# IMPULS Lesson Study Immersion Program *2013* Overview Report

January 2014

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# 1 Preface

Project IMPLUS is a project funded by the Ministry of Education, Culture, Sports, Science & Technology of Japan in 2011. The Project is housed in the Mathematics Education Department of Tokyo Gakugei University, Tokyo, Japan. The director of the project member is Professor Toshiakira FUJII, and the project members include all the faculty members of the mathematics education departmentð Professors Koichi NAKAMURA, Shinya OHTA, and Keiichi NISHIMURA. Dr. Akihiko TAKAHASHI of DePaul University joined the project as a specially appointed professor. Ms. Naoko MATSUDA (KATSUMATA) and Ms. Sayuri TAKADA also joined the project as a project staff member. In addition, this project has been supported by the external committee consists with 9 Japanese and 7 overseas researchers..

The purpose of the project is two-fold. First, as an international center of Lesson Study in mathematics, Tokyo Gakugei University and its network of laboratory schools will help teacher professionals from throughout the region learn about lesson study and will thereby prepare them to create lesson study systems in their own countries for long-term, independent educational improvement in mathematics teaching. Second, the project will conduct several research projects examining the mechanism of Japanese lesson study in order to maximize its impact on the schools in Japan.

Under these main purpose, we are working for ;

1) Research on Japanese Lesson Study to come up with ideas for establishing innovative teacher education systems for long-term, independent educational improvement in teaching mathematics.

2) Professional development to disseminate ideas for establishing innovative teacher education systems for long-term, independent educational improvement in mathematics teaching. Workshops and institutes would examine how to implement ideas for Lesson Study and innovative ideas for professional development in various schools with different systems and cultural back ground in order to prepare them to create in their own countriesôsystems for long-term, independent educational improvement in teaching mathematics.

3) Facilitate opportunities for researchers, administrators, and practicing school professionals throughout the region to exchange their ideas to improve their education systems for teaching mathematics.

The IMPULS lesson study immersion program was designed to give mathematics education researchers and practitioners from outside Japan an opportunity to examine authentic Japanese Lesson Study in mathematics classrooms. The major purpose of this program is for us to receive feedback on the strengths and weaknesses of Japanese Lesson Study and to discuss how to improve mathematics teacher professional development programs. To accomplish this, we invited leaders of mathematics education to immerse themselves in authentic Japanese lesson study, especially school-based lesson study, and to observe mathematics research lessons in elementary and lower secondary grades.

The Lesson Study immersion program started in 2012. This yearǿs program was held in Tokyo and Yamanashi in Japan from June 23, 2013 to July 5, 2013. In total 16 mathematics educators (9 form U.S., 5 from U.K and 2 from Australia) including mathematics education professors, principals of school and teachers of mathematics. Project IMPULS invited 14 participants and others joined at thier own expenses.

Two of the IMPULS overseas support committee members, Dr. Makoto Yoshida (President of GER and Director of Center for Lesson Study in William Paterson University) and Dr. Tad Watanabe (Professor of Mathematics Education at Kennesaw State University) interpret lesson and post lesson discussion observed. Almost all lesson plans were translated by Dr. Tad Watanabe and distributed before observation. And two of external evaluation committee, Dr. Rebecca Perry (Senior Research Associate, Mills College) and Minori Nagahara (Boston College) gave us useful feedback with objective evaluation of program. We would like to take this opportunity to thank all of our overseas support and evaluation committee, cooperative schools which kindly welcomed our visiting and all concerned professionals for their hard work.

# 2 Contents of Program

This program is designed for deeper understanding of Japanese school-based lesson study and it consist of these contents below.

1) Basic lecture on Japanese mathematics lesson and lesson study (1 day)
2) Observation of research lesson and post lesson discussion (7 classes)
3) Discussion among participants, Q/A and review session

Detailed schedule is shown as below.

| Date | AM | PM |
|---|---|---|
| 23, June | Arrival day | |
| 24, June | Opening Ceremony<br>Workshop "Mathematics teaching and learning in Japan, and lesson study"<br>Welcome dinner party | |
| 25, June | School Visit<br>Koganei Junior High School<br>Preparatory orientation for student teaching<br>Lesson Observation and Discussion Grade 7 ǫLetters in Algebraic Expressionsǫ | |
| 26, June | School Visit (1)<br>Public Elementary School<br>Daisan terajima Elementary School (School based LS) | |
| 27, June | Move to Yamanashi by bus | Courtesy call for local board of education<br>Cultural trip to Takeda shrine to see Sangaku |
| 28, June | School Visit (2)<br>Public Elementary School<br>Oshihara Public Elementary School | Observation School-Based Lesson Study |
| 29, June | School Visit (3) (4)<br>University of Yamanashi Model Junior High School<br>Observation preparatory nation-wide Lesson Study | Move to Tokyo by bus<br>Reflective discussion and Q/A (in the bus) |
| 30, June | Free | |
| 1, July | School Visit (5)<br>Public Elementary School<br>Matsuzawa Elementary School (School based LS) | |
| 2, July | Reflection & Discussion | School Visit (6)<br>TGU attached school<br>Koganei Junior High School (Specially Appointed LS for Fuzoku teachers) |

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|---|---|---|
| | | <Lesson> Grade 8 " Algebraic Expressions " |
| 3, July | School Visit (7)<br>Public Elementary School<br>Sugekari Elementary School (School based LS) | |
| 4, July | Reflection & Discussion | Farewell dinner party |
| 5, July | Departure day | |

Participants made 7 group to make observation report for each research lesson.

Lesson Observation (1)

Research Lesson Observation Form
By Denise Jandoli and Francesca Blueher

Names: Denise Jandoli and Francesca Blueher
School: Daisan Terajima Elementary School Grade: 4 Date: 26 June 2013
Teacher: Daisuke Nagatani

**What are the primary lesson goals?**

In the previous dayôs lesson, students were given nine quadrilaterals. They were expected to sort these geometric figures by examining their constituent parts and distinguish them based on clear rationales. In todayôs lesson, students are expected to discuss their rationales explicitly for sorting the quadrilaterals. They must look at the figures from a specific perspective and use the important factors from that perspective to organize the given figures. It is hoped that students can experience that there are things that become visible when they clarify their perspectives using their prior learning.

Moreover, during the whole class discussion, students are expected to understand that there are diverse ways of reasoning as they share each otherôs ideas. In addition, by reasoning logically as they try to understand each other, students are expected to further their ability to reason. Students should also understand the definitions of trapezoids, parallelograms, and rhombi.

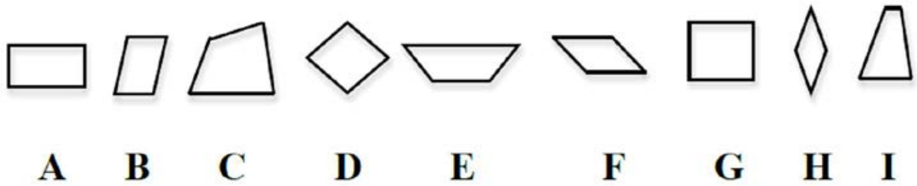
**Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?**

The name of the unit that this lesson is located within is, ñLetôs explore various quadrilateralsò. The goals of the unit include students understanding the meaning of perpendicular and parallel lines and how to draw them, understanding the definition of trapezoids, parallelograms, and rhombi and how to draw them, and, finally, understanding the characteristics of diagonals of quadrilaterals. The unit is composed of 15 lessons, broken up into 4 sub units. This lesson is the 2$^{nd}$ lesson within the 3$^{rd}$ sub unit, called ñVarious quadrilateralsò, and is the 7$^{th}$ lesson of the whole unit.

Grade 4 Research Lesson ï ñSorting Quadrilateralsò

| **Start &End Time** | **Lesson Phase** | **Notes** |
|---|---|---|
| | 1. Introduction, Posing Task |  |

1:30 The lesson began with the teacher saying, ñLets remember what we discussed yesterday.ò The students recalled that their activity from the previous day had to do with grouping 9 quadrilaterals based on clear rationales.




The teacher announced, ñToday weõll look at the different ways of sorting and look at each others to see if they make sense.ò He then wrote the lesson title on the board, ñLets make groups of quadrilaterals.ò Students take out their quadrilaterals from previous day as the teacher puts them on the board.

1:34 The teacher split the board into 3 columns, which represented the number of groups into which the students sorted their quadrilaterals. The columns were titled, ñ2 groupsò, ñ3 groupsò, and ñ4 groupsò. The teacher then instructed the students to place their green magnetized name cards underneath the appropriate column title, signifying how many groups they created for the quadrilaterals the previous day.

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| | | 1:37 The teacher said, ñHow can we make 2 groups or 4 groups? You might be wondering how you can make that many. If you donõt understand, put your name underneath that group.ò Students got out of their seats again and placed their yellow magnetized name card underneath the group number in which they did not understand how to create. The teacher explained, ñIf you understand throughout the lesson, take your yellow name card off the board.ò<br><br><br>1:38 The teacher said, ñMake sure when you explain, you want to get others to understand how you made that grouping.ò |
| | 2. Independent Problem-Solving | *The lesson observed was part two of a full lesson, therefore, the independent problem solving time occurred the previous day when the students created their own grouping rationales for the 9 quadrilaterals. The intent of the second lesson was to have a whole class discussion in order to make the students' rationales explicit and deepen their ways of mathematical thinking as they listen to each other's ideas.* |
| | 3.Presentation of Studentsô Thinking, Class Discussion | In preparing for the research lesson, the teacher had multiple sets of the 9 quadrilaterals in large manila envelopes. These envelopes were located near the blackboard so he could have easy access to them during the presentations. He also had a document projector set up next to the two chalkboards so that students could come up and show how they used their compass and set square. |

The teacher chose to have students present their quadrilaterals by number of groups sorted. He chose to focus on the "2 Group" category first, using 3 student's work.

1:38 The first student was asked to come to the board to show how he grouped his 9 quadrilaterals. The student said, "I put A (rectangle) and G (square) in one group and then the rest in another group." The teacher responded with, "I see some people nodding. Raise your hand if you had the same way of thinking." He then faced the student who was standing at the board and asked, "What did you pay attention to when you made the grouping? Remember, we can't just say that they look alike." The student responded with, "Right angles (pointing to A and G) and no right angles (pointing to the rest)."

1:43 Still focusing on the same grouping rationale as the previous student, the teacher called up a second student to come up to the document projector to model how he used his set squares to prove that the first grouping had all right angles.

1:45 The teacher supported the students' discovery about A (rectangle) and G (square) by asking the class, "What do we call lines that form right angles?" The class responded with, "Perpendicular." The teacher then placed a blue magnetized card on the board next to this first grouping rationale that read, "Right Angles".


1:48 The teacher asked the class, “Anybody else make 2 groups but focus on different things?” A new student came up and sorted his shapes into two groups. The first group had A (rectangle), G (square), and E (trapezoid) and the second group had the rest of the shapes. The student attempted to model his rationale up at the document projector but was unsuccessful because he spoke very little.

The teacher asked the class, “How did he make these two groups?” No one answered the teacher so he then asked the student up at the board if he could give the class a hint. The student responded with, “I used a compass.” A student in the class guessed, “Maybe he compared the lengths?” The student up at the board responded with, “I looked at the opposite ones.” The teacher asked the class, “What does he mean by opposite ones?” A student in the class guessed, “Opposite sides?” At this point, there was a lot of confusion so the teacher instructed the class to look at their shapes, take out their tools (compass and set square), and see what the student meant by “opposite”.

The class still could not figure out the student's rationale for grouping the shapes this way. The teacher ended up demonstrating to the class up at the document projector in order to make the student's rationale clear.

1:55 He finally explained, "In shapes A, G, and E, the lengths from one corner to the opposite corner are equal." Meaning, the student's rationale for grouping the 9 quadrilaterals this way was that the lengths of the diagonals were equal in shapes A, G, and E (the term diagonal was not mentioned in the lesson, rather "opposite corner lengths").

1:56 The teacher then called up a third student to present his grouping rationale. This student also only created 2 groups. He put D (rhombus), G (square), and H (rhombus) in one group and the rest of the shapes in another group. The student explained that he focused on the lengths of the sides. He explained that in the first group of shapes he created, all of the side lengths are the same.

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| | | 2:00 The teacher agreed and put a green magnetized card on the board next to this student's grouping that read, "All 4 sides have the same length". The teacher recalled that another student made the same grouping and used markings (tic marks) on the sides to show that the sides are the same length. |
| | | 2:04 The teacher asked the class, "Did anybody else focus on the lengths of the sides?" A student in the class responded with, "Yes, but I made 3 groups." He put D (rhombus), G (square), and H (rhombus) in one group, shapes B (parallelogram), E (trapezoid), F (rhombus), and A (rectangle) in a second group, and shapes C (quadrilateral), and I (trapezoid) in a third group.<br><br>The teacher asked the class, "What does this mean? How did he make these 3 groups?" The teacher ended up explaining that in the first group, all of the sides are equal, in the second group, there are 2 pairs of equal sides, and, finally, in the third group, none of the sides are equal.<br><br>2:07 A student then said he also made 3 groups but used a different rationale. He made the same exact groups as the previous student but he moved shape E (trapezoid) to the third group with C (quadrilateral) and I (trapezoid).<br><br>He explained that in the first group, all the shapes have the same side lengths, in the second group, opposite sides are the same length, and in the third group, he placed the rest of the shapes. |

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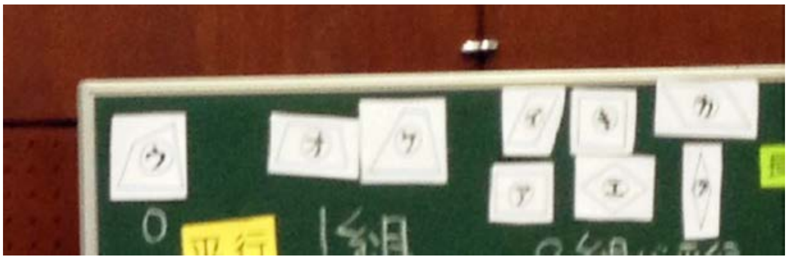
| | | 2:10 A student participated and said that she was able to create 4 groups.<br><br>She explained that she had the same grouping as the previous student but put shape E (trapezoid) in a group by itself. The teacher remarked, "This shape E is going back and forth in different groups." A student explains, "One pair of opposite sides is the same length, but the other pair is not!"<br><br>2:18 Finally, a student shared how she created 3 groups with her shapes. Her rationale was based on parallel sides. She made one group of shapes with 2 pairs of parallel sides, a second group with one pair of parallel sides, and, finally, a third group with no parallel sides.<br> |
| | 4.Summary /Consolidation of Knowledge | 2:22 The teacher summarized the boards and the different grouping rationales. He explained that different numbers of groups could be created "depending on what we look at."<br><br>2:24 The teacher then announced, "Tomorrow we're going to focus on parallel sides."<br><br>2:25 End of lesson. |

**What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?**

When we walked into the large conference room that held the post lesson discussion, we were met with groups of teachers talking excitedly about the lesson. Each table group of teachers had a large sheet of paper divided into quadrants, blue and pink stickies, and colorful markers. The quadrants were divided into what went well and areas of improvement for the teacher and the students. Teachers wrote their comments down on the appropriate colored sticky, then put the sticky in the corresponding quadrant. Whenever the teachers were asked to discuss in their groups, they were engaged and focused on the lesson. After the group discussion on the lesson, the sheets were posted on the wall for all to see. One teacher from each group presented their poster to all post lesson discussion participants. The moderator was excellent in facilitating the discussion in a timely manner. After the group presentations, the knowledgeable other spoke at length about the lesson and whether the goals of the lesson had been met.


Many insights were gained from the post lesson discussion. The groups of teachers seemed very focused on the teacherõs strategy of using magnetic name cards in the lesson. There was confusion on the purpose of the yellow cards. The green name cards were for children to put into the column of number of groups into which they sorted their quadrilaterals. The yellow name cards were intended to be placed into the columns in which students had questions. Most of the yellow cards were put into the ñ2 Groupsò column, none were in the ñ3 Groupsò column, and the remaining were in the ñ4 Groupsò column. Overall, the teachers thought the lesson went well, yet there was no discussion on if the goals of the lesson had been met.

The Knowledgeable Other (KO) spoke at length about the lesson and alluded to the fact that the teachers were not addressing the goal of the lesson. The goal of the lesson was for students to clearly communicate the rationale of sorting their shapes. Clear communication would entail that students understand deeply the characteristics of shapes developed from their previous course of study. The KO acknowledged that the teacher did most of the discussing about the characteristics of the quadrilaterals, especially at the end of the lesson when he realized he was out of time. Another goal of the lesson was to understand the properties of a parallelogram, rhombus, and trapezoid. The KO emphasized that students must deeply understand parallel lines to best understand these shapes. He commented that the groupings were mainly based on the characteristic of *length of sides* **not** on *parallel lines* so that perhaps a good understanding of these shapes was not yet possible. The KO also questioned putting up a total of 9 groupings instead of focusing on fewer groupings that might lead to a deeper understanding of parallel lines. The KO summed up the lesson as not meeting the goals since the students were not guided to this understanding in the lesson's discussion.

The insights from this lesson showed the importance of a deep understanding by the teacher of where the children were in their developmental progression of geometry before the lesson. The lesson also highlighted that it is essential for students to create their own understanding of geometric characteristics that is well supported by the lesson. Children cannot be told the definition of rhombus, trapezoid, or parallelogram without understanding the characteristics that go into making shapes.

**What new insights did you gain about how administrators can support teachers to do lesson study?**

Administration supported lesson study at this school by providing teachers the time to develop their lessons as part of their school day, and time and space for the research lesson and post lesson discussion. Many teachers were involved in the study of the lesson and all teachers participated in the post lesson discussion. The whole process of lesson study and the learning that it fosters will be used to inform instruction at all grade levels and throughout the school. The support from administrators in supporting lesson study is essential to staff professional development, student learning, and strengthening pedagogy.

**How does this lesson contribute to our understanding of high-impact practices?**

This lesson highlighted the importance of clearly making students mathematical thinking visible so that they could compare it to their classmate's thinking. The lack of this clarity was noted by both the teachers and KO and led to a superficial understanding of shape characteristics. A more efficient use of the blackboard with a more thoughtful presentation of student sortings could potentially lead the class to a stronger understanding of student reasoning for their quadrilateral sortings. Anticipating student responses, especially when the teacher intended to facilitate a classroom discussion, is crucial for building an understanding of where the children are in their understanding of geometrical properties.

Lesson Observation (2)

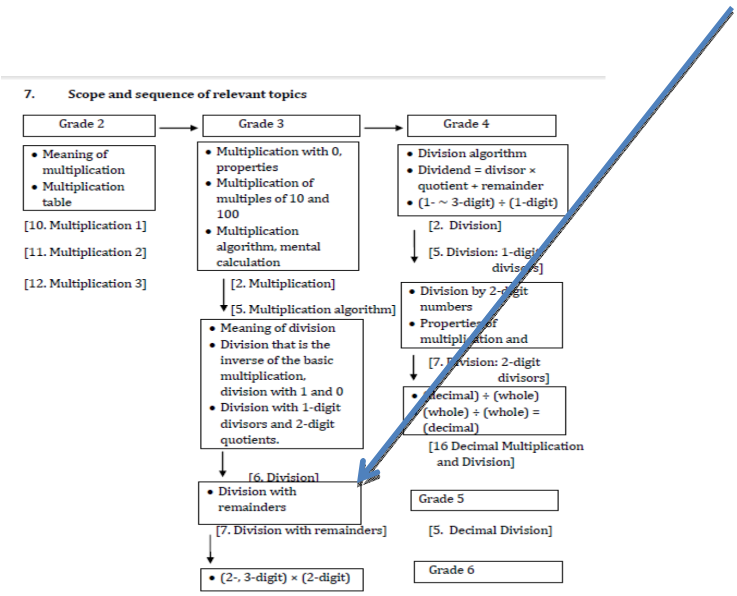
Group Members: Kathryn Palmer, David Garner and Amanda Short
Location: Showa City Oshihara Elementary School, Grade 3, Class 2
Date: 28th June 2013

Research Lesson: “Let’s think about division” (Division with remainders)

Goals of this lesson:
Students are able to think about and explain how to treat remainders appropriately according to the problem situation (round up the reminders and add 1 to the quotient).

Intention of instruction: Students have learned the meaning of remainders and how to calculate division with remainders up to the previous lesson. In this lesson, students will learn how to treat remainders (round up the reminders and add 1 to the quotient) that is different than what they have learned. Because of this reason, I am going to provide a problem situation that is close to the students’ daily life. Through thinking about the task of this lesson, I would like to students to notice that in some problem situations remainders can be treated differently and thought as a group that is added to the quotient. Through several problem-solving activities up to the previous lesson, students had to consciously think about situations where they need to find a quotient and a remainder. Because of this reason, students might come up with a wrong answer. Based on a problem situation that involves “all the students need to ride”, help students to think about how to treat the remainder. Through learning from this lesson, help students to expand their view of remainders and foster their ability to find an appropriate answer that corresponds with a problem situation.

Lesson located within the unit and overall scope and sequence: First lesson in sub-unit 2 of teaching division with remainders (7th lesson on division with remainders in all for the year, where students were asked how to treat remainders appropriately according to the problem situation (round up the remainders and add 1 to the quotient)).



| Start &End Time | Lesson Phase | Notes | |
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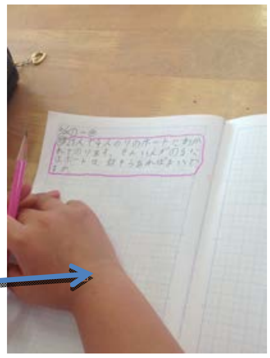
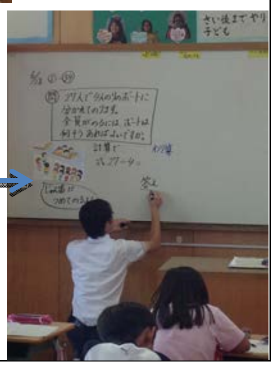
| 1:50<br><br><br><br><br><br><br>1:53 | Introduction | -Strategies to build interest or connect to prior knowledge<br><br>Teacher asks, "What are you doing; what have you done so far?"<br>Class replied "division with remainders"<br><br>Teacher writes problem on the board and class copies into journals. |   |
| 1:55 | Posing Task | Teacher shows poster and says, "What do you think?"<br>"We are riding the boats for four people."<br>"We want to think about this problem today." |   |
| | | "Using calculation, so…Can you do that?"<br>"What can we use?"<br><br>Underneath the poster the teacher wrote,<br>What's the math sentence or expression?<br><br>Student Responds: "27 divided by 4"<br>Teacher: "So, 27 ÷ 4, we understand this? I want you to think about your own ideas. Please write it in your notebook."<br>Teacher writes on the board 27 ÷ 4 |   |

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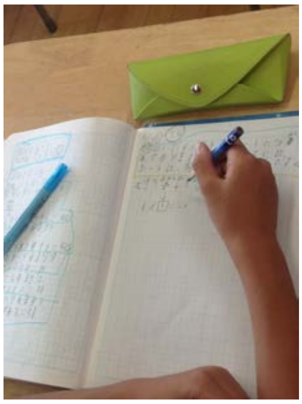
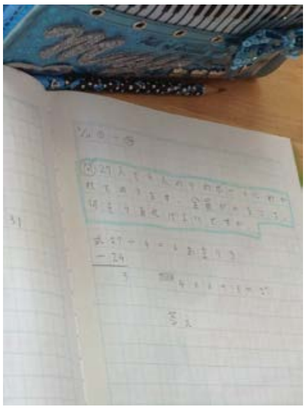
| 1:57 | Independent Problem-Solving | Student Thinking / Visuals / Peer Responses /Teacher Responses<br><br>Students begin independent work. Teacher roving around class, looking at students' work. | |
| *Selection of student responses during independent work follows.* | | | |
| | Independent Problem-Solving | 27 ÷ 4 |  |
| | | 27 ÷ 4= 6 remainder 3<br>□□<br>4 x = 6 24 |  |
| | | 27 ÷ 4 = 6 remainder 3<br>-<u>24</u><br><br>Check:<br>4 x 6 + 3 = 27 |  |

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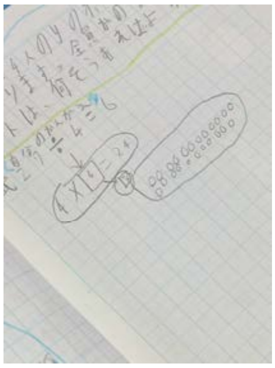
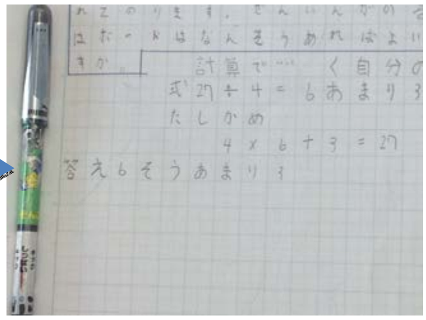
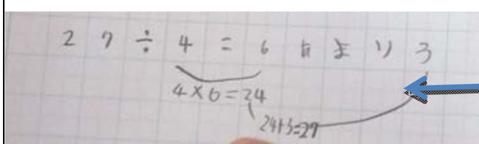
| | | 27 ÷ 4 = 6 remainder 3<br><br>4 x 6 = 24<br><br>O O O O O O O O O<br>O O O O O O O O O |  |
| | | This student calculated quite quickly the solution to the problem writing: 27 ÷ 4 = 6 remainder 3<br><br>4 x 6 + 3 = 27<br><br>answer: 6 boats, remainder 3<br><br>Then the student spent time flipping back in her notebook to the last time they did division and she looked at when they had done remainders.<br>*Did she do this to look busy or because she was checking her work?* | <br> |
| | | 27 ÷ 4 = 6 remainder 3<br>-<u>24</u><br><u>    3</u><br>Check:<br> |  |

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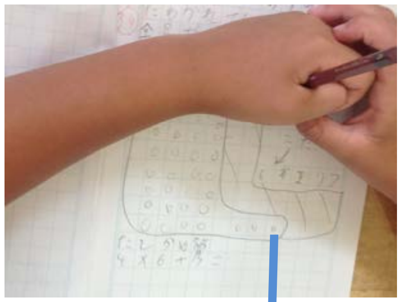
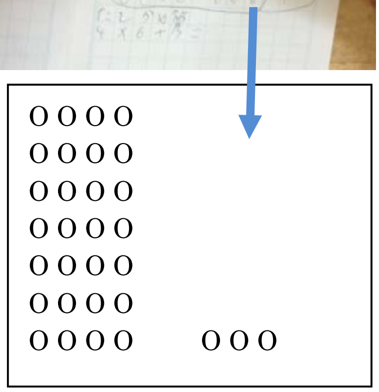
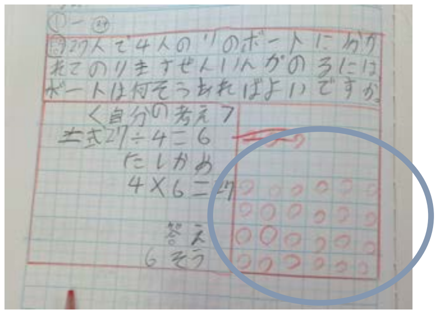
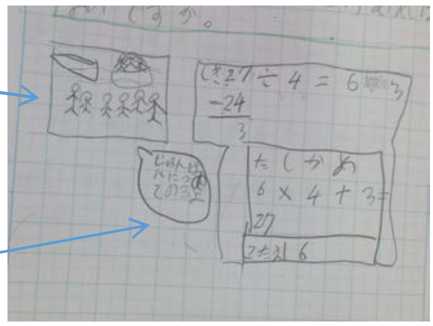
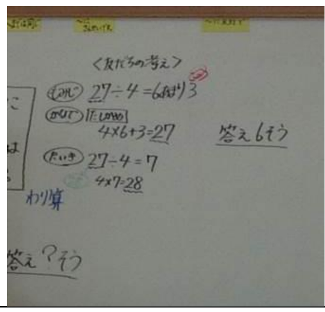
| | | In his notebook he showed 7 rows of 4 and 3 to the side – almost as if he's acting out putting kids into boats.<br>Under his drawing he wrote 4 x 6 + 3.<br><br>■ A number of children appeared to lack confidence throughout the lesson, a specific example:<br>Evidence of him trying to cover his work can be seen in this photo. | <br> |
| | | 27 ÷ 4 = 6<br>Checking 4 x 6 = 27<br><br>This image shows linking to the concept of multiplication through the array. |  |
| | | Student drew the picture and the instructions given by the teacher in their book.<br><br>27 ÷ 4 = 6 remainder 3<br><u>-24</u><br>3<br><br>Student notebook: *Fill the boats and no spaces* |  |
| 2:05 | Teacher re-direction | Teacher: "So let's have your friends to present ideas"<br><br>Students responses: "I don't know what to say,"<br>"I cannot say the answer but I can say the sentence" |  |

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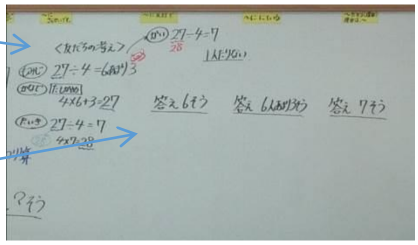
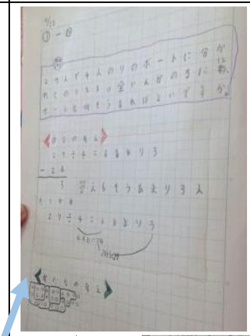
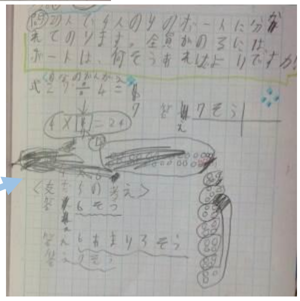
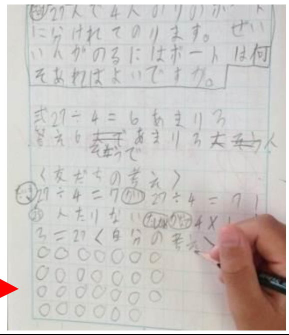
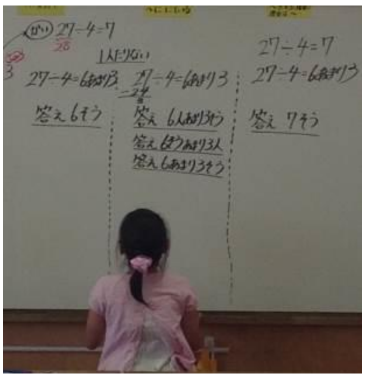
| | | Picture of board –<br>Recorded on the board – Friends ideas<br><br>Through teacher direction, 3 different answers were recorded on the board:<br>6<br>6 remainder 3<br>7 |  |
| 2:15 | Independent Work (#2) | Teacher: "So think about those and your own ideas. You can use the words, pictures, diagrams to explain."<br><br>During the second independent working time student use of diagrams increased.<br><br>Initial Working - Limited Use of diagrams<br><br>After teacher prompting – most students in the class utilized diagrams to show thinking | <br><br> |
| 2:25 | Presentation of Students' Thinking<br><br>Bansho | Teacher:<br>" I want to ask X your idea."<br>Student:<br>"I am the same as the middle one, with 6 remainder and 3 boats…6 boats, I changed my mind."<br>Teacher records 6 r 3<br>"How many of you thought about that?"<br>*1 hand went up*<br>"What do you think Amy?" |  |

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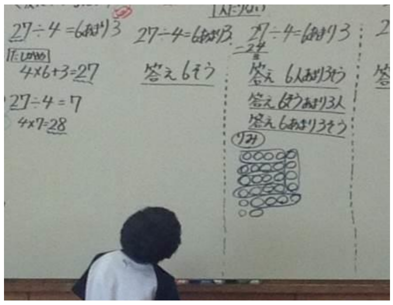
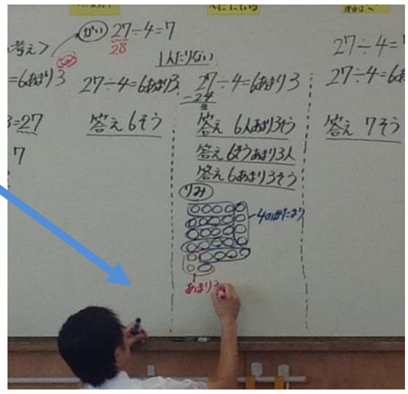
| | | Amy: "I did 6 r 3" "Can I show you?"<br>Amy comes up and talks to board and explains: "First I used multiplication to find something close to 27 and by doing so I wrote it as 24 = 6 x 4, then 27 – 24 = 3 so the remainder is 3".<br>Teacher directs her<br>"But I look at your notebook. You actually drew a picture or diagram. Can you share with your friends?"<br>Student draws array |  |
| 2:32 | | Teacher:<br>"So Amy drew this, do you understand?"<br>"Who can explain what she's doing?"<br>Students comes to board (NOTE: this students was the same student as above who cried)<br>"She made a group of 4 and made 6 of them and these 3 (pointing to remaining 3 not circled) are the remainder" |  |
| | | Teacher:<br>"So these are groups of 4. So these, I use red, are the remainder 3.<br>What is this group of 4 she is talking about?<br>How many boats do we have that can hold 4?<br>What is this 6?<br>So, the number of boats.<br>What is the 3? What does that mean?<br>So it's not just remainder 3, it means people.<br>Is this 3 boats or people?" |  |

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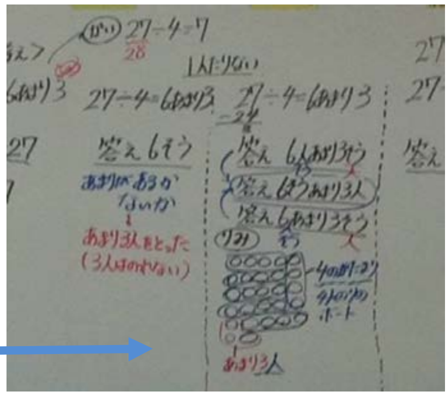
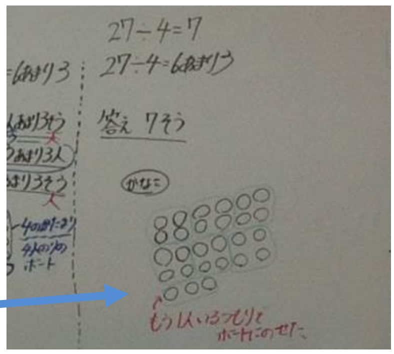
| | | Teacher:<br>"We have 3 different answers here so which is it?"<br>"So if we talk with the whole class we can correct mistakes, that's great. So you can correct yourself, that's a great thing."<br>"So, what's the difference between 6 and 6 r 3?"<br>Class responds- 1 has a remainder<br>Student: Took it ( remainder) out<br>Teacher:<br>"So they took the remainder 3 out?"<br>"What is the remainder 3?"<br>Teacher writes in red, remainder 3 taken away. |  |
| | | Teacher:<br>"So, let's think about 7 boats."<br><br>Student recording array on board and explains " there are 6 that I circled and they are boats but there are 3 left and they can't ride so if we do this all can't ride"<br><br>Teacher labeled Kanako's solution on board<br>Teacher:<br>"I forgot what she said, so who can explain again?"<br>Student:<br>"The last 3 can't ride so there is another person not here [the space left in the array to make a $7^{th}$ boat of 4 people] but we have one more boat."<br><br>Teacher writes on board – Pretend like we have 1 more person and make another boat. | <br> |

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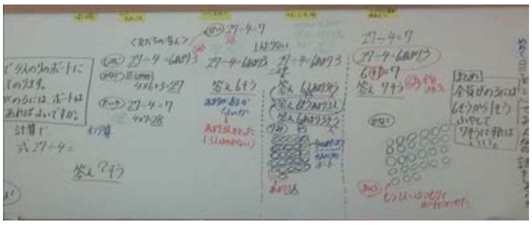
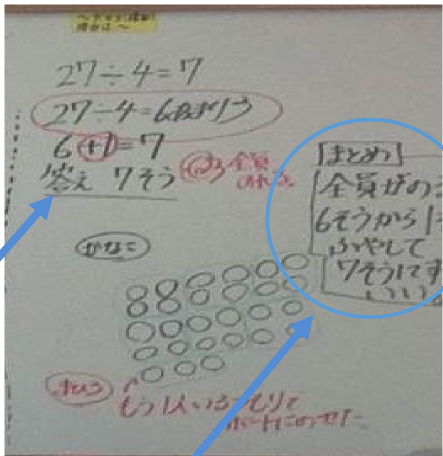
| 2:45 | Summary /Consolidation of Knowledge | Teacher:<br>“So if we look at this picture, think about your car. Do you have to have four people in it all the time?”<br>“So, is that ok? You don’t have to have another person?”<br>“So 4 X 7 and 27 is ok?”<br>“We have 27 ÷ 4 = 6 r 3 and we have three different ways. So 6 boats remainder 3, when is that correct?”<br>Student response: “when you <u>must</u> have 4 people in a boat” |  |
| 2:47 | | Teacher:<br>“So if we have a rule that we have to have 4 people. So if we change 27 to 24, the answer is 6. That’s great that you are learning to change the condition of the problem. I can’t see 7 in 27 ÷ 4 = 6 r 3, where is 7?”<br>Student responds: “28 ÷ 7 = 4”<br>Teacher: “So if we had another person, so the three people are not a remainder, what can we do?”<br>Student: “ That means you can’t put more than 4 in a boat”<br>Teacher: “So we have six boats for sure.” Records: 6 + 1 = 7<br>“What does this + 1 mean?”<br>Student response:- “ we are 1 person short from 28 but you don’t need to have another person in that boat…so another boat”<br>Student response:-“if we don’t have 7 boats then we all can’t ride”<br>Teacher:-<br>“Let’s summarise. What did you understand/learn for today;s lesson?”<br>“All people can ride.” “So what did you do?”<br>Records on board: We increased |  |

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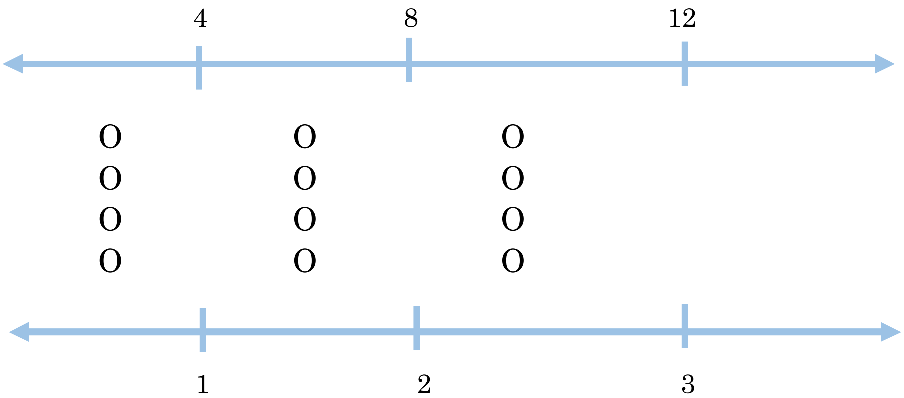
| | | by 1 boat from the 6 boats. | |
| | | Final board<br>Teacher makes the point that the last boat can go with only 3 people in it.<br>Teacher helps students think of situations for each of the three answers:<br>Answer 6) student explains: when there are 24 people, 6 boats are correct<br>Answer 6 remainder 3) When there is a rule that there must be 4 in each boat<br>Answer 7) student says: “it becomes 7 when it’s 6 remainder 3 and we need another boat and that boat has one seat remaining [empty]” |  |
| 2:54 | | Lesson Concludes | |

*What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?*

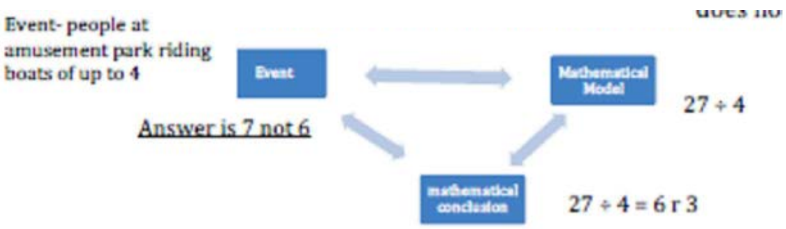
The teacher attends to precision through the planning process and anticipation of student responses:-

- The teacher, when he reflected, noticed students were just doing calculations and this reminded him to think about the context and add units.
- Prior learning e.g. the double number below is deliberately developed throughout multiplication across the school/grade levels.

