifying numbers by remainders in division), and merits of using positive and negative numbers in the context of exploratory real-life situational problems.

In this lesson, I considered that the students' learning about positive and negative numbers will almost be finished by the end of June. Consequently, I have decided to provide a topic of study that connects and bridges the content of Chapters 1 and 2. Students will think about the phenomena mathematically by utilizing positive and negative numbers, quantifying and representing the phenomena and its elements, and grasping the conditions and trends of the elements by organizing them in structures, such as tables. My goal here is to set up a problem-solving activity that becomes the initial journey for learning in Chapter 2. In this problem solving activity, I decided not to use quantities that show change over time in the phenomena. Rather, I decided to help students focus on examining, grasping, and representing the phenomena mathematically. By making this decision, the focal pint of learning is moved from "how to view numbers" to "how to view phenomena" in a more expansive, inclusive manner that (I hope) will make the transition a smoother one.

Lastly, in this lesson, students will use tables to grasp the trend of a phenomenon, which is not usually practiced in Chapters 1 and 2 of the *Mathematics I* textbook created under our school's unique curriculum.

2. Theme of the Problem Solving Activity and Reason for Setting up the Activity.

"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?":

Mathematically expressing and manipulating a game, and thinking about mathematical strategies for playing the game.

I decided to set up a problem solving activity that makes possible students' expression of the quantitative relationship of a phenomenon by utilizing positive and negative numbers, and helping students bridge the content of Chapters 1 and 2 in our curriculum. In addition, I wanted to set up a problem-solving situation wherein students utilize the table mathematically to make decisions. For these reasons, I concluded that an activity that uses game theory would be advantageous.

Basic game theory, such as zero-sum and non-zero-sum games, requires and thinking about a strategy mathematically using positive and negative numbers to express gain or loss. Because game theory itself is a study for grasping and thinking about phenomena mathematically, including problem solving and decision making, we can think of it as a way to help students foster skills of grasping and thinking about phenomena mathematically. I believe the basics of the game theory fits very well as an activity or context for this lesson and provides a fresh look at a mathematical way of viewing and grasping phenomena.

The rule for playing a zero-sum game is simple and easy for students to understand. It involves either gain or loss and a sum of gains and losses becomes zero (0). Conversely, the non-zero-sum game reveals how the complexity of gain/loss and gains and losses are not equivalent and, importantly, how the rules of this game may apply to many real-life phenomena. For example, in the real life of Grade 7 students when they are facing a conflict, it is not easy to find a situation that is zero-sum even if and when the gain is expressed with numbers. If they consider a sense of value and try to express it using numbers, they will find that it may be possible to do but others will doubt the validity of the result. For this reason and lesson, I decided to set up a game activity that connects to students' daily life and uses a zero-sum.

Given the rules for playing a zero-sum game, there are two mathematical methods to consider: the mini-max and the maxi-min strategies. The basis of both strategies involves a sense of value that both opponents' losses should be kept to a minimum. In other words, the strategies value is that of low risk and low return. If you analyze a table from a data set and this point of view, it is most likely apparent that both opponents have settled on this strategy. It is a commonly held belief this type of strategic thinking is used even if in cases of large social phenomena, such as cases involving the international relations and issues between two countries. Conversely, the strategy can be used in cases of very small data sets and phenomena, such as two individuals competing in a gain or loss situation where the situation dictates that they base their decision on the question, "What should I do if I don't want to lose as much as possible?"

On the other hand, there are situations when people need to think about the decision making from a "What should I do if I want to gain as much as possible?" strategic mindset. Usually this condition is represented by the knowledge that "if you try to gain as much as possible, your risk will go up." In other words, the strategy is high risk and high return. We can think about the strategy in the large social phenomena (macro) as well as small individual (micro) level phenomena. In this lesson's problem solving activity. In the mathematics of the classroom, students will think about a mathematical strategy for playing the game using a data set and table that students come up with to represent the phenomena. They will grapple with and apply mathematics to either the "low risk and low return" or the "high risk and high return" situations. Through participation in this game activity, students will experience a decision-making process that may apply to their individual micro-level situations, as well as apply to the macro-level social situations that include contexts such as economics and political decision-making. The intent of this lesson is to provide students the opportunity for developing a holistic skill of using mathematics and mathematical tools for thinking about, solving and tackling real-life situations.

3. About the Problem Solving Activity

"What should we think about as a strategy for playing this game?"

"In a Grade 7 class, students decided to play the following card game for a recreational activity during a homeroom period. What is the mathematical strategy for playing this card game?"

Playing "Which is lucky?" Card Game

< Rules of the Card Game >

- Play with two players.
- Each player plays the game with the following set of four number cards:

7	- 4	- 6	3

- Players stand on opposite sides of a desk to play the game.
- Players decide who will be Player A or B by using "rock-paper-scissors." The winner chooses who will become Player A and Player B.
- Players hold cards in your hands so your opponent can't see the cards, (as when you play the game, "Old Maid."
- Two players say, "Which is lucky?" together and simultaneously put down one of the cards on the desk from their hands.
- Multiply the numbers on the two cards on the desk. If the product is a positive number, Player A will receive points that are the absolute value of the product calculated. Player B will lose points that are the absolute value of the product calculated.
- If the product is a negative number, Player B will receive points that are the absolute value of the product calculated. Player A will lose points that are the absolute value of the product calculated.
- The players put down another card from their hand and play the 2nd round. The players play the game for a total of 3 rounds.
- Record the process (see Table 1 below) of each round in the game and move on to play with different players.
- When a player finishes playing with 3 opponents, students will add the points and the player with the highest number of points will become the winner of the class.

Good Luck!

This problem solving activity is developed according to the theme that is described in section 2 of this lesson plan. The following table can be constructed by thinking about how player can gain points mathematically. (See Table 1 below.)

A \ B	−6	−4	3	7
−6	36	24	−18	−42
−4	24	16	−12	−28
3	−18	−12	9	21
7	−42	−28	21	49

Table 1

The number of cards used for this game is 4 cards. If the number of cards is too small, it will be too easy to think about the strategy for playing the game, but if the number of cards is too large, it will be more complicated to think about the strategy for playing the game. Initially, I set up the game so the absolute value of numbers on the cards was small; however, I thought that the students could handle these calculations without much struggle, so I made the difference between gain and loss clearer, and decided to use these numbers. Initially, I also thought about using different sets of cards for Players A and B, but decided to use the same set of cards for both players. If players use different cards, the focus of students' discussion will shift away from thinking about the strategy they want to use to play the game to which set of cards might give a better chance of winning the game. Lastly, I adjusted the total sum of the four cards to be zero (0), so that when the sum of positive products and negative products are calculated from the 16 possible card combinations, the absolute value of the sum of the negative numbers and the sum of the positive numbers become the same. In this way there is not a different advantage for either of the players.

When you think about the strategy for playing this card game, the maximum gain for Player A from one round of game is 49. On the other hand the maximum gain for Player B is 42. It looks like Player B has a disadvantage: however, there are two cases that products become - 42 in the table (that give gain of 42 to Player B), so it is not easy to tell if Player B has a disadvantage. If both players decide they don't want to lose a lot of points, both players show "3" at the same time. In this case the product is positive 9, so students will likely see that Player B has a disadvantage. As you can see, there are many more different ways to think about the strategy for playing this card game: therefore, I think this problem solving activity helps students deepen there mathematical thinking skills.

4. Targets that Establish Using Backward Design and Evaluation Standards

The purpose of this problem solving activity is to help students bridge the content and understanding of Chapters 1 and 2. The Goal of the activity is that students grasp a real-life phenomenon mathematically and engage in deep thinking by utilizing the knowledge they learned previously about positive and negative numbers in Chapter 1. To achieve this goal, we discussed:

- What is the mathematical ability that we want to enhance through "Utilizing Mathematics" activities included in this particular problem-solving activity; and
- Which objectives of MYP's *Next Chapter* align with the mathematical ability we are looking for and want to enhance in these "Utilizing Mathematics" activities?

Objectives stated by MYP's *Next Chapter for Mathematics* are:

Objective A: Knowing and understanding
Objective B: Investigating patterns
Objective C: Communicating
Objective D: Applying mathematics in real-life contexts

In this lesson I would like to enhance students' ability to express the state of a phenomenon mathematically and logically, by reasoning mathematically using data in tables. In addition, the strategies and decision-making processes that students engage in will connect to a real-world setting. I will focus on Objectives C (Communicating) and D (Applying mathematics in real-life contexts) and strive to bring out students' mathematical engagement and the quality of learning activities.

The objectives and the assessment standards of MYP's the *Next Chapter*, inter-related and are interdependent: thus, I will use the Mathematics Assessment Criteria: Year 3 of MYP's the *Next Chapter* to assess the students' learning.

5. Plan of the Unit (Total of 3 Lessons):

	Content of Instruction	No. of Lessons
1	Students will play the zero-sum-like game, provided within the context of a real-life situation; and they will think about strategies to win the game based on their experience playing the game.	1
2	Students will think about how to mathematically organize the different cases of gains and examine the methods. Students will also think about the strategies used to play the game using the table created. **(This Lesson)**	1
3	Students think about multiple different ways to play the game and compare and contrast these strategies. The teacher helps students to make connections about what they have learned in these lessons with the real-life phenomena.	1

6. About This Lesson:

1.) The Goals of this Lesson:

Year 3, Objective C: Communicating
Elaboration of the objectives of this lesson:
• Students are able to express the gains using positive and negative numbers, and able to show the gain and loss of Players A and B. • Students are able to understand the different points of views of the players from the table, and are able to read the table effectively. • Students are able to explain how they think about the mathematical strategy for playing this game in an orderly and logical way.

Year 3, Objective D: Applying Mathematics in Real-Life Contexts
Elaboration of the objectives of this lesson:
• Students are able to examine gains in points of each other's decisions or mathematical methods and determine what strategy to use when they need to make a decision about what cards to play to give themselves a mathematical advantage. • Students are able to think about their own strategy by thinking about and identifying the other players' points of view and outcomes by going back and forth between the two player's points of view. • Students think about their own strategies using different senses of value, such as "high risk and high return," and "low risk and low return."

2.) The Goals of this Lesson:

Year 3, Criterion C: Communicating

Level	Level Descriptor	Specific Indicator
0	The student does not reach the Year 3 level described by any of the descriptors across the criteria of levels 1-8 given below.	The student does not demonstrate the criteria described at any of the descriptor levels below.
1-2	The student is able to: i. use limited mathematical language (notations, symbols and terminology) ii. use limited forms of mathematical representation to present/convey information iii. communicate using lines of reasoning that are difficult for others to understand.	The student is able to: • partially express the gains achieved in the game • explain the strategy, but the explanation is ineffective and difficult to comprehend.
3-4	The student is able to: i. use some appropriate mathematical language ii. use different forms of mathematical representation to present and convey information adequately iii. communicate by using lines of reasoning that are able to b e understood, although these are not always clear iv. adequately organize information using a logical structure	The student is able to: • calculate gains of the game • explain the strategy used, so that others can understand it ; however, the explanation is not always clear.
5-6	The student is able to: i. usually use appropriate mathematical language ii. usually use different forms of mathematical representation to present and convey information correctly iii. moves between different forms of mathematical representation with some success iv. communicate through lines of reasoning that are clear although not always coherent or complete v. present work that is usually organized using a logical structure.	The student is able to: • accurately calculate gains of the game and tries to express gains of the both Players A and B • understand the different points of views for interpreting gains recorded in the table. • explain the strategy clearly and coherently, but not consistently is a logical way.
7-8	The student is able to: i. consistently us appropriate mathematical language ii. use different forms of mathematical representation to consistently present information correctly iii. moves effectively between forms of mathematical representation iv. communicate clearly through coherent lines of reasoning that are complete and coherent v. present work that is consistently organized using a logical structure.	The student is able to: • accurately calculate gains of the game and express gains of both Players A and B • understand the different points of views for interpreting gains given in the table and provide an effective interpretation. • explain the game strategy in a clear, coherent, and logical manner.

Year 3, Criterion D: Applying mathematics in real-life context

Level	Level Descriptor	Specific Indicator
0	The student does not reach the Year 3 level described by any of the descriptors across the criteria of levels 1-8 given below.	The student does not demonstrate the criteria described at any of the descriptor levels below.
1-2	The student is able to: i. identify some of the elements of the authentic real-life situation ii. apply mathematical strategies to find a solution to the authentic real-life situation, with limited success.	The student is able to: • try to think about the gains mathematically in a limited fashion; thus, finding a strategy is also limited.
3-4	The student is able to: i. identify relevant elements of the authentic real-life situation ii. select, with some success, adequate mathematical strategies to model the authentic real-life situation iii. apply mathematical strategies to reach a solution to the authentic real-life situation. iv. describe whether the solution makes sense in the context of the authentic real-life situation	The student is able to: • chose strategies by analyzing both players' gains using a mathematical method *in a limited fashion* when they need to make decisions in the context of playing the game for gains • think about opponent's point of view and try to develop his/her own strategy.
5-6	The student is able to: i. identify relevant elements of the authentic real-life situation ii. select adequate mathematical strategies to model the authentic real-life situation iii. apply the selected mathematical strategies to reach a valid solution to the authentic real-life situation iv. describe the degree of accuracy of the solution v. discuss whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: • chose strategies by analyzing both players' gains, applying a mathematical method most of the time when they need to make decisions in the context of playing the game for gains • think about the opponent's point of view and develop his/her own strategy. • think about strategies by considering various value perspectives.
7-8	The student is able to: i. identify relevant elements of the authentic real-life situation ii. select appropriate mathematical strategies to model the authentic real life situation iii. apply the selected mathematical strategies to reach a correct solution iv. explain the degree of accuracy of the solution v. explain whether the solution makes sense in the context of the authentic real-life situation.	The student is able to: • chose strategies by analyzing both players' gains, applying a mathematical method when they need to make decisions in the context of playing the game for gains • think about his/her own strategies by considering both players' points of view, by considering the reasoning behind each move (going back and forth) • think about strategies by considering various value perspectives, such as "high risk, high return" and "low risk, low return."

3.) Focus on Enhancing Students' Ability and Support for Achieving Lesson Objectives

In this public lesson, I would like to focus on the objective "Grasping a phenomenon mathematically, expressing and manipulating the elements of the problem appropriately, and thinking mathematically by utilizing mathematics to solve it," which is part of the criterion of (2) in Heading 6 above. I have thought about how I can provide support in a way that enhances my students' opportunities and chances of demonstrating this objective. Below, the left side of Figure 2 shows the structure for improving students' mathematical activity; the right side of Figure 2 shows the primary support for achieving the objective.

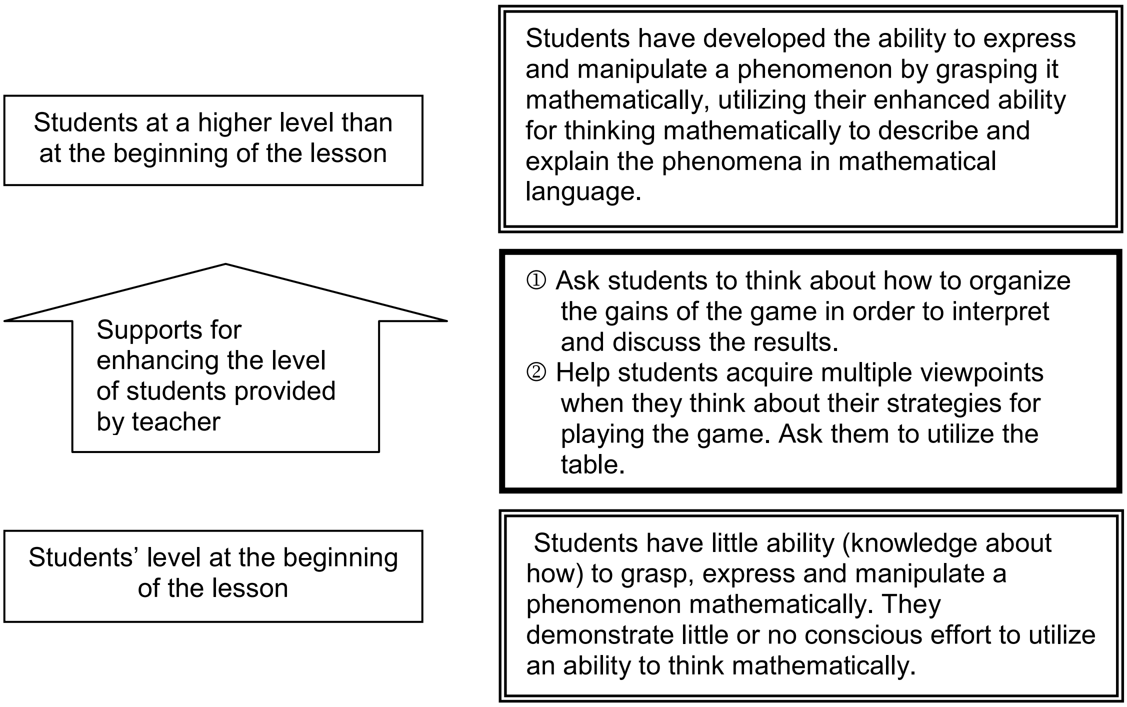


Figure 2: The structure of a lesson that enhances the students' objective

At the beginning of the lesson, it is probable that students demonstrate different levels of understanding and little evidence of having reached the objective of enhanced standards for thinking mathematically. I suspect that many of the students do not have enough knowledge or experience to demonstrate a conscious effort and the requisite mathematical abilities to organize the data of the phenomenon into a table and use the organized table to think about a strategy for playing the game mathematically. Because of the range present in regard to each student's status of learning and ability, I thought about and am including supports ① and ② to raise each student's performance standard to reach the objective above.

Given support ① ("methods that help organize the gains of the game"), I will set up the *hatsumon* for grasping the overall picture of the game and for organizing the data of gains from the game mathematically, I would like all students to experience the first step of mathematical problem solving, "grasping the total picture and organizing the data." Then, expecting that students will try organizing and expressing the gains of the game in various ways, I will ask them to examine which methods might be better, so that students can enhance their ability to grasp and express the phenomenon mathematically.

Given support ② ("strategies for playing the game"), I will ask students as they are thinking about strategies to use what that they came up with to show the gains of the game mathematically. This way, I can help students realize that mathematically organized data can be helpful for thinking about how their opponent might think about gains of the game. Using this as a base, students will become aware of and reflect on their decision making about which card he/she might want to put down next. By engaging the students in thinking about questions, such as "Do I want to gain as much as possible?" or "Do I want to lose as little as possible?," students experience thinking about many different points of view. This experience helps students to think about strategies mathematically so their demonstration of the standards and higher-level criteria is improved. Students will be thinking about strategies for playing the cards with multiple perspectives, so support ② in not only utilized in this lesson but also continuously and subsequently used through the 3rd lesson.

4.) Other Supports

① Ask students *hatsumon* that help them think whether there is a problem in their way of thinking about strategies or what idea might help them think about how to improve and make their strategies better. By asking these *hatsumon*, students are supported in seeing and understanding how necessary it is to grasp the holistic view of the game and analyze the details of the gains critically.

② Use portable blackboards to gather many different ideas, organize the data collected from students, and organize students' ideas into several categories.

③ Check if students are thinking about the gains of the game from the perspective of Player A, the perspective of Player B, or the perspective of both players. Help them become aware of important points, such as the following question: Given how unwieldy it is to show data in a diagram if we think about Players A and B separately, is it possible to show the gains from both players in one diagram? If students are wondering whether they can show both gains in one diagram, make sure they have an opportunity to share their thoughts or ideas with each other and the class.

④ Ask students to present several ideas about how to organize the gains mathematically. Compare and contrast these ideas, in order to classify and unify across ideas to deepen the analysis of the various methods. Make sure to discuss the ideas that are from the point of view of Player A, Player B, and both Players A and B.

⑤ Examine all the ideas that show the gains for playing the game and help students choose the method on which everyone agreed. Once students agree on an idea, ask them to use it to think further about strategies to play the game.

5.) Anticipated Students' Responses

① Methods that Help Organize the Gains of the Game

a.

A B

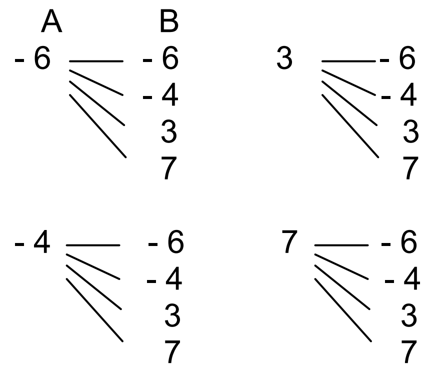
- 6 x (- 6) = 36
- 6 x (- 4) = 24
- 6 x 3 = - 18
- 6 x 7 = - 42

- 4 x (- 6) = 24
- 4 x (- 4) = 16
- 4 x 3 = - 12
- 4 x 7 = - 28

3 x (- 6) = - 18
3 x (- 4) = - 12
3 x 3 = 9
3 x 7 = 21

7 x (- 6) = - 42
7 x (- 4) = - 28
7 x 3 = 21
7 x 7 = 49

b.



c.

A \ B	−6	−4	3	7
−6	36	24	−18	−42
−4	24	16	−12	−28
3	−18	−12	9	21
7	−42	−28	21	49

d.

A \ B	−6	−4	3	7
−6	36, −36	24, −24	−18, 18	−42, 42
−4	24, −24	16, 16	−12, 12	−28, 28
3	−18, 18	−12, 12	9, −9	21, −21
7	−42, 42	−28, 28	21, −21	49, −49

★ Some students may have organized the numbers without thinking about the order in which they placed the numbers, such as from smallest to largest. Help students see that considering the order of numbers also helps them analyze the data more easily. These differences in organizing data reflect the difference in levels of mathematical thinking ability.

★ Given the different ways of organizing data listed above, the most helpful organization of the data is shown as "diagram c" above. The most mathematically involved or "elegant" thinking to the least mathematically involved thinking is as follows in order from diagrams c, d, a, to d.

② Strategies for Playing the Game

(1) Thinking about high return

i. Think about Strategies from the point of view of Player A:

I am Player A and I want to gain the most points, so I decide to put down the “7” card. However, Player B does not want to lose the greatest number of points, so Player B won’t put down his/her “7” card. If Player B thinks I will put the 7 card down, Player B may put down the – 6 card, so I should put down the – 6 card, also. → I (Player A) will put down the – 6 card.

If I (Player A) continue to think about what Player B might do, I will realize that Player B may think I will put down the – 6 card, so I will put down the 7 card. → I (Player A) will put down the 7 card.

ii. Think about Strategies from the Point of view of Player B:

I am Player B and I want to gain the most points, so I decide to put down the 7 or – 6 cards. However, if Player A puts down his/her 7 card, my loss would become greater; so the – 6 card seems like a better choice for me to put down. But Player A might suspect that I will put down the -6 card and there is a possibility that Player A will put down his/her -6 card. So, I should put down the 7 card. → I (Player B) will put down the 7 card.

If I (Player B) continue to think about what Player A might do, Player A might think I will put down the 7 card, so I (Player B) will put down – 6 to avoid the risk of losing a lot. → I (Player B) will put down the -6 card.

★ Students will notice that the logic of the decision-making process is circular. They may change their minds and start thinking about and consistently using a low risk scenario or strategy.

(2) Thinking about losing as little as possible

i. Think about Strategies from the Point of view of Player A:

I (Player A) want to lose as little as possible, so I will put down the 3 card. Whatever cards Player B puts down, my loss will be 12 or 18 points; but that is much better than a loss of 28 or 42 points. → I (Player A) will put down the 3 card.

ii. Think about Strategies from the Point of view of the Player B:

I (Player B) want to lose as little as possible, so I will put down the 3 card. Whatever cards Player A puts down, my loss will be 9 or 21 points; but that is much better than a loss of 16 or 24 points that will happen if I put down the – 4 card. → I (Player b) will put down the 3 card.

★ Students will notice that if they think about risking as little as possible, there is a distinct possibility that both players will put down the 3 card. In this case, students will recognize that being Player A is better. When playing the card game using the strategy of low risk (minimizing risk), if the circumstances become worse for Player A, he/she can always put down the 3 card to gain points; therefore, it is better to be Player A.

★ In this lesson (2nd lesson in the unit), I suspect there are not many students who will think about or use a “low risk” mindset/strategy.

6.) Flow of the Lesson

Learning Activities	T	Points to Remember for the Instruction
[Reviewing the Previous Lesson] 1． Revisiting the learning activity in which the students engaged in the previous lesson. 2． Students will think about and write down what they think they can do to improve their strategies for playing the card game. They will then present their ideas.	7	On the board, place the summary of strategies that students came up with, based on their experience playing the card game in the previous lesson. [Other Support] ① Ask students *hatsumon* that help them consider whether there is a problem in their way of thinking about strategies or what idea might help them to better think about making strategies. By asking these *hatsumon*, help students feel the necessity of grasping the complete or whole picture of the game and analyze gains critically. Also, introduce the word "gain."
[Focusing on Learning 1] 2． Students will understand the learning activity.	3	Pose Problem 1 and distribute the worksheets (No. 3).
Problem 1 (Support for achieving the objective) How can we organize the players' mathematical data of gaining points so we can better grasp the total picture of the game? Think about a method for organizing the data and try it out.		
3． Students will think about how to organize the data of players' gaining points and record the idea(s) in their notebooks.	5	Walk around the classroom and grasp what kind of ideas the students are coming up with. Classify the students' types of ideas.
4． Students will share their ideas in small groups and give feedbacks about each other's ideas. Students prepare for a presentation to the whole class. (A few students will present.)	7	Ask students to share their ideas in small groups. [Secondary Support] ② Use portable blackboards to gather many different ideas to organize the data from the students; and, then, organize the ideas into several categories. ③ Check if students are thinking about gains in the game from the perspective of Player A, the perspective of Player B, or the perspective of both players. Help them to be aware of gaining or losing points, such as: it is unwieldy to show in diagram our thinking about players A and B separately. So, is it possible to show the gains in one diagram? If students are wondering if they can show both players' gains in one diagram, make sure to share these thoughts and ideas.