4 x # groups



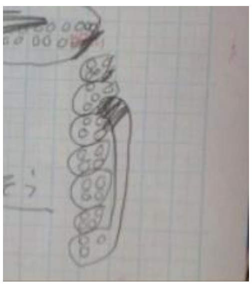
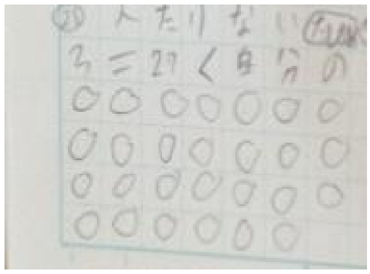
- The importance of word choice/language use in word problems is universal (across cultures). This is vital in supporting students' understanding of problems and how they can solve them independently.
- Students (and teachers) need a good understanding of multiplication before starting division. Japanese sequential order of teaching and learning, eg teach 4 processes in sequence not simultaneously, addition, then subtraction; multiplication then division always making links to prior learning whilst developing new learning
- Solving problems in real-world contexts: In this lesson, the answer isn't 6 remainder 3, the answer is 7 boats when relating back to the event (see below). In this case the mathematical conclusion does not equal the event.




- The importance of the different diagrams students use to show their Mathematical thinking.

(a) – grouping in fours (randomly)
(b) – groups of four and one group of three (remainder)
(c) – array, modeling groups of 4 (as above double number line)

(a) (b) (c)

*What new insights did you gain about how administrators can support teachers to do lesson study?*

- Knowledgeable Other ( vital- involving people from outside the school gives a much deeper insight and a different perspective and moves discussion away from mechanics/procedural discussion by sharing their deep content knowledge )
- Organizing outside observers to help create a rich and genuine debriefing
- Facilitating time for lesson study eg:-
  - the planning process
  - research lesson (dismissing students at lunchtime to release all staff in the school to attend the research lesson)

- post lesson discussion

- Commitment of principal and/or vice principal as a member of the research lesson steering committee and/or planning team
- Developing a whole school-based research lesson theme across subject areas. In this case the theme is 'Elementary career education'

*How does this lesson contribute to our understanding of high-impact practices?*

- A number of high impact practices have been highlighted through lesson study, for example;
  - Sequencing of student ideas and thinking. Through this lesson, it was obvious that a student response that was unanticipated by the teacher, was disregarded and therefore was not a part of the whole class discussion which helps to build the board work and movement towards understanding for the whole class.
  - The importance of choosing the "right" number in the problem to meet the goals of the lesson. In this case, the number 27 was selected more for the relationship to the number of students in the class, rather than the appropriate number to meet the goals of the lesson. A smaller number such as 19 would have still achieved the lesson goal, allowed students to draw a diagram more efficiently within the lesson and it is a small enough number for students to manipulate when having trouble accessing the task.
- Professor/knowledgeable other said, "students need to ask the question: "can we use mathematics to solve this problem; checking the calculation is important but needs to extend past simply checking the calculation". This illustrates the importance of deep content knowledge contributing to the post lesson discussion.
- He also highlighted the theme that the teaching of Mathematics isn't simply about calculation and process, it is about how to grow mathematically and how to grow as people. This linked to the school research goal- 'elementary career education'.
- The importance of a clear and specific goal was emphasised in the post lesson discussion. Teachers' content knowledge (built through *kyouzaikenkuu*) is vital in developing the goal, ensuring the task is aligned to the goal and appropriate to students in the class.

Lesson Observation (3)

Research Lesson Observation – June 29th
"I can figure it out without counting them all."

Sam Fragomeni
Heather Williams

What are the primary lesson goals?

*Students will be able to grasp number as "how many in one group" and "how many groups" and try to explain their ideas.*

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

*This is the first lesson in the introduction to multiplication. Prior to this lesson, students learned the benefits of repeated addition. This lesson introduces multiplication expressions and equations. Future lessons will help students to understand that they can determine the total number of objects without using repeated addition.*

| Start &End Time | Lesson Phase | Notes |
|---|---|---|
| 9:00 – 9:03 | 1. Introduction, Posing Task | The teacher activated prior knowledge by asking them if they remembered playing with the square tiles. They excitedly said "yes." He told them that today they would be creating buildings in groups using these square tiles. He showed them a nonexample. He said that there are rules: When you have two, they have to touch completely and they must use all of the tiles. To build interest, he gave them a paper that they would use to cover up their example once it is created. He stated that he would not tell them how many tiles they have and that they should keep this a secret. |

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| 9:03 – 9:07 | 2. Group Problem-Solving | The students worked in groups to create buildings using the tiles.<br> <br>Group 2:　This group initially tried to create a structure that was 15 x 2.　They found that this structure was too tall to fit onto the table, so they changed their structure to be 10 x 3.<br><br> <br>Group 3:　This group initially tried to create a building with a base of 4, but this caused them to have extra blocks.　They changed this to a structure that was 6 x 5 for their final structure.<br><br><br><br>Group 5:　This group built a structure with a base of 4.　This caused them to have 2 left over blocks, which they put on top. |

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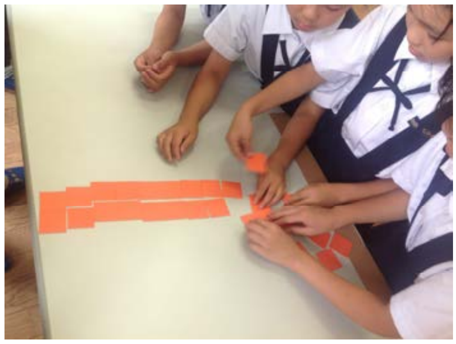
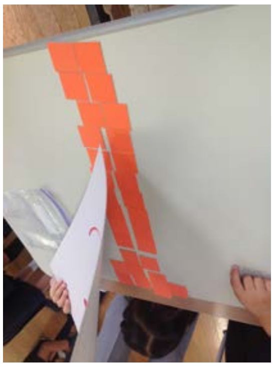
| | |    Group 7: This group initially build a structure with a base of 7 and height of 4. They had the final two tiles on the side of the shape. They abandoned this plan and went to a 15 x 2 structure so that they could have a rectangular shape. At some point, they dropped a tile (which means that they were left with 29 tiles.) This prevented them from making the shape that they wanted and they became very confused. The teacher eventually came over, found the tile, and added it to the table so that the students could complete their shape. The group couldn't fit a 15 x 2 shape on the table, so the base ended up being 4 wide w/ the rest of the tower being two wide. |

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| 9:07 – 9:12 | 3.Teacher posing of task | After the students completed their buildings, they went back to sit down. The teacher said "I have a task. Today we are going to try to guess the shape of the building and number of tiles in the other groups' buildings. Get together with your group and think about how to write hints that other groups can use. Do not use the number of tiles or the shape in your hint." The kids asked "can we draw pictures." The teacher responded that they will not be drawing the picture today and to use expressions instead of equations. |
| 9:12 – 9:22 | Group Problem-Solving | The 8 groups went back to their separate tables and each group came up with a different way of giving a hint to the other groups about what their shapes where. Each group wrote this "hint" on an individual white board that was then placed on the larger board magnetically:<br><br>This is what each group wrote:<br><br>Group 1: Across is 5 + 5 + 5 + 5 + 5 + 5 = ?<br><br>Group 2: 10 + 10 + 10 (This group initially wrote 10 + 10 + 7 + 3, but changed this before placing it on the board.)<br><br>Group 3: 3 down and 3 across<br><br>Group 4: Across 3 and down 10<br><br>Group 5: 7 fours and 2 ones, 4 + 4 + 4 + 4 + 4 + 4 + 4 + 2, 10 + 10 + 10<br><br>Group 6: 10 + 10 + 10 = ?<br><br>Group 7: 14 + ? = ?0, the tens place is 3 and the ones is 0<br><br>Group 8: 4 across and 7 lines |
| 9:22 – 9:44 | Presentation of Students' Thinking, | The teacher asked are some of these easier to understand. The students said "yes" and when prompted to name the one that was easiest, #4 was identified. The teacher took the 4th example and moved it to the other board. The teacher then asked everyone to draw this example in their notebooks using the hint given on the |

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| | Class Discussion | board:<br> <br><br>The majority of the student successfully completed this task in their notebooks.<br><br>The teacher asked "can someone come to the board and draw this building using the hint?" A student went to the board and built the building:<br><br><br>The teacher then asked the class "how many have this same building" and the majority of the class raised their hands. Two students were then called up to explain how they drew the picture using this hint.<br><br>The teacher then asked the class to identify any groups that had the same building (by looking at the other groups' hints.) Students from the class correctly identified groups 2 and 6 as having created the same building.<br><br>The teacher then asked would it be the same if he wrote 3 + 3 +3 + 3 + 3 + 3 + 3 + 3 + 3 + 3.<br><br>Next, the teacher asked if the students can draw the structure that |

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| | | group 5 made using the hint. The students agreed that this was difficult because group 5 “didn’t group it together. They didn’t make groups. They had 7 fours and a 2. That is the part I can’t figure out.” Students were asked to write the question “what makes group 4’s expression easy to make the building?” The students were then given a couple of minutes to talk about why this is the case. Students from the class decided that it was easier to draw a structure if it was “across and down.” |
| 9:44 – 9:47 | 4.Summary /Consolidation of Knowledge | The teacher then circled the first level of the building that had been created for the previous example and stated that this example was easier to understand because it is evenly 3 across. He then explained that groups make it easier and there are different ways of grouping. He showed that there were 10 threes and that this could be written as 3 x 10. “Ten groups of three is called multiplication:”<br><br><br><br>The teacher then showed that group 5 could have made their expression “easier” by writing 4 x 7 instead of what they had written.<br><br>The teacher closed the lesson by asking students to write “what made some explanations easier to understand than others in today’s lesson.<br><br>The lesson ended with the students writing their journal entries and turning them in on the table. |

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- Students seeing the merit of math is essential because they need to have a purpose for doing what they're doing.
- Teachers make a very conscious effort to extend students press math experiences so that the jump students have to make to learn new content is manageable.
- There is value in students struggling in order for them to resolve their confusion by asking meaningful questions.
- Teachers are constantly reflecting on their teaching and lessons. Even when a lesson seems to go perfectly well, teachers are still finding flaws in their student thinking and discussion.

- Students comparing and discussing each other's solutions methods is key to having a successful lesson.

What new insights did you gain about how administrators can support teachers to do lesson study?

- Administrators here are part of the lesson study process and are often times extremely knowledgeable about content and teaching strategies.
- Administrators provide time for teachers to meet and plan together.
- Administrators organize the events and allow teachers to focus solely on the lesson planning process.

How does this lesson contribute to our understanding of high-impact practices?

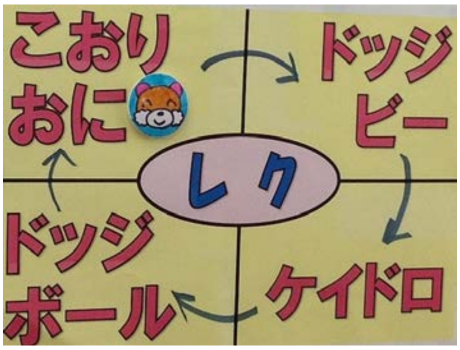
This teacher helped students link representations (words, pictures, diagrams, mathematical expressions) in this particular lesson. The teacher helped the students link the mathematical expressions to the buildings they built by making it into game-like environment where classmates had to figure out the building from the expression. The teacher also used this strategy when he circled the groups of 3 on the picture and then wrote 3+3+3+3+3+3+3+3+3+3 and then finally linked it to the math expression 3x10. This made it clear to the students where the groups were coming from and how many tiles were in each group.

Lesson Observation (4)

Research Lesson Observation Form：Utilising Remainders
Report by Aubrey Perlee, Mark Simmons, Ferida McQuillan

What are the primary lesson goals?
Students will recognize that problems may be solved by focusing on the remainders.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
Within this unit, students have begun using division with remainders. Students have seen the relationship between divisor and reminder and begun to process quotients and remainders appropriately, based on problem contexts. This was lesson 9 of 10.

| Start &End Time | Lesson Phase | Notes<br><br>(…) indicates a pause in teacher's speech |
|---|---|---|
| 10:00 | 1. Introduction, Posing Task | -Strategies to build interest or connect to prior knowledge<br>We made a few expressions which gave today's lesson number – 44, one of which was 10x4+4=44 – prefacing what was to come.<br><br>T reminds students that they'd been playing games together every Wednesday, how did we decide? We used a class survey. My teacher friend wanted to copy but her class had no Recreation Committee so she made a schedule. She places the class Recreation Schedule on the board.<br><br>T: discusses with students how we go back to the first game when one cycle is complete (adds arrow to schedule).<br>T: "What game will we play in week 8?" Teacher responds to student suggestion to count around the schedule so week 8 will be kohri<br>Class repeats the same process for week 12 (kohri again)<br>T: "Kohri, why is that?"<br>S35: "Because it's like a multiplication table, I was checking."<br>S11: "I did 4s table first but then was checking by going around" (moving around the schedule).<br>T leads the class (as they chorally respond) as she numbers each game with the week students will play it as she counts to 12. |

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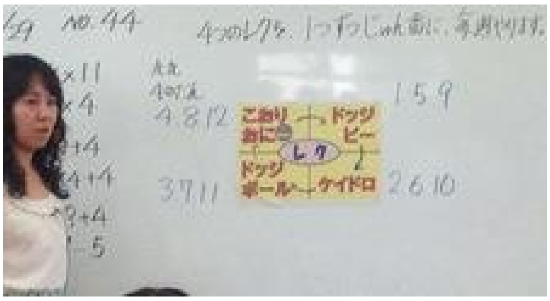
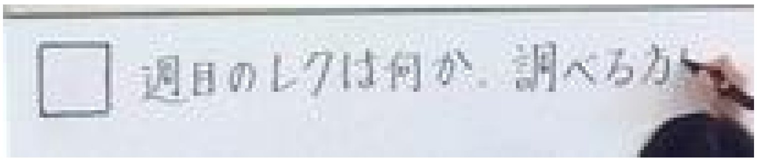
| | | T draws attention to numbers 1, 5, 9.<br>S4: "Looks like a multiplication table but skipping."<br>T writes & says "what's the game for week __"  (leaves the box blank)<br>Class explains they can't solve for an unknown week.<br>T: "Let's think about ways to find the game that we play in week 26."<br><br><br><br> |

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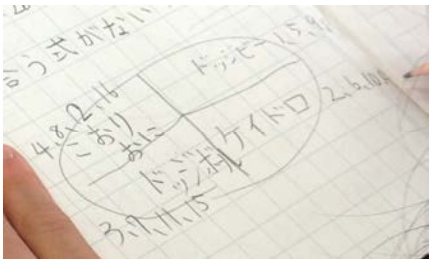
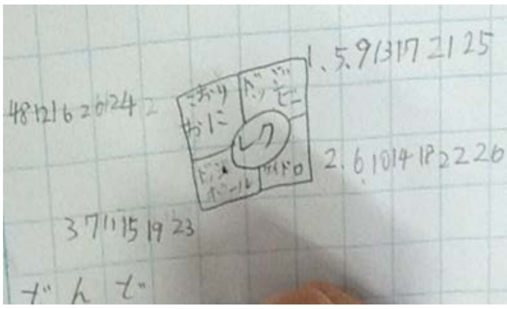
| 10:07 | 2. Independent Problem-Solving | -Individual, pairs, group, or combination of strategies?<br>Students works independently on the problem in their notebooks. The teacher walks around the classroom and checks in with various students. On her clipboard, she records student responses to determine who she will call on, and in what order for the comparison/discussion schedule. She goes back to the front and looks at a photo sample board plan she had made earlier.<br>  |
| | 3. Presentation of Students' Thinking, Class Discussion | Student Thinking / Visuals / Peer Responses /Teacher Responses<br>Photos to document chronology (use new box for each new student idea presented] |
| 10:12 | | S22: "I tried multiplication table for 4 but didn't find the answer for 26. So then I counted around 1, 2, 3, 4…until I got to 26."  Student comes up to the board and moves the magnet around the schedule.  As the child does this, the teacher writes the numbers beside each game and counts aloud up to 26.<br>T: "So what's the game for week 26?"<br>S22: "Keidro."<br>T: "Who counted it this way?"  Multiple hands go up.<br>  |

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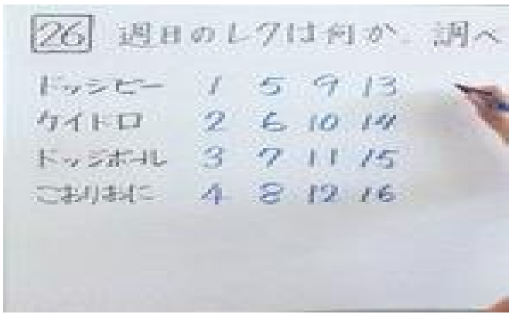
T: “I saw some people change the picture. I’m going to start drawing what they had. They wrote the name of the game (lists games).<br>
Students call out the names of each game in the correct sequence.<br>


T: “You know just by looking at this?” (pointing at the list). “Then…what did they do? (starts to write numbers up to 4 in response to students) Who can say the next one? (…) Where do we write that? (…) So 5 has to go next to 1 (…) Repeat? So can we go on now?<br><br>
T pauses at 24. Then continues to 26. “We finally got to 26, so the game for week 26 is keidro.”<br>
S27: “It looks like a calendar!”<br>
T: “So they changed to make a table like this one (writes note on the board). So what do you think?”<br>
S35: “It’s easy to understand.”<br>
T: “But that’s really tedious. It’s easy to understand but really tedious” (notes on the board that writing all of the numbers down is tedious). So if it’s tedious, is there another way we can figure out the answer more quickly and easily? Can we do it more quickly? How many think there’s a way?” (many hands raised).<br>





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| 10:19 | | <br>S35: “When we use multiplication.”<br>T: “I heard someone say multiplication. When you use multiplication tables, some weren’t sure what to do. Can you tell me the problem you had?”<br>S8: “I was doing 4s facts and looking for 4s facts that would give the answer 26. None gave it.”<br>T writes response on the board.<br>T: “Who had the same problem because no 4s fact for 26? Are you still not sure what to do? Ok, so can you explain?”<br>S20: Comes to the front and reads from notebook, “4 x 1, 4 x 2… but no 26 so I stopped right before that at 4 x 6 = 24 because 4 x 7 = 28.”<br>T: “What do you mean?”<br>S20: “I stopped at 24…26 – 24 = 2. You must go on 2 more steps. The game is keidro.”<br>T: “So how many understand what she said? (Records on board) She said 4 x 6 = 24, so what game is week 24? Students respond kohri.<br>T: “Add 2 more steps (models this). 24 is kohri, 2 more steps…so form the expression.” |
| 10:24 | | S29: “4 x 6 = 24 with a remainder of 2. It’s 2, it’s kind of strange, that’s the reason we take 2 more steps from kohri.”<br><br>T: “So what should we write here?” She writes 4 x 6 + 2 = 26. “We can make an equation like this, what can we say about this?”<br>S27: Comes up to the board and moves finger around the schedule 6 times “go around 1 is 4 x 1. Go 6 around 6 is 24. 2 more steps to the goal so you have to take 2 more steps.”<br>T: “What do you think about the explanation?...How many times do we go around?....Six times around for 6 weeks” (she notes this). “So 2 is what? It’s the remainder.” |

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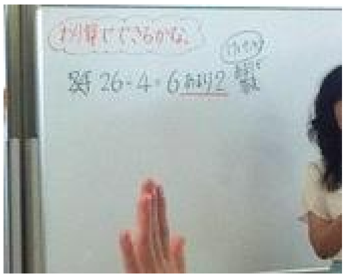
| 10:26 | | S: ñIt looks like how we check for division.ò<br>S35: “If it’s like checking the answer for division, can’t we just use division?”<br><br>T: “Who has just realized?....Who wanted to say that this was the checking equation?...So can we use division to find the answer? (She writes “Can we use division?” …there is a loud chorus of “Hai!”)<br>T: “How many of you think we can use division?...Some are not sure…..Who can explain?”<br>S06: “26 ÷ 4= 6r2 So the remainder is the answer” (teacher records this on board).<br><br><br>T: “Wait, wait, S6 said the remainder is the answer…so the remainder of 2, what does that mean? (Many students shout “Hai!”)<br>T: (Gesturing at the table of numbers) “What’s that 2 for?”<br>S28: Walks up to the board & uses magnet to go around 6 times.<br>T: Sets up a chant of 1,2,3…6 times around. So 24 is what?..24 people?<br>S28: “No, 24 weeks, so this is week 24. Remainder 2 is not 2 times around, just go 2 more steps.” …(T scaffolds his use of the schedule to count on).<br><br>T: “What does the remainder 2 mean? The remainder 2 means that from here (points to top left cell of schedule) you have to go on 2 more steps – looks like those games that we play!”<br>T consolidates the understanding, gesturing at the expression she says “26 is?<br>S: “Week number 26.”<br>T: “4 is what?”<br>S23: “Divided by 4 means there are 4 different games, so it’s the number of games.”<br>S: “To go around, you play 4 games.”<br>T: (Noting) “Number of games – there are four types of games…so we can do this division – is this ok?...so take 2 more steps…but where from?.....<br>S01: “So from kohri take two more steps.” |

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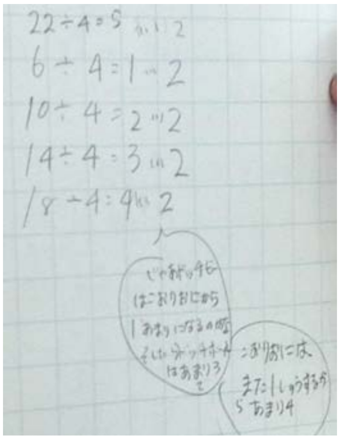
| 10:35 | | T: “Are there other games like that? Are there other numbers two more steps from kohri? Ok well, week 26 we played kohri, what about week 22?...are there others?...which ones are two steps after kohri?<br>S : “22 has 2 more steps from kohri. From 20 you take 2 more steps. All the numbers for keidro is 2 more steps.”<br>T: “So, all numbers for keidro are 2 more steps from kohri…ok (noting), so number of steps for keidro always…is it always the case that the week number for keidro is 2 more steps from kohri?...or…the number for keidro…do we always get r2?”<br>T: “Ok, so let’s check a few, 22÷4….18÷4 (gesturing at table) ..just pick one number and write the equation in your notebook, ok?...shows us that the number for keidro always gets a remainder 2. (Checks around that children are doing this)<br>S35: “Yes, we always get remainder 2 for kohri. Can we do all of them?”<br>T: “Yes”<br>S: “But we can’t divide by 2”<br>T: “Are we dividing by two?”<br>Students work independently to check for remainder 2 for all keidro numbers. T circulates the class, points at the board to help a child. After 90 seconds, T leads class in checking their answers. She records all of these on the board. PHOTO<br>T: “We haven’t done this yet, but, 2 ÷ 4 = ….” She has the class count down by looking at the pattern of the previous quotients: 5,4,3,2,1…0.”<br><br><br>T: “So, check, the number for keidro is always r 2.(Notes: the week number for keidro always has remainder 2) |

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| | | T: "So, for the other games, the same? Let's check with dodge bee…Which week do you want to try? (Students yell out answers)<br>S35: "Dodge bee 25 ÷ 4 = 6r1 r. The remainder for dodge bee is 1.<br>T: "Because four games –each time I go around four…so, 4 x 5 = 20, 4 x 6 = 24, so 25-24 = 1, so 6 r 1…so, do you think the number for dodge-bee is always r1?<br>Class responds "hai"<br>T: "So if dodge-bee is 1 and keidro is 2…what about dodge- ball?<br>S: "Dodge bee is 1, keidro is 2, and I think dodge ball will be 3.<br>T: "Try 23…23 ÷ 4 = 5 r 3"<br>S27: (Looking at the pattern of previous work on the board) "It goes around so kohri will be remainder 4."<br>T: (noting/ speaking): " Kohri…just around, so must be r4? ***Strategic pause<br>S: "No, remainder can't be 4."<br>T: (notes/says: remainder 0)…so, if we look at this we notice 0,1,2,3 dodge-bee 1, keidro 2, dodge-ball3, kohri 0. |
| 10:44 | 4. Summary /Consolidation of Knowledge | Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals.<br>T: "So, do you think we can figure out the answer very quickly? So we looked at week 26, so… (students shout)…let's…what week? (students shout)…27? Too easy? Ok. Let's try with week 31." (records "what is the game for week 31)<br>T: "What do we do?"<br>Students all working together on the same problem & yelling out responses.<br>S35: "31 ÷ 4 = 7r3 Because the remainder is 3, the game for week 31 is dodge-ball." (T inserts prompts into the students' thinking)<br>T: "So because the remainder is 3, the game for week 31 dodge-ball. So what are we checking?"<br>S18: "You have to check what's the remainder. In week 31, the remainder so we can conclude that the game for week 31 is dodge-ball."<br>S27: "What S18 said, the remainder is the answer."<br>T: "So now we know that 31 has remainder 3, we can put that in the remainder 3 team. So this is what S27, that the remainder is the answer. (Students shouting out new numbers). So you want to try with 100, we haven't done this in class yet.<br>S35: "100 ÷ 4 = 25r0"<br>T guides as class shouts responses: "I thought you split these up…maybe 4 x 10 = 40, 60 more left? Another 4 x 10 = 40…80? So how many more left? 20, so 4 x 5 = 20 and that makes 100. 40 and 40 and 20 makes 100. So the answer is 25, remainder is 0. You understand? The remainder is 0 so because the remainder is 0…the game is?<br>Class: "kohri."<br>T: "So we know we'll be playing kohri in week 100. That's the end of the |

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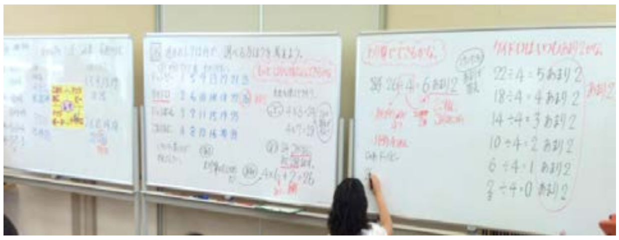
| | | lesson.　Please write in your journals."<br>Students write their summaries/reflections in their journals as she circulates.<br>  |

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- Importance of setting a clear context, in which pupils are genuinely engaged in (story telling – the pupils do games on such a schedule).
- Importance of spending time so ALL students understand the problem situation.
- Asking students to attend to efficacy, ease, speed of different methods and compare these (supported by board work).
- Working slowly and asking students to explain meanings of numbers/units in context (this is strongly supported by the curriculum which, for example, orders multiplicand by multiplier in a consistent way).
- Teachers can help many students to progress by attending to the sharing of some people's ideas and the explanation of these by others.

What new insights did you gain about how administrators can support teachers to do lesson study?

- Principals are instructional leaders, usually part of the team who writes the research lesson.　Several meetings are scheduled for the development of the initial lesson plan.
- Staffroom layout in grade level bands helps teachers to share with each other and collaborate on a daily basis
- There is always a school or district theme for research lessons, which means that teachers across grade bands/subjects are thinking about how to develop similar capacities in students.

How does this lesson contribute to our understanding of high-impact practices?

- The strong use of anticipated responses, diagrams/tables, student explanations and teacher coordinated whole class discussion, which uses small steps in student thinking to move many students deeper in their understanding.

- The teacher had a clear view of the goals for the lesson. Instead of a show-and-tell, she knew how to take student responses and move them to the next level. She knew her students and the curriculum/what they had already been exposed to.
- Students in this lesson were accustomed to thinking about "what other things have we learned before that will help us solve this problem?"

## Lesson Observation (5)

Lauren Moscowitch and Doreen Stohler

Group Observation for July 1st at Matsuzawa Elementary School

Division Word Problems- Grade 3

**Goal of the lesson:**
By comparing and contrasting partitive and quotative division problems, students will understand that there are two types of division: division to find the group size and division to find the number of groups.

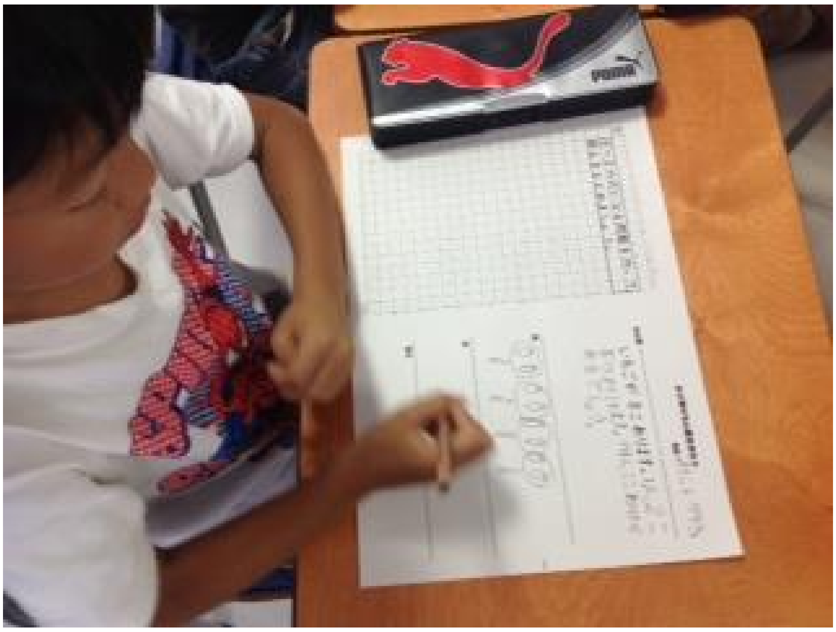
**Where is the lesson located in the unit?**
The current lesson is number 5 out of 13 lessons in the third grade unit on division. In grade 2, the students have learned the meaning of multiplication and constructed the basic multiplication table. In Grade 3, the students learn the properties of multiplication as well as the multiplication algorithm. In grade 4, they will learn the division algorithm as well as decimal multiplication and division. The grade 3 unit on division is very important for students' future work in division because Japanese teachers learned that some students were having difficulty with division in Grade 4. Their mastery of the basic multiplication facts was unsatisfactory. In this unit, we want students to be able to write appropriate expressions/equations by relating multiplication and division and making clear the type of the unknown quantity.

**1- Introduction**
Kawabata Sensei attempted to link prior work to today's lesson. She initiated the lesson by asking them to reflect on their journal. The students proceeded to read yesterday's work. Prominently displayed in the classroom were many posters explaining the previous four days' lessons.

Kawabata Sensei posed the problem to the students ("Today we are going to write story problems for division. We are going to share eight strawberries."). Initially Kawabata Sensei took all student responses and helped the students be precise in the communication of their ideas as to what the "2" meant in their answers: 2 people or 2 pieces? Kawabata Sensei asked a student to explain another student's thinking during this initial share out of ideas to clarify what the "2" meant. Once the student clarified that they would give 2 strawberries to each person, she asked "Is there another way?" At this time a student said "I think we can use subtraction." Sensei said "No, we are sharing." In reflecting on the sensei's choice of words for that studentôs response, we might have connected that studentôs thinking of repeated subtraction to partitioning. We need to link the subtraction with the division. On the board were eight pictures of strawberries. The strawberries were arranged in a 2 x4 array.

**2- Independent Problem-Solving**

**T**he students listened carefully to Kawabata Senseiôs introduction and to their classmateôs responses for approximately five minutes and ten seconds. The students were then directed to begin working on their story problems. As they began to work independently, she asked, ñDoes anyone need a hint?ò There were no responses. The studentsô worksheet was legal-sized and double-sided. There were four sections on the right half of the page, and the studentsô task detailed on the left side of the page. The first section on the right side was for writing the words to their own story problem. The second section was for a picture or other visual that matches the story they wrote. The third section was for the number sentence. The last section was for the answer. The studentsô task on the left side of the page read, ñWrite story problems that show 8÷2.ò Many students got to work right away on their story problems. The students enthusiastically used pencils, colored pens, crayons, and markers to illustrate their drawings and highlight different portions of their work. During this time, Kawabata Sensei walked around the classroom and monitored the studentsô work. She coached students as needed. We noticed Kawabata Sensei gave four students each a magnetized white board. The students were instructed to write their problems on the white board and then display it to the class. During the independent work time, many students completed one problem, but did not finish or even begin their second word problem.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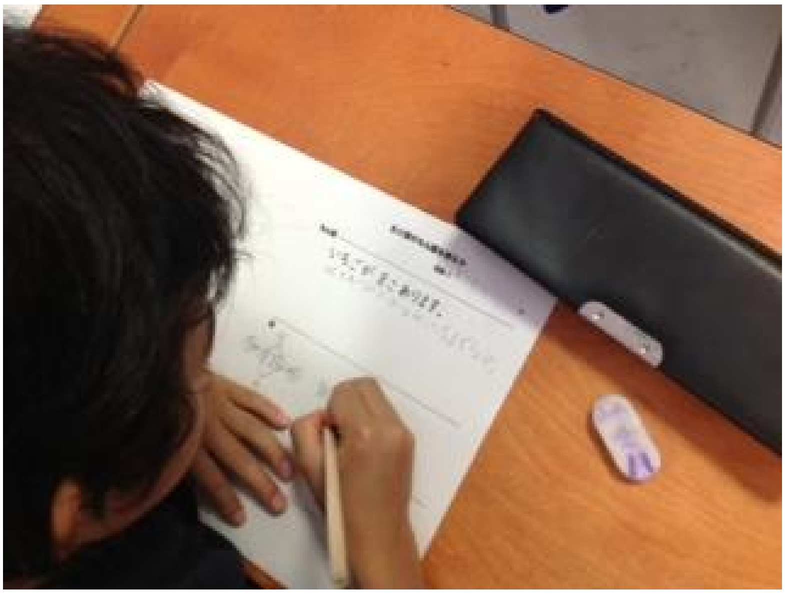