[Neriage] (*Discussing and Kneading Ideas)*) 5． Students will share their ideas for organizing the data of players' gains and discuss which idea has merit and is easiest to understand.	10	Ask students to participate in the whole class sharing discussions. Ask students who recorded their ideas on the board to explain their ideas. [Secondary Support] ④ Ask students to present several ideas about organizing the gains mathematically and to compare and contrast these ideas so the ideas can be classified and unified to deepen students' analyses of the methods. Be sure to discuss the ideas that reflect the point of view of Player A, Player B, and both Players A and B.
[Focusing the Learning 2] 6． Based on the table that shows the gaining points, students will independently think about strategies for Player A and Player B.	7	Pose problem 2 and distribute the worksheets (No. 4).
Problem 2 (Support for achieving the objective) Examine and use the table to think about the recommended strategies Player A and Player B should take.		
7． Share ideas in small groups.	9	Students share their strategies in small groups. Monitor the content of their discussions and record/summarize the characteristics of the small group discussions.
8． Students understand what they will be studying in the next lesson.	2	Ask students to hand in both worksheets they worked on during today's lesson. Inform students about the next lesson.

TGUISS Mathematics 1 Dates:__________ Name:__________________

Objective	*"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?"*

Problem Solving

What should we think about as a strategy for playing this game? (No. 3)

Problem 1: I How can we organize the players' mathematical data of gaining points so we can better grasp the total picture of the game? Think about a method for organizing the data and try it out.

TGUISS Mathematics 1 Dates:___________ Name:____________________

Objective	*"How should we grasp and express this situation mathematically? and How should we think about the mathematics and decide what to do?"*

Problem Solving

What should we think about as a strategy for playing this game? (No. 4)

Problem 2: Examine and use the table to think about the recommended strategies Player A and Player B should take.

Grade 12 Mathematics Lesson Plan

Lesson that will generate differential equations as mathematical models

Saturday, June 21, 2014 (11:10 – 12:00)
Mathematics 6 α R (7 male and 8 female students)
International Secondary School
attached to Tokyo Gakugei University
Teacher: Ren Kobayashi

1. Research Question for the Lesson

Today's lesson is a part of a series of lessons that try to address one of our goals for Grades 11/12 students, "In order to solver real-life problems, students can perform the processes such as formulating mathematical expressions, manipulating mathematical objects, interpreting the results of mathematical processes, and evaluating the outcomes." In particular, today's lesson will focus on the development of the following process: "In order to solve real-life problems, students can formulate differential equations as mathematical models, solve the equations, and interpret/evaluate the mathematical solutions." Since this lesson is the first time the students will be engaged in this process, goals of the lesson are for students to develop the idea of solving a real-life problem using a mathematical model of differential equations and to experience the way of thinking necessary to develop mathematical models. I believe the experience of using highly practical differential equations as mathematical models is an instance of a practical purpose of mathematics education.

Today's lesson is not the lesson that "explains" differential equations and their applications. Rather, the goal is to make today's lesson as the lesson in which students will "generate" differential equations as mathematical models. That is because when students generate differential equations themselves as they examine a particular phenomenon, they can understand experientially under what circumstances differential equations are useful or the meaning of differential equations themselves.

So, what should a lesson in which students generate differential equations as mathematical models look like? Differential equations are mathematical expressions that describe changes themselves. In such situations, it is necessary for a problem solver to hypothesize the status of change. How does one develop the idea of hypothesizing the status of change and describe the change mathematically? What strategies should a teacher employ so that students will develop such an idea? Those are the research questions addressed in today's lesson. I will describe specific strategies for today's lesson later, but I present three strategies as guidelines.

- Strategy 1: grounding questions in an investigation so that they will facilitate activities
- Strategy 2: reflecting on activities to extend the activities
- Strategy 3: using a mathematical content of "sequence" as a tool for students' investigation

Strategies 1 and 2 are something that must be considered in every lesson, while Strategy 3 is a particular strategy for a lesson whose aim is for students to generate differential equations. Previously, in the unit on sequences, students have learned to examine sequences by expressing a sequence in a recursive formula. Based on this knowledge, in today's lesson, I want to help students to develop the idea that, by considering a recursive formula as a difference equation, it describes the changes in adjacent terms of a sequence. Clearly, difference equations are useful themselves, but, in today's lesson, by shifting our attention from discrete changes to continuous change, it is hoped that students will generate differential equations.

Figure 1, the structure of a level-raising lesson, organizes the discussion above. Today's lesson aims to raise the level from the level of representing changes in a phenomenon using recursive formulas students have previously learned to the level of representing the changes using differential equations. To achieve this goal, a real-life phenomenon in which changes can be represented with recursive formulas will be prepared (Strategy 1). After students develop recursive formulas to represent the changes, we will re-interpret recursive formulas as difference equations (Strategies 3 and 2). In addition, by selecting the phenomena in which the changes need to be considered continuously, not discretely, (Strategy 1) we will discuss the need for shortening the unit time so that the level can be raised to consider differential equations.

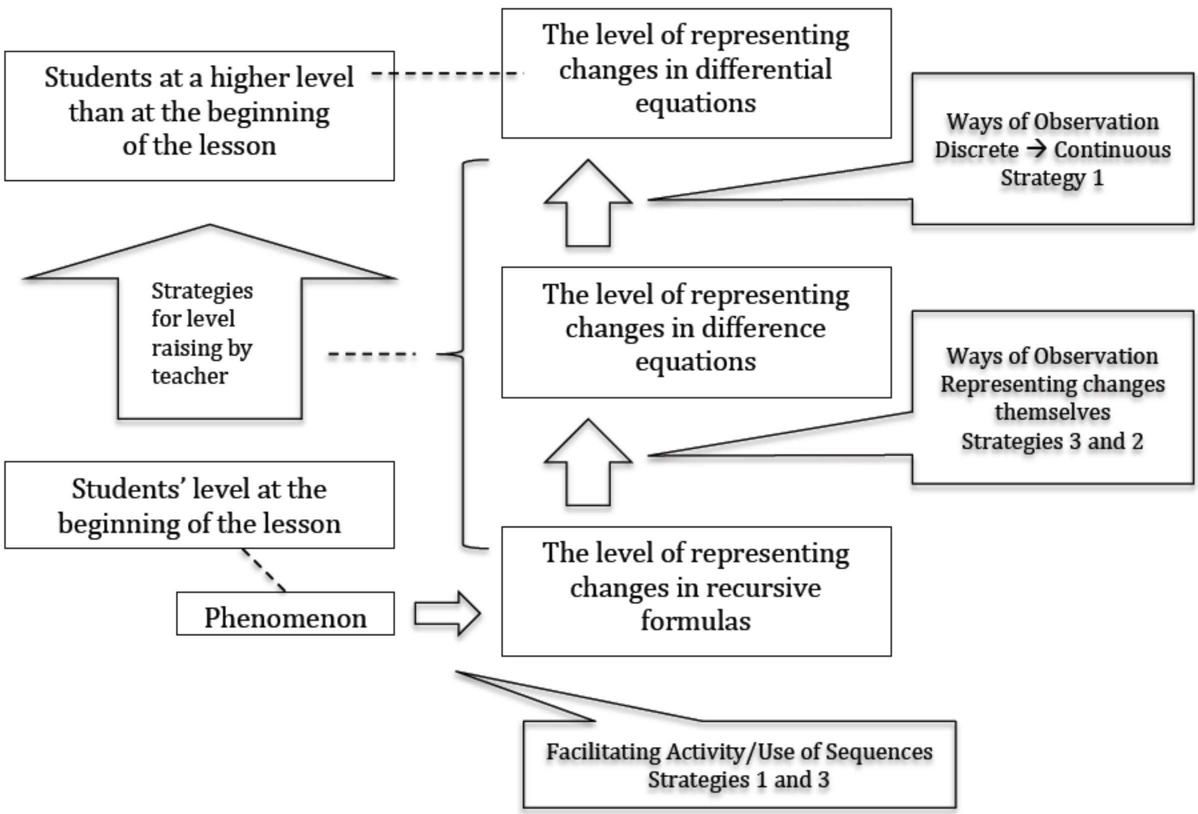


Figure 1 The structure of a level-raising lessons

In particular, today's lesson will make explicit the strategy to re-interpret recursive formulas as difference equations and the strategy to transform from difference equations to differential equations. We want to test whether or not these strategies will be effective.

2. About today's lesson

(1) Research theme and difference/differential equations

In today's lesson, I will use the following exploration task that utilizes the SIR model, the most traditional and foundational mathematical model for the spread of infectious diseases (for a more detailed discussion on the SIR model, see, for example, http://www.maa.org/publications/periodicals/loci/joma/the-sir-model-for-spread-of-disease-the-differential-equation-model).

> There is an infectious disease with the probability of infection based on a single close contact at 1.8%. Suppose an infected person entered into a population of 100,000. In this population, an infected person makes, on average, 70 close contacts in a week. An infected person is diagnosed within a week of the contact at the probability of 99% and isolated from the population. Once a person recovers from the infection, he or she develops an immunity from the disease and will have no risk of additional infection.
>
> You are the person in charge of public health in this population, and you want to encourage people to receive vaccination against this disease to avoid an outbreak.
>
> (1) Make a simulation that shows the change in the number of infected people in this population if there was no vaccination.
>
> (2) With the risk of potential side effects, it is not effective to mandate the vaccination for the entire population. Decide the minimum number of people who should receive the vaccination so that the risk of outbreak of this infection will be avoided.

Figure 2 Avoiding an Outbreak of Infection – today's task

This problem situation can be represented by the recursive formulas as follows. Let S_n be the number of people who have not been infected but susceptible to the infection (susceptible population), I_n be the number of infected people, and R_n be the total number of people immune to the infection (because they have recovered from the infection) and those who are isolated or died (removed population), each in week n. In addition, we assume that the total population is constant, $S_n + I_n + R_n = N$. If we assume that new infections only arise from the uninfected people in the population and the new removed population will come from the infected population, we can represent the situation in the following recursive formulas.

$$\begin{cases} S_{n+1} = S_n - S_n \times (I_n / N) \times 70 \times 0.018 \\ I_{n+1} = I_n + S_n \times (I_n / N) \times 70 \times 0.018 - I_n \\ R_{n+1} = R_n + I_n \end{cases} \quad \dots (2.1)$$

The graph below shows the results of the simulation based on these recursive formulas.

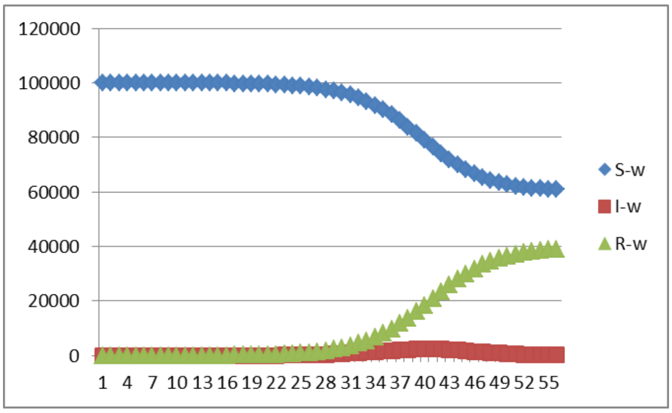


Figure 3 Results of the simulation

If we re-interpret the recursive formulas (2.1) as equations showing "weekly population change," we obtain the following set of difference equations.

$$\begin{cases} S_{n+1} - S_n = S_n \times (I_n / N) \times 70 \times 0.018 \\ I_{n+1} - I_n = S_n \times (I_n / N) \times 70 \times 0.018 - I_n \quad \ldots(2.2) \\ R_{n+1} - R_n = I_n \end{cases}$$

Then, we gradually reduce the size of unit time interval from "weekly" until unit time interval becomes "daily." Then, if we think about making the time interval approach 0, we obtain the following system of differential equations.

$$\begin{cases} dS(t)/dt = -S(t) \times (I(t)/N) \times 10 \times 0.018 \\ dI(t)/dt = S \times (I(t)/N) \times 10 \times 0.018 - (1/7)I(t) \\ dR(t)/dt = (1/7)I(t) \end{cases}$$

Moreover, if we consider $S \approx N$ at the beginning of an outbreak, the second equation above will become

$$dI(t)/dt = N \times (I(t)/N) \times 10 \times 0.018 - (1/7)I(t)$$
$$\approx 0.037I(t).$$

Under the condition, $I(0) = 1$, this equation can be solved as $I(t) = e^{0.037t}$.

(2) About mathematics

I believe that the use of the SIR infection model matches the intent of today's research lesson with the following three reasons.

First, to simulate the number of infected people, the need arises to consider not only the infected population (I) but also, at minimum, the number of the susceptible population (S). Therefore, it is necessary to consider the change of the infected population over a unit time to accommodate the movement of population from the susceptible population to the infected population. I expect students to represent these changes over time in some manners. Then, we will shift our focus on the changes themselves. We can anticipate that, as the representations of these changes are refined, difference/differential equations will be generated.