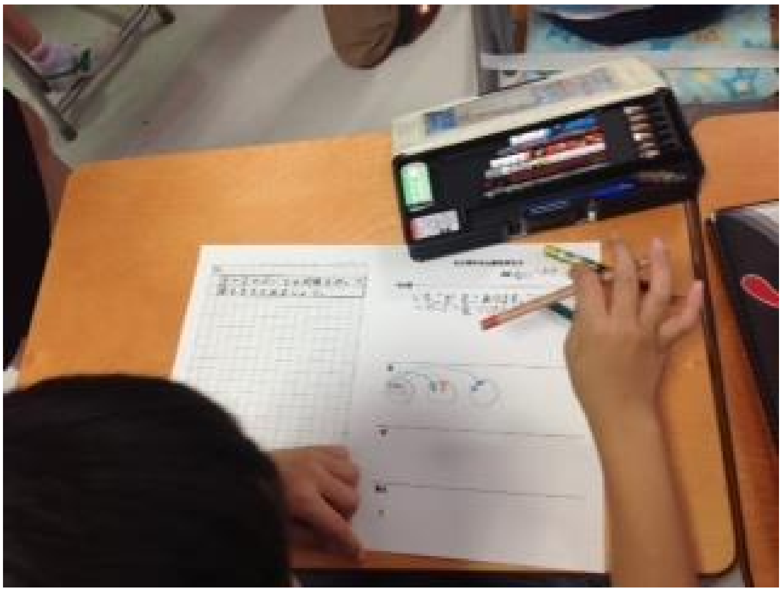

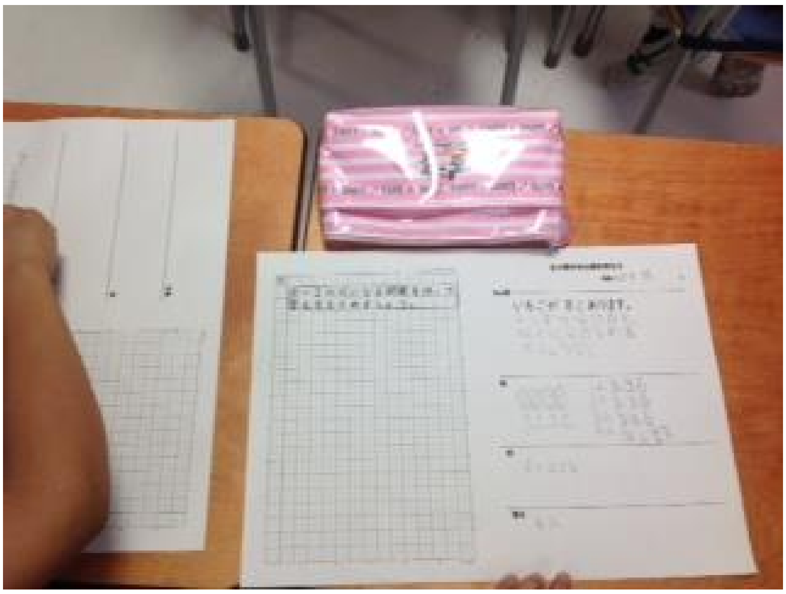

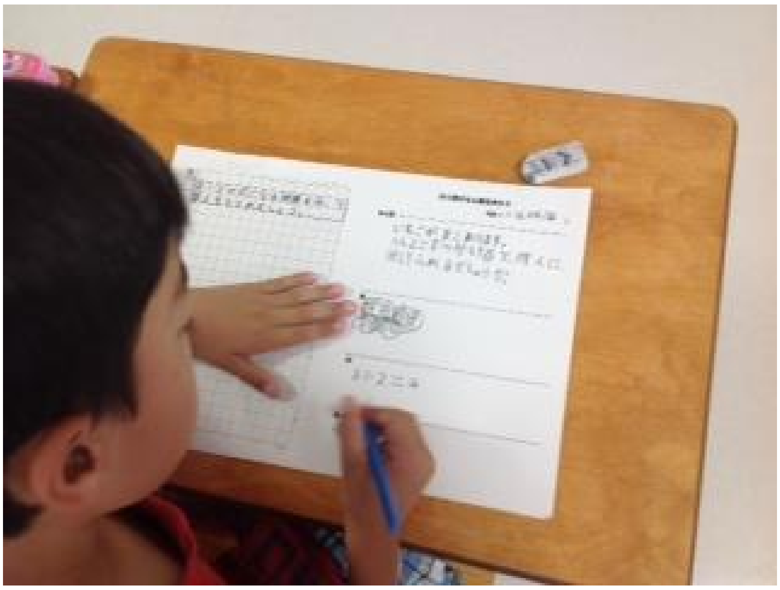

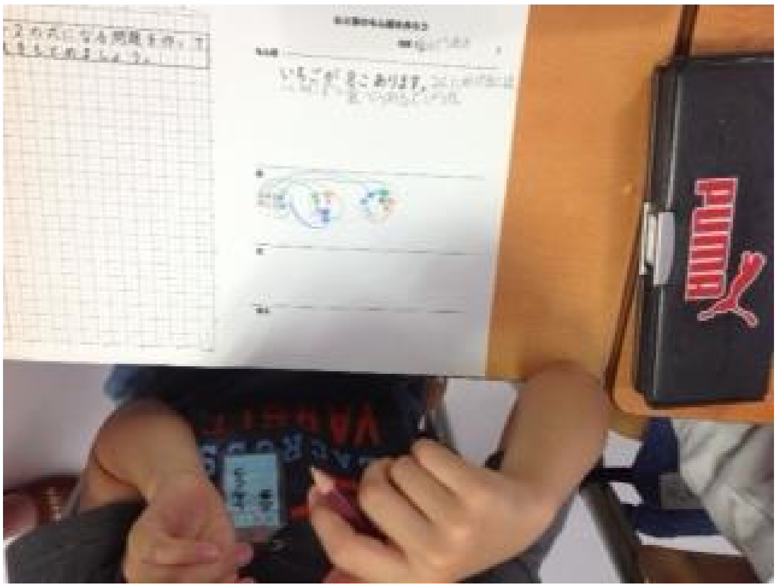
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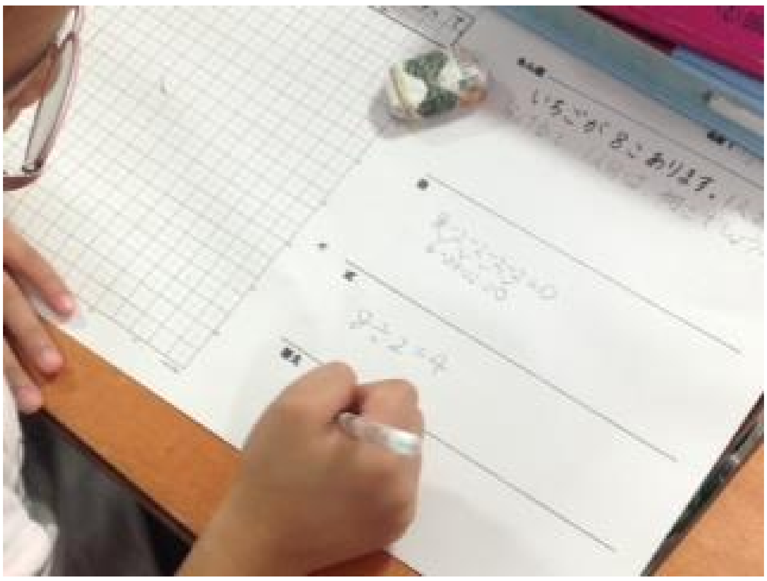
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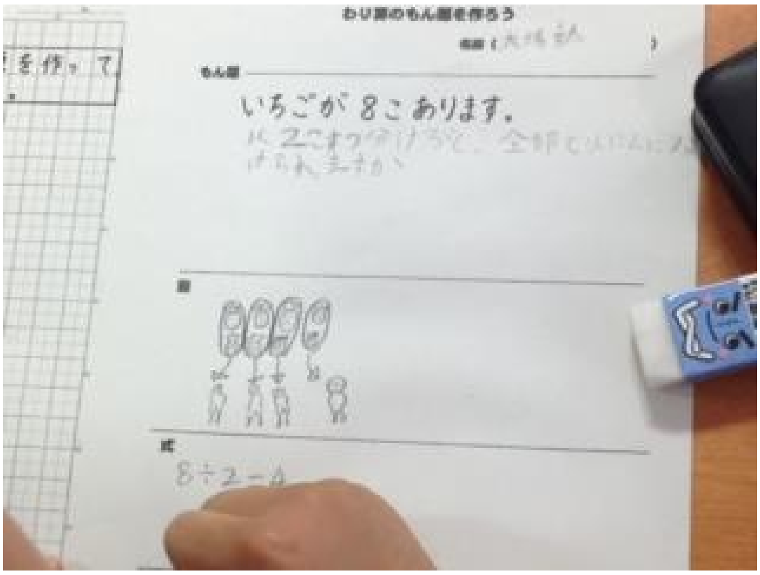
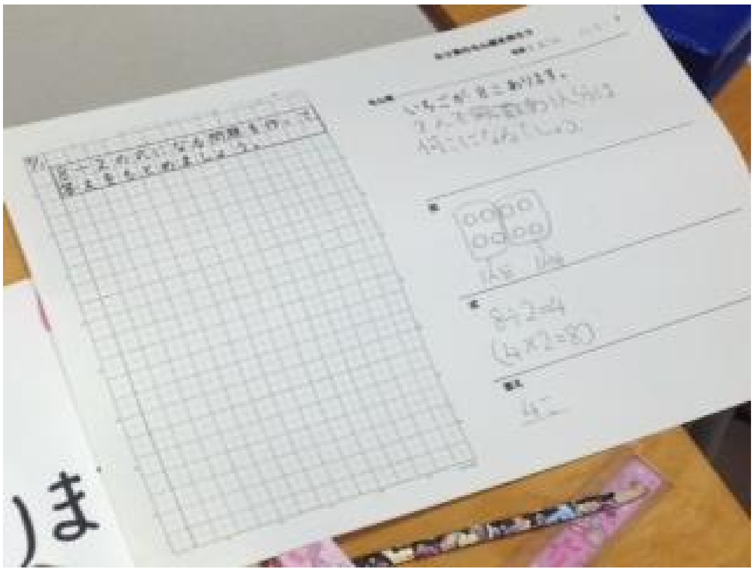
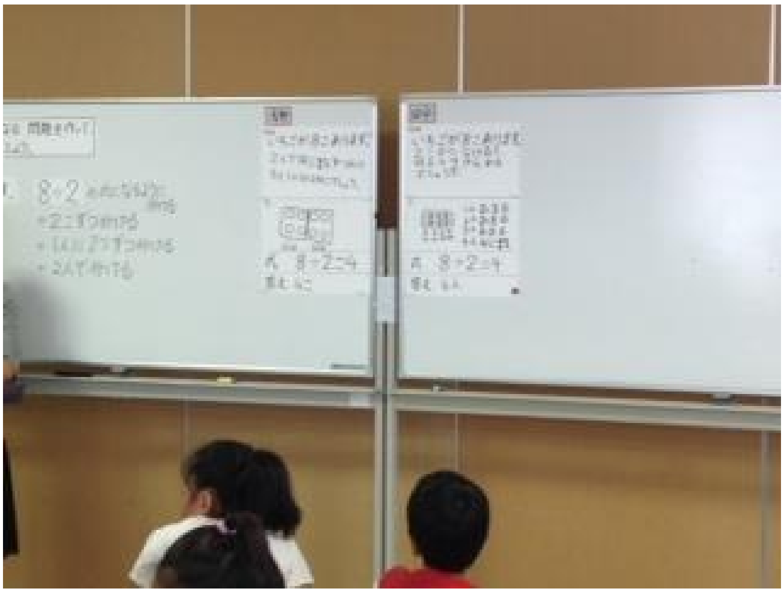


**3- Presentation of Students' Thinking, Class Discussion**

After approximately 18 minutes of independent work time, the studentsôattention was drawn back to the board for the discussion of their work.　There were two student whiteboards on the large chalkboard. On one side of the board, partitive division was represented and on the other side, quotative division was represented. Kawabata Sensei said, ñThese two are on the board already and two more are coming.　How are these alike and how are they different?



Kawabata Sensei wanted the students to match the whiteboards to the responses that the students gave earlier that were already on the board. Students raised their hands and matched one student whiteboard to a response on the board. Kawabata Sensei asked, ñWhat is similar about these two?ò A student replied, ñGive 2 pieces to each.ò Then another student clarified, ñGive 2 pieces to each person. Go one by one, give 2 pieces to each person.ò The students confirmed that the two problems were both about two people sharing the 8 strawberries. Each person has the same number of strawberries. Kawabata Sensei said, ñLook at your story. Which problem is yours like?ò The students then put their names on the board underneath the story problem that matched their own.

Each story problem was about equally represented by the class. Kawabata Sensei asked, ñWhat do you notice when you compare these problems? You have 20 seconds to talk about it with your neighbor.ò The students decided that the stories on the left described 2 people each getting 4 strawberries while the stories on the right described 4 people getting 2 strawberries each. The students also agreed that the equation and answer was the same, but the unit that goes with each answer is different. Kawabata Sensei confirmed that what is being asked in the questions is different. A student clarified what Kawabata Sensei meant when she said that the question is asking different things: in one problem there are only 2 people while in the other problem there are 4 people. Another student chimed in with, ñItôs like addition and subtraction. Thereôs just a different question.ò We believe the student was referring to fact families like 8-4=4 and 4+4=8. The student noticed that it is possible to have different stories for one equation. Kawabata Sensei confirmed that one side of the board described how many units (strawberries) while the other side described how many people. She wrote notes on the board indicating such. Kawabata Sensei asked, ñHow did you get 4 in the equation?ò The students replied with ñ4x2=8, so 8÷2=4.ò Kawabata Sensei and the students confirmed that the 2 means strawberries, the 4 means people, or groups, in this particular story, and that 8 is the number of strawberries in all.

At this point in the lesson Kawabata Sensei wrote the storyôs division and multiplication sentences on the board. She wrote:

8 ÷ 2 = Ĭ                    8 ÷ 2 = Ǐ

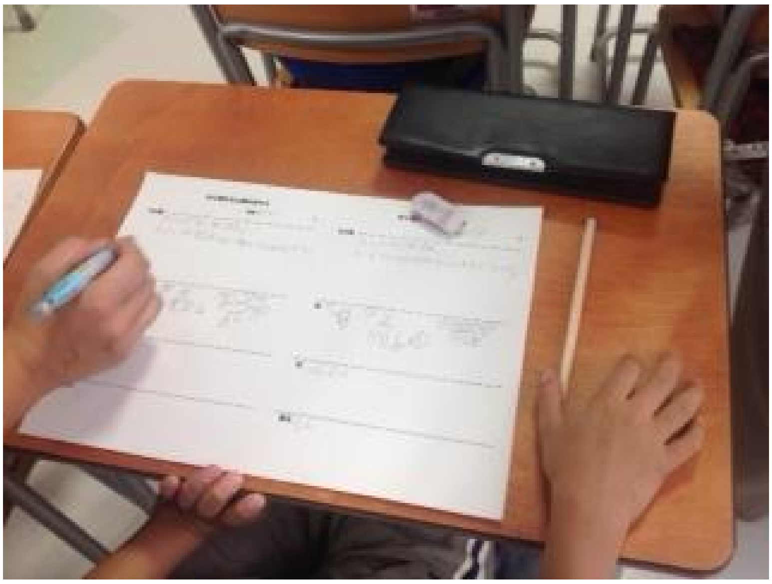
Ǐ x 4 = 8                    2 x Ǐ = 8

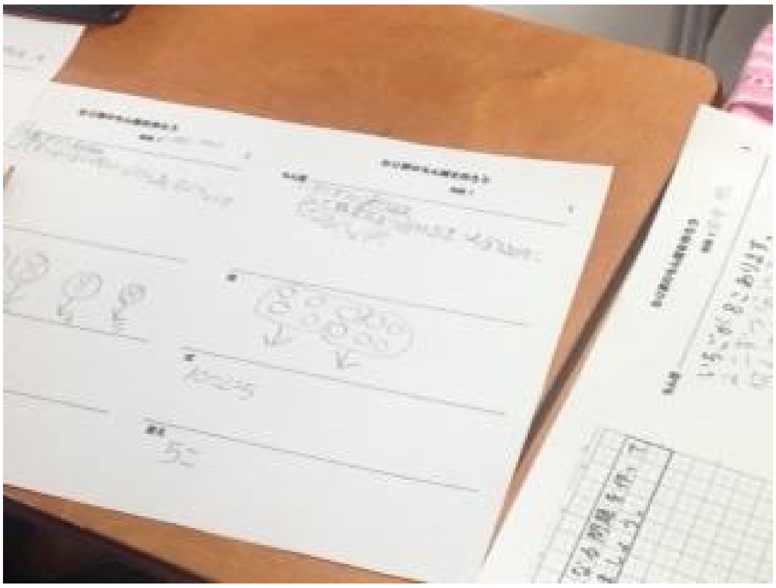
While explaining the sentences and asking the students for their input, Kawabata Sensei noticed a mistake she made. She changed the second math sentence on the left to read:

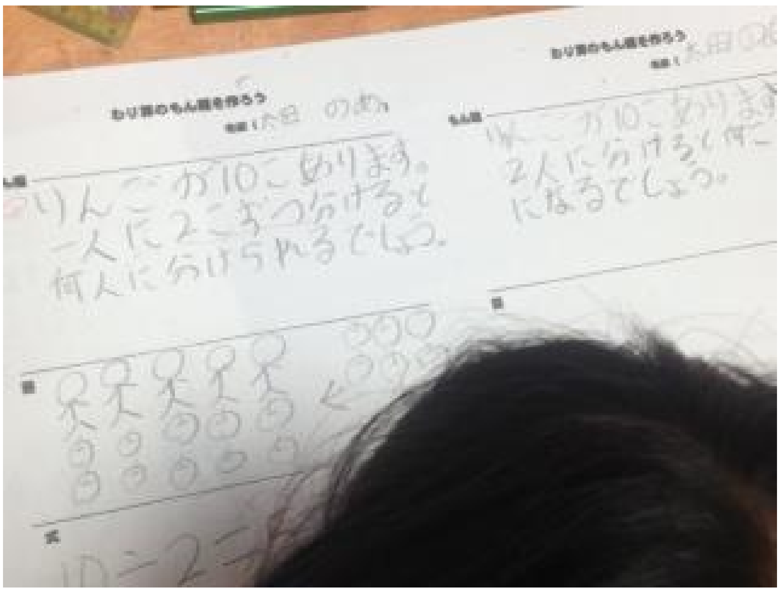
Ǐ x 2 = 8

She explained that the box in the sentences on the left is asking how many strawberries for one group, while the box in the sentences on the right is asking how many groups/people there are. At this point in the lesson, many students became confused. We speculate that in the beginning of the lesson, the students thought they understood division, but now with this new information (and the teacherôs mistake), they became unsure. The studentsô confusion was evidenced by their lackluster efforts to begin the second round of independent work.

After Kawabata Sensei finished explaining the math sentences, she instructed the students to write 2 more number stories that matched 10÷2. One boy immediately shouted out, ñI donôt think I can write 2.ò Kawabata Sensei walked around the classroom and coached as necessary. One boy in the back of the room started his story with ñI have 10 oranges.ò He was stuck at this point. Kawabata Sensei encouraged him to look at the board and decide which problem he wanted to do. She coached him through the rest of the problem and walked away. The boy continued sitting there. Another teacher came over and helped him after only a few moments.



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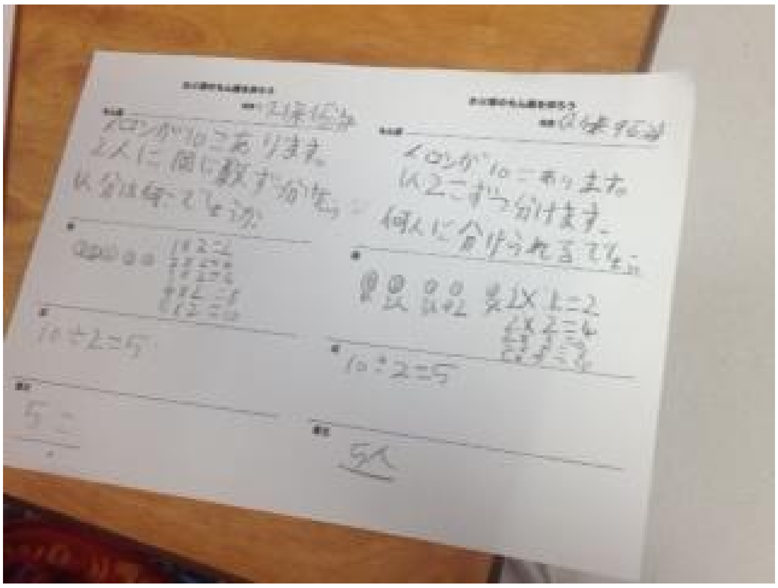
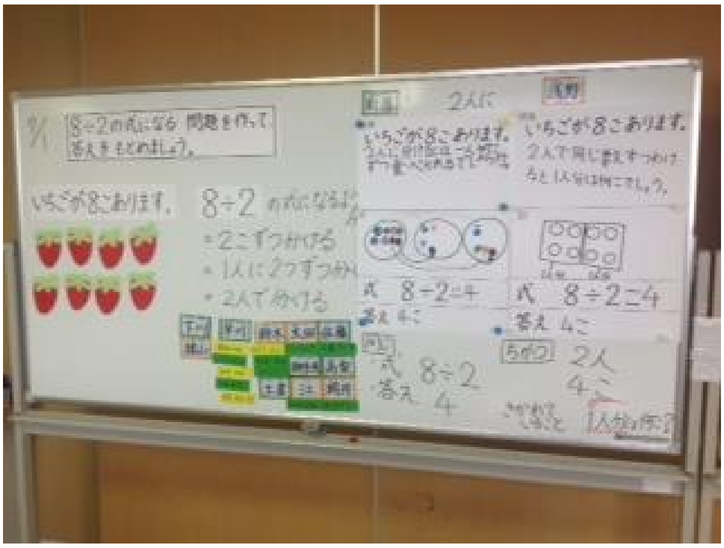
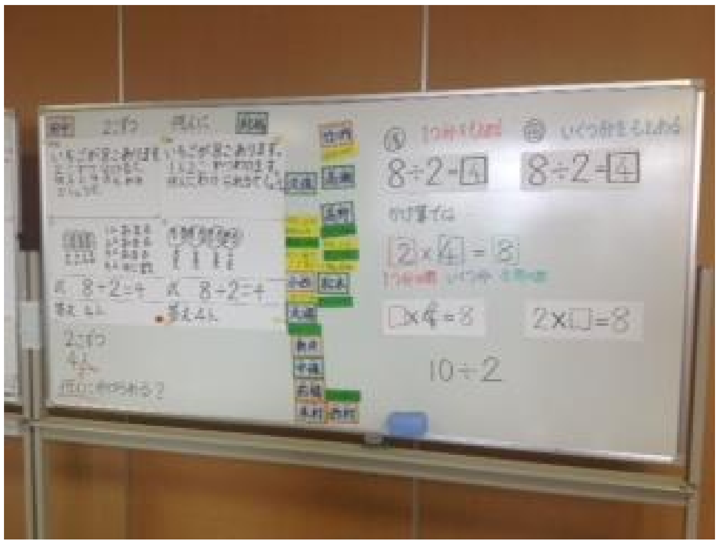


There was evidence that some students understood the goal of the lesson. The picture above shows a student who clearly understands both partitive and quotative division. This student was able to write stories that showed melons being divided in different and distinct ways.

**4- Summary/Consolidation of Knowledge**

The consolidation in this lesson was very brief. It was clear that time was running out because when Kawabata Sensei called the studentsôattention back to the board, she said, ñI know youĝre not done yet.ò She asked the students if it was easier to see the difference in the problems if you looked at the pictures? The students agreed that it is easier to see the difference when looking at the pictures to match the equations. Then, the worksheets were collected and the lesson was over. Students were not asked to write a summary or reflection in their notebooks.





**What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?**

The post-lesson discussion explained how the students learned previously that division is the inverse of multiplication. Todayȏs lesson was supposed to give the students many different opportunities to explore those inverse relationships. However, the connections were not developed yet. There was an explanation in the middle, but it took too long. We learned that the connections between multiplication and division are not apparent to the students immediately, but will take time to develop.

As stated in the lesson, the *focus* of this research lesson was making the distinction between quantitative and partitive division based on the relationship between division and multiplication. For this, students needed to think about the meaning of each equation and the meaning of the unknown quantity. In addition, the students needed to realize that the quotients can be determined by multiplication, and the difference is what quantity is to be found. But many students needed more time to develop a deeper understanding of the two types of division so as to be able to represent the problems clearly in diagrams and to be able to explain their thinking to others. Suggestions were made that might help students represent their thinking more clearly such as labeling the boxes or the 'unknown,' or allowing students to use magnetic circles (strawberries) to help them explain, and asking students to label the units of their answers.

However, it was brought out that to develop the conceptual understanding students need more opportunities to develop the ñcharacteristics of a setò rather than the ñgroups within the set.òIf we provide many examples of division problems and allow students to sort them according to characteristics, then perhaps they will more easily recognize the similarities and characteristics of each set. They will think mathematically, logically, and autonomously when presented with different division situations. In addition, the real focus should be on the meaning of the calculation itself ..what is the meaning of division?

Many additional relevant issues were brought up in the post-lesson discussion: 1) how to variate the lesson, 2) how to evaluate student understanding during and the following the lesson, 3) how to par down the written lesson plan so the important points are clear,4) what criteria constitutes understanding and, 5), the in-congruency between students' voices and the lesson goals.

What *does* constitute understanding and how will we know if students got there? Is it enough evidence of understanding that students can write sentences and draw pictures? I might be able to explain, (language), how to get to the train station, and even draw how to get there, (visual),...but can I actually get there, (experience)? All three were identified as important criteria in developing understanding during the post-lesson discussion.

The final point, (5), the in-congruency between students' voices and the lesson goals, was an important consideration for us. The desire to learn was not there. The analogy was made to approaching people who were walking on the street trying to get to the train station and giving them directions to the train station, versus stopping people walking on the street so you can tell them how to get to the train station. Will the latter group stop and listen to you? Do they care to know? The student is the subject, and we need to design the lesson so they want to know.

**What new insights did you gain about how administrators can support teachers to do lesson study?**

The administration supported teachers during the cycle of lesson study by reminding the teachers that the focus should be on assessing whether or not the students have learned the material. The administration supported the teacherôs decision to push forward with the lesson, but also reminded her that assessing the material is of foremost importance.

In general,

- Administrators may be a part of the Research Steering Committee
- Administrators help to organize monthly district research lessons
- Principals are instructional leaders involved in every step
- Lesson study is a school vision, belonging to *all* members- not just to a lesson study team.
- School research themes extend to the whole child...not a slice of content.

**How does this lesson contribute to our understanding of high-impact practices?**

The first half of this lesson was a great example of how students can learn from each other and explain each otherôs work. The teacher facilitated discussion between the students to have the students explain as much as possible. The teacher on several occasions had the students clarify or explain what another student said. This strategy is high-impact because it keep the students thinking and critiquing the work being presented. The teacher created a very warm and open environment for the students to feel comfortable expressing their ideas and presenting their viewpoints. Not every student in the class was on task, but it is possible each student learned from the discussion taking place.

By posting student work into two sections on the board, **A**, (partitive), and **B**, (quotative), she demonstrated a wise decision to focus on capturing the similarities and differences of each type of division story and avoided the technicalities of wrangling with new vocabulary at the same time.

The bansho, or story of the lesson played out across the board, helped students to continually examine both division stories and reflect back to their own work for clarification or modification. It also assisted students to advance to the second question.

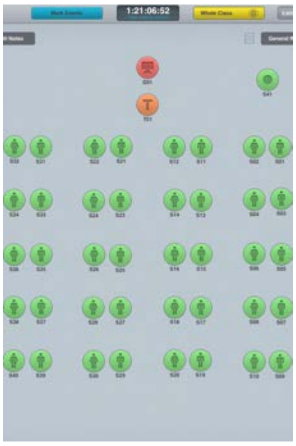
Using the Mathematical Practices established in the CCSS, there was evidence that most children made sense of the problem $8 \div 2$ in at least two different ways. Many of them worked to describe a situation where $8 \div 2$ could be used to solve a problem. They described the situation in drawings, equations and, in some instances, verbal explanations. The students modeled with mathematics as evidenced by their pictorial representation to describe the division story and subsequent mathematical equations to explain, using both multiplication and division, and even repeated subtraction. Not only did students have to reason abstractly and quantitatively, but they were asked to compare their thinking to the thinking of others and determine which thinking was most like their own. They were looking for and attempting to make use of the structural relationship of multiplication and division in solving problems.

Lesson Observation (6)

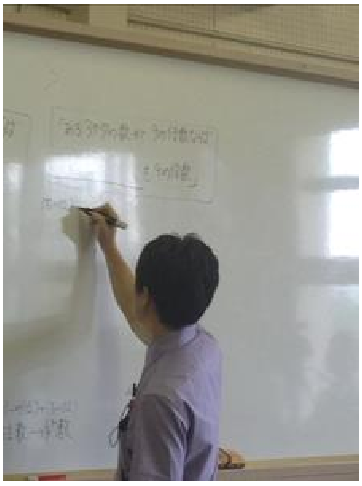
Research Lesson Observation Form (Use photos to document each section)
Koganei Junior High School Grade 8 Teacher Hideyuki Kawamura
Elnaz Javaheri & Jeffrey Glenn 2013 Study Group

What are the primary lesson goals? Students are to generalize the statement by interpreting the process of explanation using algebraic expressions and grasp the mechanism of the expression.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)? This lesson isthe third lesson in the Calculations with Algebraic Expressions




This is the seating chart for this lesson.

| Start &End Time | Lesson Phase | Notes |
|---|---|---|
| | 1. Introduction, Posing Task<br>2:22-2:24<br><br><br><br>2:24-2:26 | -Strategies to build interest or connect to prior knowledge<br><br>The teacher Mr. Kamakura asked students to re-explain if a two-digit number is a multiple of nine. He asked for Matsuda, a boy who came up with a way to represent two-digit numbers as a multiple of nine in algebraic form, to say what the exact algebraic form was that he discovered the day before.<br> <br>The student came to the board and wrote out his equations and then he explained to the class his statements. Students were taking notes on their papers. X is the variable for the tens place digit Y is the variable for the ones place digit<br>$10x + y = 9z$<br>$x + y = 9z - 9x$ |
| | 2. Independent Problem-Sol ving<br><br>2:27-2:28<br><br><br>2:29-2:30<br><br>2:30-2:30:34<br><br>2:31-2:31:28<br><br>2:32-2:32:15<br><br><br><br>2:33-2:33 | --Individual, pairs, group, or combination of strategies?<br>The teacher had the students work on this lesson individually by writing on a paper that students could take notes.<br><br>T. asked to the class to think about what you could change in this equation $x + y = 9(z - x)$. He gave them about a minute to think about this question.<br>A boy in the front row, right side (S1) of the classroom responded by saying "you can change two digits to three digits."<br>A girl in the back left center row (S28), said " I want to change the multiple of 9 to something like 3."<br>A girl in back, first row (S38), said, " I want to change the multiple number to factors"<br>The teacher said to start with the first boy's (S1) suggested change of going from a two digit to a three digit number. He wrote out a sentence in Japanese with a blank line equaling a multiple of 9.<br>T. says "you can add the digits of each place value (100's place + |

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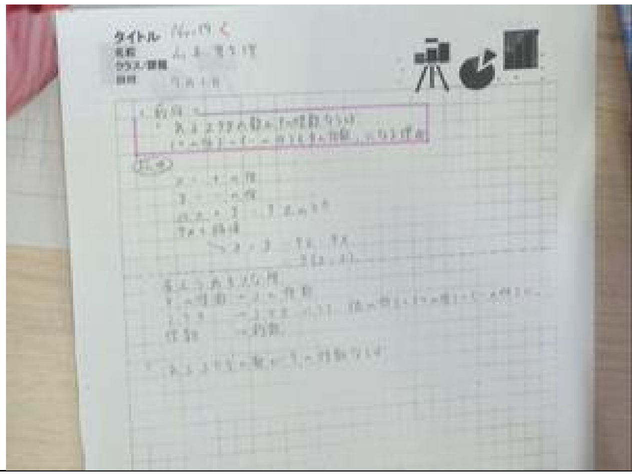
| | | 10's place + 1's place)" which was based on the boy's (S1) first suggestion. |
| | 2:33-2:35 | T. said " is there anything else you can change in this equation?" He waited for about 1 min. 42 seconds.<br>Since students were having trouble coming up with another way to change the equation, he wrote an example on the board: 189 → 1+8+9 = 18. |
| | 2:35-2:37 | "Can you think of another way to split the numbers differently?" Long pause by the teacher waiting for a student to answer. He walked around the room looking at student's working in their notebook.<br> <br> |

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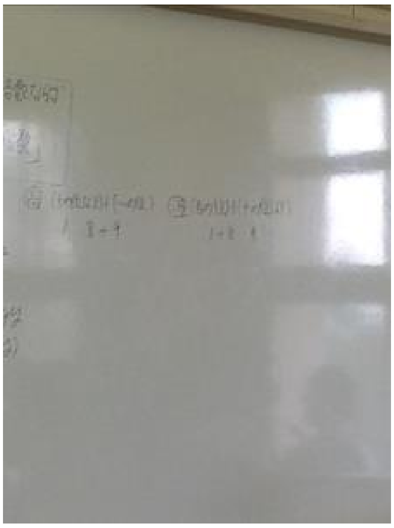
| | 3.Presentation of Students' Thinking, Class Discussion | Student Thinking / Visuals / Peer Responses /Teacher Responses |
| | 2:37-2:38<br><br><br>2:38-2:43<br>2:43-2:46<br><br><br><br><br><br><br><br>2:45-2:46<br><br>2:46-2:46<br>2:46-2:48<br><br>2:48-3:02<br>2:52-2:53:30<br>3:02-3:02<br>3:03-3:03<br><br>3:03-3:05<br><br>3:06-3:07<br><br>3:07-3:10<br><br>3:10-3:13<br><br>3:10-3:13<br><br>3:13-3:14<br>3:15<br><br><br>3:15-3:17 | T. said, "Let's look at 189. If you add the digits to see if this number is a multiple of 9, how would you do that in algebraic form?"<br>T. walked around looking at student's work.<br>T. asked "what did you write?" A student (boy, S25), raised his hand and he defined three variables, x, y, z, to represent the hundreds, tens, and ones place. He also defined the letter n to represent a whole number that can be multiplied by 9. He gave his equation $100x + 10y + z = 9n$. He used algebra to rearrange the equation to $x + y + z = 9n – 99x - 9y$ and simplified this to $x + y + z = 9(n – 11x –y)$.<br>T. said, "When you look at these equations, we look at x, y, z separately." "Are there any other ideas?"<br>A student, (girl, S6), said, "I split between the 10's and 1's place.<br>T. said, "How can we write this?" T. wrote 189 in the form of 18 + 9. Another student, (girl, S12), said, "I split 189 into 1 + 89."<br>"The 1 goes into the hundred's place plus number in the tens them are equal to 10's place."<br><br>T. said, " Can you write out math sentences for these examples.<br>T. walked around the room looking at student's notebooks.<br>T. spent 1:30 sec talking to a girl in the back row on the right side about her work.<br>T. said, "Who is having difficulty?" No on responded.<br>A student, (girl, S30), slightly raised her hand wanting to answer the question. The teacher noticed her and asked her directly to say her answer.<br>The girl said, "I am using the three digit idea and having |

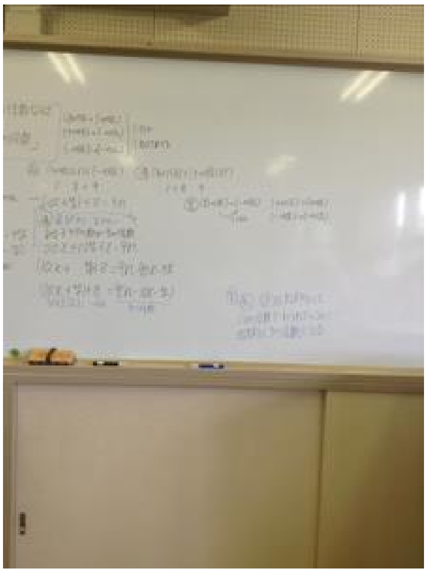
difficulty.” T. said, “Could you share your problem? What equation did you use?” The girl said, “ x + y + z = 9n” T. said, “Did you do x + y + z?” The teacher put parenthesis around the x +y on the board. The girl responded, “I made a mistake. I should have said 10x + y + z = 9n.” Then she said, “ I don’t know what to do after this.”

T. said, “What do you think about her problem? How many of you have the same difficulty?” A few hands went up to acknowledge his question.

A boy (S35), said, “I wrote 100x + 10y + z = 9n. Then he changed the equation to 10x + y + z = 9n -90x - 9y.

He simplified it to (10x + y) + z = 9 (n – 10x –y).”

A boy, (S09), raised his hand and said “Teacher, I decided to think that the hundreds place is the units place and the tens place is the tens place and the ones place is the ones place.” “If you think that way, you divide by 100, the tens place becomes the tens place and the ones place becomes the ones place.



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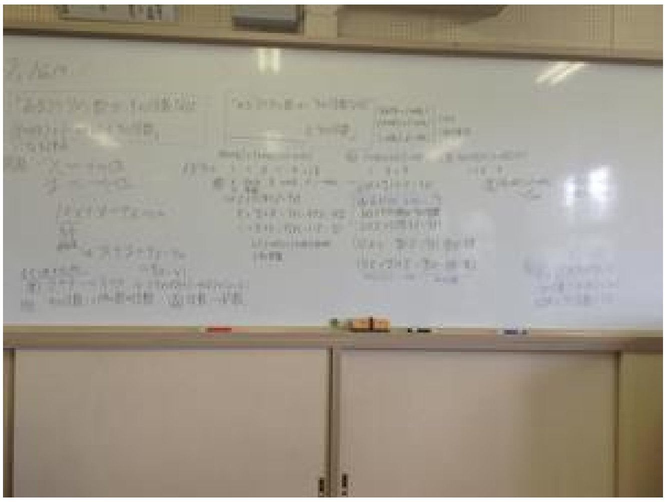
| | | A boy, (S01), said, "I am not sure. If you compare the second and third examples to the first one, as long as you decide by 10's and 100's place value, you can still maintain the idea."<br>T. said, " We haven't got any time left, we are not finished, let's stop here."<br>T. said, "Leave your sheet in the room before you return to the classroom."<br>After the lesson, some boys, about 6-7 of them, stayed after to talk to the teacher about the examples on the board. |
| | 4.Summary /Consolidation of Knowledge | In this lesson, we did not see any summary to the lesson. The white board was used to show what the students said in the lesson. The notes the teacher wrote only defined in words what the meanings of the numbers and equations meant.<br> |

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- The teacher, Mr. Kawamura, felt that the students couldn't do the transformation of equations into expression easily. He said that they go confused easily and that he should have had better transitions in the less. We see this as being a reflective teacher in the debriefing and group discussion of this lesson.
- The teacher was not able to really think about what the students were really thinking to solve this question. We see this as the teacher not able to anticipate some of their responses.
- The teacher's original idea of using the inverse of the equation was not helpful in the understanding of the transformation of these equations from a two-digit example to a three-digit example. From a pedagogical viewpoint, we and other teachers in the discussion see this as an unhelpful way to understand the goal of this lesson.

What new insights did you gain about how administrators can support teachers to do lesson study?

- The administrator has an important role in the delivery of lesson study. In this post lesson discussion, the administrator:
- supervised the discussion,
- providing the opportunity to let it be done and observed,
- he served as the special consultant giving his comments about how the lesson was taught and some points to think about to make it better.

How does this lesson contribute to our understanding of high-impact practices?

- The teacher's intention was to help students interpret expressions into equations and through the use of the two-digit example to get students to extend into three-digit examples. We refer to the Common Core Standard "look for and express regularity in repeated reasoning."
- The teacher had students raise their hand to participate in whole group discussions.
- The teacher had the students represent the concept in this lesson with algebraic symbology to model the transformations from mathematical thinking to mathematical solution.

Lesson Observation (7)

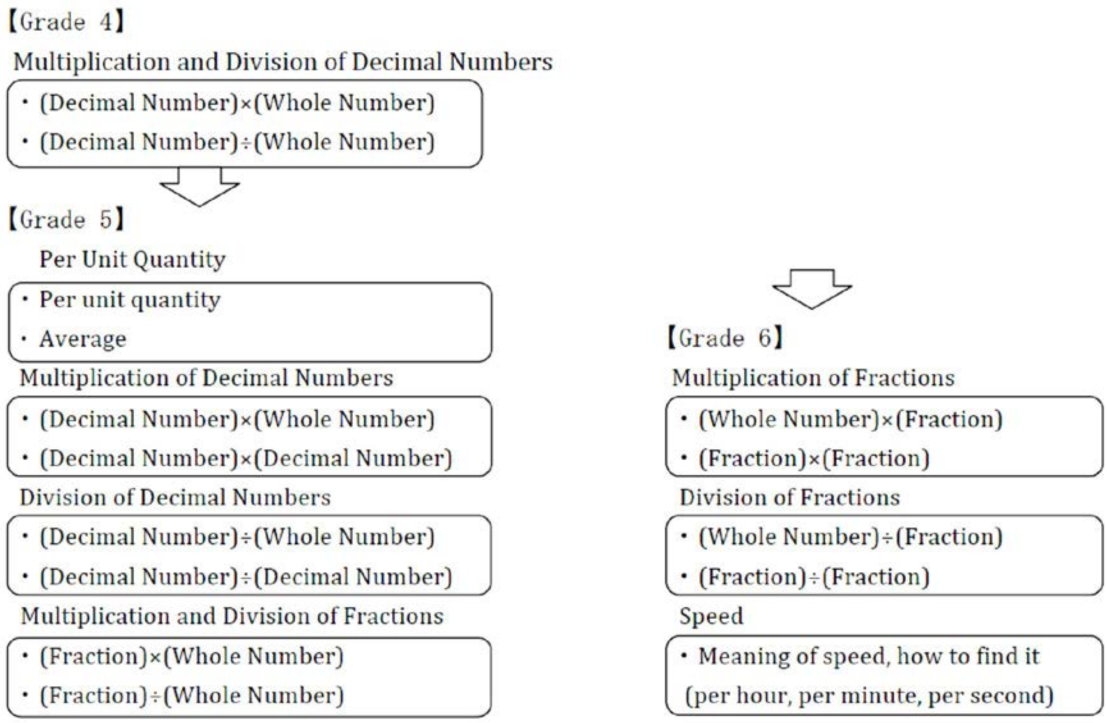
Research Lesson Observation Form (Use photos to document each section)
Meguro-ku Sugekari Elementary School Grade 6 (Class 1) 35 Students

Wed July 3, 2013, 5th period
Instructor: Koko Morita

What are the primary lesson goals?

- Students are able to think about how to compare the speed of running and explain about it by paying attention to the two quantities involved: the distance and amount of time a person runs.
- Students are able to recognize the merit for finding per unit quantity and utilizing it willingly.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?



| Start &End Time | Lesson Phase | Notes |
|---|---|---|
| | **1. Introduction, Posing Task** | -Strategies to build interest or connect to prior knowledge<br>-Exact posing of problem, including visuals |
| 0’ | | Teacher first says ñweôre going to talk about a race ï who is the fastest?ò<br><br>Teacher flashes the table ï an A3 piece of card with race times given for 3 students ï giving students a one-second glimpse of the times for A, B and C. Teacher generates interest, motivation, positive student disposition.<br><br>Teacher shows this twice, with several students calling out óCô as the fastest.<br><br>Teacher also asks ñWho is second?ò ï one student calls out ñMaybe we canôt tellò ï intrigue. T responds ñare you sure? How do you know?ò One student calls out ñwe canôt compare because we donôt know the distanceò |
| 2’ | | Teacher shows table on the board, now including the distance column, puts the date and lesson number up, and writes ñLetôs talk about the order of the speed of A, B and Cò |
| 4’ | | Students stand up one at a time to read the problem (3 times) ï the final student also adds ñdo you understand? Do you have any questions?ò ï this was addressed to the class.<br><br>S21 says ñthe distance for B and C is the same. C has a shorter time, so C is fasterò. Another student voices agreement.<br><br><br>T hides C on the displayed table.<br>ñA is faster than B ï itôs easy too - they have the same time, but B has a shorter distance, so B is slower, so A winsò<br>T moves the cover to now cover student B. ñWhat about these?ò<br>One student says ñthere is a 10m difference, and a 1 second difference ï we need to calculateò<br>Teacher: ñSo why do we have to calculate for A and C? ï you said thatò |
| 7’ | | Another student elaborates: ñwe need to calculate because both the distance and time are differentò |

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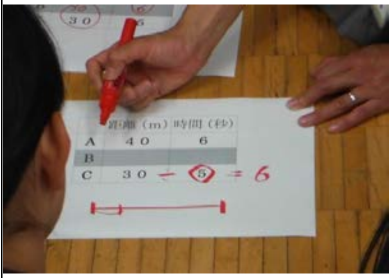
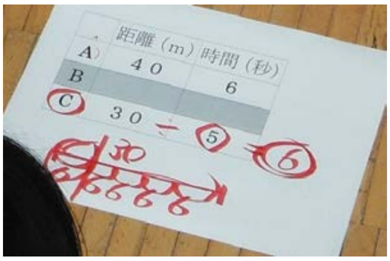
| 9’ | | Teacher: ñSo whatôs happing here?ò Writes on board: ñthe distance is different, the time is different, so we have to calculateò. The teacher reemphasises that this was something that the students had said. <br><br>Teacher gives students a choice: to stick a small printout of the table into their books; to use the available calculators.<br><br>Teacher: ñThink about A and C ï which is faster?ò |
| | 2. Independent Problem-Solving | -Individual, pairs, group, or combination of strategies?<br>-Experience of diverse learners<br>- Teacher’s activities |
| 10’ 30 ‘’ | | **Intervention group:**<br><br>After 1 minute the teacher says ñCome to the board if you have no ideasò. Twelve students do come to the front, and work on the floor. T prompts students to use the per unit time method. <br><br>Teacherôs and studentsô expectation was that students could stay as long as they liked, but should go back to their seats as soon as they understood what to do.<br><br>  Teacherôs prompts included:<br><br>ñwhat is different for A and C?ò ï Students: ñdistance and timeò ï 2 students returned to their seats after this.<br><br>ñwhat can we make the same?ò ï Students ï ñtimeò ï 3 students left.<br><br>   Teacher wrote on a copy of the table showing a line representing the distance of 30m for C. ñIf this is how far C runs in 5 |

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<td>14’</td>
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<td>seconds, how far will C run in one second?ò. She makes a mark on the line one-fifth along, and prompts the students to suggest what calculation might let us find that one-fifth distance. Once division was suggested, she wrote this between the 30 and 5 on the same sheet. All students except for three returned to their seats.<br><br>The teacher suggested to the remaining three that they go and get help from their friends ï two students left. The teacher reassured the one remaining student that she would come and check on her work later in the lesson.<br><br>This whole intervention took less than 4 minutes.</td>
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<td>10’ 20’</td>
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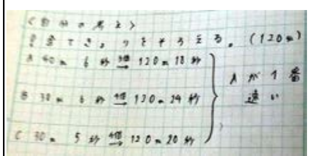
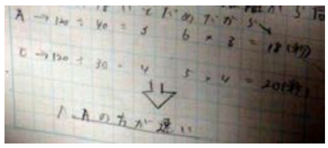
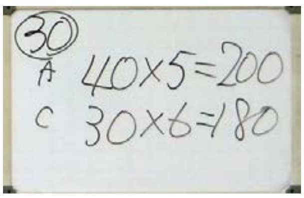
<td>Students working individually at their desks<br><br>There was a range of approaches used within the class, including those anticipated and described in the lesson plan (óanticipated student responseô = óASRô)<br><br>It is interesting to note the range of methods used to perform the calculations. Students made it clear later in the lesson that the perceived difficulty of the calculation was the main reason they chose a particular method.<br><br>
<table>
<tr><th>Method</th><th>Student work</th></tr>
<tr><td>(lowest) common multiple of distance<br><br>[anticipated student response 1]<br><br>ASR1</td><td><br>Multiplier indicated<br><br>120m identified as common multiple<br>onto using the multiplier to find the time to use for comparison</td></tr>
<tr><td>(lowest) common multiple of time<br><br>[anticipated student response 2]<br>ASR2</td><td><br><br>Identified multiplier</td></tr>
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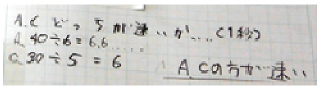
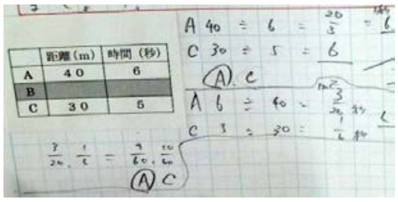
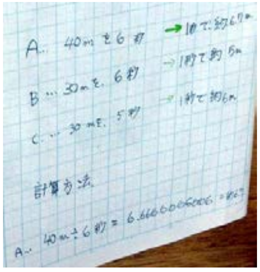
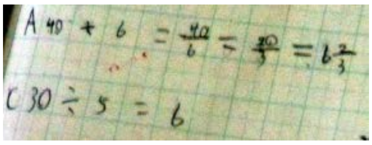
<td>Per-unit time<br>[anticipated<br>student response<br>3]<br><br>ASR3</td>
<td><br><br><br>Student moves on from per-unit distance to per-unit time, and uses equivalent fractions to compare speed of A and C.<br><br></td>
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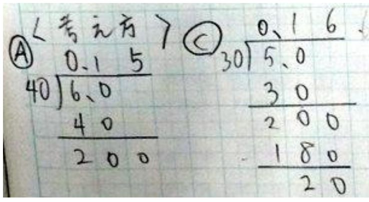
<td>Per-unit distance<br>[anticipated<br>student response<br>4]<br>ASR4</td>
<td></td>
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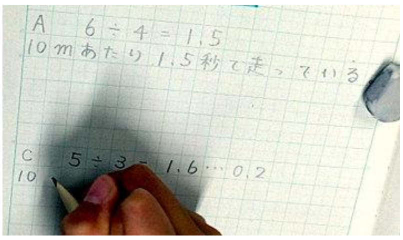
<td>Finding the time for 10 metres</td>
<td></td>
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7 minutes into the individual problem solving time, the teacher says ñIf you have done it one way, try another ï  which is easier?ò<br><br>10 minutes in, teacher is checking all student responses, and deciding on who will share their method on a large whiteboard to be displayed at the front.<br><br>This independent problem-solving phase lasted just over 10 minutes.
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<td>3.Presentation<br>of Students' Thinking, Class Discussion</td>
<td>Student Thinking / Visuals / Peer Responses /Teacher Responses Photos to document chronology (use new box for each new student idea presented]</td>
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| 21’ | ASR 1 : common multiple of distance | <br>Teacher selects student with ASR 1: whiteboard on board.<br><br>Student: ñI canôt really seeò (the writing was a little small)<br><br>Students call out “4 times as much … 3 times as much … you use 120m”<br><br>Student: ñI did the sameò<br><br>T: ñwho else used this?ò. 5 students raise their hands.<br><br>T: “what does this 120m mean? Can someone who didn’t use this method please explain”<br><br>Student comes to front “40m in 6seconds, so I thought: how long for 120m?”<br><br>T: “did you understand? What has he tried to do? Did he add 40 and 30?”<br><br>Class: “No – make 40 and 30 the same.” … “Make the distance the same.”<br><br>T: “So… 120 is 3 times as much as 40, so 6 becomes 3 times as much … 18”<br><br><br>T: “How can you tell which one ran faster, and how can you describe it?”<br><br>Many students call out that A is faster.<br><br>T: “Why? Please explain. If the distance is the same…”<br><br>Student interrupts / picks up the cue: “for A the time is shorter, so A is faster.” |
| 29’ | ASR2: common multiple of time | Student calls out ñI did time the sameò<br><br>T: “what do you mean ‘make time the same’?”<br><br>The student whiteboard with ASR2 goes onto the board.<br><br>T: “What might OO have done?”<br><br>Student: “We’re finding the lowest common multiple of the time”<br><br>T: “OK, what is it?” |

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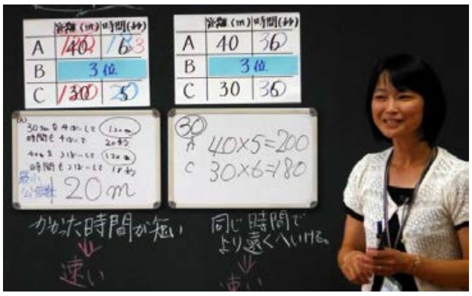
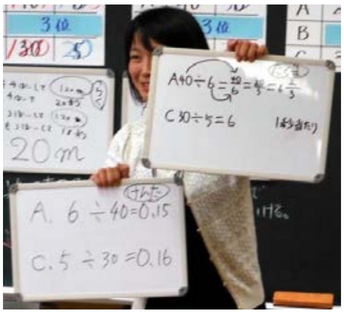
| | | Class: “30”<br><br>T: “So… to *make* 30 …”<br><br><br><br>T: “Which is faster? The smaller number was faster before – what do you think?”<br><br>S: “If you have the same time and run a greater distance, that is faster”<br><br>Teacher praised the quality of the answer – “that is a good way to think about it”<br>Student calls out “you are making different things the same” [the lesson is planned to include an opportunity for students to realise this for all 4 methods later on]. |
| 33’ | ASR3: per-unit time | <br><br>Student ñI did the time for 1 second to make the time the sameò [interesting to note the impact of the order of ASRs , with per-unit (time) following LCM (time) ï was this intentional? It is useful that students have already noticed that both ASR2 and ASR3 are making time the same. Might this be part of the reason that the teacher deviates from the lesson plan here?]<br><br>Teacher deviates from the lesson plan, showing student whiteboards for ASR3 and ASR4 together.<br><br>T: ñwhich is for one second? Which one gives the per-second method?ò<br><br>S: ñthis one ï the other says óper one metreôò T giggles.<br><br>T: (referring to the per-second method) ñDo you understand what this person was thinking? Talk to your partner and describe what you think. é Put your hand up if your partner did a good explanation. ñ<br><br><br><br>Student comes to the board: ñWe made the two the same by making them both one second. So we divided by six and then did the same for C.ò<br><br>Student: ñYou find out how many metres you run in one second.ò<br><br>T: “This is hard. Please raise your hand if you do not understand.” |

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| | | <br>The teacher explains that she wants a good explanation – “to help your friends”<br>Another student comes to the board: “So for C she ran for 30 metres – so we can do 30 divided by 5 to find out how much in one second.” The teacher scribes beneath the student whiteboard as the student gives her explanation.<br><br>Teacher is smiling, friendly: “Do you all understand? When we run, the first steps might be slower, and faster at the end, so it’s hard to get the distance for one second. I’m thinking of this as an ideal situation where we are going constant. So… how can we find out the distance for one second?”<br>Student continues: In one second A ran six and two-thirds metres, and C only ran six”<br>Class call out “A!”<br>T: “so looking at it per second, if you run more you are going faster.”<br><br><br>The teacher uses a bright chalk and makes thick highlighting beneath ASR3 – drawing students’ attention to the preferred method. |
| 42’ 30’ ’<br><br>Ends<br>44’ 30’ ’ | ASR4: per-unit distance | <br><br>Students calls out: “OO made the distance the same”<br>T: “what if the race was over one metre – we need to think about that.”<br>As in the lesson plan, students have an opportunity for paired discussion.<br><br>Teacher leads the explanation – more than previously. She highlights that the numbers shown in the original table are ‘reversed’ for the calculation. The time spent on this ASR4 was less than 3 minutes. |

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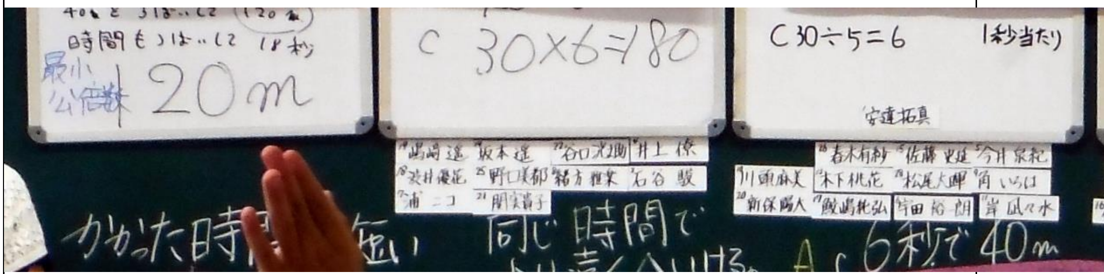
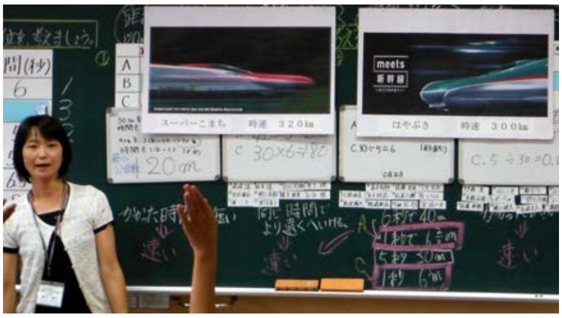
| | | Student calls out: “It’s the same as the first one” – i.e. ASR1 and ASR4 can both be characterized by making distance constant for each runner. |
| | | Note: the time spent on each anticipated student response is:<br>ASR1 8’<br>ASR2 4”<br>ASR3 10”<br>ASR4 2ãö |
| 44’ 30’ ’<br><br>47’ 30’ ’ | | Teacher moves back to the left of the board. T: ñRemember ï we had a problem at the start. How can all of these methods help? What is the same about all of these methods? ñ<br>Student calls out: ñthe time or the distance is the sameò<br>T: ñwe make distance or time the same to compare. Which one makes time the sameò<br>C: ñMethod Two and Threeò<br>T: ñdistance?ò<br>C: ñOne and Fourò<br>Students call out: ñThree and Four are per-one, One and Two use common multiples.ò<br>**Teacher was about to move on, when a student called out: “so who won?”. Teacher smiled and allowed a recap, with the class calling out: “1st” – “A!”; “2nd?” – “C!”; “3rd?” – “B!”. Students have their own value system – which they voice here; there are aspects of the lesson that they expect to happen – in this case they needed to be satisfied that they had answered the original question.** |
| 47’ 30’ ’ | | Teacher adds the results of 3 more runners – D, E and F – to the existing table, with decimal times and different distances.<br><br>Student calls out: ñOh ï this is difficult!ò<br>The teacher explained that we are not going to work this out now, but that students are to choose the method that they would actually use to find out the speed of each. Students have a magnetic name-card to place by their preferred method.<br><br>As students come to the board, the teacher says ñI will ask you why you decided on one methodò. |

| | | |
|---|---|---|
| | | The teacher hurries the kids along ï this phase lasts no more than 3 minutes. One student is allowed to take longer to decide – the teacher moves on.<br> |
| 51’ | | <br>The class votes are almost equally split three ways:<br><br>10 choose ASR2 – LCM(time)<br>11 choose ASR3 – per unit (time)<br>12 choose ASR4 – per unit (distance)<br><br>The teacher addresses the whole class, several of whom call out.<br><br>S: ñIôm using the fourth methodò<br>T: ñWhy do you think itôs easier?ò<br>S: ñBecause the calculation is easier ï Iôm not good at calculating with fractionsò<br>Other students called out: ñé but you get 6 recurringò<br><br>T: ñWhy did you choose one particular method?ò<br>S: ñMethod 3 ï when I see all of themò<br>S: ñitôs complicated to find the LCMò<br>S: ñ[Method] 3 looks easierò<br><br>Another student goes for the second method.<br>T: ñOK ï whatôs the common multiple for the six-runner race?ò<br>S: ñthe common multiple is hard to find because there are so many. I thought fractions or decimals would be easier.ò<br>T: ñSoé if we have many runners, the second method becomes more difficult.ò |
| | 4.Summary /Consolidation of | Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals. |

| | | |
|---|---|---|
| | Knowledge | The teacher has 2 large pictures of trains, with the top speed shown. |
| 55’ | |  |
| | | T: “Here’s a picture – which is faster?” |
| | | C: “the left one!” |
| | | T: “so you do know that if the number is greater it is faster” |
| | | Student calls out: “that means how far you would run if you had one hour.” |
| | | T: “So… if you have a larger number, it’s faster.” |
| | | T summarises: “When the distance and the time are different, what do we need to do…?” |
| | | Class: “make one thing the same” |
| 58’ | | T: “Great – so we can compare – make a note / reflection in your book” |
| | | Teacher circulates and reads student work during the ~2 minutes that students have to record a reflection. |
| | | T: “you may still be writing, but maybe we can share.” |
| | | S: “when we have this problem, we should make the conditions the same.” |
| | | The teacher complimented the student on the good use of the word ‘condition’ |
| | | Another student: “ it looks like Science” |
| | | S: “it makes it easier to compare” [students have picked up the teacher’s language?] |
| | | S: “if I’m not sure, I make something the same – it’s like finding the common denominator for fractions. |
| | | S: “If you are doing per-metre the number is smaller, if you do per-second …” |
| Ends<br>1h 2’ | | Another student butts in “… the number is bigger” |

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- Students have their own value system

Students' choice of preferred method was interesting, especially that around a third of the class either stuck with - or on reflection preferred - the common multiple (time) method. It is interesting to consider how students make these strategic, meta-, qualitative mathematical decisions; is it for perceived ease [common multiple of 5 and 6], sense-making [multiplication preferred to division], because there have been strong suggestions that one method / way of thinking is somehow 'better'? Mathematics educators decide which method is the most beautiful / efficient / logical / parsimonious from a perspective where we can compare many different methods. Students are typically not in the same position – they will choose their preferred methods based on what they feel, understand, what they 'know' or 'see' works. Students' initial expression of their preference will likely be visceral, in the affective domain – the capacity to describe their thinking abstractly in symbols and reason may be relatively underdeveloped. If these heartfelt views are contradicted by peers or teachers, there is a risk of causing upset. Conversely these same views can be a source of joy. Time is profitably spent in lessons on respecting, expressing and communicating these opinions and ideas, exploring the relationship between a felt 'Aha!' moment and its various mathematical representations.

- Simplification of the problem

There is a tension between the cleanness / messiness of the problem:

What are the pros and cons of

i) simplifying the necessary calculations (e.g. avoiding fractional or decimal answers) to help students 'see' the concept more clearly, and

ii) keeping the problem more 'real', so that students are more familiar with the types of numbers and situations that can arise in authentic real-world multi-factorial 'messy' problem-solving?

What new insights did you gain about how administrators can support teachers to do lesson study?

Administrators should make sure that teachers know that they do not have to 'complete' the lesson plan. Decisions about when to move on in the lesson are made by the teacher on the basis of student thinking. This raises questions of evaluation / assessment / equity in the classroom.

The post-lesson discussion was structured so that teams of teachers could write comments on coloured paper, which were then collated and displayed at the front. Used well by the chair / moderator of the meeting (this has always been a school teacher), this gave the session a clear structure, promoted inclusion of all teachers' comments, avoided duplication of points of view and gave an opportunity for a deepening of subsequent questions.

How does this lesson contribute to our understanding of high-impact practices?

Many high impact strategies were seen in this lesson.

This lesson followed the same structure as other research lessons, the 4 key anticipated answers were selected and displayed on the board, making students thinking visible. This created the structure for the class discussion, where students had opportunities to explain each other's methods. The solutions were placed in an order which aimed to scaffold students' thinking towards the desired learning goal.

The extended task with additional runners caused students to reflect on the presented methods again, helping justify the chosen divisor method by a need for efficiency and the idea of 'bigger means faster'. However students' responses suggested many had not yet come to this conclusion, and needed to experience the process of doing the calculations to fully make this transition in thinking. These calculations were planned to happen in the next lesson, demonstrating how students' understanding would continue to develop next lesson, just as this lesson had built on students' previous per-quantity lessons.

The lesson ended with students being shown pictures of two high speed trains; students were engaged and excited about the concept of speed and their future learning.

# 3 Reflection Journals

Invited IMPLUS participants were requested to write a reflective journal about mathematics teaching and learning in Japan and Japanese lesson study.

# IMPULS Lesson Study Reflection

**Amanda Y. Short**
**4th Grade Teacher, Montezuma Elementary, Albuquerque, NM, USA**

The IMPULS program has given me much to think about. As I reflect on my incredible experience a few themes come to mind.

Theme one: 27 vs. 23 (Oshihara Elementary School Grade3 Lesson).

This lesson illustrated for me the vast amount of attention and contemplation that Japanese teachers put into every detail of the lessons they use and present for lesson studies. Having participated in lesson study for 3 years, I understood the importance of planning lessons in great detail but I did not have a true understanding of what this detail truly looked like.

At first glance the difference between 27 and 23 did not seem that important to this lesson. I could understand why the teacher chose 27 - so that his students could more easily relate to the problem. And I could understand why the curriculum used 23 ï so students would not get caught up in the calculation and give them more time to work out the use of the remainder. But was it really a big deal? Both seemed fine. As the lesson progressed longer and longer past the time it should have finished, I began to understand why it was important to think deeply about the reasoning behind choosing 23. And as the post-lesson discussion highlight, the teacher also understood the ñgravityò of his choice in 27.

In sum, not only was this meticulous attention paid to details in each lesson, this attention was paid to details in the units of study as, well as the curriculum development, as well as how teachers were taught to become teacher. This deliberate attention to detail is something I hope to use in my classroom and with my colleagues as we plan our yearõs lessons.

Theme two: Student Thinking vs. Teacher Thinking.

Much of what I love about lesson study is the deliberate look at student thinking and how it matches (or doesnõt match) teacher thinking. In the planning of a lesson the teacher and team discuss anticipated student responses. This, in and of itself, is a great way to better your practice ï by thinking about what you believe the students will do you can better plan your lesson. Of course when your anticipated responses donõt match what the students do, this also become a great way to better your practice. The post-lesson discussion from Oshihara Elementary School Grade3 Lesson illustrated this. The teacher made a comment about how the students wanted to discuss a point he did not want to discuss but he allowed them to do it anyway ï bringing his point in later in the lesson. What a powerful way to allow students to develop their own mathematical thinking and understanding!

Through this experience and in previous experiences with lesson study I have heard the phrase: the students were trying to figure out what the teacher was thinking instead of the teacher trying to figure out what the students were thinking. I have begun to realize how this was a major difference between the Japanese style of teaching and the US style of teaching. US teachers are constantly trying to get students to understand what they are thinking. As a teacher of (a mere) 8 years I had never heard this concept discussed or contemplated until I began looking at lesson study. What an important difference! This seems to be a major change we need to work toward in the US. As a teacher I hope to spend more time understanding what students are thinking and how I can orchestrate the building of their mathematical understanding instead of trying to push them to guess what Iõm thinking.

Theme three: Teachers as Scholars and Researchers.

It became very evident through the IMPULS program how revered scholars/teachers are in Japan. This is also a major and important difference between the US and Japan! What a concept to be respected for your scholarly opinion as a teacher. This attitude would completely change our school system in the US. How refreshing to see teachers discussing and researching their practice ï just as any other professional does. This was very evident in the Yamanashi Fuzoku lessons. Here teachers were proposing what they believed would be good changes to the national curriculum ï a huge difference from the US where curriculum is developed by publishers wishing to sell a product, making the label ñresearch basedò a joke. The classroom as a laboratory is so important and are not a part of the US system. Before my trip with IMPULSE I believed this position was for people working at Universities or who had PhDs. As we saw more and more lesson study lessons in Japan it became very evident how valued and important this research and study was to the craft of teaching. It is my dream that some day the US will value the opinions and research of teachers as scholars! This would take a major cultural change in our current education system. Until then I will promote discussions in my own school and hope to start a study group to deepen our own teaching understanding as well as our own mathematical understanding.

Our current education system will be using the Common Core State Standards to teach depth, not breadth of Reading and Math curriculum. I have a strong distrust of the common coreõs political interest (for example, corporationõs conflict of interests in supporting the CCSS) I do believe project IMPULS has helped me do this with lesson study. Before traveling to Japan I had a very superficial understanding of lesson study and all the integral details that fit together to create a nation-wide system that works! I greatly deepened my understanding of lesson study and teaching through problem solving with this IMPULS experience. This was most evident to me in: Japanese lesson practitionersõ attention to details on every level, attention and study of student thinking and how this betters their practice, and viewing teachers as scholars and researchers of their profession. It seems overwhelming to me to think about *implementing* (I use this word with caution because in our current education system it is used inappropriately) this kind of change across our nation or even my very large school district but I can think about helping make these changes at my grade level and my school and hopefully over time this will growé

**Reflective Journal**

**Amy Rouse, Head of Mathematics & Assistant Head Teacher.**
**Swanwick Hall School, Derbyshire, UK.**

The invitation to attend the immersion project happened at a key moment in my progression as a teacher, and as a leader of professional development for others. I arrived in Japan with two key questions; how do I teach problem solving effectively, and in achieving this support my department in doing so? How do I run lesson study effectively within my school?
The following reflection will outline the key learning points and practises which I believe will help me tackle these questions.

## Teaching Effective Problems Solving

In Japan there is a shared national responsibility of the curriculum and how it is taught. Professors and teachers work together to research, propose, and plan change, through lesson study and in doing so share best practise. A national text book enables teachers to guide students through a learning route that is proven to be successful, supporting progression for all students, creating consistency between schools, and giving teachers the tools they need to teach effectively through problem solving. In result students develop good mathematical understanding, repetition is not necessary.
Teachers in the UK are inexperienced at teaching problem solving. For most, it was not part of the education they received, and they have not been provided the support needed in order to teach problem solving effectively. Changes in examinations are fast and pressures for results ever greater, problem solving lessons expose gaps in studentsô understanding and hence cause teachers to shy away from these types of lessons, despite knowing their worth and importance. Teachers need the knowledge of how to create problem solving lessons that have structure; they also need support in choosing problems that build on studentsô previous experiences and as well as suited to their current understanding. I believe we are able to do this by learning from the Japanese.
Problem solving lessons in Japan have the following fixed structure:
*Time to grasp the problem* - time is spent ensuring students understand the task by ósetting the sceneö, gaining studentsô interest and explaining the context. The problem is often read several times and a picture may be used to help prompt studentsô exploratory thinking. Students must fully understand the problem in order to model it mathematically; hence problems are often simple in nature.
*Pursue* ï This is individual problem solving time in which each student tackles the problem themselves, using a method they have chosen. The teacher will circulate looking for the anticipated responses which will facilitate the whole class discussion.
Students are confident learners and rarely need reassurance from the teacher. The teacher may notice studentsô misconceptions but will leave these unchallenged, allowing the learner to develop their thinking, until they are ready to tackle misconceptions themselves, with true understanding. The teachersô and learnersô quietness during this time allows uninterrupted individual thinking, producing work which is a true reflection of each individuals thinking and understanding at that point.
*Deepen & Rise* ï This section dominates the lesson. The conversation is focused on the studentsô methods, the studentsô thinking and explanations. The teacherös role is to structure and facilitate this discussion by anticipating the studentsô methods and planning a learning route.
The order of the shared anticipated answers is of great importance and should scaffold students understanding towards the desired learning goal, allowing students to move from less efficient, concrete methods, to models which use mathematics in a sophisticated way, and prepare for future

learning. Students will normally find that their response is one of the chosen methods, helping them access the planned route of thinking.
This frequent sharing of work expands studentsô problem solving repertoire. It also develops a culture of inquisitive learners; students understand that there is no one correct way to solve a problem and all methods are valued.
The questioning used places responsibility of learning on the student by using their explanations to explain their methods which are presented on the board. ôDo you understand what he/she said?ô ôHow do you know?ôôWhat were they thinking?ôôWhy?ôôWhat does this number represent?ôôWhat do you thinkô ôWho has the same?ô These questions encourage reflection and aim to deepen understanding. It is not uncommon for questions to be posed and left unanswered.
Board work is planned with precision and is used to make studentsô learning visible, reflecting the order of discussion to be had. As the discussion gains depth, notes are added around the studentsô work. Mistakes or misconceptions are left on the board but are corrected in a different colour showing that mistakes are valued as a learning tool, deepening the discussion to be had. Simple diagrams or actions are often used to support methods and help explanations. Equations or expressions are annotated to explain what they represent. Attention is paid to every detail. Nothing is removed from the board during the lesson, allowing students to look back at the journey of the learning and also act as a structure on which to students can build their new ideas.
Summarise & Expand ï Learning is linked back to original problem, an essential part of the lesson where students use the Mathematical models they have created to answer the original contextualised problem. Students also look forward to the next stage of their learning. Finally students make a reflective note in their journal giving the teacher an insight into their understanding and thought processes.
The structure of lessons is routine to Japanese teachers but for teachers in the UK, I believe, it is the key to our progression in delivering effective problem solving. Although, there are some cultural differences to bridge. For example: The time students spending listening is unlikely translate without issues. We will need to approach this time in a creative way, aiming to find ways for all students to be actively involved for periods of this time.
A question I still ponder is; what is the best action to take, when it is realised that the desired learning needs more time than the allocated lesson allows? Does the whole problem solving process need to happen within one lesson? Can students be left to reflect on a problem until the next lesson? Would this be as effective?

**Running effective lesson studies**

Japanese teachers are trained to observe lessons. From the simple ideas such as logging the timings of the lesson, to becoming an experienced evaluator, who plans observations around questions prompted by the lesson plan. Observation training is not a common practise in the UK, I will need to work with my inexperienced observers from basic skills.
During my stay in Japan I found that the way I conducted my observations changed rapidly. During my early observations I was overcome by cultural differences and routines, but these were soon understood and then made way for deeper reflections. Detailed lesson plans, which gave the sequence of events and often the detail of discussion, meant only a few notes were needed when something unexpected happened or the teacher changed their plan. This enabled time to be spent reflecting on what was happening. What was making the lesson successful and what could be improved. I also found myself constantly asking the questions; ôHow do I normally do this?ôôHow

can I apply this in my culture?ô and found myself making many notes for my future professional development, more so than I have ever done from lesson observations in the UK or alternative training.

Working with Matthew Lewis, we produced the report on the last lesson, on speed. This felt like perfect timing and an opportunity to put together all I had learnt. Matthew and I spent time discussing the lesson plan in detail, anticipating successes and potential issues. This gave our observation distinct foci, which we discussed in depth as we put together our record and reflection of the lesson. During this final cycle, I felt that I truly understood the essence of Japanese lesson study. The importance of attention to detail, and the dedication required for the reflective thinking which allows us to progress.

Post lesson discussions where organised in many different ways. Some as a whole group but many had several smaller groups to enable observers to be more involved in the discussion. Effective discussions had a predetermined structure. This was sometimes done by all observers contributing to make a shared collection of ôpositives and needs to improvesö. All post lesson discussions where chaired in a skilled manor which created an organised discussion.

During discussions many teachers would have chance to contribute and give their opinion, often ending their observations with a question. Many of these questions were not answered but left open, as a point of reflection for all involved, and as points as development for the school and planning team.

An aspect of many of the pre- and post- lesson discussions that surprised me was the attention to detail. For example, the nationally understood use of ordered multiplier and multiplicand or at a micro level in lessons ôWhy did you use the number 27?ô This level of scrutiny and shared responsibility allows an already excellent education system to continue to improve.

**Challenges**

Having observed many lesson studies I now feel I understand the main processes of lesson study, although I recognise I still have much detail to learn, as well as some challenges to overcome. A key part of the post lesson discussion is the knowledgeable other. In the UK there is the lack of experienced knowledgeable others who are accessible to schools. There are also issues of teachers feeling they are going to be ranked or judged when people of seniority enter their classrooms and the stress that this causes. A possible solution for this is to reflect the Japanese system and ensure that senior staff are part of the planning team and therefor have joint responsibility for the lesson.

Another challenge is the time taken to plan lessons, in Japan many hours are spent researching and developing the lesson plan. I expect that we will need to develop this level of sophistication in planning over time, whilst proving to teachers that this large planning commitment is of great worth.

Finally I will be running cross curricular lesson studies and in this aspect, I am still exploring territory which is quite unknown to myself.

**Thank You**

Being a part of the immersion program has given me the tools I need to address my key questions successfully; it has also provided me with a professional network which will help sustain my progression. There is much I have learnt and many questions I still have to explore. I plan to read more about problem solving and lesson study but in order to truly understand, I must explore the questions I have myself, experience the outcomes, listen to others and reflect.

The experiences and insights I gained in Japan have also given me a whole new perspective of education and a wealth of ideas, and for that I am very grateful. I believe this immersion study will influence my practise for the rest of my career, and in turn the practise of those I work with. Thank you so much.

IMPULS Lesson Study Immersion Program Reflective Journal

Aubrey Perlee

The experiences that I had as a result of the IMPULS Lesson Study Immersion Program were amazing. It was a once in a lifetime opportunity and I truly feel privileged to have been apart of it. To experience lesson study, in the place where it originated, day after day, was thrilling. I had hoped to learn how lesson study worked in its truest & purest form. Seeing and living this gave me a foundation for understanding and a model for what I should be striving for within my own classroom.

One of my first observations was that kids are kids, no matter what country they live in! For some reason, I had this perception that Japanese students were different somehow from my own students; smarter, more focused or more ñroboticò. Japanese children laugh, run, yell, play and goof off. It sounds a little ridiculous but it was refreshing to see that they are silly and goofy, just like all of my own students.

The board work was my biggest takeaway from the trip. I was particularly intrigued by the relationship between the teacher, student responses and the board work. The teacher poses a problem & students write it in their notebooks. As students work to solve the problem, the teacher walks around & notes the mathematical thinking/strategies used by the students. During the comparison/discussion portion of the lesson, the teacher to call on students, in a specific order, based on the strategy each child used to solve the problem. The teacher records each childôs thinking on the board & has already pre-determined exactly where each strategy should be recorded on the board. The board work is a development of student responses. The order in which students are chosen to share out is strategic because each response builds in mathematical sophistication and brings the studentsô thinking to a higher level. At the end of the lesson, the board work is a representation that tells the mathematic story from the day. In one lesson I observed, the teacher would reference a picture she had of what she anticipated/wanted the board work to look like. This does not mean that the board work is static or that a teacher would force it to look a particular way. Certainly, the teacher is teaching the students not the curriculum. If student responses take a different direction than the teacher originally anticipated, the board work would be

If you had asked me before the trip if I would have ever described board work as beautiful, I would have quickly replied ñnoò. I humbly stand corrected. I saw some gorgeous board work that almost felt like a work of art. To improve my own practice this year, I have an additional white board that was added to my classroom that I am using strictly for mathematics. I am sure that it wonôt necessarily be pretty, but I plan to improve my board work day after day.

The teachers in Japan are incredibly accurate at anticipating student responses. There is a deep knowledge of their students and what their previous experiences with mathematics have been, even at the beginning of the year. A challenge of using lesson study within my own classroom is vast experiences with which my students bring with them to school and the teaching of mathematics. There is no consistency with curriculum, which makes it very difficult to anticipate student responses. Teaching kindergarten (and it being the first real introduction to school) makes it even trickier! This upcoming school year I want to focus on my students and their mathematical thinking/reasoning.

The beginning step of taking their thinking to a higher level is to be able to anticipate their responses to begin with!

I was also extremely impressed with the professionalization and content knowledge of all of the Japanese teachers I encountered. During the school-based lesson study research lessons, all of the teachers participated by giving feedback and suggestions during the post lesson discussion. The Art teacher and Kindergarten teacher were commenting on the mathematics that occurred in the 6th grade lesson and making suggestions for improvement. In the U.S., the attitude of most teachers is that they teach their one grade level/subject and that is their specialty. While much of that has to do with the difference between the teacher training process between the two countries, it is clear that the teachers in every school share the responsibility for teaching ALL of the students, not just their own. This is a sentiment that is very different to me and to teaching in the U.S. It is something that I shared with my colleagues as one of my biggest takeaways from this experience.

Upon returning from Japan I was fortunate enough to participate in the Chicago Lesson Study Conference where I was able to use all of my experiences from Japan to work with my kindergarten team to write a research lesson to address teaching and learning problems within our curriculum. Last year, my school began using lesson study as a tool to address teaching and learning problem of using student note taking and teacher board work to address the Common Core State Standards for Mathematical Practice. This year, the kindergarten & 4th grade teams are each presenting research lessons for the entire staff to observe during two of our Professional Development Days. This way all teachers, regardless of grade/subject/previous participation with lesson study, will be able to experience it. I hope that it will give other teachers insight as to the lesson study process as well as ignite a genuine interest and spark in participating in their own research lessons. Lesson study is done primarily in mathematics within the U.S. and also within my school, it can be used for all subject areas. I hope non-mathematics teachers at my school will see this.

When I first began lesson study a year ago, I really had no idea what it was. I taught a research lesson before ever observing one. Before my trip to Japan, I had only observed one, live research lesson. This IMPULS trip was so beneficial to me because now I know what lesson study looks like in Japan and Iõm clear about the direction that I need to take as a teacher to help improve student learning within a lesson study context.

# IMPULS Immersion Program 2013 – Reflection

*David Garner – Numeracy Coach and Assistant Principal, Creekside K-9 College, Victoria, Australia*

The opportunity afforded me by the IMPULS Immersion Program, 2013 cannot ever really be fully measured. Such was the depth of ñimmersionò and learning, that it changed my thinking and perspective immediately and profoundly. For me, this was an invaluable opportunity to be a part of the study of teaching and learning (Lesson Study) in the home of ñlesson studyò. Whilst I had had experience taking part in multiple research lessons in Australia with Deakin University as both a planning team member and observer, as well as leading a research lesson in Mathematics at my school, Creekside K-9 College, nothing could ever quite do justice to being a part of the whole process/practice in Japan itself.

At the conclusion of the program a number of key features of lesson study and Mathematics education in general stood tall in my mind as significant and noteworthy;

**Effect of Lesson Study on teachers' content knowledge**

Through all our observations of planning meetings, research lessons, post-lesson discussions and our own reflective meetings there is a very clear and deep understanding of the continuum of learning in the teachers and educators of Japan. I personally was drawn to lesson study as a practice of professional learning to help build teachersô content knowledge for Maths teaching. In those lessons whose teachers and planning teams had an obvious depth of knowledge for Mathematics teaching was evidenced through:

- Detailed lesson plan ï including quality *hatsumon*, detailed and thoroughly considered student anticipated responses, clear and concise rationale for the units/lessons, and a sequential and detailed unit plan;
- Classroom practice ï *bansho* and *hatsumon*. In those lessons that were most effective in achieving the lesson goal, the *hatsumon* used to facilitate discussion and the exquisite *bansho* steered students towards achieving the lesson goal. The two lessons at the Yamanasi School (in Year 2 and 3) were the greatest evidence of effective *hatsumon* and *bansho* – exemplars of the practice in fact.

**Involvement of "whole system" in practice of Lesson Study**

The Involvement of principals and assistant principals in the process of lesson study, often as members on the planning team and steering committee, as well as observing the lesson and actively participating in the post-lesson discussion was striking. For the practice of lesson study to be effective in a school and across districts, commitment from the administration at schools is vital. A concept schools in Australia are yet to embrace fully. Victorian schools have long espoused the importance of Instructional Leadership and I see no better way to partake in this than to be committed and actively involved in lesson study in school.

Whole system commitment to lesson study was also evidenced by:

- Alignment of schools with universities ï accessing research-based practice, knowledgeable others; professors as principals;
- Involvement of pre-service teachers in lesson study practice

**Significance of the lesson plan**

I have felt at times in Australian research lessons and planning teams, the significance of a detailed and well-researched lesson plan has not been appreciated. It was clear from our journey through Lesson Study in Tokyo and Yamanashi that the best lesson plans produced the best lessons and the worst lesson plans produced the worst lessons observed. Those lessons which were most ñsuccessfulò in achieving the lesson goal/s were accompanied by lesson plans which:

- Were more detailed, well-articulated in terms of background information, conveying the whole school goals and vision through the goals of the lesson and rationale;
- Generally displayed the level of content knowledge of the teacher through *hatsumon*, anticipated responses, outline of overall unit, clearly articulated lesson goals.