Next, by examining the initial stage of an outbreak, a first order linear differential equation, $dy/dx = ky$ will be generated. This form of differential equation is very concise and simple to solve, yet it is can be used to express a variety of phenomena. Thus, knowing this form of differential equation is useful in itself. Moreover, because this form of differential equation can be used to represent a variety of phenomena, we can use the assessment problem (see Appendix) to assess whether or not the goals of the research lesson are achieved. We will discuss the assessment problem later.

Finally, I believe that students can experience the practical use of differential equations through this exploration task. This situation is an actual example of situations in which differential equations are used. Students can experience how useful differential equations in real life can be as they consider mathematically ways to avoid or minimize the outbreak of an infection.

(3) Unit Plan

Differential equations are discussed toward the end of an approved textbook for Mathematics III as an extension topic. That is because the textbook discussion includes how to solve differential equations. However, the intent of today's lesson is not "solving differential equations" but "representing in differential equations" as a mathematical model. Thus, this unit is positioned as "an application of differentiation." Thus, it is assumed that students have already learned differentiation in Mathematics III.

This unit will have the total of 4 lessons. Today's lesson will be the third lesson of the unit. It is anticipated that, by today's lesson, students have generated a system of recursive formulas (2.1). Moreover, by assuming $S_n \approx N$ at the beginning of an outbreak, the second equation in (2.1) can be transformed as $I_{n+1} = 1.26I_n$. In other words, the number of people in the infected population, I_n, increases as a geometric sequence (discrete exponential function).

Table 1 Unit Plan

Lesson	Activity	Goals
1	Independent problem solving (in groups) of the exploration task.	Students will be able to represent the phenomena in recursive formulas and run the simulation based on the formulas.
2	Sharing and discussion of the solution of the exploration task. Examination of the changes at the initial stage of an outbreak (discrete).	Students will be able to characterize the changes mathematically using recursive formulas.
3 (Today)	Examination of the changes at the beginning of an outbreak by re-interpreting the recursive formulas as difference equations (continuous).	Students will be able to describe the changes in the phenomena and generate differential equations.
4	Determine the vaccination rate based on the boundary conditions.	Students understand the process of mathematical modeling.

(4) Specific strategies to help students generate differential equations as a mathematical model

Of several instructional strategies developed for this unit, mainly the following two specific strategies will be employed in today's (lesson 3 of 4) lesson.

Strategy 3-(1):	Ask students what they can observe about the weekly changes in the number of infected people.
Purpose 3-(1):	To help students re-interpret the recursive formulas to generate difference equations.

As a part of the simulation to develop a preventative strategy, students will investigate the weekly increase in the number of infected people at the early stage of an outbreak. By asking about the increase instead of the number of infected people, weekly change will become students' focus. Although there are various ways to investigate the increase, in the end, I would like them to discover that "weekly increase in the number of infected people is proportional to the number of infected people at the beginning of that week" based on the second recursive formula of (2.1) (with the assumption $S_n \approx N$).

Strategy 3-(2):	Ask students if we can investigate changes in smaller time intervals than one week.
Purpose 3-(2):	To generate differential equations from difference equations.

Based on the simulation using weekly changes in infected people, we can only discuss weekly preventative strategies. However, ideally, we would like to be able to investigate the changes in the number of infected people in much shorter time intervals, and ask students to think about ways to use shorter time units. In reality, we can probably make observations of the number of infected people every day, and we can use daily rates of change as approximations of instantaneous rates of change.

3. About students and their previous experiences

Students in Mathematics 6 α (Mathematics III) are students at an advanced standing, and they are very competent mathematically. They have many experiences of investigating phenomena in contexts mathematically, and they have the disposition to approach challenging problems independently. They are eager to tackle problems that pique their interests. Moreover, when the teacher asks a question to the whole class, several students are willing to share their ideas without being called upon. Moreover, they take other students' ideas and explanations seriously. One of the characteristics of the students in this grade is that they can write an in-depth reflection of their investigation.

In May of last year, 8 of the 15 students in the class experienced a 2-lesson Integrated Study unit in which they simulated the changes in population of living entities. In that particular unit, they developed the following ways of observing the population changes applying sequences. One way is to consider that the yearly increase in the population of sheep is proportional to the size of the population (discrete Malthusian model). The other is to consider that the yearly increase is proportional to the product of the total population and the number of survival population (discrete logistic model). However, we did not focus on the differences themselves since they have yet to learn about differentiation. Thus, what they considered was the yearly increase, not the rate of change in population over a unit time of one year. It is unknown how that experience will impact students' learning in the current unit. I plan to distribute those 8 students across the groups for this exploration.

4. Assessment Plan

After the completion of this unit based on the plan discussed above, the assessment task, Carbon-14 dating method, will be used to assess individual students' learning of the process, "In order to solve real-life problems, students can formulate differential equations as mathematical models, solve the equations, and interpret/evaluate the mathematical solutions." The evaluation rubrics are as follows.

Table 2 Evaluation Rubrics Perspective B. Processes and Reflection (B1)

0	Has not reached any of the levels below.
1-2	Students represented a concrete phenomenon using differential equations.
3-4	Students represented a concrete phenomenon using differential equations, and they can develop conclusions using mathematical manipulations.
5-6	Students represented a concrete phenomenon using differential equations, and they can develop conclusions using mathematical manipulations. Moreover, they can reflect on the process of manipulations and the conclusions and evaluate them.

As for the question of whether or not the level of activities students engaged as a class was raised will be assessed based on students' worksheets and also by keeping a record of students' remarks during the lesson.

5. Plan for Today's Lesson

(1) Goal of the lesson

Students will generate differential equations as a mathematical model and use them to represent changes in a phenomenon.

(2) Flow of the Lesson (condensed version)

T: teacher questions; S: Anticipated response by students; ○ teaching strategy

Time	Instructional content/main *hatsumon* and anticipated student responses	Instructional points of consideration
5	**1. Review of the previous lesson and presentation of a new task** T0: We learned that the change in the number of infected people at the early stage of an outbreak is a geometric sequence (discrete exponential function). **T1: In order to prevent an outbreak, we need to minimize the weekly increase of the number of infected people. Based on the simulation of an early stage of an outbreak, what can we say about the increase in the number of infected people?**	Ask the question so that the need for investigating the weekly increase in the number of patients.
10	**2. Independent problem solving in groups** S1-1: Analyze the data --- calculate the weekly increases and examine their differences and ratios.	

	S1-2: Analyze the graph --- calculate the weekly increases and make predictions based on the shapes of the graphs. S1-3: Analyze the general term --- using the equation for the general term, $I_n = 1.26^{n-1}$, generate the expression, $I_{n+1} - I_n$. S1-4: Analyze the recursive formula --- using the formula, $I_{n+1} = 1.26I_n \cdots ①$, generate the expression, $I_{n+1} - I_n$. ○ If S1-4's response does not come up, ask students if we can use the recursive formula. S1-5: It changes as a geometric sequence. / It is similar to the changes in the number of infected people. S1-6: It is 0.27 times as much as the number of infected people at the beginning of the week. S1-7: It is proportional to the number of infected people at the beginning of the week. ($I_{n+1} - I_n = 0.27I_n \cdots ②$) ○ If S1-7's response does not come up, ask students what kind of a functional relationship might exist between the increase ($I_{n+1} - I_n$) and the number of infected people (I_n).	After some time is passed, call upon a group with S1-4 (if this idea has not come up, S1-3) which are analyzing equations and ask them what they can observe.
10	**3. *Neriage*** **T2: If the weekly increase in the number of infected people is proportional to the number of infected people at the beginning of the week, what happens to the increase when the number of infected people become 2, 3, ... times as much?** S2: It also becomes 2, 3, ... times as much. **T3: Conversely, let's verify that In will become a geometric sequence if we assume the weekly increase in the number of infected people is proportional to the number of infected people at the beginning of the week.** S3: If $I_{n+1} - I_n = aI_n$, then $I_{n+1} = (1 + a)I_n$. So, it is a geometric sequence. **T4: So, at this point, we can conclude that In we derived in the previous lesson becomes a geometric sequence, if we assume that "the weekly increase in the number of infected people is proportional to the number of infected people at the beginning of the week."**	Interpret what it means for the weekly increase is proportional to the number of infected people at the beginning of the week. Emphasize that the assumption is focused on the increase, $I_{n+1} - I_n$, not I_n, and by examining $I_{n+1} - I_n$ we can investigate the change.

20	**4. Further exploration** **T5: We have been examining the weekly change in the number of infected people. However, in order to make our preventative strategy as accurate as possible, we want to investigate how the number of infected people changes more closely. What can we do?** S4: We should use a shorter time period as a unit than "weekly." **T6: What would be the ideal unit time interval?** S6-1: If we know the number of infected people at each instant, the graphs will become continuous. S6-2: Is it necessary for us to know the number in each instant when we are working with the number of people? S6-3: Realistically, daily numbers may be the best we can get. S6-4: I think it is enough if we know daily numbers.	Emphasize that the graphs are dot plots and there are gaps in between points.
	○ If the key words such as "continuous" or "instant" do not come up, remind students that I_n is a function of the number of weeks, *n*, thus discrete. Then ask students, ideally, if we can think of it as a function of time, *t*.	Make sure students understand that instantaneous changes are idealized notion.
	T7: Let's first explore the changes in the number of infected people at the beginning of the outbreak. How will the recursive formulas change?	We can anticipate that it will be an exponential function.
	S7-1: I think we can use 10 for the daily number of close contacts. S7-2: I don't think the proportion of susceptible people who will be infected by a contact will remain the same. S7-3: If we consider I_n represents the number of infected people on the n^{th} day, $I_{n+1} = I_n + N \times \frac{I_n}{N} \times 10 \times 0.018 - (1/7)I_n \fallingdotseq 1.037 I_n \cdots ③$ **T8: What do we need to do if we consider *I*(*t*) as a function of *t* and examine the change continuously?** S8-1: We need to shorten the unit time interval even further from "daily." S8-2: I think we can even think in terms of "hourly." S8-3: Maybe we can express the intervals as *h* or Δt, we can take the limit of the function.	Re-confirm the conditions. The rate of removal may be difficult to figure out. If the whole class discussion does not develop, give students to discuss it in groups.

	S8-4: Maybe we can consider that the instantaneous rate of increase in the number of infected people is proportional to the number of infected people at that moment. ○ If S8-4's idea does not come up, remind the students that we have already seen the situation of making the intervals (the amount of increase) approach 0, and ask them how we represented it.	Give students time to discuss this in groups if necessary.
	T9: Based on the recursive formula ③ and if Δ*t* << 1, then, lets' represent the relationship between *I*(*t* + Δ*t*) and *I*(*t*) and consider what happens Δ*t* → 0. S9-1: $I(t+\Delta t) = I(t) + N \times \frac{I(t)}{N} \times 10\Delta t \times 0.018 - \left(\frac{1}{7}\right)\Delta t I(t)$ ···④ If we transfer $I(t)$ to the left hand side and letting $\Delta t \to 0$, $dI(t)/dt = 0.037I(t)$. S9-2: If we assume that the instantaneous rate of increase of the number of infected people is proportional to the number of infected people at that moment, we can say $dI(t)/dt = 0.037I(t)$. ○ If S9-1's idea does not come up, re-interpret the left hand side of equation ③, $I_{n+1} - I_n$ as the average daily rate of change, and ask students how to express the instantaneous increase in the number of infected people. ○ Even if S8-4's response does not come up in response to T-8 above, make sure we interpret equation ④ as done by S9-2.	Ideally, S8-4's idea would come up, but if it does not, make use of the idea like S8-3's and move the discussion forward. If any group uses an time interval unit that is less than 1 hour and write a recursive formula, we can make use of it to think about Δt.
5	**5. Summary of the lesson and the task for the next lesson** T9: If we mathematically represent the assumption, "the rate of change in the number of infected people is proportional to the number of infected people," it will be $dI(t)/dt = aI(t)$. In this particular problem situation, we could express it as $dI(t)/dt = 0.037I(t)$. Therefore, $I(t)$ is a function whose derivative will be (approximately) 0.037 times as much as the function itself. In the next lesson, let's try to find the function that satisfies this condition.	The label, "differential equation," will be given after the conclusion of the exploration in the next lesson.

Appendix 1: Assessment Problem

In archeology, Carbon-14 dating method is used to determine the age of ancient clay pots or ancient remains. The nuclei of radioactive Carbon-14 are naturally unstable, and, without any external influence, they emit radioactive rays and disintegrate into different nuclei at a constant rate over a fixed time. Carbon-14 in the atmosphere is created by the cosmic rays, and its concentration is virtually constant. The concentration of Carbon-14 in living organisms also remains constant, because of photosynthesis in plants and food chains in animals, while they are still alive. However, once the living organism dies, Carbon-14 in the body will continuously disintegrate without absorption of any additional Carbon-14 from outside.

A seed was discovered in a clay pot. It was determined that the number of Carbon-14 nuclei is 4.2×10^{10}. It is known that a seed of the same plant today contains 6.0×10^{10} Carbon-14 nuclei. We want to estimate about how many years ago the clay pot was being used.

(1) The time it takes for the number of nuclei to become ½ of the original number is called "falf-life." Show that the half-life of Carbon-14 is constant.

(2) It has been measured that the half-life of Carbon-14 is approximately 5730 years. About how many years ago can we estimate that this clay pot was being used?