**Questions arising from IMPULS Immersion Program**

Whilst the IMPULS Immersion Program has played a significant role in cementing my ideas around lesson study practice and the teaching and learning of Mathematics, it has also left me with a number of reflective or hanging questions that are unanswered and open for discussion and consideration as I embark on the implementation of my learnings back in Australia:

- Are these lessons ñtypicalò of an everyday lesson? What does a week of Maths learning look like for any particular class?
- How much autonomy do teachers have over the content taught?
- Do the structured problem solving tasks and lessons observed as part of the lesson study practice allow for differentiation and maximum engagement by ALL students?
- Is my view of differentiation consistent with that of Japanese educators?
- How did the studentsô written reflections in their notebooks show learning/thinking? (this was very hard to gauge due to the language barrier)
- How can I continue to implement a ñlesson studyò approach to team planning on a weekly basis?
- How can I lead the implementation of a whole school model for teaching and learning that shows the link between: Whole school ethos → school research theme → lesson and unit goals} CLEAR and CONSISTENT?
- Is there a need for more scripted curriculum in Australia? (there was a clear understanding continuously articulated that specific things HAVE been learned a specific year levels in Japan)
- Can we get schools and universities to align for a whole system commitment to lesson study?
- What place does articulating the learning intention to students have in a structured problem solving lesson?
- How much emphasis do we place on note taking during the teaching and learning of Mathematics?

**What now for me?**

My immediate thought as I hit the airport to head home was ï what now? I must say I felt immediately invigorated to get back into the classroom more to model problem-based lessons. I feel that for me to lead and coach people in this I need to be continually polishing the stone myself. In Term 4 (November and December) this year, I will be leading another team of teachers in planning and implementing a research lesson. This program has re-focussed me and inspired me to pay attention to those components I have mentioned above more and set a high standard for the entire lesson study.

Finally, a huge thank you to Tokyo Gakugei University for facilitating this priceless learning opportunity. Thank you to all the international compatriots from US and UK ï it is vital we keep sharing how we implement this in different cultures. Sensei Takahashi, Sensei Fujii, Sensei Watanabe and Sensei Yoshida ï your knowledge and wisdom in an area of teaching and learning that we are all so passionate about is inspiring.

# 2013 IMPULS Lesson Study Immersion Program Reflective Journal

**Denise Jandoli**
**Bernardsville Middle School, New Jersey, U.S.**

I would like to begin by expressing my gratitude to Project IMPULS and all parties involved for allowing me to participate in the 2013 Lesson Study Immersion Program and making it such a wonderful, well-rounded learning experience for all participants. I distinctly remember on my last day in Japan, as I packed my suitcase and sorted through the lesson plans and notes I had acquired from the seven school visits, I wondered how I was going to describe my two week, career-changing excursion. I had experienced more professional growth in those two weeks than any one person might encounter in an entire career. I had taken in a new culture in a way that few people get the privilege to. I had professional development that mathematics education researchers and practitioners from outside Japan only hear or read about. Now being back in the United States, feeling rich with new knowledge about my profession and having had time to reflect, I finally have the opportunity to describe my journey.

Being a 6th grade mathematics teacher in the U.S., I find it most compelling to begin my reflection with the mathematics teaching that I observed in Japan. One of the many observable differences between U.S. and Japanese teaching is the role of the teacher in a lesson. In The Teaching Gap, Stigler and Hiebert explain,

> [American] teachers act as if confusion and frustration are signs that they have not done their job. When they notice confusion, they quickly assist students by providing whatever information it takes to get the student back on track... [Japanese teachers] often choose a challenging problem to begin the lesson, and they help students understand and represent the problem so they can begin working on a solution... Rarely would teachers show students how to solve the problem midway through the lesson. (J. Stigler & Hiebert, 1999, p. 93)

The difference described above, highlights one of the many key aspects of the student-centered instructional approach that Japanese teachers utilize called structured problem solving. The lesson phases for practicing this problem solving approach consist of (1) posing and understanding the task, (2) individual problem solving, (3) presentation of studentsô solutions and class discussion (neriage), and (4) summary/consolidation of knowledge. All of the research lessons observed throughout my school visits in Japan incorporated these four lesson phases and were models of teaching mathematics through structured problem solving.

I started learning about teaching mathematics through problem solving in the summer of 2012 when I attended, ñJapanese Structured Problem-Solving as a Resource for U.S. Elementary Mathematics Teachersò, hosted by the Mills Lesson Study Group, in Chicago, IL. Japanese mathematics educators (Akihiko Takahashi, Tad Watanabe, and Makoto Yoshida) and U.S. researchers (Catherine Lewis and Rebecca Perry) worked closely with us educators from all over the United States, to develop, test, and refine approaches for teaching grades 3-5 mathematics through problem-solving. I took part in two cycles of lesson study in my school district to test and refine problem-solving materials that originated in Japan. My experience with this project had sparked an undeniable drive for learning more about teaching mathematics through structured problem solving and incorporating the practice into my daily lesson planning. Being presented the opportunity to go to Japan to observe authentic Japanese Lesson Study that focused on teaching mathematics through problem solving, a practice that I had been researching extensively over the past school year, I knew it had the potential to be an extremely humbling and inspiring experience.

In the seven school visits, I had the privilege to observe a variety of passionate, novice elementary and lower secondary school teachers. Some better than others, engaged their students in meaningful problem solving and critical thinking with rich, probing problems that allowed for a variety of solution strategies. After independent problem solving time, the presentation and comparison of the studentsô solutions strategies became the focus in the class discussion. This lesson phase allowed the students to reach the lessonõs goals by the teacher drawing out the key mathematical ideas from the studentsô posted and presented solution strategies, which is much different from the traditional U.S. approach to a class discussion. Many U.S. teachers save the comparison of studentsô solution strategies for the end of the mathematics lesson, if time permits. The discussion tends to be focused on the teacherõs ideas and solution strategies or, simply, a ñShow and Tellò of the studentsô strategies. Itõs a difficult and foreign concept for some teachers in the U.S. to step aside and allow his or her students to make sense of a problem on their own and have their ideas and solutions sculpt a lesson.

In Japan, true to the teaching through structured problem solving approach, I watched the students always be in the spotlight during the class discussion. The teacherõs role, after choosing and sequencing responses to be presented by the students, was to orchestrate the class discussion through neriage (ñkneadingò studentsô ideas) and use the blackboard, student notebooks, and discussion to help students develop their mathematical thinking. By anticipating studentsô possible responses during the lesson planning and taking note of each studentõs approach throughout the independent problem solving time, these Japanese teachers were able to create an effective, dramatic flow of strategies, solutions, and ideas. With the flow of answers going from concrete to abstract and student learning remaining the center of focus every step of the way, it became evident how essential this lesson stage is for developing studentsô mathematical practices and how much it contributes to students becoming proficient users of mathematics. In the research lessons I observed, students of all grade-levels understood that in mathematics, itõs a vital task to construct viable arguments, critique the reasoning of others, compare their strategies to those of their classmatesô without any judgment or hint of insecurity, and, of course, record their learning through independent reflection. The studentsô teachers, who all partake in lesson study, carefully nurture all of these mathematical practices in every lesson.

My trip to Japan made it apparent that lesson study is a must-have in order to see teachers grow professionally. It is vital to incorporate collaborative work with colleagues into my school district in order for teachers to learn a variety of teaching strategies and problem solving methods that can be used day to day in the classroom. The focus for teachers should be on growing together as educators, rather than on independent work and research with little to no communication with colleagues. Currently, there are no lesson study groups established in my school district, besides a small group that I am a part of, as a result of my participation in "Japanese Structured Problem-Solving as a Resource for U.S. Elementary Mathematics Teachers" Project. The challenge will be to motivate my fellow colleagues to partake in a lesson study cycle. Between incorporating the new Common Core State Standards and the new teacher evaluation system, the teachers in my school district already feel extremely overwhelmed. Another challenge that I will encounter when establishing lesson study at my school will be finding common planning time with colleagues. Since lesson study is currently not a focus in my district, it will be a huge challenge finding that common time throughout the school day to do research, plan, and reflect collaboratively. Lastly, a personal challenge that I will inevitably battle with this school year will be realizing that the creation of something so important and detailed takes time and I am going to have to start small in order to actually reach my goals of using lesson study in my school district.

My hope is to create a lesson study structure that's inviting and nonthreatening to both my fellow colleagues and administration. By starting small and designing a good entry-point, for example, board writing or journaling, others will instantly realize that lesson study goes hand-in-hand with growing professionally as an educator as well as enhancing student learning. My goal is to make lesson study an exciting, innovative way to adjust to and successfully implement all of the new changes that are currently taking place in my school district, that way it doesn't seem like another major change, rather a way of adjusting to all of the new changes. Since the purpose of lesson study is to work collaboratively, focus on student learning, and engage in meaningful *kyozai kenkyo*, study and exploration of instructional materials, implementing lesson study should feel like a natural practice to begin while incorporating the CCSS into the curriculum. In the end, no matter how close I get the lesson study in my school district to look like the authentic lesson study that I observed in Japan, I am creating and being a part of something that I truly believe in. As long as I set goals and attempt to implement some form of collaborative work with the teachers in my school district, I will grow as an educator and my students will reap the benefits.

In summary, with my trip to Japan being over and a new school year right around the corner, I find myself anxious to implement everything I learned. I have a brand new outlook on both teaching and mathematics, along with friends from across the world who will forever be a teaching resource. We share priceless memories of our experience together in Japan that will forever bond us. To have had such unique, thought-provoking professional development in such a small amount of time, I am extraordinarily grateful! Thanks again to the entire IMPULS team and all of the teachers and students from the schools we visited for such an unforgettable, life-changing journey!

**References**

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York: Free Press.

# Project IMPULS Reflections

Doreen Stohler
K-6 Elementary Math Coach
Hamden, Connecticut

**Lesson Study Immersion Program 2013**

*International Math-teacher Professionalization Using Lesson Study*

In reflecting on my experience in visiting Japan and observing research lessons in various schools, I am still filled with awe and gratitude in having been given the opportunity to do so. It is one thing to hear and read about lesson study in Japan, and another to finally experience it. I have a deeper understanding, a greater appreciation and a more global perspective on teaching and learning.

**Japanese culture**.

**Teaching and learning is respected**. When I observed the teaching and learning process in Japanese schools, I noticed some striking differences from the same type of scenarios taking place in the United States classrooms. Whether the learning was taking place in the interaction between teacher and student, or student and student, or student to class, it appeared that students themselves believed the conversation...or neriage taking place, to be of value and important to their learning. This is based on my observations of students' attention, their posturing, their preparedness, the calling out to be recognized while raising hands, their eagerness to share and willingness to risk explaining others' thinking. The record of their learning unfolding on the board and the subsequent document of work recorded in their notebooks, was deliberate and intentional in not only what was written, but in how it was written; neat, color-coded, classmates' ideas labeled with the author's name. Learning is important and valued.

I observed that same sense among the teachers I observed. Not only is teaching valued and respected but Japanese teachers have a strong sense of self-efficacy. They have a high regard for the process of learning and devote hours to planning, discussing and evaluating how well they believe they have affected learning and have moved students' thinking. At least from my observations among educational venues, teaching in Japanese culture is a well-respected profession and the master teacher is highly revered. Mastering teaching is developed over years of continued practice and study. Teaching is important and valued. The teacher has the capacity to make a tremendous impact on the learning taking place in the life of a student.

**A common vision is the autonomous development of the whole child, and this responsibility is shared by all.** Japanese society allows for children as young as 6 or 7 to travel independently by foot, or by taking a train and/or public bus to get to school each day. The health and physical well-being of students' bodies and minds is encouraged through the provision of balanced and nutritious meals prepared fresh by a school dietitian and students from an early age are taught to swim. I observed teeth-brushing as part of the school routine. I observed unicycles and stilts available for the recreational development of balance and independence. I observed the development of life skills through raising chickens, growing vegetables, and learning to sew. Respect for one's environment, as well as control over it, is fostered by holding students responsible for meal set up and clean up, and regular maintenance of the school. All of these areas are part of the common vision of developing the whole child, and are the collective responsibility of all members of the school and community.

**Lesson Study Reflections**

**Collaboration extends beyond a "Lesson Study Team."** The lesson study 'team' is far more encompassing than a group of a few grade-level or mixed grade-level teachers. Membership is on many levels, depending on the purpose and may include administrators, University professors, the school community and teachers from other districts. Organizing a multi-layered infra-structure to support and sustain lesson study is a challenge for me to address within my school and district.

**The presentation by the 'knowledgeable other' following the post-discussion is powerful in raising deep questions about the progression of math learning.** These 'mini lectures' were insightful in examining the intended goals of the math lesson in relation to the overarching math concept being developed. The comments and questions presented often raised new ways of looking at how student thinking is developed and how easily the development of *real understanding* can get derailed.

**Japanese Lesson Study 'pilots' or may be said to be the 'testing ground' of proposals of the National Course of Study and therefore has the potential to effect wide-spread change.** Teachers use the recommendations in the national COS to begin generating research units and lessons. They examine curricular materials and the sequence of math-concept acquisition through the grades. This kyozaikenkyu is essential to good lesson planning. Once the research cycle has occurred, the findings resulting from the post lesson discussion are considered 'research based' and may be used as evidence to either support or modify the national COS. Inherent is the value placed on the research conducted through lesson study. With the national CCSS making its debut in the United States, Lesson Study could be a valuable tool in providing on-going feedback.

**The greatest challenge appears to be in generating quality hatsumon and neriage.** One challenge in the research lessons I observed was the teacher's ability to ask meaningful and strategic questions to elicit the ideas necessary to move the mathematical thinking along. 'Hatsumon' is important because the way a problem is posed influences studentsôlearning significantly. One can not lead by questioning if they don't know where they are going. Knowing the right question to ask, to whom and when, requires flexibility and a keen awareness of the mathematical progression of the lesson. While teachers prepare good questions in advance based on anticipated responses, student responses and paths of thinking aren't always predictable. It requires that the teacher *listen carefully* and *observe closely* to what the student is thinking at the moment and respond in a way to provoke his or her mathematical thinking.

The second challenge of research lessons I observed was the neriage ... kneading the collective thinking of students to meet the lesson objective(s) and to move student thinking closer to the goals of the unit. Again, this requires careful observation of how the students respond to the teacher's questions and react to the tasks during the lesson. It is knowing which students you will ask to share work, in what order they will be asked to share, in what arrangement their work will be incorporated in the bansho, and what questions will be asked so students can see relationships among the solutions and make generalizations.

In lessons I observed where the hatsumon and neriage were strong, the lessons were amazing.

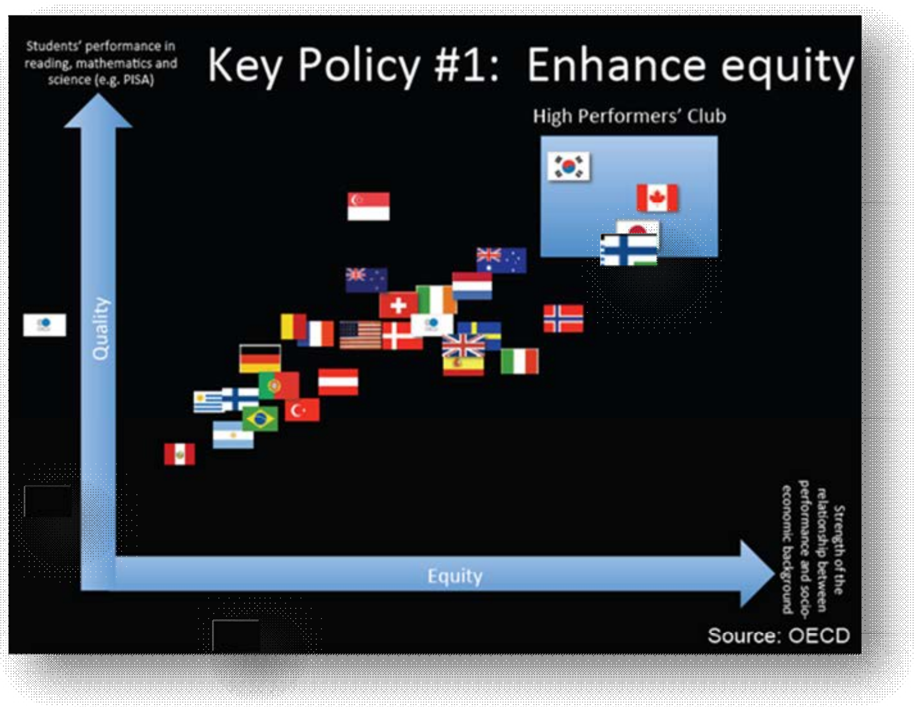
**IMPULS Lesson Study Immersion Programme 2013 Reflective Journal**

*Elnaz Javaheri*

**Advance Skill Teacher of Mathematics, Heartlands Academy, Birmingham, England**

A week before my visit to Japan I went to Kingôs College London for the education departmentôs annual talk by Pasi Sahlberg about the education system in Finland. There I was shown the graph below which represents the most high performing countries in the world, one of which was Japan It was interesting for me to see that Finland and Japan are both part of highest performing countries. During the talk Pasi expressed the basis of the education model in Finland;

1. More collaboration, less competition
2. More prevention, less repair
3. More evidence-based reforms , less experimentation with children
4. The children must play
5. Less test, more trust




I started to get very excited about my visit and wondered whether the Japanese education foundation would be the same as Finish or different since both of which are high performers?

On the first day of my visit we were given ñThe guide of the course study for High/Primary School Mathematicsò The following lines from the guide book took my attention; ñParticular attention must be paid to achieve basic knowledge and skills, and to nurture study the attitude to willingly pursue learning in order to lay foundation for lifelong learningò. Besides that, it was mentioned; ñfollowing are characteristics in Japanese education, Child Centred approach, Well-sequenced curriculum and Improvement through Lesson Studiesò. I tried to focus and find evidence for what was expressed in the book during my visit. Below are my findings;

1. Child Centred approach
    - Healthy diet

A healthy diet is the basis for successful learning. In every school we visited the lunch was simple but nutritious. Almost everywhere lunch included soup, rice, vegetables, fish, a bottle of milk and a piece of fruit. Students were happy about the lunch provided for them as it was evidence from the fact that there were no left overs on their plates.

Figure 1 shows students serving each other lunch

- Game

Imagination and creativity develop through studentsô engagement in childhood games. Japanese students start school at the age of 6 or 7 to allow them longer to play before formal education starts. I was fortunate to observe how talented Japanese students are in playing their traditional games and making Origami[1]. In addition there was always a game at the beginning of the lessons I observed, to engage the pupils.

[1] *Origami* is the traditional Japanese art of paper folding, The goal of this art is to transform a flat sheet of paper into a finished sculpture through folding and sculpting techniques

Figure 3 shows students playing Chuzara

Figure 2 shows Origami made by Japanese students

- Trust/responsibility

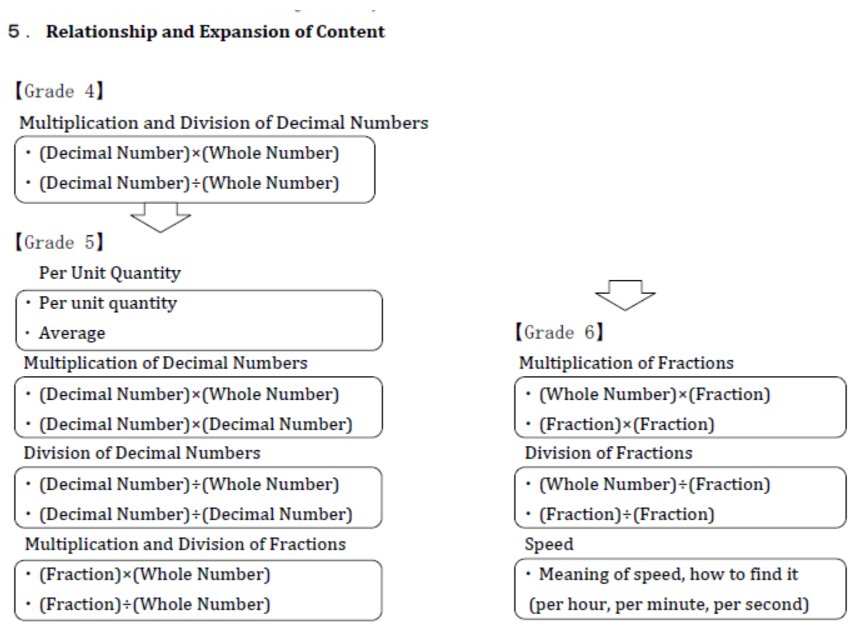
There is a big sense of trust between teachers and students. I observed a Design and Technology lesson where students aged of 9-10 were making things out of wood by using hammers and saws. In my view, this demonstrates their culture is fit for such a free environment whereas in the UK this may be limited due to health and safety concerns. Another example is that students take responsibility to serve their own lunch in a collaborative environment.

Figure 4 shows how students get ready to serve

All the things I observed under Child Centre approach reminded me of Finnish education elements of more ótrustôand óchildren must playô

2. Well-sequenced curriculum

In every lesson plan I was given, the unit of the work from grade 1 to grade 8 was mapped in detail. I could understand *the gradual progression* of every topic from grade 1 to grade 8. On the other hand, in the UK the spiral curriculum requires teachers to deliver larger blocks of the curriculum in one year and for this to be repeated in subsequent years.




3. Improvement through lesson study

I learnt in Japan that there are three levels of teaching.
Level 1: Teachers can tell students important, basic ideas of mathematics such as facts, concepts, and procedures.
Level 2: Teachers can explain the meanings, reasons of the important basic ideas of mathematics in order for students to understand them.
Level 3: Teachers can provide students with opportunities to understand these basic ideas, and support their learning so that the students become independent learners.

In Japan, educationists believe that in order to develop expertise, teachers should plan the lesson carefully, teach the lesson based on the lesson plan, and reflect upon the teaching and learning based on their careful observations. Japanese teachers and educators usually go through this process using the óLesson Studyôformat.

First, they need to establish a óResearch Committeeô including the head teacher of the school and teachers from different experience levels. Secondly, the óResearch Committeeô decides on a goal such as ‘Designing lessons that students become absorbed inô and also form several sub-groups that engage in a lesson study cycle. See the mapping below:

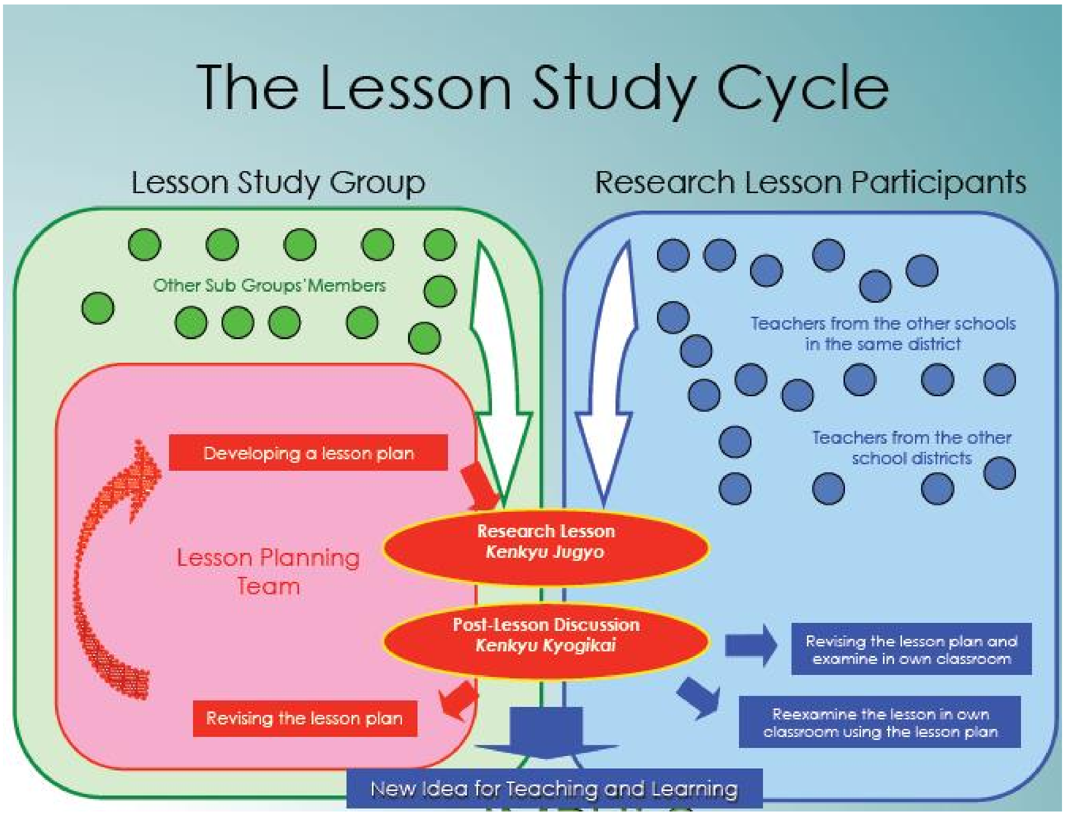
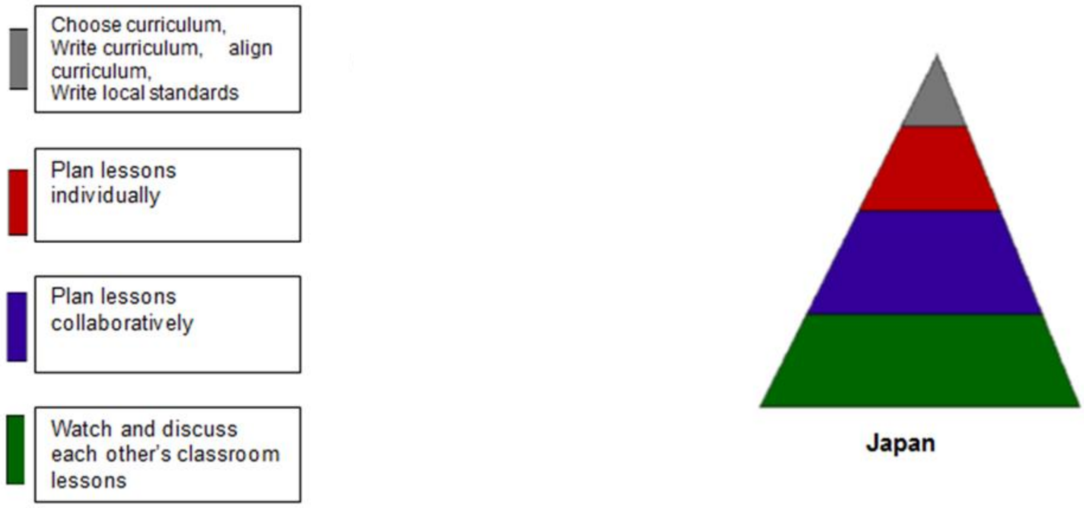



**Figure 5 shows the lesson study cycle, extract from IMPULS presentation**

Later on during my visit I realised the findings of each lesson study observation were to be sent to the Department of Education in Japan. Based on the results sent to them, the education committee review the textbooks and curriculum. This process makes teachers use textbooks comfortably since they are aware that the textbooks are tested and adjusted so many times to fit the purpose. The elements of lesson study reminds me of ớmore evidence-based reforms (such as lesson study), less experimentation with childrenôin Finnish education.

"Lesson Study" Implementation Comparison between Japan and UK

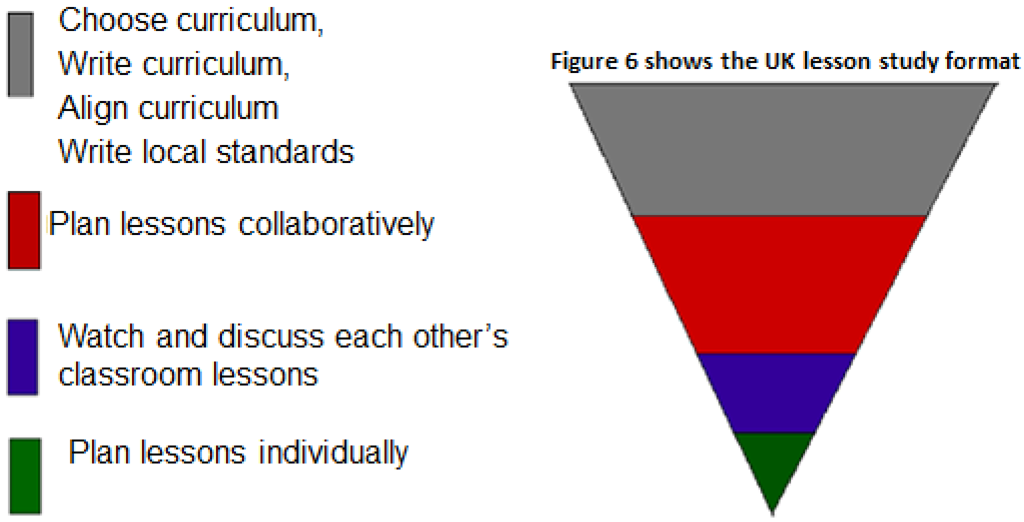
Before my visit I came across the chart below and I never understood the implications fully until my visit. The diagram shows the relative amount of time spent at each stage of the lesson study process. During my visit I realised how specific the designed curriculum in Japan is and therefore how much easier it is for teachers to choose what they want to focus on in each observation. Also every school explained to us that they meet at least three times for planning and sometimes each meeting would last up to 2 hours.




Every post lesson discussion was held immediately after the lesson observation with the presence of someone from higher education that is called the óKnowledgeable otherô. The post lesson discussion was there to guide teachers and was always summarised through general guidance points given by the óKnowledgeable otherô. Some of the post lesson discussions were very productive not only to us but also to the Japanese participants. Below are some of the points I learnt from the discussions;

1. It is not always necessary to come up with óExcitingô hooks, but the mathematical situation itself can result in an engaging interaction between teachers and students
2. There are no perfect lesson plans as lessons often do not go the same as the plan. Having a flexible approach is more important than having a perfect lesson plan.
3. We can improve studentsô questioning abilities by developing their understanding of the topics.

From my own experience the same chart based on the lesson study cycle in the UK would look like this;




I realised in the UK like Japan, we spent more time on planning lessons together rather than individually. Even though we knew how valuable the post lesson discussion was, we were unable to allocate sufficient time due to other teaching commitments. As with any other country involved in ñLesson Studyò, we found the process time consuming. However, I would like to point out that itôs a long term investment which would be worth the effort.

Finally I would like to refer to the chart I put at the beginning of the report regarding the high performing countries in the talk by the Finnish educationist .The lessons I learnt from Japan are ;

1. Collaboration
2. More evidence-based reforms such as lesson study , less experimentation with children
3. The children must play
4. More trust

The above points were similar to those Pasi Sahlberg mentioned as the foundation of education in Finland. What really strikes me here is, even though both countries have got totally different cultures, the foundation of their Education is the same. During my 4 year teaching experience, and from what I have seen, read and heard, the foundation of any successful education model have the above elements in their model. Therefore, whilst cultural differences will influence the application, I hope that one day all countries will incorporate these principles in the design of their education systems.

I would like to express my sincere appreciation to the team of óIMPULSô without them this project would have happened. I would like to also take the opportunity to express how much I appreciate the time and effort all the staff and principals of the schools who opened their doors to us so warmly and hospitably. Also my warm regards and thanks go to the graduate students who were so caring and made this visit memorable.

**IMPULS Lesson Study Immersion Programme Reflective Journal.**
**Dr Ferida McQuillan**
**Head of Maths at Heartlands Academy, Birmingham, England.**

I would like to thank IMPULS for inviting me to participate in the Lesson Study programme; it has been a unique and invaluable experience for me. It was invigorating to work with so many enthusiastic and intelligent colleagues overseas. I wish to reflect on the following aspects of the Japanese education and teacher training systems which stood out for me during the visit.

I was impressed by the idea that Japanese students decide by the end of their schooling if they wish to become teachers; these students attend specialist teaching universities where the emphasis is on training them to become excellent practitioners. In each teaching university, there is a school within the campus, and further schools attached to the university. This allows the trainees to have the opportunity to observe experienced teachers and begin to develop their own skills even before graduating. In addition, the principal of the school within the university campus is always one of the faculty professors.

Teachers are assigned one of 3 clearly structured levels of competency:

- Level 1 Teacher ï Teachers who introduce pupils to basic concepts and procedures
- Level 2 Teacher ï Teachers who can explain the meaning and reason of the important basic ideas of Mathematics
- Level 3 Teacher ï Teachers who provide opportunities for students to understand these basic ideas and support their learning so that they can work independently

On average, it takes a teacher at least 10 years to reach Level 3.

The Japanese system also differs in the way schools are funded. Funds come partly from central government but also directly from local taxes; this raises the schoolôs profile in the community and encourages the children to take pride in their school. All the schools we visited emphasised a well-balanced diet and all round fitness so that the pupils are ready and fresh for learning. To reflect this, every school we attended had a swimming pool.

Some schools do not have a uniform. Students select what they want to wear whilst meeting the expected standards. This gives them a certain amount of freedom, but also helps them develop the ability to take responsibility for their choices.

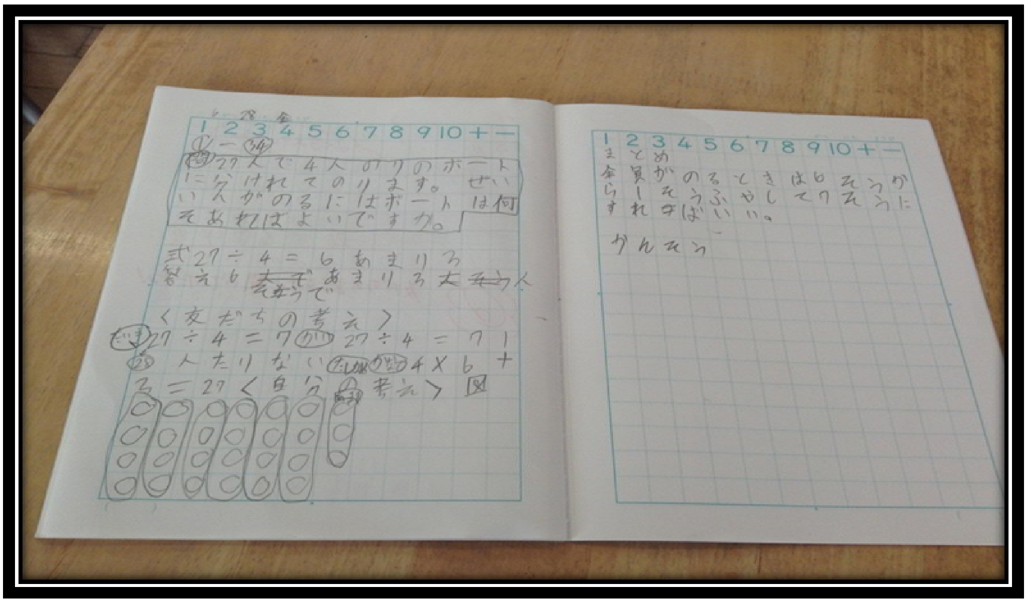
Research lessons are designed with a district or school goal in mind. This means that all participating teachers across the grade bands and subjects are thinking about how to develop similar capacities in the students.

Every school has a study committee, consisting of the Principal and a group of teachers. Members meet at least three times for every lesson to plan, evaluate and improve the lesson. In addition, after the lesson has been delivered there is a post lesson discussion involving the Principal, the study committee and the teacher delivering the lesson. This includes reflecting on what did and did not work in terms of the impact on student progress. A ďknowledgeable otherô is also invited to observe the lesson, attend the review and advise on future developments.

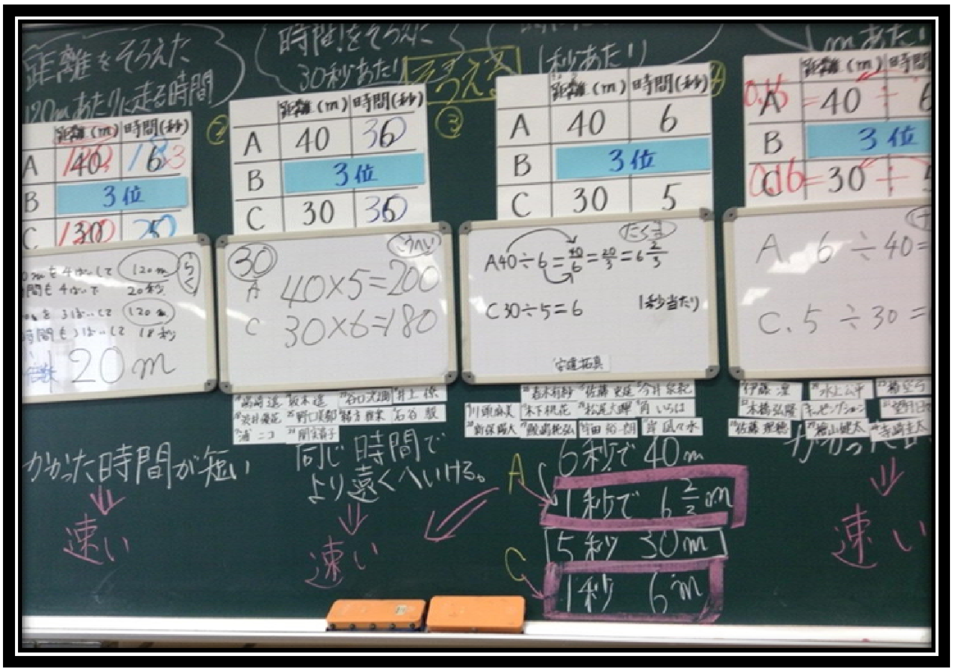
I observed many lessons during my visit to Japan and noticed that the focus of most lessons was how to help students work through mathematical problems, and how diagrams can be used to scaffold the interpretation of results. Skilful questioning helped pupils to consolidate their previous knowledge and think logically in order to solve new problems. There was constant dialogue between teachers and students. Students were encouraged to come to the board and share their work with the rest of the class.

The classes consist of a mixed ability of students; consequently, in some lessons not all pupils progress at the same pace. However, this is picked up when the teacher reads the studentsô journals and remedial help is given where necessary. It is essential that the thinking of the teacher and the student is in the same direction so that progress can be made when students are incurring difficulties. Anticipating the childrenôs questions is an important part of the planning process.

Much is made of the use of interactive whiteboards in England. In Japan, all the work is written during the lesson, with the teacher having planned the final board in advance. Mistakes are simply crossed out so that the board details the journey made during the lesson. Not only does the blackboard serve to communicate ideas to the pupils, the studentsô own attempts are also showcased to the rest of the class as well.



At the end of every lesson, pupils write a reflection in their notebook. Unfortunately (because the children were writing in Japanese), I was unable to understand what they had written, and so the schools have agreed to send us translations of a sample of lesson reflections in several notebooks.




What I liked about the Research Lessons was the fact that it brings together so many different levels of experience, and the success of the lesson rests with the whole study committee rather than just the individual teacher. It is also a valuable tool in helping inexperienced teachers to develop much quicker from the knowledge sharing of their colleagues. It was good to see the whole school were working towards the same goal. I am looking forward to applying these lessons from Japan within my own department.

Finally, I would like to thank all the organisers and student teacher helpers who made our stay so enjoyable. As mentioned earlier, this proved to be an invaluable experience for my colleagues and I. Furthermore, I would like to express my gratitude for the great hospitality and welcome the Japanese students and teachers showed us throughout our visit. A lot was learnt during our trip, and I am looking forward to implementing many ideas I experienced as soon as possible at my own school.

## IMPULS Lesson Study Immersion Program Reflective Journal

**Francesca Blueher**
**Montezuma ES**
**Albuquerque, New Mexico United States**

I am still aglow from the magnificent experience of the IMPULS Lesson Study Immersion Program I recently attended in Japan. It was a privilege and honor to be introduced to the Japanese culture through its schools, its philosophy of educating the *whole* child, its embrace of lesson study, and the welcoming teachers and students. As I reflect on all I learned about Japan's rich, robust educational system, I am grappling with how to take these essential values back to the United States and incorporate them into our schools.

The values to which I most connect are the Japanese elementary education system's goals of nurturing the gifts that children bring to school through an equitable education for all students, and supporting teachers as learners, researchers, and thinkers. These core values were in abundant evidence at the school campuses that we visited, embedded into the lessons we observed, they drove the developmental progress of children, and were the vision that formed the foundation of its curriculum. Japanese public schools at the elementary level, understand the vital importance of educating every student's physical, mental, creative, and intellectual aspects. All elementary students, beginning in Kindergarten, are educated in the arts, home economics, energetic play, swimming, English language as well as core academic subjects.

Vibrant examples of student handiwork were on display at all the elementary schools we observed. I saw expressions of the children's creativity in the lavender being dried in preparation for bouquets, colorful murals, songs and dances, calligraphied poetry sprinkled throughout the hallways, and hand crafted paper decorations being tied to trees for the upcoming star festival. Children's physical needs are nurtured in the many juggling, top, and baton clubs offered during school; swimming lessons that are given during the day in most elementary schools; and frequent, unsupervised play time held outside throughout the day.

As part of the children's physical, social, and intellectual development, schools weave opportunities for student responsibility throughout the day. At lunchtime, this value is particularly highlighted. Children, beginning in 1st grade, are responsible for getting their class' meal, serving dishes, and cutlery, putting it on a cart and taking it to their classroom. Several children don chef hats and aprons and serve lunch to their classmates after a thanks is given to those who worked hard on preparing the meal. The meal is served from and eaten on non-disposable dishes. Music and singing usually accompanies the lunch. Children are then responsible for clean up. Remaining food is either composted for the garden or put in the chicken coop or rabbit hutch. Nothing is thrown away as there are no paper napkins, plastic utensils, or food packaging. There are no trashcans in the classrooms! The end of lunch brings another thanks, a song (perhaps played on the piano which is in every classroom), and then restoration of the room by the children. Getting their classroom back in order and keeping it clean is important to the class environment as there are no custodians in Japanese schools.

The Ministry of Education, school administration, teachers, and teacher education programs are intensely focused on ensuring that all elementary school children receive this well rounded, active, intellectually stimulating, environmentally responsible, thoughtful education. Getting to see these values put in action was thrilling to observe and was starkly different than the values currently found in education reform policies in the United States. Here, we attempt to quantify every aspect of our children's learning in the annual standardized tests in Reading and

Math.

So what do these education values that the Japanese schools hold have to do with lesson study? The Japanese education system has found that lesson study incorporates the values of its education system by providing a way for teachers, student teachers, principals and educational administration to study and learn from rich, robust, developmentally appropriate, harmonious, problem based lessons. Teachers bring what they have learned and observed to their classrooms by studying the pedagogy, content, and educational values embedded in the lesson study process which fits in with the goals of educating the physical, intellectual, and artistic natures of children. The premise of lesson study research is to meaningfully educate all children so that they make developmental progress in their individual learning trajectory. Schools strive to achieve their goals of nurturing student curiosity, creativity, intellectual pursuit, and harmony through the texts of well researched lessons that are observed and discussed by the entire school staff. These research lessons are integral towards the goals and visions of education.

The schoolsôand districtsôgoals for educating children are clearly stated in every research lesson. These are some of the explicitly stated visions and goals that were drawn from the research lessons we observed:

- compare their (students) own ideas and those of others and recognize their strengths
- awareness of value of self, autonomy and independence, creative thinking, relationship between self and society
- nurture a disposition to generate and solve their own questions
- creative reasoning ability

The post lesson discussions revolve around whether the lesson just observed met the lessonôs goals. If the goals were or were not met is vitally important to the teachers and administration as it informs and fuels their future instruction.

In Japan, I learned that Lesson Study has the robust support of the teachers, school administration, colleges of education, and Ministry of Education. This support is essential in providing a meaningful education for all children and the professional development of the teachers. Bringing back this wealth of learning from the IMPULS Immersion Project to my school and district will be challenging. I am exhilarated and excited about this challenge, but how can I bring this back to my school and district whose values at this time emphasize student achievement in standardized tests, quantifying student learning, and evaluating teachers and schools based on Reading and Math test scores?

I feel my first priority is to develop awareness in my schoolôs staff and principal about the deep learning of teachers, administration, AND students which can be achieved in the practice of studying classroom lessons. I would also like to make a presentation to district administration about lesson studyôs potential to inspire robust, developmentally appropriate education for all children. Developing a basic awareness of the possibilities of lesson study for student and teacher learning could possibly set the stage for lesson study as a core value in our schools.

I am so deeply appreciative for the experience to immerse myself in the culture of lesson study and the country of Japan. The greatest gift I got was realizing that it was possible to create a system that sets out to educate every aspect of children and to see it brought to life. If I can bring even a small part of this to our challenged school system, I will feel like I have made enormous steps in improving the education of our children. My respect and admiration for the Japanese values has inspired me to no end. To value the developmental stages of children, to respect the creativity and resourcefulness of teachers and students, and to always maintain a greater vision of nurturing future curious human beings is admirable.

**IMPULS Immersion Program**

**Heather Williams**
**Teacher, Volusia County, Florida, USA**

Many thanks go out to the IMPULS professors and grad students as well as the schools, teachers, and students who were so welcoming and provided this once-in-a-lifetime opportunity to all of the participants to visit a most beautiful country and observe some outstanding math lessons. The two week immersion experience in Japanese classrooms provided me an in-depth look into authentic Lesson Study processes, mathematics teaching and learning, as well as the culture in Japan.

Reflecting on the Lesson Study process I saw during my experience, I have to say I feel quite validated in what my Lesson Study team is doing. I think we have done a good job implementing Lesson Study as true to the authentic process as possible; although, we do have a long way to go in terms of reflecting, debriefing, and getting teachers onboard. It was very encouraging to see so many teachers involved in Lesson Study, as it is *the* professional development opportunity for teachers. The support from administration at the school level all the way to the district and state levels is like nothing I could imagine happening here in the US. We still have principals who havenâ heard of Lesson Study or refuse to provide time and resources to let teachers participate. I believe the support of administration is integral to successful Lesson Study. It is a whole school effort to produce meaningful research lessons. Another aspect of Lesson Study I enjoyed seeing in Japan was the involvement of pre-service teachers in the process. Giving these student teachers the opportunity to see great teaching will only make them better teachers themselves. During the lessons and debriefings, I was surprised to find out the in depth and wide range of content knowledge held by teachers, principals, and knowledgeable others. Although the process of debriefing was slightly different in each school, the essence was similar: all teachers involved wanted to be better teachers. The discussions were very reflective and focused, creating more questions for the participants than answers! Lesson study truly highlights the importance of carefully observing students and seeing through the eyes of children in the learning process.

This Lesson Study immersion program allowed me to observe a number of top-notch lessons by superior math teachers. The most important thing this experience has taught me in the realm of teaching mathematics is the flow of the lesson. The lessons I observed became true mathematic stories from the beginning problem solving problem to the very end summary. The lessons were beautiful models for the flow of what lessons should be. I appreciated and see the importance in giving students mathematically rich problems to solve. In each lesson, the bansho unfolded as the key piece to providing students and teachers with meaningful neriage. I was impressed at how, with seamless effort, students were able to talk about their own solution strategies as well as compare them with classmatesâ Each research lesson abounded with hatsumon which helped guide the students through the lesson and reflect on their own thinking. I have gained some insight from this immersion experience how to better summarize lessons for and with students. This was made possible, I think, from the math journals kept by students. They have been taught from a very early age what is important to record in their journals. Also, a number of times I saw students using their journals to go back to previous lessons either to refresh their minds or look up a key piece of knowledge they learned in prior lessons. It was made so very clear the importance of a coherent curriculum. Students were not forced to make big jumps in their learning, rather small jumps they could tie to prior understandings. I think one way this is made possible is through the extensive teaching guides as well as the textbooks available. The textbook publishers seem more interested in providing coherent curriculum rather than making a buck off of ten different versions of the same shallow text. Although these curriculum resources are important to the teaching of mathematics for teachers, I think the biggest reason teaching mathematics is done so well in Japan is the practice of teacher reflection. We, US teachers, need to become more reflective in our daily teaching.

I am often asked if it isn't the teaching so much as the culture that makes the successful lessons I saw possible. I have to say that I am a little torn about the answer to this question. There seems to be a profound difference in the amount of respect there is in schools. Students respect the teacher as well as the lessons being taught. Teachers respect the students as children and acknowledge the need for students to be kids (as reflected in the number of playful incidents that happened in classrooms which would have been immediate referrals in the US). There seems to be a more clearly defined line between a time for work and a time for play than there is in our classrooms in the US. I believe this is a major cultural difference that contributes to the success in teaching and learning. There is also a grand responsibility on students and their schoolwork both at school and home as well as on teachers to create classrooms of thinkers and learners.

Overall, the impact of this IMPULS immersion program is grand. Because so many teachers from so many different parts of the world were included, Lesson Study will continue to be implemented and encouraged by all of us. Again, I thank the IMPULS staff and everyone involved in making this program a worthwhile experience. I look forward to continuing professional conversations with all of my new colleagues and friends!

# IMPULS Lesson Study Immersion Program 2013- My Reflection

Kathryn Palmer

Network Numeracy Teaching and Learning Coach and Consultant, Melton, Victoria

The immersion program was exactly that, an immersion into the *Lesson Study* process in Japan. Not only did I develop a deeper understanding of how *Lesson Study* is implemented but also a good understanding of the school system in Japan.

*Lesson Study* is an ongoing, collaborative, professional development process; teachers systematically examine their practice in order to become more effective instructors. (*The Trends in International Mathematics and Science Study* TIMSS) video study identified *Lesson Study* as a powerful, ongoing process for improving the quality of teaching in mathematics. (Stigler & Hiebert, 1999). Our opening day of the program focused on *Lesson Study* as a *'Nice to Have'* or a *'Must Have'*. By the end of the program I could see the benefits of a ó*Must Have'*. This program highlighted to me a number of considerations that need to be taken into account to improve our teaching practice in mathematics in Australia:-

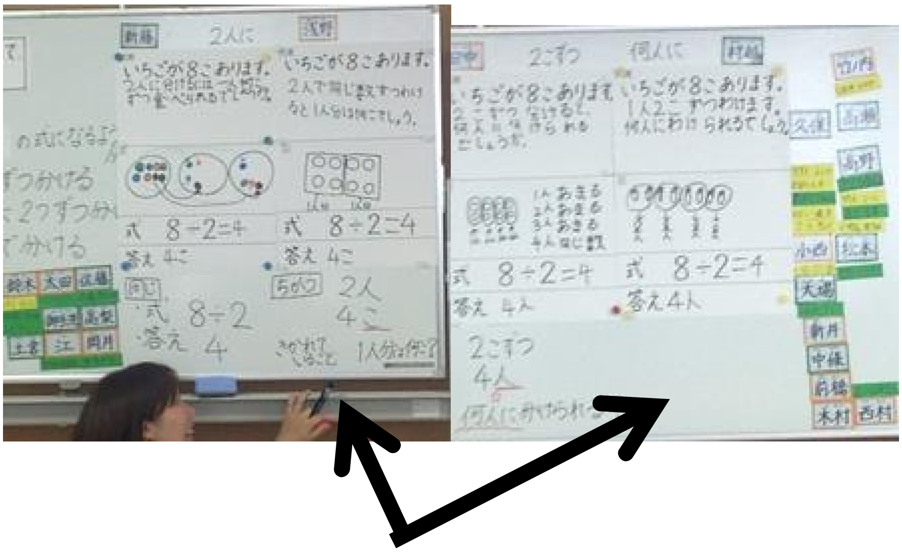
1. Development of teacher content knowledge and the associated understanding of the continuum of learning that this provides. This program highlighted to me a vehicle to use to improve this by getting teachers together to focus on the study of teaching and learning. Where schools become a place where teachers not just students learn. I think we have much to learn about how to develop sequential problems and make connections to prior learning. The Japanese textbooks and curriculum documentation make clear developmental links to the progression of learning and support the teacher to present a problem without first demonstrating how to solve the problem, allowing the students to find out for themselves.

2. Working collaboratively. The entire process of *Lesson Study* is that of collaboration.
   -Inclusion of the leadership (principal and other school leaders) as instructional leaders into developing curriculum.
   - School working together to develop a long term goal for *Lesson Study*.
   During our program we were fortunate enough to observe different types of research lessons; school based research lessons and district research lessons which took place on a Saturday so as many teachers as possible across the district could attend the research lessons.
3. Inclusion of pre service teachers in *Lesson Study* whilst they are still training at University.
4. Support of Knowledgeable Other in post research lesson discussions.
   During the observation of post lesson discussions it became evident the important role the Knowledge Other plays in this process. The teacher discussion is important but the input of this expert gives teachers a much deeper understanding of the mathematics and gives insights into the lesson that the teachers donôt see or know and donôt have the expertise to raise during their post lesson discussion.

While teachers cannot engage every day in such deep lesson investigations, conducting it for the purpose of a research lesson leads to a deeper understanding of the curriculum and the mathematical content and goals underpinning it, as well as the importance of matching problems to both the mathematical goals of the lesson and studentsô knowledge.

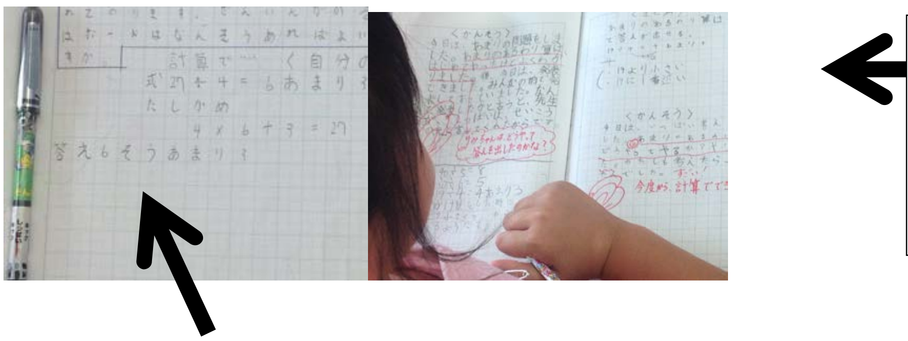
The *research lessons* in mathematics that we observed were based on structured problem solving. During research lessons we saw four stages: teacher posing the problem; students solving problems individually, in pairs or small groups; whole-class discussion which involved detailed board work to capture student thinking; and clear summing up.

1. The whole class discussion involves the teacher selecting students to share their solutions in a specific order and all shared solutions are clearly displayed on the board and discussed.



The teacher has 4 students share their different solutions for the division problem, using both partition and quotient.

2. The summing up or Matome is a vital component where students write in their notebooks to clarify their understanding. The teacher collects and reads these and develops knowledge of each students thinking. I noticed on numerous occasions students referring back in their notebooks to prior learning related to the new problem they are working on.



Then the student spent time flipping back in her notebook to the last time they did division and she looked at when she had first done remainders

This student calculated quite quickly the solution to the problem writing: 27 ÷ 4 = 6

Continuing the learning in Australia:-

In 2012 and into this year I was involved with Deakin University on a project, *Implementing structured problem-solving mathematics lessons through lesson study*. The project worked with two Year 3/4 teachers from each of three schools from a Melbourne school network to explore ways in which key elements of Japanese Lesson Study could be embedded into Australian mathematics teaching and professional learning. Teachers were supported not only by members of the Deakin research team, but also by a key leading teacher at each school (e.g. a curriculum specialist or numeracy coach) as well as myself the network numeracy coach ï a total of ten participants. Each *Lesson Study* cycle involved two cross-school teams of three teachers and two leading teachers or coaches planning a research lesson on the same topic during four two-hour planning sessions. Each team was supported by two of the university researchers. One member of each team taught the research lesson in front of observers, with both teams participating in the post-lesson discussions. Key staff at each school, together with all interested teachers who could be released from their classes at the time of the research lessons, as well as other professionals such as numeracy coaches and leadership teams from other network schools, and mathematics educators, were invited to observe the lessons and take part in the post-lesson discussions.

This project became the vechicle to build teacher pedogogical content knowledge. The collaboration between the schools and teachers involved, support from both our university collegues at Deakin and the network all resulted in teachers involved being more reflective of their practice. The professional learning of teachers is an ongoing process of knowledge building and skill development in effective teaching practice. I am working closely with the network to continue this process but am having to overcome the challenge of releasing teachers without funding and changing school priorities.

Since returning to Australia I have shared my experiences of *Lesson Study* at the Australian Association of Mathematics Teachers Conference during both a workshop with my collegues from Deakin University and during a keynote adress I delivered. I am sharing again at the annual Mathematics Association of Victoria Conference in December. I am extremely passionate about the teaching of mathemtics and found like minded associates at the IMPULS immersion program. I look forward to continuing the learning with them and continue to build understanding of *Lesson Study* .

Lauren Moscowitch
Harlem Village Academy, New York, NY
Reflective Journal

The Japanese Lesson Study Immersion Project was an eye-opening experience for me. I learned not only about mathematics education, but about cultural traditions in Japanese schools, cooperation between teachers and administration, and about deep reflection and improvement practices. Japanese Lesson Study should have a permanent place in my home school in order for all teachers to work together cooperatively and continually improve our practice.

Japanese elementary schools are remarkable places. I was immediately struck by the sense of community that all members share as soon as we entered a school building. We saw various building lay outs and sizes, but all the schools we visited shared a love for children, education, and Japan, as well as an overall sense of joy. One example of the love for Japan and a sense of joy that I saw was when Oshihara Elementary School welcomed me into their music classroom and put on an impromptu concert of various Japanese traditional and folk songs. Being foreign made no difference; we were wholly embraced by the schools we visited and made to feel welcome. As a kindergarten teacher, I appreciate how independent young Japanese children are. By Western standards, having children use saws and hammers is dangerous, but the Japanese children handled those tools in stride. First graders prepare their lunches and clean up after themselves. They sing together and share the joy in each studentsô successes. Japanese elementary schools are places where children succeed and thrive because the atmosphere is so conducive to learning.

There are many strengths to Lesson Study that are of benefit to my home school. The first strength of Lesson Study that I witnessed is the collaboration between teachers. This trip was my first experience in reading a research lesson plan, seeing a research lesson, and listening to a post-lesson discussion. Teachers from different grades and experience levels worked together to set a goal, research a lesson, write and re-write the lesson several times, conduct the lesson, and finally critique the lesson in a supportive environment, all for the benefit of the students. Japanese teachers know their students well and utilize questioning effectively. Since teachers change grades frequently throughout their careers, teachers have a broad understanding of what is expected at each grade. The teachers share their knowledge with each other during the research phase as well as the planning phase of the lesson. The extended collaboration between the teachers is made even stronger by the fact that the administration joins in the planning and discussion as well. Each individual school is a very powerful place because the school is not compartmentalized by teachers and administrators. A school is a group of professionals working together for the benefit of the children.

Another strength of Lesson Study was the research teamôs efforts to create a research lesson with a strong rationale. I was not able to see a research team at work, but I was able to read the research lesson plans as well as re-writes that the team created the day before the lesson was executed. The entire school knows the goals of the lesson. It is powerful that the entire school is aware of the goals, so that each team member can work towards the goal. Similar to Western Backward Planning, having the goals created first ensure that the goals have a higher percentage of getting met.

The role of Knowledgeable Other was completely new to me. This position has many strengths for the entire educational team. It allows for an outside member of the educational community to offer feedback and expert advice for the teaching teams. The entire school is present at the feedback session, so the knowledge is shared throughout the whole school. The Knowledgeable Others who I witnessed were able to offer advice that was specific enough to address the concerns of the lesson he/she watched, while still being general enough for all the teachers to find useful for their own practices. One concern that I had regarding the Knowledgeable Other was that his/her speech often included information that was not directly pertinent to the day's lesson, but rather was information about the district or a new program that would likely be implemented. I'm sure the information was useful, but at the end of a long day and after observing a research lesson, perhaps that information would be best disseminated at another time.

A definite strength of Japanese Lesson Study was the detailed reflection that I witnessed several times in all the schools we visited while on the Immersion Program. The hard work that was put into each lesson would not have been as effective if not for the reflection period that followed each lesson. The teachers were thoughtful in their reflections. With grace and humility, the teachers realized that their lessons were not perfect, and were open to hearing feedback and advice. It struck me how much the feedback was about the lesson, not the teacher. The feedback sessions were very respectful to the teacher and never were made to feel like an attack on the individual. The lesson was a team effort, and the feedback was also directed at the team. I have one question about the presence of the team during the feedback sessions: Where were the team members during the post-lesson discussions? The teacher who taught the lesson was the only representative on the panel. If the emphasis is heavily on team work, shouldn't the team be represented on the panel, not just the teacher who taught the lesson? If the entire team planned the lesson, not just one individual, I would have liked to see more input from the team during the post-lesson discussions. I'm sure there is a cultural or other explanation for the team's presence being absent during the post-lesson discussions, but to my Western eyes, I do not understand the sudden switch to the individual.

The knowledge that I gained over this trip will stay with me for a lifetime. I feel privileged to have experienced the Japanese way of life for a brief while. I received a rare glimpse into Japanese teacher education and daily classroom life that will influence the way I plan my classroom this school year and for many years in the future. Thank you for the experience and the knowledge.

**Reflections arising from the IMPULS 2013 immersion project: Mark Simmons, PGCE Maths Lecturer, University of Nottingham**

I will begin as others have by thanking sincerely all of those involved in the work of the IMPULS project. My participation has resulted in an explosion of reflections on my part, a few insights (I hope) and predictably, a host of new questions. The majority of my reflections concern the contrasts I perceive between Japanese and UK maths education. I also have a few reflections upon the process of lesson study as I have witnessed it in Japan and the possibility of ôborrowingô from it in a UK context.

The aims and purposes of an education system are many and I believe that only some of these are ever made explicit to the people who participate in the system. Essentially, I would propose, most education systems serve to reproduce the nature of the society in which they are situated (although I would not wish to deny that they might also have transformative potential).

I come from a context (England) in which society, in my view, is structured with a small elite in positions of political and financial power, a large majority whose daily efforts serve to provide for themselves but also to enrich and protect the elite and an impoverished minority. This minority experience a daily struggle with the facts of poverty, and their over-representation in the prison population serves to help motivate the majority. Peopleøs (misguided) belief in a meritocratic nature for this structure is sustained through a variety of mechanisms, education and the media being two of the most important.

The English education system serves to reproduce this structure by ósortingô the majority using high-stakes testing (thus bolstering the meritocratic myth) and reserving the position of the elite by maintaining for them access to an entirely separate network of schools, the óindependent sectorø. The roots of this system of structuring reach back into history for hundreds of years. I would argue that without some grasp of the nature of the societal context of an education system, any study of aspects of that system or its daily practices will tend to raise more questions than it answers. I would value the opportunity to learn much more about Japanese history and culture to help contextualise my learning about its education system.

It was my overwhelming impression that elementary education in Japan was rooted in thoughtful practice with a long story of accumulating wisdom and that Lesson Study was an important component of this. Japanese secondary education remained much more of a mystery. I was told that there were high-stakes examinations to gain admission to a secondary school and that there were three types of high school (which I guessed would probably be held in different esteem by the populace). I was told that setting by óabilityô was used in high schools. It sounded as though there was a great deal of out-of-school tutoring in preparation for these entrance exams and those for university entrance. All of this sounded (depressingly) reminiscent of English secondary education and made me wonder whether the egalitarian ethos which appeared to pervade elementary education was not representative of the Japanese education system as a whole.

I asked Makoto about the Learning theorists whose ideas underpinned education in Japan. I was impressed by the readiness of his answer and the degree to which I felt that I could see the influences he explained to me in the lessons we observed. I understood the stress on context for learning to come from Dewey; Learning through problem-solving from Polya (a theorist rarely mentioned in my experience in England); Careful staging of the introduction of new concepts from Piaget (eg leaving probability until late due to its completely abstract nature); Combination of enactive (use of manipulables), visual, linguistic and symbolic representations in a lesson and its óBanshoô from Bruner; Careful management of next steps based upon prior learning (ZPD) from Vygotsky ï as well as the communal construction of mathematical knowledge and understanding through classroom discussion (though discussion in Japanese lessons rarely resembled discussion as I would expect to see it in England). I was also particularly struck by something which Akihiko said regarding the shift a teacher can make from seeking to understand what is inside childrenôs heads to having them seek for what is inside his/her own ï that learning stops at this point. This would cast serious doubt on a majority of English maths lessons at some point or another, I think. The latest iteration of the English National Curriculum for mathematics seems to be based on a belief that repetitive practice will lead to fluency, which will in turn lead to understanding. There is a lot of early introduction of difficult concepts ï presumably because the earlier something is introduced, the more time you have in which to practise. Most English teaching contains a great deal of ótellingô followed by practice.

It is important to note here that I left Japan thinking that most elementary school lessons there followed the problem-solving approach we had seen in all of the Lesson Study lessons ï a colleague called this into question on my return and I am unsure now ï perhaps this is something which could have been made clearer?

The structure of the UK maths curriculum as studied by most secondary-aged pupils is what I would describe as a óflat spiralô in that all topics in the curriculum are covered each year by nearly all pupils, and the vast majority make little progress in their understanding ï hence they do it all again the following year. This contrasts sharply with what I saw of Japanese elementary school where the curriculum very carefully and gently slopes upwards, always seeking to build upon prior knowledge, not repeating all of last yearôs work, neither seeking to cover all of the difficult aspects of a topic just because that topic had been begun, so that the majority (I hope) make progress year-on-year. Clearly there is a very significant contrast too between the Japanese ethos (at elementary school at least) of keeping everybody together and the English ethos of strong differentiation and separation by óabilityô at every stage. These have different strengths and weaknesses. I can see how the English system helps to stratify members of society from an early age. I can also imagine that it may serve to help identify those people capable of elite mathematical thinking from a relatively early age to try to ensure that these people do indeed go on to become the leading mathematical thinkers of the nation. It is less clear to me what the Japanese system is setting out to achieve if there is, as it would appear, such a shift in ethos at the transition to secondary school. Is it simply that Japanese educators want to help achieve a more egalitarian starting point before the setting system begins its work? If so, what are the costs and benefits of this to the individuals and to the society?

Teacher professionalism appeared to be another area of rich contrast. Mathematical Knowledge for Teaching (MKT) appeared far better developed in Japanese elementary school teachers than in their UK counterparts at either primary or secondary age. My own field of work is in the education of secondary mathematics teachers in England. Their degree studies rarely contain education as a focus and their subject knowledge is usually highly context-embedded and often very shallow and superficial. Trying to deepen the MKT of my student teachers in the course of their one year PGCE is a challenge indeed ï particularly since schools are often happy with what Sugiyama (2008) would refer to as level 1 teaching ï telling the pupils how to do things, because you know how to do them. Lesson Study clearly had a part to play in the ongoing deepening of the MKT of Japanese school teachers with the More Knowledgeable Other apparently playing a key role in this, and as yet there is no similar mechanism in English education. In fact, the current English government appear to see teaching as more of a craft than a profession. They seem to want new teachers to learn solely from practising teachers, reducing (to zero?) the role of University ITE tutors ï perhaps the closest equivalent to a More Knowledgeable Other that we currently have, (given that most maths education professors in England have limited input into teacher education.) The Japanese view of the teacher of less than ten yearsô experience as a novice sends powerful messages regarding the nature of the profession and the reflectiveness required to continue to improve as a professional over years.

One of the most powerful factors (there are several) which contributes to poor quality (ólevel 1ô) teaching in English maths classrooms is the use of national examination results for teacher accountability through school performance league-tables and performance-related pay. I was somewhat alarmed to hear a school district superintendent (?) apparently hinting that more of this was on the horizon for Japanese teachers. I would strongly urge that this be resisted!!

On Lesson Study formats, I felt that the roles which appeared most crucial were those of chair and More Knowledgeable Other (MKO). However preliminary discussions were conducted, it was important for the chair to have the skill to identify key points arising from them which were relevant to the research question set for the Lesson Study (LS). The chair could then manage the contributions of teaching staff and visitors to maintain a discussion focused upon the research question. There seemed to be three major obstacles to this:

1. Teachers getting slightly ósidetrackedô and talking too much about the instantaneous pedagogic decisions made in this lesson by the classroom teacher (as opposed to decisions made by the planning team in structuring the plan). Having said this, there was often significant value in these discussions for the development of pedagogy by the teachers present
2. MKOs taking too much time talking in very vague terms, again sometimes off-topic as well. Chairs need to feel empowered to interrupt the MKO at times?
3. Other guests such as district administrators who had their own agendas which were not directly linked to this LS. Chairs would need to feel empowered to interrupt and perhaps re-direct these speakers.

It may be culturally very difficult to imagine the chair person dealing with these difficulties in the manner I have outlined. I suspect that to a large extent the post-LS drinking is a useful compensatory forum in which several peoplesô opinions of any unhelpful contributions may be informally aired and further productive discussion ensue.

Careful and reflective group lesson design, multi-observer study and extended post-lesson discussion are elements which I hope I may be able to employ in my own context, although institutional support/funding for this is likely to be necessary for long-term sustainability.

Once again, many thanks. I hope to see you again. Kanpai!!

**Matt Lewis: Reflective journal on mathematics teaching and learning in Japan and Japanese Lesson Study**

In anticipation and throughout my time on the immersion programme in June 2013, I was aware that aspects of what I experienced might seem so different that the only likely conclusion would be to attribute perceived differences in mathematics teaching and learning to differences in culture. This would be unhelpful, as my intention was and still remains to be able better to understand mathematics education for all learners, regardless of their cultural setting. ñTeaching lessons is a very cultured eventò is a true statement, but my hope and belief is that a culture of teacher professionalism can transcend national-cultural characteristics. Although the programme was described early on as ñan opportunity for you to engage with óauthenticô lesson studyò ï which it certainly was ï the interest of the IMPULS group in our views made it clear that this was also a mutual learning opportunity. In this brief reflective journal I will sometimes compare and contrast my experience on the immersion programme with my experience of the English education system, especially mathematics education. The journal is roughly divided into three significant themes that emerged during and since my time visiting Japanese schools and experiencing mathematics Research Lessons: the moral purpose of education; Japanese teachersô rigour in designing lessons; the impact of Lesson Study on teachersô practice.

We were very privileged to be able to witness the introductory lectures given to undergraduate pre-service teachers ahead of their first in-school placement in Koganei Junior High School. These are selected quotes from the three speakers who addressed the cohort of a few dozen pre-service teachers:

- ñThink about what is the philosophy of education in the school. A key word is autonomy.ò
- ñWhen working in the school, consider ówhy are we teaching our subjectô? Think about what we are doing in education ... Think carefully about how and why you are hereò
- ñBuy the textbooks; you may be confident in your knowledge, but learning and teaching are not the same thing. The textbook states things simply, but you have to study the materials to understand the thinking behind the textbook.ò
- ñWe are helping students learn to live life ï you have to care for them, to protect their life.ò
- ñTake the time and opportunities to interact with students.ò
- ñIn your thirteen days here, you should work on your communication and trust, relationships with each other, and the connections between your subject and students.ò
- ñYou are part of the school: be involved, contribute to the beauty of the school. Please take care of yourselves.ò

Many of us visitors were moved by the words that were said. We were struck by the strong focus on the purpose of education, the importance of considering the whole child, and the repeated urge for teachers to think, consider, reflect on their role and responsibility. None of these considerations are alien in the English education context, but they are very rarely the focus of attention. Reflecting on this, I feel that much public and in-school discourse in England is concerned with the *what* of teaching rather than the *why*. It is very valuable carefully to explicitly consider the moral purpose of education. Not all teachers will agree on what that is, but knowing that we are working together makes teachers more accountable to each other and militates against superficial teaching with the sole aim of maximising test scores; considering questions such as this is an aspect of the high level of teacher professionalism that was seen in Japan.

In the Japanese context these considerations are explicit throughout the education system. At the national level, the law states that:

> ñparticular attention must be paid to achieve basic knowledge and skills, to cultivate thinking, decision making and expressing ability to solve problems by using those knowledge and skills, and to nurture the attitude to willingly pursue leaning in order to lay foundation for lifelong learningò.

The statement on learner attitude is much stronger than equivalent phrases in the English equivalent document. This national document is then translated via prefectures to schools, who have overarching óresearch themesô that often touch on learner disposition, for example to develop

> ñcharacters and abilities necessary for learning toward harmonious living.ò

This in turn informs mathematics lesson planning, such as this example from one of the lessons we experienced on the immersion programme:

> ñMathematics is a discipline which is taught systematically. Therefore, it is important that each student develops his or her own questions. Students can then recognize each otherõs strength from ñdifferencesò in their ideas and experience ñunderstanding.ò We consider this to be the learning toward harmonious living.ò

This represents teachersô efforts to be empathetic with students, to describe learning from the studentsô own perspective, and brings me on to the second striking feature of Japanese teaching and learning, the rigour with which lessons are designed. A common ï and in my view very successful ï feature of all of the research lessons that we saw was teachersô explicit documentation of anticipated student responses which then became an integral part of the learning. A crude three-part generalisation of the approach used to designing lessons is:

Identify clear lesson goals, research themes ⇨ Anticipate student responses to the given activities, prompts ⇨ Use the student responses to achieve the lesson goals

Planning lessons in this way leads to coherent learning experiences, where the teacher can skilfully direct studentsô attention to key aspects of what students themselves have said in that lesson. Students are empowered inasmuch as they feel that this lesson is all about what they are thinking ï what they say is highlighted, their work is visible and often central on the main display. The careful lesson design leads to an approach reminiscent of Vygotskyõs Zones of Proximal Development ï the teacher is always leading students to take the small step just beyond where they currently are.

My final section in this brief reflection is on the variety of approaches used in the administration of Research Lessons, especially the post-lesson discussion. In one lesson the teachers positioned themselves, crouching, amongst the students, so that a report could be given on every individual student. This was an exception, with observing teachers in all other lessons tending to stand at the edge of the classroom when the teacher was addressing the class. In every lesson seen the teacherõs colleagues were prominent in the post-lesson discussion, each presenting questions and comments from their perspective. Three examples were seen of teachers being organised into groups during the post-lesson discussion, having time to write comments on coloured paper which were then displayed and the basis for the discussion itself. There were differences even within these similar organisational procedures. One school used blue paper to list shortcomings, yellow for suggestions for improvement and pink for strengths, with each piece of paper displayed on a teacher-student continuum. Another school used white paper for comments on evaluation, pink for criteria / standards and green for instructional materials. The different categories used had an impact on the post-lesson discussion, and give an indication of the subtly different outcomes expected by each school. This was interesting from my perspective because it was evidence of decisions that each school had made regarding ótheirô lesson study ï the teachers involved (or maybe the principal and school research committee) are empowered to modify the Lesson Study approach itself. This was a significant realisation for me; in my context Lesson Study is a new approach, and the starting point has been to work from a template: ñthis is how to run a post-lesson discussionò. Now considering the extent to which we can make decisions about how to organise the post-lesson discussion makes me realise the extent to which the post-lesson discussion is actually a lesson with the teachers as learners. The *hatsumon* / activity is the reporting of comments following the lesson; there is an attempt to anticipate responses ï very challenging, but in the two examples above the learners are directed to respond in a particular way, to direct the flow of the learning towards the lesson goal ï a reflection on the research theme. The comments from the *koshi* are functionally similar to the *matome* ï the summary at the end of problem-solving lessons, pointing out what we have been learning in this process and what questions we might ask to continue our learning.

Just as the koshiôs comments end the post-lesson discussion, so this is my final comment in this reflection. There was a suggestion from some of us visitors that it might be effective to allow the teacher and/or planning team to respond to the koshi comments during the post-lesson discussion. Looking back at the koshi summaries, I am pleased that there was not such a response. The comments are highly reflective, and deserve proper consideration before responding; two examples are:

ñChildren have their own value system ï that was evident in the choices they made todayò

ñIf students want to get to the station, do you say óIôm an expert at getting to the station?ô ... If we say ódo you want to know the meaning of ñdivideòô students will not respond well ï we need a context, another way in.ò

To leave these questions hanging, without a definitive answer, is powerful: to give an answer would deny an opportunity to learn. Responsibility is now placed on teachers to reflect on these questions, in exactly the same way that pre-service teachers are urged to reflect on their practice, an indication of the impressive extent to which there is a consistent and coherent approach to teaching and learning in the Japanese system.

Summing up my overall sense of mathematics teaching and learning in Japan and Japanese Lesson Study, the overriding feature is the thoughtfulness, rigour and care shown by teachers, school leaders. This is not a peculiarity of the Japanese national culture ï it is a highly prized professional culture that is achieved through hard work and appreciative recognition of that hard work better to understand mathematics education.

I send here my sincere thanks to all of the IMPULS administrators and facilitators, fellow visitors, and above all the schools and teachers who invited us into their classrooms ï thank you for the inspiration, which I hope will help us make a difference in the work that we do.

**Sam Fragomeni**
**Lesson Study Immersion Experience Reflection**

I would like to begin by expressing my gratitude for the opportunity to participate in the 2013 Lesson Study Immersion Program. It has truly been an enlightening experience and I will take back the lessons I have learned to the United States in the hope of making an impact on many students through what I have learned.

I am the Principal of a school in New York City that has implemented the translated version of the Japanese national math curriculum in its Kindergarten this year. I arrived in Japan hoping to see how the Japanese math curriculum is philosophically approached in Japan. Additionally, my hope was to see how professional development (lesson study) is being implemented in Japanese schools so that I could hopefully replicate some form of this in my own school. I am satisfied with my level of completion for these two goals and I look forward to creating a plan to use the knowledge that I have gained in this program.

**Curriculum & Implementation**

The approach to curriculum in Japan is very different than what I am used to in the United Stated. There is an agreed upon set of mathematical concepts that will be taught in each grade and all 6 text book manufacturers use this scope & sequence to create their books. From what was described, it seems as though there is very little variation between these texts in Japan. In the United Stated, our major textbook companies have major philosophical differences in how mathematics is approached. I did not see a single teacher directly interacting with on one of the student textbooks during my visit. This is very different form the typical American approach and it differs from the way in which we attempted to interact with the Japanese Kindergarten curriculum in my school this year. It seems as if, retrospectively, our teachers interacted too much with the textbooks that we were given for our Kindergarten curriculum.

I think that this leads to one of the two biggest realizations that I had with regard comparing how we implemented this curriculum in our school with how it should have been implemented: We often taught this curriculum in a way that prevented teachers from having the freedom to adapt lesson in a way that would be beneficial to students. All of the lessons that we witnessed this week involved teachers making adaptations in order to make the lesson fit their particular set of students in a way that would be beneficial to them. It is somewhat intuitive that this is what is best for students, but we often shied away from this in our implementation of the curriculum for the sake of sticking to what is written.

The second big realization I had was the way in which lessons are driven. Japanese teachers seem to rarely present information in a purely instructional way. Instead, they act as facilitators as they ask question after question to lead the students down a path to learning. When we were presented with this new curriculum a year ago, we knew that questioning had to be an important part of it, but I donât think that we correctly interpreted how this would look. Hopefully, through coaching and teaching model lessons, I can help the teachers in my school to correctly implement this form of questing in their lessons.

One additional item that was ñclarifiedò for me during this trip was what good board organization looks like. I have, for years, been hearing about the importance of board organization and have been seeing examples of this to some degree. I donât think I have ever seen an example of this in the United States that has been as effective as any of the 7 lessons we witnessed while in Japan. I have a much different view now of what board organization looks like. My hope is that I can exemplify this for the teachers that work in my school.