Graph 2-1 Assessment problem, "Carbon-14 dating method"

Appendix 2: Worksheet

The following worksheet was handed out to the students when the task was initially posed.

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TGUISS6 α Date______________ Class______________ Name__________________

Let's prevent an outbreak!

Suppose a person who is infected by a particular infectious disease entered into a population of 100,000. When an uninfected person comes into a close contact with an infected person, there is a probability of 1.8 % that the uninfected person will be infected. A person who was infected by this disease can infect other people for 7 days after he or she is infected. When a person becomes no longer infectious, i.e., when he or she is recovered, the person develops immunity from this disease. Finally, in this population, an infected person makes, on average, 70 close contacts in a week.

You are the person in charge of public health in this population, and you want to encourage people to receive vaccination against this disease to avoid an outbreak. When you conducted a survey, it was found that, in this population, an infected person makes, on the average, 70 close contacts with others.

(1) Make a simulation that shows the change in the number of infected people in this population if there was no vaccination.

(2) With the risk of potential side effects, it is not effective to mandate the vaccination for the entire population. Decide the minimum number of people who should receive the vaccination so that the risk of outbreak of this infection will be avoided.

Grade 4 Mathematics Lesson Plan

Teacher: Masaki Tsuruta

1 Name of the Unit Let's make quadrilaterals

2 About the Unit

In this unit, students will learn about the relationships of lines – perpendicularity and parallelism – and quadrilaterals such as parallelograms, trapezoids, and rhombuses.

The National Course of Study position the topics in this unit, perpendicularity/ parallelism and quadrilaterals, as follows.

(1) Through activities such as observing and composing geometrical figures, to help pupils pay attention to the elements that compose geometrical figures as well as their positional relationships, and deepen their understanding of geometrical figures. a. To understand the relationships such as parallelism and perpendicularity of straight lines. b. To get to know parallelograms, rhombuses and trapezoids.

The aim of this unit is that through activities of observing and constructing geometric figures, students will understand the relationships such as perpendicularity and parallelism and quadrilaterals such as parallelograms, rhombuses, and trapezoids. In particular, students will understand the characteristics of geometric figures by using position relationships of their sides (perpendicularity and parallelism) or diagonals (their lengths and the way they intersect with each other).

In Grade 2, students have learned about the foundational ideas for perpendicularity and parallelism in the unit, Triangles and Quadrilaterals, by observing and constructing rectangles and squares. In addition, in the Grade 3 unit, Triangles, they learned about classes of triangles, isosceles triangles and equilateral triangles, by focusing on the lengths of sides in triangles. In those units, students have used "the number of vertices or sides," "the length of sides," and "the size of angles" as lenses to observe geometric figures.

In this unit, what is important is for students to use "perpendicularity" and "parallelism" as new lenses to re-examine familiar geometric figures and discover new properties of quadrilaterals. In particular, by focusing on "parallelism," students will understand that if there is a pair of parallel sides in a quadrilateral, it will be a "trapezoid," and if there are two pairs of parallel sides in a quadrilateral, it will be a "parallelogram."

In the 2013 National Assessment, there was a question about how to draw a parallelogram. The question asks students which property of parallelograms was used to draw the parallelogram. In drawing a specific geometric figure, it is important that students grasp the characteristics of the geometric figure and use them to guide their drawing. When the students in this classroom were asked to draw triangles, not many were connecting the characteristics of triangles with their methods of drawing. From this perspective, it is important for these students to learn to draw geometric figures by making use of characteristics of the geometric figures. Through the study of this unit, it is my hope that students will not only understand the relationships of lines such as perpendicularity and parallelism and quadrilaterals such as parallelograms, rhombuses, and trapezoids, but also enrich their sensitivity toward geometric figures.

In teaching this unit, I will have students first use their intuitions to sort different positional relationships of lines instead of immediately using rulers or set squares. Then, by using protractors or set squares, students will more carefully examine the relationship of the given lines. It is my hope that students will be able to use perpendicularity and parallelism as they discuss geometric figures. Moreover, I want students to grasp characteristics of various quadrilaterals by using the relationships of their sides or other constituent parts. To do so, we will incorporate activities of sorting and organizing quadrilaterals. Then, through activities to draw particular quadrilaterals to match the way they sorted and organized quadrilaterals, I want them to understand their characteristics. Moreover, by using those characteristics to draw quadrilaterals, I want them to be able to connect steps of drawing and characteristics of quadrilaterals. In addition, students can discover additional characteristics by manipulating cut out quadrilaterals. Through these concrete activities, I would like them to develop intuitive understanding of congruence and symmetry so that they can be utilized in their future study.

In their study of geometric figures, students have only considered those segments that are visible, that is, sides of polygons. In contrast, diagonals is invisible until students imagine the segment that connects vertices that are opposite of each other. In this way, diagonals involve more abstract aspects and that may create difficulty in some of them. However, when students realize that diagonals can be used to identify properties of geometric figures, I hope that they will understand merits of diagonals.

By studying geometric figures, when students encounter various geometric figures in their daily lives, they may be able to sort them based on the properties learned in this unit or they may recognize the beauty of geometric figures themselves. I hope to enrich students sensitivity toward geometric figures as they realize that there are so many geometric figures around us.

3. Relationship to the school-based research

The research theme for the last academic year was "Elementary school career education that lays the foundation toward autonomy: developing lessons that will raise students ability to reason logically with anticipation and to express themselves." We concluded that we need to incorporate "the ability to reason logically with anticipation and to express themselves" in our lessons and make full use of them.

In order to raise the ability to reason logically with anticipation and to express themselves," students need to recognize the good points about their own ideas or mistakes contained in them as they try to express their ideas. By repeatedly engaging in such reasoning, students will develop the ability to think logically and further their ideas. Therefore, it is important that students will share their ideas and learn from each other.

In the study of geometric figures, there are many activities in which students explore properties of certain geometric figures. Those activities provide opportunities for students to express their ideas - why did they sort shapes in that way, or why their ideas make sense. It is hoped that students will come to share many ideas that originated from individual students and further develop their sensitivity toward geometric figures.

(1) How students might "reason logically with anticipation and texpress themselves" in this unit

As we progress through this unit, it is hoped that we can observe students doing the following.

- Students understand what it means for 2 lines to be perpendicular or parallel, and they can draw such lines.
- Students can sort quadrilaterals based on the perpendicular/parallel relationships of their sides.
- Students can express their ideas using the properties of quadrilaterals.
- Students can recognize good points of their friends' ideas and try to incorporate them in their own reasoning.

(2) Strategies to raise students' "ability to reason logically with anticipation and to express themselves"

① How to pose the task

Until now, we have been examining geometric figures by focusing on their constituent parts such as the length of sides. In the "grasp" stage of the lesson, students will learn about parallelograms, trapezoids and rhombuses by including parallelism as an additional view point. Thus, I plan to use questions such as "I wonder if we can use what we have learned so far to sort these shapes" and "What should we focus on?" to help students develop ideas they can use to complete the task. By sharing different ideas uttered by individual students with the whole class, I plan to help students understand the mathematical purpose of the task.

② During independent problem solving

I want students to understand which of the ideas they have learned can be used to complete the task. I will ask students to explain the reason for their ideas through questions like "Why did you decide to use that idea?" as they complete the task. In the same way, I will encourage students to record their ideas in the notebooks so that the reason behind their ideas will be clear. In addition, if students make mistakes or change their ideas, I will encourage them to record their new ideas separately instead of erasing the previous ones and writing the new ideas over them. This way, students can more easily see the changes in their reasoning later.

③ During the whole class discussion

In order to critically compare and contrast various ideas, students must first share their own ideas clearly to other students. To do so, they must first fully understand their own ideas. Therefore, I will ask students to look back on their own ideas and organize their thoughts so that they can present their ideas in a way that will be easy for others to follow. I plan to have students form pairs and share their ideas with their partners. In this way, students will have an opportunity to revise the way they present their ideas to the whole class.

Furthermore, I want students to grasp the similarities and differences between their own ideas and those of their classmates as we engage in the whole class discussion. It is hoped that this will motivate their future learning as well applying what they learned in different contexts.

4. Goals of the Unit

○ Through activities of observing and constructing position relationships of lines or various quadrilaterals, students will come to understand the perpendicular and parallel relationships of lines, parallelograms, rhombuses, and trapezoids. They will enrich their sensitivity toward and ways of observing geometric figures. • Students will pick out 2 perpendicular lines or 2 parallel lines in their surroundings and identify parallelograms, rhombuses, and trapezoids. They will think about situations those figures may be utilized. [Interest, Eagerness, and Attitude] • Students will be able to identify and represent properties of various quadrilaterals based on the position relationships of their sides and other constituent parts. Students will be able to grasp properties of diagonals for various quadrilaterals. [Mathematical Way of Thinking] • Students will be able to draw 2 perpendicular lines, 2 parallel lines, parallelograms, rhombuses, and trapezoids. [Mathematical Skills] • Students will understand the meaning and properties of 2 perpendicular lines, 2 parallel lines, parallelograms, rhombuses, and trapezoids. They will enrich their sensitivity toward geometric figures. [Knowledge and Understanding]

5. Unit Plan (15 lessons)

sub-units [# of lessons]	No.	Main Activity	Assessment Criteria
1 [2]	1	• Investigate how 2 lines can intersect. • Learn the meaning of the term, "perpendicular."	• Students are investigating the way 2 lines are intersecting by focusing on the angles formed by them. (Interest, Eagerness, and Attitude) • Students understand the meaning of lines being "perpendicular."
	2	• Based on the meaning of perpendicularity, students will think about ways to draw perpendicular lines using a pair of set squares. • Draw perpendicular lines.	• Students can explain the method of drawing perpendicular lines using a pair of set squares by focusing on the right angles in the set square pieces. (Mathematical Way of Thinking) • Students can draw perpendicular lines by using a pair of set squares. (Mathematical Skills)

2 [4]	1	• Students will explore how lines may be arranged. • Learn the meaning of the term, "parallel."	• Students understand the meaning of lines being "parallel." (Knowledge and Understanding)
	2	• Students will explore the width of a pair of parallel lines. • Students will summarize that the distance between a pair of parallel lines is constant. • Students will explore angles formed by parallel lines and a line intersecting them. • Students will summarize that a line will intersect parallel lines forming angles of equal measurements.	• Students understand that the distance between a pair of parallel lines is constant. They also understand that a line will intersect parallel lines forming angles of equal measurements. (Knowledge and Understanding)
	3	• Students will think about ways to draw parallel lines using a pair of set squares.	• Students can explain how to draw a pair of parallel lines based on the idea of equal corresponding angles. (Mathematical Way of Thinking) • Students can draw a pair of parallel lines using a pair of set squares. (Mathematical Skills)
	4	• Students will think about ways to identify perpendicular and parallel lines by using the grid lines as a guide.	• Students understand how to identify perpendicular lines or parallel lines using the grid lines as a guide. (Knowledge and Understanding)
3 [7]	**1***	• **Students will sort various quadrilaterals using perpendicular or parallel sides.** • **Learn the meaning of the terms "trapezoids" and "parallelograms."**	• **Students can sort quadrilaterals using a variety of view points. (Mathematical Way of Thinking)** • **Students understand properties of various quadrilaterals. (Knowledge and Understanding)**
	2	• Investigate characteristics of parallelograms by using 2 pieces of cut out parallelograms. • Summarize the properties of parallelograms.	• Students can identify and explain properties of parallelograms by focusing on the position relationships of their sides, length of their sides or their angle measurements. (Mathematical Way of Thinking) • Students understand the properties of quadrilaterals. (Knowledge and Understanding)

	3 4	• Think about ways to draw parallelograms. • Draw parallelograms using the properties of parallelograms. • Work on application problems.	• Students can explain how to draw a parallelogram based on the properties of parallelograms. (Mathematical Way of Thinking) • Students can construct parallelograms. (Mathematical Skills)
	5	• Learn the meaning of the term, "rhombus." • Summarize the properties of rhombuses and draw rhombuses.	• Students can explain properties of rhombuses by focusing on the position relationship of the sides, length of the sides and the measurement of their angles. (Mathematical Way of Thinking) • Students can draw rhombuses. (Mathematical Skills)
	6	• Explore characteristics of lines drawn by connecting vertices of various quadrilaterals. • Learn the meaning of the term, "diagonal." • Summarize the characteristics of diagonals in various quadrilaterals.	• Students are thinking about and understanding the relationships among quadrilaterals based on the characteristics of diagonals. (Mathematical Way of Thinking) • Students understand the properties of diagonals for various quadrilaterals. (Knowledge and Understanding)
	7	• Investigate the two triangles obtained by cutting a rectangle or parallelogram along a diagonal. • Make various quadrilaterals by putting together pieces obtained by cutting a quadrilaterals along a diagonal.	• Students can make various quadrilaterals by using 2 congruent triangles. (Mathematical Skills) • Students understand that the two triangles obtained by cutting along a diagonal are congruent. (Knowledge and Understanding)
4 [2]	1 2	• Work on unit exercises.	• Students can solve problems by utilizing what they learned in the unit. (Mathematical Skills) • Students have basic understanding of materials discussed in the unit. (Knowledge and Understanding)

* Today's lesson

6. Today's lesson

(1) Goals of the lesson

- Through activity of sorting quadrilaterals from a variety of view points, students will attend to parallel sides and understand properties of trapezoids and parallelograms.

(2) Date/Time Monday, June 23, 2014, 1:50 - 2:35 (Period 5)

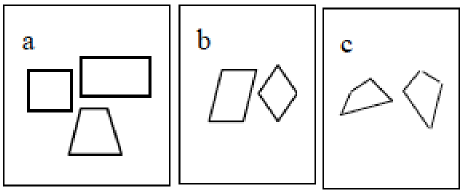
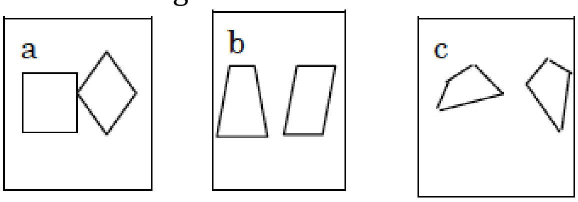
(3) Location Oshihara Elementary School (Showa Town), Grade 4 Room 1

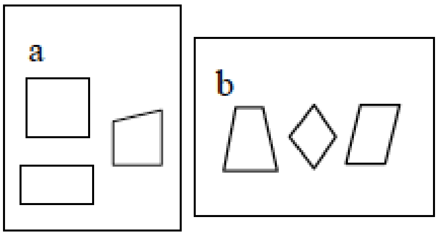
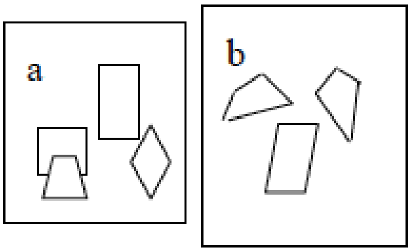
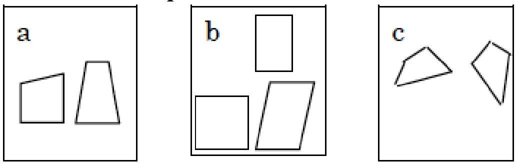
(4) Purposes of the lesson

In previous lessons, students have learned about perpendicularity (2 lines intersect to form right angles) and parallelism (2 lines that are perpendicular to a common line). By understanding these relationships, students have learned to draw lines that are perpendicular or parallel to each other. In today's lesson, using parallelism as a viewpoint, students will learn about trapezoids and parallelograms. Therefore, I want students to make full use of what they have been learning in previous lessons. As students sort quadrilaterals, I anticipate that students will use a variety of criteria. Therefore, as students express how they sorted quadrilaterals, I would like them to use words and diagrams so that they can make their explanation easier for other to understand. Moreover, by incorporating the activity to think about other students' sorting strategies, I want students to recognize the similarities and the differences between their own ideas and those of other students.

(5) Flow of the lesson

Steps	Content and Task	Instructional considerations	Assessment
Grasp (10 min)	1 Understand the task (1) Look at triangles (previously learned) • It's an isosceles triangle. • It's an equilateral triangle. (2) Think about the quadrilaterals they created in previous lessons. (3) Have students share what they noticed. • There are squares. • There are rectangles.	• Display some of the quadrilaterals students created in the previous lesson. • Make sure students understand that some of them have names while others do not -- this will help them plan their strategies.	

	2 Understand the task for today's lesson Let's sort these quadrilaterals and make some groups. (1) Develop a plan. • I think I will make a group with sides of equal length. • Some of them have sides that are parallel. • Some have right angles and others don't.	• Remind students to think about what they have been studying so that they will have ideas what to focus on.	
	3 Independent problem solving (1) Sort quadrilaterals based on own criteria • Based on the appearances  a: They look close to squares. b: They have slanted sides. c: Their shapes are different. • Use the length of sides  a: The length of sides are all equal. b: Opposite sides are equal length. c: The length of sides are all different.		

Explore (10 mi)	• Based on angles  a: There are right angles. b: There is no right angle. • Based on similarity  a: The parts match up when these are folded. b: The parts don't match up when these are folded. • Based on parallel sides  a: A pair of opposite sides are parallel. b: Two pairs of opposite sides are parallel. c: None of the sides are parallel.		

Deepen (15 min)	4 Whole class discussion (1) Explain what they used as criteria to sort. (2) Compare the shared ideas and discuss which previously learned ideas are being used to sort the quadrilaterals. • If we look at the length of sides, the square and the shape of a diamond have 4 sides that are equal. • When we grouped based on right angles, we could make several groups. • We also got several groups when we sorted based on parallel sides, but the groups we made were different from those based on right angles. (3) Think about the groups based on parallel sides using the ideas we already learned. • We call quadrilaterals with a pair of parallel sides "trapezoids," and quadrilaterals with two pairs of parallel sides "parallelograms."	• Have students other than the one that shared their groups to explain how the quadrilaterals were sorted. • Have students think about why we cannot make good groups by simply focusing on the length of sides of angle measurements. • Let children know that because we use parallel sides as the criterion, we call them parallelograms or trapezoids.[1]	• Students can explain the reason for their sorting in an easily understanda ble way. (Mathematic al Way of Thinking) [notebook]

[1] The Japanese word for trapezoids literally means the shape of a footstool/pedestal.

Summarize (10 min)	5 Summarize the lesson (1) Using the words for summary, distinguish the given trapezoids and parallelograms. (2) Write a journal entry • I understand what we focused on to sort the quadrilaterals. • I understand the difference between trapezoids and parallelograms. • I didn't think parallelism we learned will be useful in today's lesson. I want to see how else we can use that idea.	• Make suggestions so that students can incorporate the following point in their journals. ⌘About ways of explanation that were easy to understand. • We will discuss the inclusion relationship in the next lesson.	

(6) Assessment for today's lesson

- Did the students attend to parallel sides and come to understand properties of trapezoids and parallelograms through activity of sorting quadrilaterals from a variety of view points?

References: *Omitted*

Results of the Readiness Test for the Unit

Problem	Score	Main incorrect answer
[1] Draw an isosceles triangle whose side lengths are 5 cm, 6 cm, and 6 cm.	84 %	No response
[2] Write the numbers in the [ ]. (1) One right-angle measures[2] [ ] degrees. (2) Two right-angles measures [ ] degrees. (3) One whole turn = [ ] degrees.	 (1) 100 % (2) 96 % (3) 92 %	 (2) 18 degrees (3) 36 degrees
[3] Name the following figures (not included). a: rectangle b: square c: isosceles triangle d: equilateral triangle	 a: 68 % b: 68 % c: 84 % d: 76 %	 a: quadrilateral[3] b: quadrilateral c: equilateral triangle d: isosceles triangle
[4] Name the triangles we can make by putting together a pair of set squares as shown below (figure not included). a: equilateral triangle b: isosceles triangle	 a: 64 % b: 80 %	 a: triangle b: triangle
[5] What are the measurements of angles *a* and *b*? *a*: 60 degrees *b*: 120 degrees	 *a*: 68 % *b*: 48 %	 *a*: 120 degrees *b*: 60 degrees
[Question on topics not yet discussed] [6] Answer the following questions based on the figure below (figure not included). (1) Which lines are parallel to each other? (*c* & *d*) (2) Which line is perpendicular to *a*? (*e*)	 (1) 12 % (2) 24 %	 (1) *a* and *e*; no answer (2) *b*; no answer

A Readiness Test was administered before the unit. Although students were able to do well on drawing an isosceles triangle or identifying the degree equivalences of one and two right-angles (Problems 1 & 2), they were not as successful in naming shapes or problems involving vertical angles (Problems 3, 4, an 5). Therefore, in this unit, as we will re-examine isosceles triangles and equilateral triangles by clearly focusing on the measurements of their sides and angles. In addition, I want to make sure students understand that the measurements of vertical angles are equal by actually measuring them, as well as calculating to determine missing angles.

[2] "Right-angle" here is used as a "unit."
[3] The Japanese word for quadrilaterals literally means "four-angle shape."

Grade 5 Mathematics Lesson Plan

Wednesday, June 25, 2014
Period 3
Koganei Elementary School
attached to Tokyo Gakugei University
Grade 5 Classroom 1 (39 studnets)
Teacher's Name: Kishio Kako

1 Name of the Unit: Division of decimal numbers

2 Unit Plan

Lesson 1 Represent a problem situation in an expression in the form of Whole Number ÷ Decimal Numbers (1 decimal place, 300 ÷ 2.5).
Lesson 2 Think about ways to calculate 300 ÷ 2.5 (including the use of the division algorithm).
Lesson 3 Represent a problem situation in an expression in which both the dividend and the divisor are decimal numbers (1 decimal place), and think about ways to calculate.
Lesson 4 Represent a problem situation in an expression in which the dividend is a decimal number with 2 decimal places by a decimal number with 1 decimal place, and think about ways to calculate.
Lesson 5 Represent a problem situation in which the divisor is a decimal number less than 1, and think about ways to calculate.
Lesson 6 Think about the meaning of the remainder whey dividing by decimal numbers.
Lesson 7 Understand how to approximate the quotient when dividing decimal numbers.
Lesson 8 Understand that division of decimal numbers can be used to make multiplicative comparisons.
Lesson 9 Understand that division can be used to make multiplicative comparisons even when the quotients become decimal numbers.
Lesson 10 Understand that we can make comparisons using both subtraction and division. (Today's lesson)

3 Proposal in today's lesson

As we examine various achievement test results, we notice that the success rate for problems involving *wariai* is rather low. There was a problem involving *wariai* in the 2013 National Assessment, and its success rate was 76.9%. In each of the 2008, 2009, 2010, and 2012 National Assessments, there was a problem involving *wariai*, and their success rates were 55.1 %, 57.1 %, 57.8%, and 58.7 %, respectively. Although it is difficult to pinpoint the cause for these low success rates, I believe we need to think about teaching that will help students understand the ideas of *wariai*.

When we compare numbers or quantities, we can do so by either using the difference or bai (the quotient). *Wariai* is used when we compare using the quotient. For example, think about the following problem. On which day did the basketball player do better shooting free throws, yesterday when he made 6 of 8 attempts, or today when he made 7 of 10 attempts? If you just consider the number of free throws made, it looks like the player did better today since he made 7 free throws today while he only made 6 free throws yesterday. However, since the player did not make the same number of attempts, we cannot simply compare the number of successful free throws. In this situation, we can use the number of attempts as the base quantity to calculate *bai* (how many times as much is the number of successful free throws as the number of attempts) for each situation and compare them. Yesterday: 6 ÷ 8 = 0.75 Today: 7 ÷ 10 = 0.7 Therefore, the player was more successful yesterday than he was today. When we compare two situations using the idea of *bai*, we consider *bai* (the quotient) as *wariai*. Therefore, I believe that students will understand the idea of *wariai* more deeply if they have opportunities to compare situations using the idea of *bai* before they receive the formal instruction on *wariai*.

We use *bai* to make comparisons when the base quantities are not equal. Thus, in the problem situation above, if the player made the same number of free throw attempts yesterday and today, we can just compare the number of successes. For example, suppose the player attempted 10 free throws on both days and he was successful 6 times yesterday and 7 times today. Then, we only need to compare 6 successes with 7 successes, and the comparison using subtraction is sufficient. In children's everyday life, it is rare that they encounter a situation where they must compare using *bai*. For example, if they want to compare who can ran faster, they simply compare the time. If they want to know how much their test scores improved from the last test, they will just look at the difference in the scores. If students lack experiences to compare using *bai* in their everyday situations, then I believe it is necessary to intentionally set up learning situations in which they can experience the comparison using *bai* before the formal study of *wariai*.

4. About the task in this lesson

The task being used in today's lesson is based on the problem on p. 56 of the textbook published by Tokyo Shoseki. The original problem is as follows.

(Problem) The prices of a notebook and a pen in 1980 and 2005 are as shown below. Which item's price increased more from 1980 to 2005?

Notebook: 80-yen in 1980 → 120-yen in 2005;
Pen: 50-yen in 1980 → 90-yen in 2005

Based on this task, I created a new task in which students can more easily consider the two ways of comparison explicitly depending on cases. To do so, as a part of the task, I included a case in which the comparison using the difference is sufficient (that is, a case in which the base quantity will be the same). The following task will be used in today's lesson.

(Problem) Takashi's parents raised the monthly allowances for Takashi and his brothers. Whose allowance can we say was raised most?

	Before	After	
Takashi	500-yen	700-yen	(−)200 (×) 1.4
Younger brother	500-yen	600-yen	(−)100 (×) 1.2
Older brother	2000-yen	2200-yen	(−)200 (×) 1.1

We will start with the comparison of Takashi and younger brother so that students can make use of the comparison using the difference. Then, the information about their older brother will be presented, and students will be asked to think about whose allowance, Takashi or his older brother, was raised most. Students will realize that this comparison is difficult because the base quantities are different. I will then guide them to consider the idea of using *bai* to make comparisons. In this way, students will understand that there are two ways to make comparisons, one based on the difference and another based on *bai*. By including the case where the use of the difference is sufficient, I believe it becomes easier to examine the cases in which different ways of making comparisons should be used.