**Lesson Study**

My school has been involved in lesson study for many years at our middle school level. We have tried various versions of lesson study with the hope of getting our teachers inspired by what they can learn from the process. It has been very difficult get this to catch on in our schools, mostly because of the time involved in doing it well. Our school already has an extended-day model and teachers routinely work 11 ï 12 hour days in order to properly plan their daily instruction. In order to plan lesson study lessons and teach them, we have to add hours onto this already taxing schedule.

My hope is that, in the upcoming year, we can implement lesson study at our elementary level in the upcoming year. My plan is to make lesson study an optional component for two full cycles during the school year. I would be a member of the groups myself with the hope of inspiring more teachers to participate and to disseminate the information that I have learned during my time in Japan.

One major takeaway from my experience in Japan is that lesson study can be implemented in different ways according to what you are trying to accomplish. We went to lesson study events that only involved some of the teachers in a single school as well as one that had hundreds of teachers come from around the surrounding areas. My school has, in the past, attempted lesson study on a large scale by inviting teachers from around the area to attend. These events ended up being very large with several lessons taught in one day. I think that, in an effort to be more successful, I will attempt to start lesson study groups on a much smaller scale this year in our elementary schools. I believe I will be successful if I begin with an optional lesson study group and only invite the teachers in our school to begin with. If I can make this type of lesson study consistently successful, then I will attempt to increase the scale over time.

**Summary**

The United States (as well as my school) has a long way to go in order to come close to what Japan is accomplishing in mathematics education as well as teacher professional development. It would be overly ambitious to try to mimic the Japanese system on a large scale as I donôt think Americans are quite ready for it. I do plan to use the knowledge that I gained on this trip to make improvements to my schoolôs mathematics approach and teacher professional development. Again, I am very grateful to have been granted this opportunity and I hope to do this program justice in its goals when I return to the United States to use what I have learned.

# 4 External Evaluation of the Program

External evaluation was done by Dr. Rebecca Perry and Ms.Minori Nagahara as below.

**2013 Immersion Program Evaluation Report**

**Minori Nagahara, Boston College, Boston, MA, USA**
**Rebecca Perry, Mills College, Oakland, CA, USA**

**December 5, 2013**

## Background

In June and July of 2013, Project IMPULS of Tokyo Gakugei University, organized a ten-day lesson study immersion program designed to familiarize an international group of educators with authentic Japanese lesson study and Japanese mathematics education. As part of the program, participants conducted seven school visits in Tokyo and Yamanashi prefecturesð touring school facilities, interacting with students and teachers, and observing research lessons and post-lesson discussions. Prior to the school visits, the participants attended lectures on lesson study and its role in the Japanese education system. In addition to activities related to lesson study and mathematics education, program participants had opportunities to experience Japanese culture and explore local tourist sights.

Sixteen educators from three countries (Australia, Great Britain, and the United States) and six U.S. states (Florida, Illinois, Michigan, New Jersey, New Mexico, and New York) took part in the 2013 lesson study immersion program. All participants had prior experience with mathematics lesson study (50% of the participants had up to a year of experience with lesson study and the other 50% had two or more years of experience with lesson study). Most of the participants had no previous direct exposure to Japanese lesson study or mathematics instruction. One participant had prior experience with Japanese lesson study through participation in the Japan Exchange and Teaching Programme (JET) program as an English instructor.

Drawing on multiple forms of data including participant surveys administered before and after the program, field notes, participant reflections and video recordings of discussions, this report summarizes the learning that resulted from the program. A summary of findings and recommendations for improvement are followed by comparisons with the 2012 lesson study immersion program and daily learning highlights.

## Executive Summary of Findings

ñI was so thrilled to be able to experience lesson study, in the place where it originated, day after dayò was a comment written by a participant on the survey given at the conclusion of the program, summing up how many of the participants felt about their 10-day experience. As evidenced by thoughtful and enthusiastic participant responses on surveys and in daily journals, as well as during discussions and informal conversations, this program offered participants an invaluable opportunity to learn about various facets of Japanese lesson study, mathematics teaching and learning, and the Japanese education system (See Figure 1 showing mean participant ratings of learning of 25 program elements).

This was the second year that IMPULS has funded and organized this immersion experience, and this year's program benefited from recommendations that emerged from the 2012 program, which included the following suggestions: 1) more time for reflection, 2) opportunities to see other stages of lesson study (rather than just the lesson and post-lesson discussion), 3) visiting a wider range of schools, and 4) limiting the number of program participants. As was the case for the previous year's program, participants were generally very pleased with their experience. The number of participants in the current year's program was significantly smaller than in last year's program, and this allowed participants to move around the classroom more freely during lessons, taking photos and detailed notes. While many participants in both years reported that they wanted to see examples of different stages of lesson study, particularly the planning stages, this was difficult due to time constraints. In order to address this need during this year's program, professors Fujii and Takahashi set aside time before each school visit to give participants the context of the lesson they were going to observe, introducing the school, its research focus, and some background information regarding how the lesson was developed. When possible, photos, videos, and documents from planning sessions were shared and explained. While this was not the same as being present for a planning session, this gave participants some idea of how a lesson was developed over time. As was the case in last year's program, participants were given extensive plans for each lesson along with other relevant documents highlighting research themes and structures for lesson study for schools and districts, which many participants found to be helpful. One area in which improvement was not as clear from the previous year's program, had to do with the time that was allocated for reflection. Participants in both years of the program felt that more time and structures were necessary during the program in order to process all that was being learned.

Seven school visits in two different prefectures and opportunities to observe research lessons in a range of grade levels allowed participants to witness the extent to which lesson study is ingrained and sustained as a valuable professional development model in the Japanese education system. The majority of participants responded positively when asked about the program itinerary, expressing their satisfaction with the number of school visits. When asked if they felt too busy during the program, many participants responded that they expected as much from an immersion program and were not surprised or displeased with the schedule.

While program participants were all familiar with lesson study in the context of their own countries and schools, many reported that observing Japanese lesson study was an eye-opening experience. The program served to highlight the potential of lesson study to help teachers develop knowledge, skills, and dispositions that are necessary for high quality teaching and learning in mathematics. Summarized below are two general categories of learning that emerged through the program: 1) Learning about lesson study and 2) Learning about effective teaching practices.

**1. Learning About Lesson Study**

*a) Lesson Study in Japan*

Through a combination of lectures, seminars, observations, and discussion, participants were provided an overview of how Japanese educators approach lesson study, how unit and lesson plans are developed over time, and how lesson study is supported by organizational structures and practices. On average, ñlearning how lesson study is conducted in another countryò was rated highest of 25 program elements addressed in the post-program survey (See Figure 1). In particular, participants were interested in the concept of *kyozai-kenkyu* and its role in lesson planning. Through observations and discussions, it became clear to the participants that planning for instruction was an iterative process that involved acquiring and refining mathematical content knowledge through research, teaching, reflection, and dialogue with colleagues and knowledgeable others. This was emphasized throughout the program, not only in lectures and discussions led by the program facilitators, but also through observations and interactions with Japanese practitioners of lesson study. One participant put it this way:

> ñThe conceptual development described in the unit plan section of the lesson plan indicates the kyouzai-kenku that the planning team has engaged in. It is notable that despite the clear attention that the team put into this aspect of the planning, nonetheless the óknowledgeable otherô suggested that the conceptual development could be made still more clear and apparent. This contributes to my thinking about engaging in lesson study myself ï it is very important to strive for a rigorous and clear view of the mathematical content; the clearer we are about the conceptual development, the better we will understand the content ourselves, and the more likely we are to create opportunities to help students understand in the same way.ò

Along with acknowledging the importance of mathematical content knowledge in lesson planning and teaching, participants reported that their own content knowledge was enhanced during the course of the program as evidenced by a significantly higher rating on the post-program survey when asked about the extent to which they agree with the statement, ñI have strong knowledge of the mathematical content taught at my grade level.ò[2] Cultivating a deep understanding of mathematical content knowledge and ongoing study of mathematical principles and concepts are expected of Japanese educators from the time they are student teachers throughout their professional lifespan. This emphasis helps to ensure that when teachers come together to plan, execute, and reflect on lessons, their discussions are substantive and constructive, identifying practices in need of improvement and offering a variety of solutions. Many participants noted that the focus on content knowledge that they witnessed in Japan is missing from their professional development experiences in their respective countries and suggested it as an important way to enhance their practice of lesson study when they return to their schools.

A theme that emerged for some of the participants was that in Japan, learning to teach mathematics well is something that takes time and practice, with long-term goals in mind (e.g. student curiosity, motivation, risk-taking), and that while lesson study may not produce immediate quantifiable results as is required by some educational systems today, the learning that results for

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[2] Pretest mean = 4.38, posttest mean = 4.63 on a 5-point Likert scale, $p < .05$.

both teachers and students is well worth the time it takes. Throughout the program, many of the participants considered the differences between Japanese approaches to lesson study and mathematics pedagogy and the approaches in their countries, as evidenced by the following comments:

> ñThe strengths of using lesson study in our district would be many. I think the content knowledge and pedagogy could potentially be strengthened. Our children would get a more thoughtful well-rounded education. Before we can begin to embrace lesson study though, our system needs to reflect on why we are educating our children.ò

> ñThe long term approach that Japanese teachers take using lesson study shapes this curriculum to help students based on the research from lesson study. Lesson study also helps with teacher professional development. PD done by using the lesson study observations and comments from outside experts and fellow colleagues makes teaching better because teachers examine what works for students and what does not work.ò

As educators who had been learning about lesson study and practicing it in their own school contexts prior to the immersion program, the participants were generally familiar with the different components that comprise a lesson study cycle. A number of participants reported, however, that what struck them during this program was that all of the research lessons and post-lesson discussions had unique characteristics, taking into account the culture of the school and its teachers and students. In particular, the different structures and organization of post-lesson discussions gave participants the opportunity to consider how what they were observing might apply in their own schools, as the comments below (from two different post lesson discussions) reveal:

> ñThe post-lesson discussion was organized in a way that I have not seen, which was interesting. Teachers worked in groups of 5 or 6, writing on coloured post-its to describe, discuss and agree strengths, shortcoming and suggestions, organized on a large sheet of paper in a continuum from óteacherô to óstudentô. All teachers seemed fully engaged, and each group (three) presented their agreed comments. Although this was enthusiastically engaged in, these groupsô comments were later criticised for being relatively superficial in comparison to the óknowledgeable otherôs comments. An ideal scenario would be both to capture teachersô energy and also deepen the thinking ï the moderator / facilitator / chair of the post-lesson discussion might play a key role in establishing a situation such as this.ò

> ñThe moderator did a good job of asked [sic] pointed questions of the observers and did not hesitate to abandon a question if it was clear that the question wasnôt going to be fruitful. He also called direct attention to the components of the lesson that obviously needed to improve, such as the timing. I was also fascinated by the fact that the different observing teachers had apparently been assigned to watch specific students so that they could report back on how each of these students performed during the lesson. I feel like experiencing this model of lesson study and seeing how the moderator worked in this situation helped to inform me of a new way to drive discussion in my own school.ò

On the post-program survey, the item asking how much had been learned about ñorganizing a successful post-lesson debriefing sessionò was rated relatively high among 25 program elements (See Figure 1). While some participants reported that the post-lesson discussions, with multiple speakers and the limitations of simultaneous interpretation, were not always easy to follow, it seemed that participants found them useful and thought provoking in considering the possibilities and limitations of debriefing research lessons.

*b) Lesson Study and Building a Professional Community*

Another aspect of lesson that became clear through school visits and lesson observations was how it helped to create and sustain robust professional communities with the ability to provide a rich, coherent mathematics curriculum to students across grade levels. Participants reported high levels of learning about ñorganizational/structural supports for lesson studyò and ñsupporting participants to have powerful and effective lesson study experiencesò through the program (See Figure 1). Structures and supports for lesson study were evident in the amount of time and resources that were invested in lesson study cycles at the school and district levels.
Beyond that, many of the participants commented on the commitment shown by teachers, administrators, and district-level leaders to create communities of practice where ongoing learning is the norm. The comments below capture how some participants reflected on lesson study as a means for meaningful professional collaboration:

> ñThe strengths of using lesson study include team collaboration and a shared sense of responsibility. Classrooms do not operate independently using lesson study, but are team efforts where teachers from different grades and backgrounds work together to make a stronger lesson.ò

> ñLesson study enables us to work together as peers to develop our skills. If is far more effective then a lot of the professional development sessions currently provided to teachers. The reason for this is [sic] that is allows staff to work collaboratively on planning in detail and researching their subject. Ideas are then shared and seen by known peers, not watched on a video or described but seen and experienced. The post lesson discussion allows teachers not only the time to reflect but provides a structured rich discussion supporting teachers to develop. We are trying to improve teaching so lets [sic] do it by teaching!ò

As was highlighted in the comment above, many participants expressed their belief in the value of improving teaching by actually teaching, a theme that was also emphasized by Professor Takahashi throughout the program. Participants felt strongly that lesson study is just as relevant for experienced teachers as it was for novices. By bringing together teachers, administrators, and knowledgeable others from outside the school, lesson study created both collective and individual opportunities for learning, as the comments below assert:

> ñI believe that our more experienced teachers can truly learn a lot from lesson study. The major strength is that it seems to meet everyone on the level where they are, so it can benefit teachers regardless of their experience.ò

> ñLesson study is a must-have in order to see teachers grow professionally. It is vital to

incorporate collaborative work with colleagues into my school district in order for novice teachers to learn different teaching strategies and problem solving methods that can be used day to day in the classroom. The focus for teachers in every school should be growing together as educators, rather than independent work and research with little to no communication with colleagues.ò

By observing firsthand how lesson study is conducted in Japan and having the opportunity to process those observations in light of their own experiences with lesson study, many of the participants felt that this program reinforced the assertion that lesson study should not be considered a luxury, but a necessity for professional learning, particularly in the area of mathematics teaching.

**Learning About Effective Teaching Practices**

In addition to learning about lesson study, many participants expressed excitement about witnessing effective teaching practices that helped students think deeply about the math problems they were presented. During the course of the program, there were opportunities to observe lessons on a variety of topics in classrooms with a range of different teaching styles. Some lessons involved the use of manipulatives, while others were more discussion-oriented. Some involved group work while others revolved around whole class instruction. In other words, not all lessons approached problem solving in the same way. Participantsôcomments after lessons often pointed out differences between lessons and these differences often prompted insightful discussions. Across many lessons, participants were struck by the *bansho* (the teacherøs board work) that was involved, and its effectiveness in supporting student thinking and guiding students through the lesson. One participant characterized *bansho* as creating a ñstory of the lesson being taughtò pointing out that students should be able to track the lesson using the board and see the progression of ideas. Other participants were also fascinated by *bansho* as the comments below reveal:

ñThe board work is what stuck out to me the most. The extreme thought that is put into what is put on the board, the placing of each thought, etc. The board work was beautiful. Also the accuracy with which teachers anticipated student responses was amazing. There was such a knowledge of their students, even at the beginning of the year!ò

ñIn my own classroom I would like to experiment with bansho. Having seen clear and succinct board organization, I realize the power that it has for guiding studentsôlearning. I would like to make the board work in my classroom more visible so it can be a powerful learning tool for the students.ò

ñThe teacher of this lesson very deliberately created the backboard layout so students could easily move from less-sophisticated ideas to highly-sophisticated ideas around the idea of division. This lesson was very deliberately and craftily constructed to ñtrickò students into moving toward thinking about division and this transition was very evident in how the board was laid out.ò

ñThe board in todayøs lesson was organized brilliantly in order to display the flow of the lesson so that students could clearly understand where they started and where they ended up. The left side of the board consisted of student examples of the buildings that they created and the right side included repeated addition problems and eventually a multiplication expression. The students were asked to write in their journals multiple times during the lesson in order to directly interact with what was being taught at that moment.ò

*Bansho*, as some of the participants pointed out in the comments above, proved to be much more than a visually appealing accessory to the lesson or even merely a record of the lesson. Effective *bansho* provoked student thinking and prompted students to articulate and critically consider their own ideas as well as those of their classmates. Students were often asked to consider alternative solutions to a problem by imagining and explaining their classmatesôreasoning. As one participant commented:

ñI like how the teacher asked one student to say an answer and a different student to explain the reasoning behind the answer. Utilizing this technique will help with creating a whole class discussion and allows the students to critique the reasoning of others. I would like to begin having a student come up to the front of the class and speak to his or her classmates when explaining a solution strategy, rather than to just me. By stepping aside and allowing the student to address the class, naturally, the discussion will transition into student-to-student talk, rather than student-to-teacher talk.ò

Observing and reflecting on effective practices including *bansho*, questioning, and classroom discussions may have contributed to the participantsôlearning about ñstrategies for making students' thinking visible,ò ñstudentsô mathematical reasoning,ò and ñhow to build a classroom learning communityò items which all received a high rating on the post-program survey (See Figure 1).

**Recommendations for Program Improvement**

Generally, the participants of this program were pleased with what they gained professionally as a result of this program, and consequently, had a limited number of suggestions for improvement. When asked about the number of lesson observed during the program, 93.8% of the participants responded that it was just right while 6.3% responded that there were too many. When asked if the program felt too busy, 12.5% strongly disagreed, 31.1% disagreed, 37.5% were neutral, and 18.8% agreed. Participants offered the following comments:

ñI appreciate the immersion experience. We were supposed to be busy!ò

ñI didn't feel like the schedule was too busy, but if a lesson were dropped, I don't think the program would be lacking either.ò

ñThe itinerary was busy, but appropriately so I felt. One less lesson perhaps would have been no detriment to the program. I wanted to know more about the secondary phase (High school teaching) in Japan, and about the balance of problem solving lessons to lessons of other types in elementary schools...though I have only realised this now!!ò

> ñone or two lessons too many. Substituting these with more time in group discussion would have been very beneficial.ò

> ñI think the program gave us enough experiences and a varied amount, with both visits to country/city areas, secondary and elementary schools; public/ private and open lesson studies.ò

> ñI think that, perhaps, some of the post--lesson discussions could have been skipped and we could have had our own post--lesson discussion instead. It was difficult to listen to these discussions sometimes.ò

One area in need of significant improvement is the amount of time on the agenda for discussion about the lessons and post-lesson discussions observed. On some of the days, time for reflection was scheduled prior to school visits, but oftentimes, these sessions were cut short by previews of the lessons that were going to be observed that day. Many participants reported that they wanted more time to process observations through in-depth discussion. Some of their comments are as follows:

> ñHaving our own post-lesson discussions and hearing the experts we were with discuss the lessons before and after [would be helpful in getting more out of the program]. This would be a good model for how to discuss a lesson in a post discussion as well as helping me to prepare before the lesson in what to look for during the lesson.ò

> ñI would have liked the opportunity to have had more of our own separate post lesson discussions. Listening to the translated discussions was useful but at times difficult to follow. So instead, for maybe 3 lessons, we could have had our own discussion listening to one of the professors comments too, which we always wanted to here [sic]. Instead informal conversions went on, on a ad--hoc basis, but this meant we did not hear everyone's ideas.ò

> ñHaving time to discuss with each other at the end of the lesson study workshop was helpful to organizing everything we saw and learned over the 10 days. Maybe having a discussion time in the middle of the itinerary would be helpful for the candidates to organize their thoughts from the first half of school visits.ò

> ñI would've liked more time to reflect on the observed lessons with the Lesson Study Immersion Program Group, especially with Akihiko, Tad, and Makoto.ò

> ñI would have liked a little more time with just the immersion group after each post lesson discussion to develop our own post lesson discussion.ò

Given the time constraints during the program, small group discussions may prove to be more manageable than whole group interactions after each lesson and post-lesson discussion. Using daily reflections to guide discussions after lessons may also prove to be useful. If daily discussions are logistically unrealistic, having one discussion session for the first part of the program and one for the second may suffice, as one participant suggested.

Other suggestions from participants included opportunities to 1) plan a research lesson, 2) observe a planning meeting, 3) analyze student work, and 4) select lessons for observation by grade level, though it was acknowledged that these would be difficult to fit into the agenda. When asked about other aspects of the program including accommodations, meals, and communication with program staff on the post-program survey, participants were generally satisfied.

**Comparisons with the 2012 Immersion Program**

The 2013 IMPULS lesson study immersion program was different from the 2012 program in a number of ways. First, it was a much smaller program of sixteen participants compared to last yearôs program with forty-two participants. This much smaller sample of participants in 2013 needs to be considered when drawing conclusions from the statistical analysis. In general, the participants of the 2013 program had more experience with mathematics lesson study (50% with at least one year of experience and the other 50% with two or more years of experience) whereas most (73%) of the 2012 participants had no prior experience with lesson study.

As shown in Figure 2, there were statistically significant differences between participantsô reported learning between the 2012 and 2013 immersion programs, according to participant responses on the post-program surveys. With the exception of item h. ñanalyzing/studying curriculum materials,ò where the 2012 participants rated their learning higher than the 2013 participants, the other eleven items were rated higher in 2013. Item h. ñanalyzing/studying curriculum materialsò was one of the lowest rated items on the 2013 survey, suggesting that there may have been less emphasis on this element compared with the other elements. This may also have to do with the fact that most of the study of curriculum materials took place during the first two days of the program during the lectures, and were not as strongly emphasized during the rest of the program. In comparing and contrasting the 2012 and 2013 post-program survey item regarding the participantsô learning of the 25 program elements, it is important to note the differences between the two groups. It is possible that the higher ratings on the post-survey resulted from differences in background knowledge between the two groups. A comparison of pre-program and post-program results on this item for both years may help to provide a more accurate picture of what participants learned through the program. As the pre-program survey for 2013 did not include this item, this comparison is not possible.

When asked about the extent to which they agree with the statement, ñI am interested in the mathematics taught at many grade levelsò on the post-program survey in 2012, participants on average, expressed greater agreement. It should be noted, however, that for the 2013 participants, there was no statistically significant difference on this item between the initial survey and the post-program survey, indicating that the program did not affect the participantsô attitude on this item. Differences between the two years of the program may be attributed to differences in participant backgrounds and experiences as educators.

There were also similarities between the 2012 and 2013 programs, particularly with regard to what the participants felt were the most significant areas of learning. For both years, the following elements were consistently rated the highest among 25 areas of learning: 1) how lesson study is conducted in another country, 2) a typical school day at a Japanese elementary school, 3) organizational/structural supports for lesson study, and 4) how lesson study is conducted in different educational contexts (e.g., schools, districts, etc.). For participants in both years, the program clearly provided meaningful exposure to the Japanese educational system and to lesson study. For participants who were relatively new to lesson study, the program offered an overview of this professional development model in its country of origin, and for participants who were familiar with lesson study in their own schools and countries, the program provided opportunities to compare and contrast their experience of lesson study with Japanese lesson study. During both programs, participants reflected on how their learning might be applied in their own educational settings.

Participants in both years of the immersion program commented extensively (both during the program and in written reflections) about practices in Japanese mathematics education designed to support student thinking and problem solving. In particular, participants were fascinated by teachersô board work (*bansho*) that supported students in their problem solving and highlighted key areas of learning as the lesson progressed, and by the practice of *neriage* in which teachers facilitated extensive conversations with students, encouraging students to think deeply about their ideas and solutions as well as those of their classmates. Many participants felt that these practices were relevant for mathematics instruction in their own classrooms and schools.

**Daily Learning Highlights**

**Day 1, June 24**

**Activities:** Opening session (introductions and logistics); lectures on mathematics teaching and learning in Japan, lesson study in Japan, and teaching through problem solving and kyouzai-kenkyu; welcome reception

The participantsô enthusiasm for learning about Japanese mathematics instruction and lesson study was evident during the opening session as they introduced themselves briefly and shared what they hoped to gain through the program. Introductions were followed by a series of lectures by Dr. Akihiko Takahashi. The first two lectures focused on mathematics teaching and learning in Japan, making the case that in order for teachers to develop expertise in teaching mathematics, it is essential for teachers to have opportunities to plan, carry out, and reflect on lesson plans. Lesson study, argued Dr. Takahashi, is how Japanese teachers improve their teaching of mathematics. Consequently, lesson study should not be considered a luxury, but rather, a necessity for professional growth, and ultimately, for improvement in student learning. Dr. Takahashi also described the characteristics of school-based lesson study and cross-district lesson study.

The third lecture introduced the idea of ñkyozai-kenkyuò a process through which teachers study various teaching materials (e.g. textbooks, curriculum guides, manipulatives, and worksheets), consider different teaching methods, and anticipate student misconceptions. Using examples from Japanese mathematics textbooks, Dr. Takahashi explained how Japanese mathematics instruction emphasizes problem solving and that through kyouzai-kenkyu, teachers were able to develop lessons and units that facilitate studentsô problem solving and deepen their understanding of mathematical concepts.

**Day 2, June 25**

**Activities:** orientation session for student teachers at Koganei Junior High School, lesson observation (Grade 7), preview of next dayôs research lesson

During the second day of the program, the participants had the rare opportunity to observe a group of student teachers that had gathered for an orientation to prepare for their student teaching placements at Koganei Junior High School starting in the fall semester. The principal of Koganei Junior High School welcomed them and gave them a brief overview on the history as well as the culture of the school. This was followed by some logistical announcements and a brief lecture encouraging the student teachers to study curriculum material during the summer months to build their content knowledge and inspiring them to think deeply about what it means to teach and learn.

Following the orientation, the program participants observed a meeting for student teachers in the mathematics department with the mathematics faculty at Koganei Junior High School. During the meeting, the student teachers were asked to come up with different solutions to the problem that the Grade 7 students were going to be asked during the research lesson that day and there was a discussion of potential student responses and misconceptions. The participants then observed the research lesson titled, ñHow many stones?ò from the mathematics unit on the use of letters in algebraic expressions.

During the lunch break that followed, many participants commented on the lesson they had observed, mentioning a range of topics including high levels of student engagement and focus during the lesson as well as the teacherôs questioning skills. For most of the participants, this was a first look at Japanese mathematics instruction and it served to heighten their interest in both lesson study and Japanese mathematics instruction generally speaking.

The afternoon session was used to give participants some context for the lesson they were scheduled to observe the next day at Daisan Terashima Elementary School. Professor Toshiakira Fujii, who served as a ñknowledgeable otherò at the school, gave participants an overview of how the next dayôs research lesson had been planned. Following Professor Fujiiôs presentation, Professor Takahashi gave a brief tutorial on the use of Lesson Note, an iPad application used for lesson study observations.

**Day 3, June 26**

**Activities:** School visit & school-based LS observation (Daisan Terashima Elementary School)

Having observed a lesson from a mathematics unit titled, ñLetôs explore various quadrilaterals,ò the participants made the following observations. First, the teacher was intentional about making student thinking visible during the lesson. He asked students to explain their thought processes as they sorted quadrilaterals into groups. Students were also asked to consider and verbalize how their classmates grouped the figures differently.

> ñHe divided the board into sections of two groups, three groups, and four groups and he arranged the shapes to visually show what that student was thinking when they explained how they sorted the figures. He also had a student use an overhead display off to the side to highlight why one figure fit into a group or didnôt fit into a group and why.ò

> ñ[The teacher] clearly displayed several students' groupings of quadrilaterals on the board for all to see. Students were invited to explain others' work at the board for all to hearé [He] prompted the students to say how their work different or similar to other students' work.ò

While it was clear that the teacherôs aim was to foster discussion among students and to get them to consider their own ideas as well as those of their classmates, many participants also commented that the lesson was largely teacher-led and that this contributed to some of the studentsô lack of engagement with the lesson, as their comments below illustrate.

> ñé studentsô mathematical reasoning was made visible and compared, by the teacher, to classmates thinking. When these two students were asked to come up to the document projector to share their strategy on sorting their quadrilaterals it was unclear if the other students listening to their presentations were understanding the reasoning behind the sorting.ò

> ñThe lesson was very teacher-talk heavy, so many students became unengaged. The teacher could have easily called on more students to come up to the front to show what they have done and to explain their work.ò

Though many of the participants seemed to feel that this was not a particularly strong lesson, specifically in the area of student engagement and participation, these critiques were raised and discussed during the post-lesson discussion. As one participant noted:

> ñThere were very high expectations of student commitment, but the lesson was too teacher-led for the students to be able to build their capacity to solve challenging mathematics. There was a useful pedagogical clarification in the post-lesson discussion by the óknowledgeable otherô who suggested that the teacher could have stepped to the side, so that when students spoke they would be addressing their own work on the board rather than reporting to him. The implied sense of student ownership of the lesson could contribute to building their mathematical resilience.ò

**Day 4, June 27**

**Activities:** Travel to Yamanashi, sightseeing

On this day, the participants spent time being immersed in different aspects of Japanese culture. The day began with a bus trip from Tokyo to Yamanashi with stops at tourist sites including Ohinohakkai near Mount Fuji and Takeda Shrine in the city of Kofu. Participants also experienced local Japanese cuisine (Ho-tou noodles).

**Day 5, June 28**

**Activities:** Reflection & discussion; school facility tour and lunch with students, followed by school-based LS and post-lesson discussion at Oshihara Elementary School; Japanese-style dinner at Hotel Fuji with teachers from Oshihara Elementary School

Prior to observing a school-based lesson study on division with remainders, Professors Makoto Yoshida, Takahashi, Tad Watanabe, and Fujii offered an overview of the dayôs lesson, providing the participants some context around the teaching of multiplication and division in Japan. Professor Yoshida described how multiplication is taught in Grade 2 and division in Grade 3. Professor Takahashi explained to the participants that in Japan, students are expected to master single digit multiplication by the end of second grade (taking approximately a month and a half to learn the times tables from one to nine) and that this mastery helps with their exploration of division. Multiplication and division are taught sequentially rather than simultaneously. Professor Fujii then described the research lesson for the day, emphasizing that students would be asked to solve the following real world problem using mathematical modeling:

> Grade 3, Class 2 is going to an amusement park. 27 students are divided into groups to ride boats that can hold 4 passengers each. How many boats do we need if everyone rides in boats?

He highlighted the idea that even though the mathematical conclusion to 27 ÷ 4 is 6 remainder 3, this answer needed to be interpreted in order to become the solution for the real life situation regarding the number of boats required and that this extra step of interpretation was something new for the students.

During the lesson, the participants noted that the teacher was successful at using the blackboard as well as the student journals in order to facilitate student thinking and problem solving as evidenced in the following comments:

> ñStudentsô methods and thinking were displayed on the board, which is well organised to show left-to-right progression, although it was suggested at the post-lesson discussion that the pictures used to set the context should have been kept on the board throughout. The teacher repeatedly asks students to óthink about your ideasô, and encourages the use of multiple representations: óuse words, pictures, diagrams, mathematical sentences, expressions and equations to show your thinkingô.ò

> ñThe high impact strategy from today's lesson that I found most highlighted was the use of the blackboard and journals to promote metacognition. The teacher really emphasized "let's write this down...lets think about this". He also used phrases like "what do you think about this...what do you think someone else was thinking when they solved this problem?" These questions, I think, helped students think about their thinking, especially as to how it compares to others'.ò

> ñ[The teacher] also set up the bansho for the students to construct arguments and critique the reasoning of others. He was deliberate in setting up the board with three possible answers to encourage the students to figure out which was the right answer.ò

Participants were also impressed at how the teacher encouraged different ways to represent and solve the problem at hand. As one participant noted, ñstudents were told they could use diagrams or words to represent their thinking, providing alternative avenues to make thinking visible.ò The dayôs lesson also prompted many of the participants to consider the *use of time during a lesson*. In this particular lesson, there were two independent problem-solving periods for students during the lesson, once in the beginning and then again toward the middle of the lesson. While many pointed out that the lesson was too long, it was also acknowledged that problem solving is time consuming, and that in some cases, it takes longer than originally planned. Two of the participants offered the following thoughts:

> ñThe teacher definitely needs to be more precise with the timing of the lesson and plan how to use his time more efficiently. This will come with more reflection, thought and

experience. However, even though the lesson was running long, the teacher did not get flustered and skip steps of his plan for sharing out. In my own classroom when things are running long, I am tempted to skim over things, which obviously impedes the learning of the students. The teacher did not do this and instead validated each student's voice and thinking. He went back to the answer that a student had given at the beginning of the lesson, 27 divided by 4 = 7. I do think it was valuable for student learning to forge through the lesson instead of cut it short in the interest of time.ò

ñI would like to experiment with using two independent problem-solving times during a lesson. I understand that it is something I have to use sparingly and accordingly because it is not appropriate for all teaching through problem solving lessons. Time management must be a big focus when I choose to do this since it takes up more class time. I like how it allows studentsô solutions to evolve and gives them time to analyze whether or not their solution is actually answering the original task.ò

Overall, the participants appeared to enjoy their visit to Oshihara Elementary School with its unique, environmentally friendly facilities and friendly faculty and students. As one participant wrote in his reflection:

ñI was fascinated by the culture at todayôs school. It was clear that a lot of time has been put into setting up a very positive, loving school culture. I was impressed with every single class that I saw and I saw sizeable difference between the culture of this school and the culture of the school that we observed two days ago. I would be interested to hear how much of this culture came about through overt action as opposed to random chance.ò

**Day 6, June 29**

**Activities:** District-wide lesson study & post-lesson discussion at University of Yamanashi Model Elementary School; travel back to Tokyo

Sixty-eight percent of the participants rated the second lesson observed this day as being the most professionally informative. The focus of this lesson was to help students understand that focusing on the remainder of a division problem can be useful. Students were presented with the following scenario: There are four games on the class recreation chart (dodge bee, keidro, dodge ball, kohri) and the four games are rotated from week to week. Letôs think about ways to find out which game we will play during week 26. Participants were impressed by the seamless flow of the lesson and the way the teacher led the students through the problem, encouraging students to find a variety of ways (e.g. making lists, using addition, subtraction) to find the solution. The following survey responses sum up the reasons why participants felt that this lesson was informative:

ñThe teacher's bansho was extremely well organized. She moved fluidly among various representations of student strategies for solving the problem to solidify their thinking, from counting the games, referring to the student--generated table, to writing mathematical equations. She moved from the concrete to abstract many times, checking in with students along the way about what the numbers mean to confirm their understanding. Student thinking moved from counting to equations. Students were eager

> to solve more equations than needed as they noticed a pattern and were able to make generalizations.ò

> ñThe teacher was so skilled at anticipating student responses and bringing student thinking up to a higher level through her questioning. Her board work was stunning, a beautiful representation of the mathematics that took place during the class.ò

> ñThe teacher was able to skillfully lead the students to understand that each of their ideas was important and useful for solving the problem but she was able to get them to see that there are more efficient methods too.ò

Several of the participants noticed that during the post-lesson discussion, the teacher was not the only person responding to the questions raised by the audience. In the other research lesson observed up to this point in the program, the teacher typically responded to questions on their own, without input from others who were involved in developing the lesson. As one participant put it:

> ñThe post lesson discussions included the teacher who taught the lesson but also the head of the math department and they worked together to answer questions raised by the audience. I felt this was better than other post lesson discussions because more than one person was responsible for the lesson so more than one person should answer for it.ò

**Day 7, July 1**

**Activities:** Reflection; school-based LS at Matsuzawa Elementary School; optional *hanseikai* with Japanese teachers

Upon arriving at Matsuzawa Elementary School, the participants discussed the lessons that were observed while in Yamanashi. One question that was raised had to do with the quality of post-lesson discussions:

> ñMy question is about the teacher professional development that we witnessed in each of the lessons in Yamanashi. Was there good opportunity for the teachers to develop or for the audience as well during that post-lesson discussion, because I think Iôve missed it. I didnôt see those opportunities. I thought there could have been more of a to and fro, to hear opinions.ò

Professor Takahashi responded to this question by emphasizing that post-lesson discussions are limited because of time constraints, but that the conversations that started there ñdefinitely triggered the teacherôs interest and many teachers continued discussionò beyond that meeting. He gave an example of a conference at which he presented the day before and how he participated in a conversation with some of the teachers from the lesson study in Yamanashi.

Another question was raised about whether the teacher who taught the lesson should be the only one responding to the questions during the post-lesson discussion. Professor Takahashi pointed out that it depends on the school whether the teacher is the sole respondent or if the planning team members respond as well. He made the distinction that in Japan teachers are aware that during a post-lesson discussion, the ñteachingò rather than the ñteacherò is being evaluated, and therefore, the teacher who taught the lesson does not feel like they are being attached when they are the only one responding to questions.

After this discussion, the participants observed a lesson on division, making a distinction between quotative and partitive cases. Students were asked to write word problems that can be solved by the equation 8 ÷ 2. The students were first presented with an example of 2 people sharing 8 strawberries and making groups of 2 with 8 strawberries. Several of the participants took particular notice of how the teacher collected student artifacts during the lesson as revealed in the comments below:

> ñI would like to collect evidence of learning and/or struggling more often. I like how the teacher is now able to read through each studentôs word story and track the studentsô progression/thought process within the one lesson. She is now able to adjust the next dayôs lesson accordingly.ò

> ñI really liked the teacherôs use of the worksheets that used the two different expressions (8 ÷ 2 and 10 ÷ 2) on them. The sheets provided a space for students to create mathematical stories around the expressions. There was also a space for them to draw a model of their story. This would be excellent evidence to collect of student learning.ò

This lesson also prompted some of the participants to consider how they might approach this lesson differently. One participant made the following suggestion in her daily reflection:

> ñI would use make the lesson into two days. One day, students would explore what kind of stories they could write where 8/2 is necessary to find the answer. This presents an interesting problem to solve. Then I would compare student work and group them according to similar characteristics. I would summarize the lesson at this point by identifying similar characteristics. The next day I might present another division story and again gather characteristics of each group. At this point, perhaps the students can make generalizations.ò

**Day 8, July 2**

**Activities:** Reflection, observation of a school-based lesson study at Koganei Junior High School

Professor Fujii started the morning session by revisiting the previous dayôs lesson and pointing out areas that needed improvement. He pointed out that, in particular, the teacher needed to consider timing her questions to students more carefully during lessons because some of the questions were asked prematurely and resulted in students giving responses that did not help to move the lesson forward. He then explained the lesson planning process of the research steering committee at Matsuzawa Elementary School using videos of planning sessions and meetings. As with the day before, a participant questioned why during the planning committee did not answer questions along with the teacher during the post-lesson discussion, and once again, Professor Takahashi clarified that for Japanese teachers, it is a given that lessons are planned collaboratively and that it is not necessary for the whole committee to speak in order to acknowledge that fact. Following Professor Fujiiôs presentation, Professor Yoshida explained the lesson plan for the lesson that would be observed in the afternoon.

Seventy-three percent of participants rated this lesson on algebraic expressions as being the least professionally informative during the program. For many of the participants, this lesson stood out in stark contrast to the rest of the lessons observed during the program, particularly in the lack of student engagement and understanding. The following comments summarize many of the participantsôresponses to the lesson:

> ñBeginning a lesson by standing in front of the room and asking questions to which nobody responds leads me to wonder how well thought out the lesson plan was. The silence was painful. There was minimal interaction among students or between the teacher and students.ò
>
> ñIt was so clear that the teacher was teaching the lesson plan and not his students. No one was engaged whatsoever. He would ask questions and it was so silent in the room, you could hear a pin drop. He was not reaching the children and instead of changing his plan, he pushed forward. I am not sure that any child left that classroom having learned anything new.ò
>
> ñThere was a lack of communication between the students and the teacher. It was a very quiet lesson and the teacher did not use any type of effective questioning when walking around during independent problem solving time. He did not help his students make sense of the problem, so they did not persevere in solving it. No tools were used throughout the lesson and there werenôt any viable arguments for other students to critique. It was a rather boring lesson to observe and seemed poorly planned.ò

As one participant pointed out, during the post-lesson discussion, the lesson was characterized and critiqued as a ñ prolonged period of independent problem solvingò and many other participants commented that the lack of high-impact strategies in this lesson served to emphasize their importance. For some of the participants, this lesson also raised questions about the differences between elementary and secondary math instruction in Japan. For example, participants wanted to know why the elementary lessons observed appeared to be much more hands-on and attentive to studentsô needs whereas the secondary lessons appeared to be much more teacher-driven and less interactive. A participant also wondered whether or not professional development was different for elementary and secondary teachers.