5. Today's lesson

(1) Goal of the lesson

By considering the two ways of making comparisons, one based on the difference and another based on *bai*, students will understand that the comparison using *bai* is more appropriate when the base quantities are different.

(2) Flow of the lesson

<table>
<tr><th>Main Activities (T → hatsumon; C → examples of student responses)</th><th>○ Instructional consideration
□ Assessment</th></tr>
<tr><td>1. Understand the task
T: Takashi's parents raised the monthly allowances for Takashi and his brother. Takashi's allowance was raised from 500-yen to 700-yen. His younger brother's allowance was raised from 500-yen to 600-yen. Whose allowance can we say was raised most?
<table>
<tr><th></th><th>Before</th><th>After</th></tr>
<tr><td>Takashi</td><td>500-yen</td><td>700-yen</td></tr>
<tr><td>Younger Brother</td><td>500-yen</td><td>600-yen</td></tr>
</table></td><td>○ Make sure students understand that if you keep money in a bank for a while, you will receive both the amount you deposited and the interest you earned.

○ Guide students to realize that the initial amounts are the same. Help students to pay attention to the equality of the base quantity as the condition for using the differences to make comparisons.</td></tr>
<tr><td>2 Understand the way to compare using the difference
C: Takashi received a 200-yen raise and his younger brother got a 100-yen raise. Since Takashi's raise was greater, his allowance was raised most.
T: How did you figure out 200-yen and 100-yen?
C: I calculated 700 – 500 = 200, and 600 – 500 = 100.
T: So, you compared the difference between 700-yean to 500-yen and the difference between 600-yen and 500-yen, didn't you?</td><td>○ If the idea of bai is suggested, we will also discuss it. As we discuss the idea of bai, make sure to have students think about which quantity is compared to which – which is the base quantity – so that they can understand how to determine bai.</td></tr>
<tr><td>3. Understand the main task
T: Actually, Takashi also has an older brother, and his allowance was raised from 2000-yen to 2200-yen.
<table>
<tr><td>Older Brother</td><td>2000-yen</td><td>2200-yen</td></tr>
</table></td><td>○ Make sure that students realize that they were using the differences to compare.</td></tr>
</table>

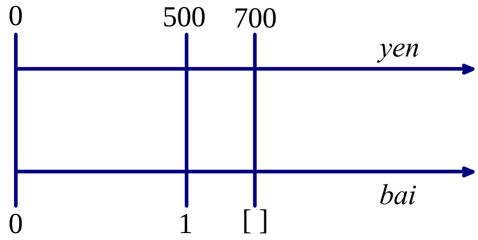
T: So, whose allowance can we say was raised most, Takashi or his older brother? C: I think it is the same since Takashi's allowance was raised 200-yen, from 500-yen to 700-yen, and his older brother also received a 200-yen raise from 2000-yen to 2200-yen. So, it's the same. C: But their allowances weren't the same at first, 500-yen and 2000-yen. So, I don't think it is a good idea to compare using the differences.	○ If we look at the differences, it may appear that Takashi and his older brother received the same raise. Bring students' attention to the fact that their initial allowances were different and the comparison based on the differences might not work well in this situation.

How should we compare if the initial amounts are different?

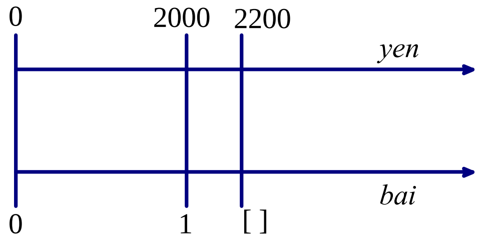
4. Independent problem solving C: If we consider the initial allowance amount as the base quantity, we can calculate how many times as much (*bai*) is the new allowance.	○ If a large number of students do not think about using *bai*, keep the independent problem solving time short. ▯ Students can think about the way to make comparison other than using the differences. (Mathematical Reasoning)
5. Whole class discussion C: I calculated *bai* by dividing the allowance after the raise by the original allowance amount. Takashi: 700 ÷ 500 = 1.4 Older brother: 2200 ÷ 2000 = 1.1 When you compare 1.4 times as much and 1.1 times as much, I know 1.4 times as much will be more. So, I think Takashi's raise was greater.	○ If the idea of *bai* does not come up, ask students how many times as much is each of the brothers' allowance is compared to the original allowance amount.

C: I used double number lines.

Takashi



Older brother



T: When we compared Takashi and his younger brother, we used the differences. On the other hand, when we compare Takashi and his older brother, you would rather use *bai* to compare them, wouldn't you?

6. Summarize the differences in the cases

<u>T: To compare Takashi and his younger brother, we used the differences. To Compare Takashi and his older brother, we used *bai*. I wonder when we can use the differences to compare and when we should use *bai*.</u>

C: When the initial allowance amounts are the same, we can compare with the differences, but if the initial allowance amounts are different, we should use *bai*.

7. Journal writing

○ When using *bai* to make comparisons, it is better to write double number line representations one directly above the other, lining up 1's on the number line for *bai*.

○ To help students more easily understand that the initial allowance amounts are the base quantities and we are comparing the raised allowances, use double number line representations to highlight the relationships between the base quantity and the compared quantity.

○ Make sure students understand that we are using *bai* to make comparison.

○ Generalize when to use comparison based on the differences and when to use *bai*. (The underlined question on the left is *hatsumon* for that purpose.)

□ Students can reason that comparison using *bai* is more appropriate when the base quantity is different. (Mathematical Reasoning)

○ If there is time, compare the allowances of younger and older brothers so that students can experience the situation where the conclusions we reach using the two methods may be opposite.

References: *omitted (because all documents are available only in Japanese)*

2014 Japan Trip - Initial Survey

Welcome

About the Survey
This survey will be given to all participants in the 2014 Japan Lesson Study Immersion Trip to provide data on the usefulness and impact of the trip. The trip is a collaborative effort of Project IMPULS (Tokyo Gakugei University, Tokyo, Japan), Global Education Resources, L.L.C. (NJ, USA), Mills College Lesson Study Group (CA, USA), and the Lesson Study Alliance (IL, USA).

Please complete the survey by **June 13, 2014.** Time needed to complete the questionnaire is approximately 30 minutes. We welcome your comments and invite you to give us your feedback at the end of the survey. Some of you who completed the GER application process may have already answered similar questions, but we ask you to also complete this survey so that all trip participants have provided the same information. Please feel free to cut and paste here the information you previously submitted.

Some questions are marked with an asterisk. These are required questions that enable us to understand whether our survey has reached the intended participants. All other questions not marked with an asterisk are voluntary. If you come to a question you do not wish to answer, simply skip it. We hope that you will answer as many questions as possible.

Your answers will be kept strictly confidential. The research is conducted under stringent DePaul University and government regulations designed to safeguard study participants. Identification codes are used only for follow-up purposes; your name will never appear on the questionnaire. Results for the survey will be reported only in summary or statistical form, so that neither individuals nor their organizational affiliations can be identified.

Please do not print or share any portion of this questionnaire.

Thank you for contributing your time and thoughtful responses to this research effort. We hope that you find the questions professionally meaningful and interesting.

For Further Information
If you have any questions about the survey, please contact Nell Cobb, IMPULS evaluator at ncobb@depaul.edu.

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1. Please enter the unique participant ID number emailed to you.

If you cannot locate this number, please contact Nell Cobb (hit the back button for contact information). *

2. Please describe your experiences with lesson study to date, including: a. Number of years you have been involved in lesson study; b. Content area (e.g., math, English/ language arts) of lessons you have experienced; c. Number of times you have observed and participated in lesson study; d. Whether these experiences were within your home country or in another country.

3. What do you think are the strengths/ benefits of using lesson study in your local context(s) (e.g., district, school, university setting)?

4. What do you think are the challenges to using lesson study in your local context(s)?

5. Please describe how your current organizational contexts use lesson study for educational improvement.

6. Please describe how you hope to use lesson study for educational improvement in your current organizational contexts after this trip.

7. Please select and rank in order of importance the **five** items from the previous question that you believe will be **most professionally useful** for you **within the next year**. Please remember to rank **only 5 items**.

a. Mathematics content	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
b. How to build students' problem solving ability	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
c. Evaluating a lesson on the basis of a written lesson plan	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful

d. How lesson study is conducted in another country

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

f. Collecting data on student thinking to inform instruction

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

g. Strategies for making students' thinking visible

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

h. Analyzing/studying curriculum materials

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

i. Ways to build connections among educators at multiple levels of the education system

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

j. Anticipating student responses

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

k. Writing a useful lesson plan

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

1st Most Useful

l. Supporting participants to have powerful and effective lesson study experiences	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
m. Organizational/structural supports for lesson study	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
n. Students' mathematical reasoning	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
o. Differentiating/ offering support for struggling learners	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
p. Cultural influences on mathematics teaching and learning	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
q. Organizing a successful post-lesson debriefing session	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
r. A typical school day at a Japanese elementary school	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
s. Developing mathematics units and lessons	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful

t. Strategies for working effectively in a lesson study group

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

u. My own country's approaches to mathematics instruction

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

v. Analyzing written student work/ responses

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

w. Analyzing and interpreting verbal student comments

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

y. How to build a classroom learning community

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

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8. Four teachers were discussing the way they believe mathematics is learned by students. To their surprise, no two of them agreed on the principal way mathematics is learned, although each suggested that intellectual processes were necessary.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about the way mathematics is learned. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "To learn mathematics, students have to practice, practice, and practice. It's like playing a musical instrument--they have to practice until they have it down pat."

SUSAN: "The most important thing is reasoning. If students can reason logically and can see how one mathematical idea relates to another, they will understand what is taught."
BARBARA: "The primary thought process in learning mathematics is memory. Once students have the facts and rules memorized, everything else falls into place."
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures--right or wrong--and discover things for themselves, they will understand the mathematics and how it is used."

points MARY

points SUSAN

points BARBARA

points DENISE

Please write about your view of how students learn mathematics.

9. Four teachers were discussing the role of problem solving in students' learning of mathematics. To their surprise, no two of them agreed on the role of problem solving in mathematics learning.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about problem solving in mathematics. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: “Problem solving is like any other skill in mathematics. Students have to practice, practice, and practice. It's like playing a musical instrument--they have to practice until they have it down pat.”
SUSAN: “The most important thing in problem solving is to develop logical reasoning. Problem solving helps students learn to reason logically and can see how one mathematical idea relates to another. Thus it helps them understand mathematics.”
BARBARA: “Students should first master the prerequisite facts and skills of mathematics before they are assigned problem solving. Problem solving should emphasize the application of these facts and skills to real life situations.”
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make

conjectures--right or wrong--and discover things for themselves, they will understand the mathematics and how it is used."

points MARY

points SUSAN

points BARBARA

points DENISE

Please write about your view of the role of problem solving in students' learning of mathematics.

10. Please indicate how well each of the following statements describes your current attitude. (Circle ONE for each statement.)

	Strongly disagree	-	Neither disagree nor agree	-	Strongly agree	Not applicable
a. I enjoy learning about mathematics.	○	○	○	○	○	○
b. I have learned a lot about student thinking by working with colleagues.	○	○	○	○	○	○
c. I have strong knowledge of the mathematical content taught at my grade level.	○	○	○	○	○	○
d. I have good opportunities to learn about the mathematics taught at different grade levels.	○	○	○	○	○	○
e. I think of myself as a researcher in the classroom.	○	○	○	○	○	○
f. I have learned a great deal about mathematics teaching from colleagues.	○	○	○	○	○	○

g. I am always curious about student thinking.	○	○	○	○	○	○
h. By trying a different teaching method, teachers can significantly affect a student's achievement.	○	○	○	○	○	○
i. I am interested in the mathematics taught at many grade levels.	○	○	○	○	○	○
j. I would like to learn more about the mathematical content taught at my grade level.	○	○	○	○	○	○
k. Working on mathematics tasks with colleagues is often unpleasant.	○	○	○	○	○	○
l. I find it useful to solve mathematics problems with colleagues.	○	○	○	○	○	○
m. Japanese mathematics teaching approaches are not likely to be useful outside of Japan.	○	○	○	○	○	○

11. Please indicate your current position (Check ALL that apply.)

- ☐ a. Teacher, Elementary (K-5)
- ☐ b. Teacher, Secondary (6-12)
- ☐ c. Teacher, Post-secondary (college faculty)
- ☐ d. Coach, or other out-of-classroom position for K-5
- ☐ e. Coach, or other out-of-classroom position for 6-12
- ☐ f. School or district education administrator other than coach
- ☐ g. Education researcher
- ☐ h. Other: Please specify ____________

12. How many years of teaching experience do you have?

____

13. Please list any grades to which you have ever taught mathematics.

14. Please add any comments or feedback you have about this survey.

Thank You!

Thank you for taking our survey. Your response is very important to us.

2014 Japan Trip - Post Survey

Welcome

About the Survey
This follow-up survey is for all 2014 Japan Lesson Study Immersion Trip participants, with the purpose of providing data on the usefulness and impact of the trip.

Please complete the survey by **July 18, 2013**. Time needed to complete the questionnaire is approximately 30 minutes. We welcome your comments and invite you to give us your feedback at the end of the survey.

Some questions are marked with an asterisk. These are required questions that enable us to understand whether our survey has reached the intended participants. All other questions not marked with an asterisk are voluntary. If you come to a question you do not wish to answer, simply skip it. We hope that you will answer as many questions as possible.

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Thank you for contributing your time and thoughtful responses to this research effort. We hope that you find the questions professionally meaningful and interesting.

For Further Information
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---

1. Please enter the unique participant ID number emailed to you.

If you cannot locate this number, please contact Nell Cobb (hit the back button for contact information).
*

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2. After the trip, what do you now think are the strengths/ benefits of using lesson study in your local

context(s) (e.g., district, school, university setting)?

3. What do you think are the challenges to using lesson study in your local context(s)?

4. Please describe how you hope to use lesson study for educational improvement in your current organizational contexts after this trip.

5. How much did you learn about each of the following during the immersion trip to Japan?

	Not at all	-	Some	-	A lot
a. Mathematics content	○	○	○	○	○
b. How to build students' problem solving	○	○	○	○	○
c. Evaluating a lesson on the basis of a written lesson plan	○	○	○	○	○
d. How lesson study is conducted in another country	○	○	○	○	○

e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)	○	○	○	○	○
f. Collecting data on student thinking to inform instruction	○	○	○	○	○
g. Strategies for making students' thinking visible	○	○	○	○	○
h. Analyzing/studying curriculum materials	○	○	○	○	○
i. Ways to build connections among educators at multiple levels of the education system	○	○	○	○	○
j. Anticipating student responses	○	○	○	○	○
k. Writing a useful lesson plan	○	○	○	○	○
l. Supporting participants to have powerful and effective lesson study experiences	○	○	○	○	○
m. Organizational/structural supports for lesson study	○	○	○	○	○
n. Students' mathematical reasoning	○	○	○	○	○
o. Differentiating/ offering support for struggling learners	○	○	○	○	○
p. Cultural influences on mathematics teaching and learning	○	○	○	○	○
q. Organizing a successful post-lesson debriefing session	○	○	○	○	○
r. A typical school day at a Japanese elementary school	○	○	○	○	○
s. Developing mathematics units and lessons	○	○	○	○	○
t. Strategies for working effectively in a lesson study group	○	○	○	○	○
u. My own country's approaches to mathematics instruction	○	○	○	○	○
v. Analyzing written student work/ responses	○	○	○	○	○
w. Analyzing and interpreting verbal student comments	○	○	○	○	○
x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)	○	○	○	○	○
y. How to build a classroom learning community	○	○	○	○	○

6. Please select and rank in order of importance the five items from the previous question that you believe will be most professionally useful for you within the next year. Please remember to rank **only 5 items**.

a. Mathematics content

1st Most Useful
2nd Most Useful
3rd Most Useful
4th Most Useful

b. How to build students' problem solving ability	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
c. Evaluating a lesson on the basis of a written lesson plan	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
d. How lesson study is conducted in another country	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
f. Collecting data on student thinking to inform instruction	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
g. Strategies for making students' thinking visible	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
h. Analyzing/studying curriculum materials	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
i. Ways to build connections among educators at multiple levels of the education system	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
	1st Most Useful

j. Anticipating student responses	2nd Most Useful 3rd Most Useful 4th Most Useful
k. Writing a useful lesson plan	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
l. Supporting participants to have powerful and effective lesson study experiences	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
m. Organizational/structural supports for lesson study	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
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p. Cultural influences on mathematics teaching and learning	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
q. Organizing a successful post-lesson debriefing session	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful

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t. Strategies for working effectively in a lesson study group	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
u. My own country's approaches to mathematics instruction	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
v. Analyzing written student work/ responses	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
w. Analyzing and interpreting verbal student comments	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful
y. How to build a classroom learning community	1st Most Useful 2nd Most Useful 3rd Most Useful 4th Most Useful

7. If you remember the items you chose before the trip, comment on any changes made to your responses after the trip. Why do you now consider some knowledge more or less useful than before the trip?

8. Please review the following three problems, and provide your ratings again after the trip.

Four teachers were discussing the way they believe mathematics is learned by students. To their surprise, no two of them agreed on the principal way mathematics is learned, although each suggested that intellectual processes were necessary.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about the way mathematics is learned. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "To learn mathematics, students have to practice, practice, and practice. It's like playing a musical instrument--they have to practice until they have it down pat."
SUSAN: "The most important thing is reasoning. If students can reason logically and can see how one mathematical idea relates to another, they will understand what is taught."
BARBARA: "The primary thought process in learning mathematics is memory. Once students have the facts and rules memorized, everything else falls into place."
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures--right or wrong--and discover things for themselves, they will understand the mathematics and how it is used."

points MARY

points SUSAN

points BARBARA

points DENISE

Please write about your view of how students learn mathematics.

9. Four teachers were discussing the role of problem solving in students' learning of mathematics. To their surprise, no two of them agreed on the role of problem solving in mathematics learning.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about problem solving in mathematics. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: “Problem solving is like any other skill in mathematics. Students have to practice, practice, and practice. It's like playing a musical instrument--they have to practice until they have it down pat.”
SUSAN: “The most important thing in problem solving is to develop logical reasoning. Problem solving helps students learn to reason logically and can see how one mathematical idea relates to another. Thus it helps them understand mathematics.”
BARBARA: “Students should first master the prerequisite facts and skills of mathematics before they are assigned problem solving. Problem solving should emphasize the application of these facts and skills to real life situations.”
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points MARY

points SUSAN

points BARBARA

points DENISE

Please write about your view of the role of problem solving in students' learning of mathematics.

10. Please indicate how well each of the following statements describes your current attitude. (Circle ONE for each statement.)

	Strongly disagree	-	Neither disagree nor agree	-	Strongly agree	Not applicable
a. I enjoy learning about mathematics.	○	○	○	○	○	○
b. I have learned a lot about student thinking by working with colleagues.	○	○	○	○	○	○
c. I have strong knowledge of the mathematical content taught at my grade level.	○	○	○	○	○	○
d. I have good opportunities to learn about the mathematics taught at different grade levels.	○	○	○	○	○	○
e. I think of myself as a researcher in the classroom.	○	○	○	○	○	○
f. I have learned a great deal about mathematics teaching from colleagues.	○	○	○	○	○	○
g. I am always curious about student thinking.	○	○	○	○	○	○
h. By trying a different teaching method, teachers can significantly affect a student's achievement.	○	○	○	○	○	○
i. I am interested in the mathematics taught at many grade levels.	○	○	○	○	○	○
j. I would like to learn more about the mathematical content taught at my grade level.	○	○	○	○	○	○
k. Working on mathematics tasks with colleagues is often unpleasant.	○	○	○	○	○	○

l. I find it useful to solve mathematics problems with colleagues.	○	○	○	○	○	○
m. Japanese mathematics teaching approaches are not likely to be useful outside of Japan.	○	○	○	○	○	○

11. Please select the research lesson and post-lesson discussion that you feel was most professionally informative for you.

June 26 Quadrilaterals (Grade 4, Daisan Terashima Elementary School)
June 28 Division with Remainders (Grade 3, Oshihara Elementary School)
June 29 Multiplication (Grade 2, University of Yamanashi Elementary School)
June 29 Division with Remainders (Grade 3, University of Yamanashi Elementary School)
July 1 Division (Grade 3, Matsuzawa Elementary School)
July 2 Algebraic Expressions (Grade 8, Koganei Junior High School)

12. Please explain why you selected this lesson and post-lesson discussion. **What** about the lesson and post-lesson discussion was informative for you?

Copy of

13. Please select the research lesson and post-lesson discussion that you feel was least professionally informative for you.

June 26 Quadrilaterals (Grade 4, Daisan Terashima Elementary School)
June 28 Division with Remainders (Grade 3, Oshihara Elementary School)
June 29 Multiplication (Grade 2, University of Yamanashi Elementary School)
June 29 Division with Remainders (Grade 3, University of Yamanashi Elementary School)
July 1 Division (Grade 3, Matsuzawa Elementary School)
July 2 Algebraic Expressions (Grade 8, Koganei Junior High School)

14. Please explain why you selected this lesson and post-lesson discussion as the least professionally informative for you. What specifically was missing?

New Page

15. Please comment on the number of lessons you observed during the program.

Too few	Just right	Too many
○	○	○

16. Please indicate the degree to which you agree or disagree with the following statement: There were too many items on the itinerary, and as a result, the program felt too busy.

- ○ Strongly disagree
- ○ Disagree
- ○ Neutral
- ○ Agree
- ○ Strongly agree

17. Other Comments:

18. What changes to the trip itinerary might have helped to deepen your own learning about lesson study and mathematics teaching and learning?

New Page

19. Please indicate your level of satisfaction with the following aspects of the program:

	Very Dissatisfied	Dissatisfied	Neutral	Satisfied	Very Satisfied	Not Applicable
Accommodations (Hotel Mets)	○	○	○	○	○	○
Meals (Hotel Mets)	○	○	○	○	○	○
Accommodations (Hotel Fuji)	○	○	○	○	○	○
Meals (Hotel Fuji)	○	○	○	○	○	○
Communication with program staff prior to arrival	○	○	○	○	○	○
Communication with program staff during the program	○	○	○	○	○	○

20. Other comments:

21. Please add any comments or feedback you have about this survey.

Thank You!

Thank you for taking our survey. Your response is very important to us.

July 11, 2014 Niikkei

日本流に学び 教師力UP

教員同士の「授業研究」海外が注目

「やる気どう引き出す」自国でも実践

授業の質を向上させるため教員同士が学び合う授業研究。日本特有の研修方法が海外から注目を集めている。6月には欧米の教育関係者が続々と来日。「子供たちのやる気をどうすれば引き出せるか」。"日本流"を身につけようと、授業や議論の行方に熱い視線を注いだ。

米英などから視察

「ビオトープに45匹いたオタマジャクシのうち、27匹がカエルになっていました。残りのオタマジャクシは何匹でしょうか」。6月中旬、東京都世田谷区立松沢小学校で開かれた2年生の算数の公開授業。見学に訪れた米、英、豪など計15人の外国人教諭らが、同時通訳を受けながら熱心に教員と児童約30人のやりとりを見守った。

45分間の授業で出された問題は「45引く27」の計算1問だけ。「先に十の位から引き算すれば、どうだろ」「5から7は引けないから十の位から借りてきて……」。教員は児童らが考えた様々な意見に耳を傾け、丁寧に板書していく。

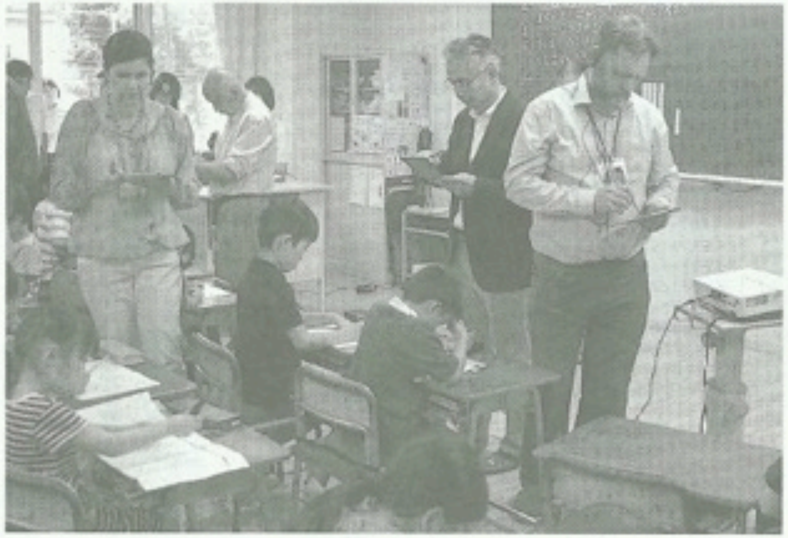
授業後は全校の教員が改善点などを探る意見交換会を開催。授業の速記録を基に「もっと子供たちのいろんな考え方を引き出せたのではないか」「時間配分にミスがあった」などと検討を重ね、最後は教育の研究者が講評した。

解法から自分で考え答えを導き出す日本式の算数授業。解法を教えて演習問題を与える方法に慣れた外国人教諭らは、物珍しげにタブレット端末で授業や議論の風景を撮影したり、児童のノートをのぞき込んだり。

意見交換会の後には、互いに「子供たちのやる気をどうやって引き出しているのか」「自国に持ち帰って日本式の授業を実践してみたい」といった質問や感想を述べ合った。

東京学芸大の藤井斉亮教授（教育学）ら数学科教育学研究室のグループが研修を企画した。2011年から文部科学省の助成で始まった「国際算数数学授業研究プロジェクト」の一環で、外国人教諭の受け入れは今回で3回目。約10日間の日程を組み、都内など7つの小中高校の授業や授業研究を視察した。

藤井教授によると、授業研究は明治期以降受け継がれてきた「日本の伝統文化」。教員らが学年ごとに数週間かけて案を練り、効果的な授業を目指して全校の教師で議論し合う。世界でも珍しい取り組みで、藤井教授は「学習指導要領と



日本の算数の授業に海外から関心が高まっている（6月、東京都世田谷区の松沢小）