**Day 9, July 3**

**Activities:** School-based lesson study and post-lesson discussion at Sugekari Elementary School, optional *hanseikai* with Japanese teachers

Professor Yoshida started the morning session by giving participants an overview of the dayôs lesson. The topic of the lesson was speed and the participants spent some time, sharing their experiences as well as asking some general questions they had about the lesson plan. The principal joined the meeting soon thereafter, introducing himself and his school, and explaining how the research team developed the lesson plan over time. Highlighting that there were only 7 classrooms in the school, the principal described the faculty, as a close-knit community in which there were ongoing conversations about teaching and learning.

During the lesson, students were asked to use different strategies to compare the following students and draw conclusions about their order with regard to speed: Student A, who ran 40m in 6 seconds; Student B, who ran 30m in 6 seconds; and Student C, who ran 30m in 5 seconds. Generally, the participants felt that this was a well-organized lesson in which the students used the board effectively and the teacher anticipated student responses well, as the following comments suggested:

> ñboard writing was well organized and supported students making connectionsò

> ñBecause the teacher anticipated the studentõs responses, she was able to choose specific student solutions to go up on the board, as well as, the appropriate order to discuss them. The teacher also organized her board very well. The presentation of mini whiteboards allowed students to compare their own thinking with their fellow classmatesúò

> ñMorita Sensei had clear and concise bansho. Her board organization and presentation was extremely clear for the students to follow and use as a reference tool. The four strategies were clearly labeled with clear examples of how to use each strategy. The students can reflect on the dayõs work by copying into their journal and using their notes to assist them in the future.ò

Another area that caught the participantsô attention was the teacherõs ability to differentiate instruction for students who were struggling. Of all the lessons observed during the program, this lesson seemed to address the needs of struggling learners in a way that inspired participants to think about their own teaching, as the comments below suggest:

> ñThe teacher encouraged students struggling with starting the problem to come to the front and she gave them further clarification and direction for the taskò

> ñThe element I would like to experiment with in future lessons would be the way she provided help to students who couldn't get started. I really liked that the students could leave when they felt capable of solving the problem on their own.ò

**Day 10, July 4**

**Activities:** Independent work/reflection, whole group reflection, farewell dinner

Professor Takahashi began the session by revisiting the goal of the program which is to ñreceive feedback on the strengths and weaknesses of Japanese Lesson Study and to discuss together how to improve mathematics teacher professional development programs.ò Participants were then given sheets of paper to brainstorm positive and negative aspects of 1) Japanese mathematics instruction and 2) lesson study as an approach to professional development. The participantsô written comments were used to guide the discussion as participants explained what they had written and engaged in dialogue with each other and with the facilitators.

Some of the positive aspects of lesson study/Japanese mathematics instruction that were mentioned included:

- An expectation of deep content knowledge for teachers

- A clear curriculum with scope and sequence
- Ongoing learning for teachers with the help of knowledgeable others from outside the school
- Teachers building on prior teaching and prior knowledge
- Developing the ability to incorporate and build on student responses during the lesson
- Principals as instructional leaders with a vested interest in teacher learning
- Collaboration among faculty members

Some of the more challenging aspects of lesson study/Japanese mathematics instruction and areas in need of improvement were identified as follows:

- The need to better structure post-lesson discussions for participation
- The whole planning committee should be responsible for the post-lesson discussion instead of just the teacher
- Differentiation for struggling learners
- The need for more low-risk ways to share ideas (instead of whole class discussions)
- The potential for too much emphasis on following the lesson plan rather than responding to studentsôneeds during the lesson

Figure 1. Reported Mean Learning about Program Elements (Posttest Rating)

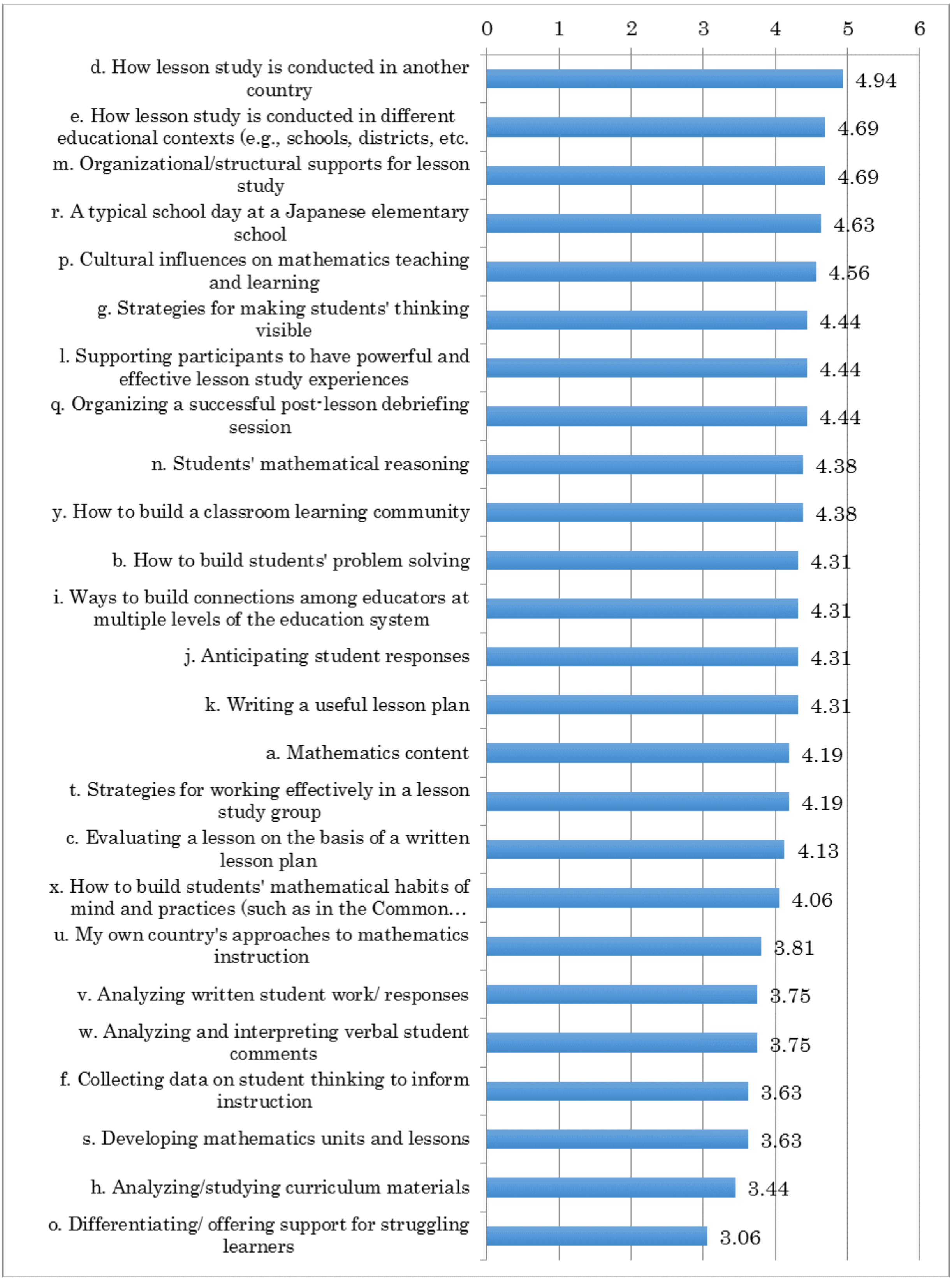
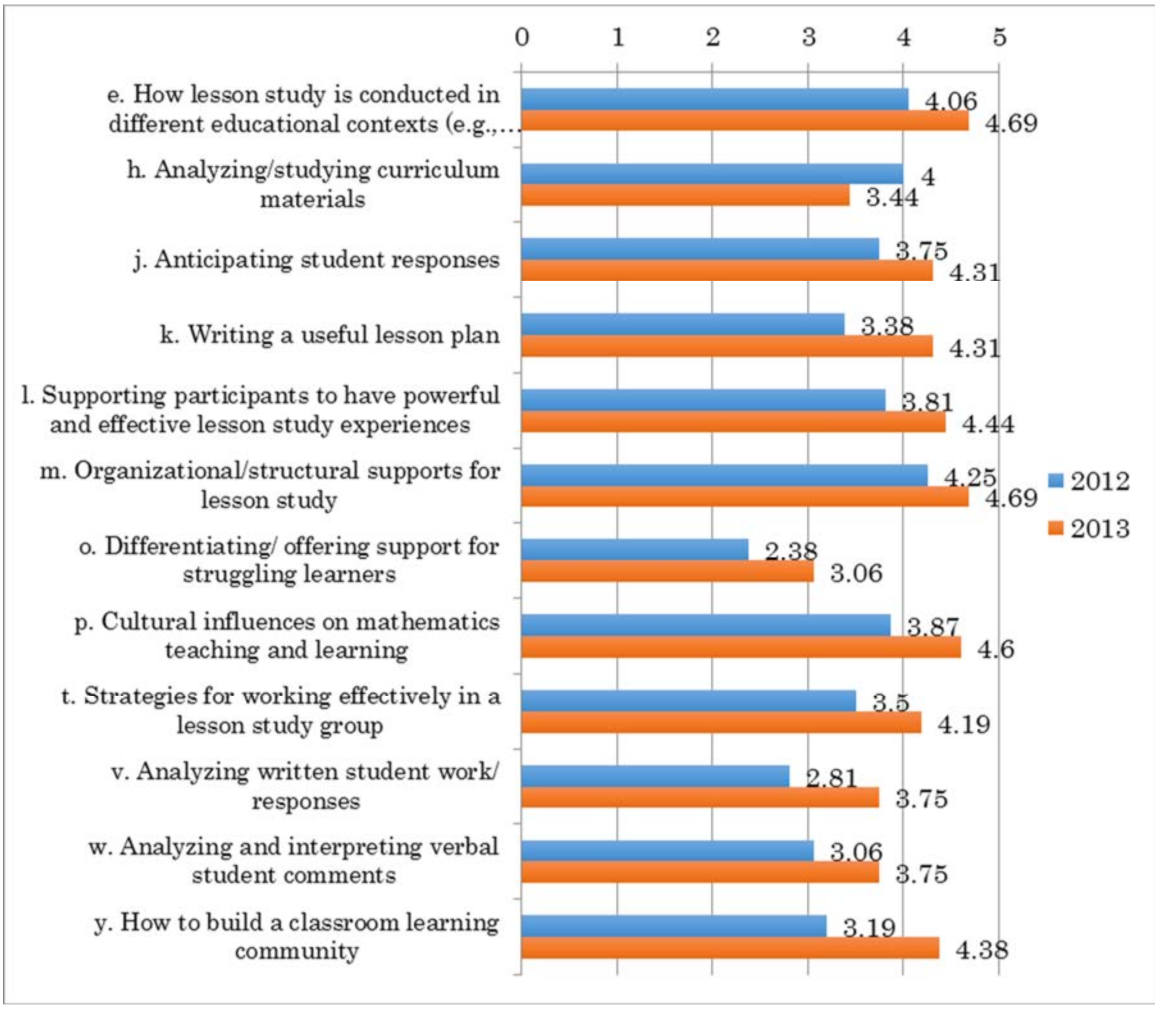


Figure 2. A Comparison of 2012 and 2013 Learning About Program Elements (Statistically Significantly Posttest Differences Only)



*Annex ;*

*(1) List of participants*

*(2) Lesson Plan*

*(3) Questionnaire for external evaluation and research*

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International Math-teacher Professionalization Using Lesson Study

# List of Participants for Lesson Study Immersion Program 2013 June 24 - July4 in Tokyo, JAPAN

| | Name | | Country | School/ Department |
|---|---|---|---|---|
| 1 | Ms | Amanda Y. Short | U.S.A | Grade 4 teacher Montezuma Elementary in the Albuquerque Public Schools (APS) district |
| 2 | Ms | Amy Rouse | U.K | Head of Maths, Swanwick Hall, Derbyshre |
| 3 | Ms | Aubrey Perlee | U.S.A | Kindergarten teacher Dr. Jorge Prieto Math & Science Academy in Chicago Public Schools |
| 4 | Mr | David Garner | Australia | Numeracy Coach and Acting Assistant Principal  Creekside K-9 College |
| 5 | Ms | Denise Jandoli | U.S.A | Grade 6 mathematics teacher Bernardsville Middle School |
| 6 | Ms | Doreen Stohler | U.S.A | District K-6 Mathematics Coach Bear Bath K-6 Elementary School, Hamden Public School District |
| 7 | Ms | Elnaz Javaheri | U.K | Maths Teacher, Heartland Academy, Birmingham |
| 8 | Ms | Ferida McQuillan | U.K | Head of Maths Maths Teacher, Heartland Academy, Birmingham |
| 9 | Ms | Francesca Blueher | U.S.A | an Instructional Coach at an elementary school Montezuma Elementary, Albuquerque Public Schools |
| 10 | Mrs | Heather Williams | U.S.A | a classroom teacher- grades 2 and 3 Pathways Elementary in Volusia County School District |
| 11 | Mr | Jeffrey Glenn | U.S.A | assistant teacher  International Academy of Bloomfield Hills |
| 12 | Mrs | Kathryn Palmer | Australia | Melton Network Numeracy Coach |
| 13 | Ms | Lauren Moscowitch | U.S.A | lead kindergarten teacher Harlem Village Academy Leadership in East Harlem |
| 14 | Mr | Mark Simmons | U.K | Lecturer, Faculty of Social Sciences, Head of Postgraduate Certificate of Education (PGCE), University of Nottingham |
| 15 | Mr | Matt Lewis | U.K | Advisory Teacher, Secondary Mathematics at London Borough of Barking and Dagenham |
| 16 | Mr | Samuel Fragomeni | U.S.A | Director of Elementary Schools and Lead Principal  Harlem Village Academy Leadership Elementary |

# Grade 7 Mathematics Lesson Plan (Condensed Version)

Tuesday, June 25, 2013, 11:40 ~
Tokyo Gakugei University Affiliated Koganei Junior High School
Grade 7, Classroom A (20 boys & 20 girls) Teacher: Koichi Kabasawa

1. Unit: Letters in Algebraic Expressions

2. Title of the lesson: How many stones (as an introduction of algebraic expressions)

3. About mathematics in the lesson

Today's problem involves determining the total number of stones used in the game of *Go* arranged in patterns. he trength f his roblem s hat t aturally enerates iverse ays f observing and reasoning, which will make it possible to have a rich activity of interpreting algebraic expressions. Sometimes, the same algebraic expression represents different reasoning processes. Therefore, it is particularly suited to naturally discuss the idea of generalization.

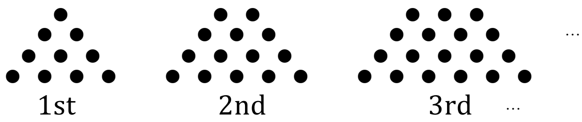
In elementary schools, students have learned about using letters such as *a* and *x* in expressions in place of symbols, like ☐ or Δ, or words that represent numbers and quantities. They have also studied the idea of substituting specific numbers in letters to evaluate expressions. Since the beginning of the lower secondary school, we have been using letters in algebraic expressions frequently. Therefore, students should not have much problem using algebraic expressions with letters to represent their reasoning in today's lesson. However, as it has been reported in the existing literature, understanding variables and the meaning of algebraic expressions with letters may be a significant challenge to students. Therefore, I will approach this topic carefully and thoroughly.

Keeping these things in mind, in today's lesson, we will engage in a mathematical communication activity that requires students to represent own reasoning in algebraic expressions and interpret others' algebraic expressions. Through this activity, it is expected that students will become aware of the idea of variables and understand that an algebraic expression represents both the process and the result.

4. Goals of the lesson

- Students will be able to generalize own ideas and represent them using algebraic expressions, and they understand the meaning of algebraic expressions.
- Students will be able to interpret algebraic expressions in the context of a particular phenomenon.
- Students will solve the problem using diverse ways of observing and reasoning, and they realize the usefulness of using letters.

5. Flow of the lesson

| | Learning Activity | Anticipated Responses | Evaluation (O) and Considerations (*) |
|---|---|---|---|
| **Opening** | • Display the problem<br><br>"*Go* stones are arranged as shown in the picture. Find the total number of stones in the 5th arrangement." | <br>• Draw the arrangements completely.<br>• Can determine the total number without drawing the complete pictures. | O Students are trying to find ways to determine the total number.<br><br>* In order not to influence students' reasoning, post the picture instead of drawing these patterns on the board.<br><br>* For those students who are drawing complete pictures, ask "Do you have to draw them all?" |

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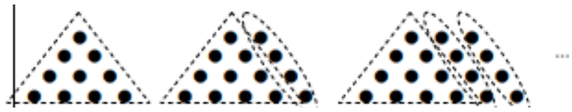
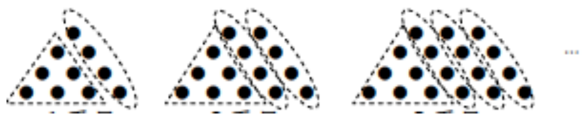
| Development | • Individual problem solving<br><br>"Write an expression to represent how you figured out the total number of stones." | • Possible student responses<br><br>1. 10 + 4 × 4 ← There are different reasoning with the same expression<br><br>10 + 4 × (5 − 1), 10 + (5 − 1) 4<br><br><br><br>1st 2nd 3rd …<br><br>2. 6 + 4 × 5, 6 + 5 × 4<br><br><br><br>1st 2nd 3rd … | ○ Students can represent their reasoning processes using expressions.<br><br>* If a student found a way, encourage him/her to think about other ways and represent them in expressions.<br><br>* Evaluation points for students' expressions.<br><br>(1) Is the student paying attention to the difference between the multiplier and the multiplicand?<br><br>(2) Is the students thinking about variable like use of numbers?<br><br>(3) Is the student paying attention to the way the total is increasing (in relationship to methods 1 and 2)?<br><br>* In particular, focus on (1) and (3). For example, ask students to explain which parts of their expressions represent the position number or the number of stones. |

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| | • Comparing and discussing solutions<br>○ Have students share their expressions.<br>○ Have students explain other students' expressions. | 3. 3 × 7 + 5<br><br>1st 2nd 3rd …<br>4. 8 × 4 − 6, 4 × 8 − 6<br><br>1st 2nd 3rd …<br>5. {5 +(5 + 3)} × 4 ÷ 2 ➔ trapezoid or parallelogram<br><br>1st 2nd 3rd …<br>6. {5 + (5 + 3)} × 2, {(5 + 1) + (5 + 2)} × 2<br>➔ rectangle<br><br>1st 2nd 3rd …<br>7. Without thinking about generalizing, simply group stones at random.<br>★ After we study calculations of algebraic expressions, we will know that all of these will be simplified to the same algebraic expression, "6 + 4*x*." | * Do not discuss the benefit of variable like use of "5" in (5 1) here.<br>○ Students can explain others' ideas using tools like diagrams.<br>○ Students can make sense of others' ideas.<br>* Help students attend to the benefit of keeping "5" (position number) in the expression, like (5 1).<br>* Write students' methods on the blackboard so that they can become aware of varying quantities and introduce the use of letters as variables. |

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| | "If we want to find the total number of stones in the 100th arrangement, which expressions would you use? Find the total number of stones in the 100th arrangement."<br><br>"What characteristics do you notice about the expressions you thought about using?"<br><br>• Write appropriate expressions with letters | (Example) 6 + 4 × 100<br><br>✜ It is possible for some students to argue that simply adding the number of stones in the four rows if we are finding the total number for the 100th arrangement.<br><br>• We just need to change "5."<br>• The position number is included in the expression.<br><br>$5^{th}$ 6 + 4 × 5<br>$100^{th}$ 6 + 4 × 100<br>position 6 + 4 × (position)<br>$x^{th}$ $6 + 4 \times x$<br><br>• Anticipated student responses<br><br>1. $10 + 4 \times x$, 0 $x \times 4$, $10 + 4 \times (x - 1)$, $10 + (x - 1) \times 4$<br><br>2. $6 + 4 \times x$, $x \times 4$<br><br>3. $3 \times (x + 2) + x$<br><br>4. $(x + 3) \times 4 - 6$, $4 \times (x + 3) - 6$<br><br>5. $\{x + (x + 3)\} \times 4 \div 2$ ➔ trapezoid or parallelogram.<br><br>6. $\{x + (x + 3)\} \times 2$, $\{(x + 1) + (x + 2)\} \times 2$<br><br>➔rectangle | * Depending on student responses, ask "Which expression is easier to use to determine the total number of stones for any position number?"<br><br>* Have students pay attention to variable like use of numbers.<br><br>* Write expressions with specific numbers and expressions with words so that students can become aware of varying numbers, then introduce letters as variables.<br><br>* Have students find the total number of stones for specific position numbers.<br><br>* For simple cases, have students substitute the numbers in various expressions to verify that they all result in the same number. |
| Summary | • Summarize the lesson<br><br>"What are some of the strengths of using letters in expressions?" | • We can calculate the total number of stones if we substitute the position number in the letter.<br>• We can interpret other people's reasoning from their expressions with letters. | * If there is enough time, ask students which algebraic expression is the best. |

Grade 4 Mathematics Lesson Plan

Wednesday, June 26, 2013, Period 5
Daisan Terajima Elementary School
Grade 4, Classroom 1
Teacher: Daisuke Nagatani

Research Theme Nurturing students who can think on their own:
Exploring teaching approaches that incorporate critical thinking

Rationale for the research theme

This academic year is the third year of the full implementation of the new National Course of Study. Our goal is to develop "solid learning" in our students through improvements of teaching. An ongoing challenge is we can improve ourselves so that students will acquire and master "basic and foundational knowledge and skills" and develop the abilities to "reason, judge, and express own ideas" ecessary o se heir nowledge nd kill heir veryday ituations.

During the last academic year, we focused our attention on helping students to develop their problem solving ability. Our focuses on lesson improvement ere n tudents' bility o nderstand problems and their independent problem solving. As a result, students are developing the disposition to solve problems independently. This year, we are also examining the way to improve group discussion by attending to students' language development.

When students encounter a problem, they must interpret the problem. They must also think about how they might tackle the problem. They need to think about how to express their ideas, as well as listening to other students' ideas and make sense of them. We hope to nurture students' ability to reason and express themselves by letting students engage in these rich language activities and becoming aware of diverse perspectives.

**Vision of Ideal Students**

[Lower Grades] As they listen, they can compare their own ideas and those of others and recognize their strengths.

[Intermediate] As they listen to others ideas, they can judge the viability and recognize their strengths.

[Upper Grades] Students can interpret their own and others ideas in light of mathematical power and deepen their understanding of the strengths of various ideas.

Mathematical power hear includes usefulness (easy to use), conciseness (simple), generalizable (can be used in many settings), accuracy, efficiency, extendable (applicable) and beauty.

In he termediate epartment, e mphasize hinking gically nd eveloping rguments with clear rationale. Our goal is for students to experience strengths of each other's ideas through language activities in which they will examine the viability of each other's ideas.

## 1 Name of the unit

"Let's explore various quadrilaterals"

## 2 Goals of the unit

1. Students will understand the meaning of perpendicular and parallel lines and how to draw them.
2. Students will understand the definition and properties of trapezoids, parallelograms, and rhombi and how to draw them.
3. Students will understand the characteristics of diagonals of quadrilaterlas.

## 3 Evaluation standards

[Interest, Eagerness, and Attitude]

- Students are trying to find perpendicular and parallel lines, trapezoids, parallelograms, and rhombi in their surroundings.
- Students are trying to examine perpendicular and parallel lines, trapezoids, parallelograms and rhombi based on geometric properties they have learned previously.

[Mathematical Way of Thinking]

- Students can categorize quadrilaterals based on perpendicular and parallel sides, and they can reason about their properties.

[Mathematical Skills]

- Students can draw perpendicular and parallel lines sing et quares.
- Students can draw trapezoids, parallelograms, and rhombi using compass and set squares.

[Knowledge and understanding]

- Students understand the meaning and properties of perpendicular and parallel lines.
- Students understand the meaning and properties of trapezoids, parallelograms, and rhombi.
- Students understand the meaning and properties of diagonals.

**4 Relationship between this unit and the research theme**

(1) About the unit

The topics discussed in elementary schools that relate to this unit are shown in the figure below.

In his nit, e ill xamine he elationships f lines lane. tudents ill arn he meaning of perpendicular and parallel lines. Students are also expected to develop the basic understanding of properties of parallel lines. The goal is not to simply teach the vocabularies. Rather, through concrete activities, students are expected to deepen their understanding and observe their surroundings using their knowledge of perpendicular and parallel elationships and identify perpendicular and parallel lines.

As we explore quadrilaterals, we will emphasize activities to carefully observe, compose and decompose the given figures. Through those activities, we will make it more explicit about the features that define the basic geometric figures (trapezoids, parallelograms, and rhombi). An important goal is for students to investigate properties or sides and angles that are the constituent parts of quadrilaterals. Students will also investigate properties of diagonals of quadrilaterals.

Furthermore, throughout the unit, students will continue to master drawing geometric figures using appropriate tools. This is an important basic/foundational idea. Students are expected to refine their ability to draw geometric figures as they engage in the activities of drawing perpendicular and parallel lines using set squares or drawing various quadrilaterals using ruler, compass, and protractor.

**Gr. 2A**

**6. Triangles and Quadrilaterals**

- Meaning of and sorting triangles.
- Composing triangles and quadrilaterals.

**Gr. 2B**

**11. Rectangles and Squares**

- Definition of right angle.
- Definition of rectangles, squares, and right triangles. Composing and soring shapes. Tiling with right triangles.

**Gr. 3B**

**13. Triangles and angles**

- Definition of and basic properties of isosceles and equilateral triangles.
- Drawing circles using a compass.
- Definition of angles, and comparing the sizes of angles.

**Gr. 4A**

**3. Angles and Angle Measurements**

- Measuring and drawing angles.

**Gr. 4A**

**7. Quadrilaterals**

- Definitions and basic properties of perpendicular and parallel lines, and drawing perpendicular and parallel lines.
- Angles formed by parallel lines and a line intersecting them.
- Definitions and basic properties of trapezoids, parallelograms, and rhombi. Drawing various quadrilaterals.
- Investigating how diagonals of quadrilaterals intersect.

**Gr. 5A**

**1. Angles of Geometric Figures and Congruence**

- Definition and properties of congruent figures, and drawing congruent figures.
- Sum of angles in a triangle.
- Sum of angles in polygons.

**Gr. 6A**

**1. Symmetric Figures**

- Meaning of and basic properties of line and point symmetries and line of symmetry. Drawing symmetric figures.
- Corresponding points, sides, and angles.
- Examining geometric figures using properties of symmetries.

**Gr. 6B**

**9. Scale Drawings**

- Meaning and basic properties of scaled drawings, and how to draw scaled figures.
- Meaning, representation, and use of scales.

(2) Current state of students with respect to mathematics

There are many students who like mathematics. Mathematics is the second most favorite subject according to a survey of our class. Specially, there are many students who enjoy calculations, but there are students who still lack computational fluency. There is lso endency hat heir arning is ompartmentalized new idea is not connected to what they have learned previously.

In he tudy f eometric hape, tudents re truggling o aster kills uch s rawing traight lines nd sing ools like ompass and protractor, in part because their manual dexterity is still developing. There are many students who do not feel good about making accurate observations and grasping spatial relationships accurately, in part, because they struggle making use of their prior knowledge and observing geometric figures from multiple directions. However, students are becoming aware of the importance of drawing geometric figures accurately, and their skill levels are improving.

There are students who often give up thinking when they get stuck. On the other hand, there are students who are eager to share their ideas with others. In particular, we have been using student name cards during the whole class discussion, and students appear to enjoy the discussion time.

The first goal in today's lesson is for all students to independently solve the main problem and participate in the whole class discussion with their own ideas. Furthermore, the whole class discussion will be enriched by incorporating diagrams into this language activity. It is hoped that students' ability to reason and express themselves will be nurtured through sharing and examining the viability of each other's ideas. The experience should enjoyable for students.

(3) Teacher's perspective (on the unit and the topic)

There are many students who feel less confident about the study of geometric figures than the study of numbers. When observing geometric figures, we must observe them from multiple perspectives. Multiple perspectives here include spatial relationships f igures, omparison f heir shapes, directions from which we view the figures, and so forth. I believe a part of difficulty students feel is ecause hey have to identify quantities and relationships among figures which do not appear to share any commonality – their lengths are different, they are place at different positions, and their orientations are different.

In oday's sson, tudents re xpected o istinguish igures s hey ake heir oints f iew explicit. They will sort figures that look different or the same on the surface. They must look at the figures from a specific perspective and use the important factors from that perspective to sort the given figures. It is hoped that students can experience that there are things that become visible when they clarify their perspectives using their prior learning.

Moreover, during the whole class discussion, students are expected to understand that there are diverse ways of reasoning as they share each other's ideas. In addition, by reasoning logically as they try to understand each other, students are expected to further their ability to reason.

## 5 Relationship to the vision of ideal students

< As they listen to others ideas, they can judge the viability and recognize their strengths.>

(1) Independent roblem olving

In rder o olve roblems dependently, tudents ust ocus n onstituent arts f geometric figures and their spatial relationships. Students should solve problems independently identifying and relating the previously arned elationships parallel nd erpendicular) nd eatures such as lengths of sides. To support individual problem solving, students will be reminded of what they have learned in previous lessons and how they have been able to use their prior knowledge to solve new problems. I elieve uch n xperience ill e elpful heir uture tudy s ell.

(2) Use of geometric figures

"It looks like ----" is not a rationale for a viable argument. To help students examine geometric figures and identify the reasons for distinguishing them, actual shapes will be provided so that they can work with them concretely.

(3) "Based on the chosen perspective, articulate the rationale for their judgment logically."

For those students who are unsure about how to express their reasoning, provide them a sample sentence template: "I made groups of **** and ####. I used whether or not ---- is/are ••••."

By examining the rationale and the actually sorted figures, erify hether r ot here s ny logical law r ontradiction." hrough his ctivity, e ant tudents o xperience he trengths each other's ideas. (Critical Thinking)

## 6 Unit plan (15 lessons)

| # | Goals | Activity | Main evaluation standards |
|---|---|---|---|
| Sub Unit 1: How lines intersect | | | |
| 1 | • Students will understand the definition of perpendicular lines.<br>• Students can distinguish lines hat re perpendicular from those that are not. | • Explore how 2 lines intersect.<br>• Create perpendicular lines y olding apers.<br>• Distinguish lines hat re perpendicular from those that are not. | • Students are trying to explore how 2 lines intersect. (Interest, agerness, nd Attitude)<br>• Students are thinking about how 2 lines intersect. (Mathematical Way of Thinking)<br>• Students understand the definition of perpendicular lines. Knowledge and Understanding) |

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| | | | • Students can distinguish lines that are perpendicular from those that are not. (Mathematical Skills) |
| 2 | • Students can draw perpendicular lines. | • Investigate ays o raw perpendicular lines.<br>• Draw perpendicular lines. | • Students can draw perpendicular lines by using set squares. (Mathematical Skills)<br>• Students understand how to use set squares to draw perpendicular lines. (Knowledge and Understanding) |
| Sub Unit 2: How lines are arranged | | | |
| 3 | • Students will understand the definition of parallel lines. | • Explore how 2 lines may be arranged.<br>• Create parallel lines by folding papers. | • Students are trying to explore parallel lines. (Interest, Eagerness, and Attitude)<br>• Students are thinking about ways 2 lines may be arranged. (Mathematical Way of Thinking)<br>• Students understand the definition of parallel lines. (Knowledge and Understanding) |
| 4 | • Students will know properties of 2 parallel lines.<br>• Distinguish lines that are parallel from those that are not. | • Investigate he idth f parallel lines.<br>• Explore angles formed by parallel lines and a line intersecting hem. | • Students understand the properties of parallel lines. (Knowledge and Understanding)<br>• Students understand that corresponding angles formed by a line tersecting arallel lines re ongruent. (Knowledge and Understanding)<br>• Students can distinguish lines that are parallel from those that are not. (Mathematical Skills) |
| 5 | • Students can draw parallel lines.<br>• Students can identify parallel lines in their surroundings. | • Investigate ays o raw parallel lines.<br>• Draw parallel lines.<br>• Look for perpendicular and parallel lines in their surroundings. | • Students can draw parallel lines sing et quares. (Mathematical Skills)<br>• Students are trying to find perpendicular and parallel lines heir urroundings. (Interest, agerness, nd Attitude) |

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| Sub Unit 3: Various quadrilaterals | | | |
| 6 | • Students will distinguish figures based on clear rationales. | • Draw various quadrilaterals on dot paper.<br>• Sort quadrilaterals. | • Students sort quadrilaterals by examining their constituent parts. (Mathematical Way of Thinking) |
| 7 | **TODAY'S LESSON**<br>• Students will discuss their ideas y aking heir rationales explicit.<br>• Students will understand the definitions of trapezoids, parallelograms and rhombi. | • Discuss the rationales for their sorting.<br>• Students will learn the definitions of trapezoids, parallelograms, and rhombi. | • Students can discuss sorting of geometric figures with clear rationales. (Mathematical Way of Thinking)<br>• Students understand the definitions of trapezoids and parallelograms. (Knowledge and Understanding) |
| 8 | • Students will summarizes the properties of trapezoids, parallelograms and rhombi. | • By organizing the properties visually, students will understand the properties of each type. | • Students understand quadrilaterals based on their constituent parts. (Knowledge and Understanding) |
| 9 | • Students will draw trapezoids and parallelograms. | • Think about ways to draw trapezoids and parallelograms and actually draw them. | • Students can draw trapezoids and parallelograms. (Mathematical Skills) |
| 10 | • Students will draw rhombi. | • Think about ways to draw rhombi and actually draw them. | • Students can draw rhombi. (Mathematical Skills) |
| Sub Unit 4: Diagonals | | | |
| 11 | • Student will understand the definition of diagonals.<br>• Students will explore properties of diagonals in various quadrilaterals. | • Draw diagonals.<br>• Investigate iagonals f various quadrilaterals. | • Students understand the definition and properties of diagonals. (Knowledge and Understanding) |
| Summary of the unit | | | |
| 12 | • Students will recognize that quadrilaterals are used in various situations in heir urroundings. | • Look for quadrilaterals in their surroundings. | • Students are trying to find quadrilaterals in their surroundings. (Interest, Eagerness, and Attitude) |
| 13 | • Students will work on "Review" and solidify their understanding. | • Work on "Review." | • Students understand the content of the unit. (Knowledge and Understanding) |
| Mathematical Activities | | | |
| 14 | • Students draw quadrilaterals using the properties of circles and diagonals. | • Draw quadrilaterals using circles. | • Students can explain properties of quadrilaterals using their diagonals. (Mathematical Way of Thinking) |

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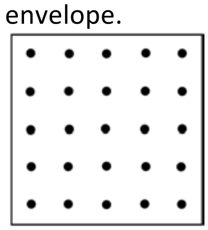
| | • Students will make various shapes using Tangrams. | | • Students are trying to make various shapes by using the characteristics of each piece of Tangrams. (Knowledge and Understanding) |
| 15 | • Students will design tiling patterns using parallelograms, trapezoids, and other quadrilaterals. | • Design tiling patterns using quadrilaterals. | • Students are thinking about ways to tile using quadrilaterals. |

## 7 Goals of today's lesson (Lessons 6 & 7 /15)

- Students will discuss how they sorted quadrilaterals by making clear their rationales.
- Students will deepen their ways of mathematical thinking as they listen to each other's ideas and verify the rationales and the results of sorting match.

## 8 Flow of the lessons (Lessons 6 & 7 / 15)

Flow of Lesson 6

| | Content and Main *Hatsumon* | Anticipated Responses | Evaluation | Instructional teps |
|---|---|---|---|---|
| Understanding the problem | ○ Draw quadrilaterals.<br>1. By connecting 4 dots on dot paper, draw quadrilaterals.<br>• Shapes that are enclosed by 4 straight segments are called quadrilaterals.<br><br>[general quadrilateral, parallelograms (2), trapezoids (2), square, rectangle, rhombus (2)]<br><br>2. Let's make groups of quadrilaterals. You will be sharing how you grouped them, so be prepared to explain your sorting. | 1.1 Students will draw quadrilaterals using dot paper.<br>1.2 Students are not sure how to use dot paper.<br><br>2.1 Students understand the task.<br>2.2 Students don't understand the task. | ○ Students can connect 4 points on dot papers using straight segments. (Worksheet) | Prepare sets of cut out quadrilaterals for each student (general quadrilateral, 2 different parallelograms, isosceles rapezoid, general trapezoid, 2 types of rhombus, rectangle, and square). Prepare a set to be used on the blackboard.<br><br>Students will put away their dot papers in an envelope.<br><br>2.2 Individually work with these students. |

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| Plan | 3. Have you thought about how you might group these shapes? Use your tools like set squares, compass, and protractor, instead of just ow hey ok. | 3.1 I'm going to look at their sides.<br>3.2 I will look at their angles.<br>3.3 I will compare their sizes.<br>3.4 I'm not sure what to do. | 3.1 & 3.2 They are able to look at the constituent parts of the quadrilaterals.<br><br>3.3 & 3.4 They are unable to look at the constituent parts of the quadrilaterals. | ○ Help students clearly identify what feature to focus on.<br>○ Remind students to use tools such as set squares and protractors instead of just focusing on the appearances.<br>3.3 & 3.4: Help students to look at the constituent parts of quadrilaterals. |
| Individual roblem olving | ○ Sort the given quadrilaterals.<br><br>4. Let's make groups with these quadrilaterals. Make sure you can explain why some quadrilaterals belong in the same group.<br><br>• Tell students to write their answers in heir otebooks.<br><br>a. rectangle<br>b. parallelogram<br>c. general quadrilateral<br>d. rhombus<br>e. trapezoid<br>f. parallelogram<br>g. square<br>h. rhombus<br>i. rapezoid | 4.1 Based on parallel sides.<br>(a, b, d, f, g, h), (e, i), and (c)<br><br>4.2 Based on right angles<br>(a, g) and the rest<br><br>4.3 Based on the lengths f ides<br>(g, h) and the rest<br><br>4.4 Based on their appearances<br><br>4.5 Based on parallel sides<br>(b, g) and the rest<br><br>4.6 Cannot explain the rationale<br><br>4.7 Cannot make groups | 4.1 4.3 Students are able to sort quadrilaterals with clear rationales.<br><br>4.4 Students' verification is incomplete.<br><br>4.5 Grouping does not match the rationale.<br><br>4.6.1 Students cannot tell what to focus on.<br>4.6.2 Students are sorting at random.<br><br>4.7.1 Students do not know what to do.<br>4.7.2 Students do not know what to focus on. | ○ Encourage students to use tools such as set squares, compass, and protractor.<br><br>4.1, 4.2, & 4.3 Encourage them to think about different ways of sorting.<br><br>4.4 Encourage students to use tools such as set squares, compass, and protractor.<br><br>4.5 Check what follows from the rationale.<br><br>4.6.1 Ask them what they can focus on.<br>4.6.2 Suggest what to focus on.<br><br>4.7 Suggest students to focus on right angles. |

Flow of Lesson 7

| | Content and Main *Hatsumon* | Anticipated Responses | Evaluation | Instructional teps |
|---|---|---|---|---|
| | ○ Know how to sort quadrilaterals with clear rationales.<br>5. From your notebooks, here are the ways you made groups with these quadrilaterals.<br>• based on the number of groups | 5.1 I think my grouping is there.<br><br>5.2 I wonder if you can make groups that way. | 5.1 Students are listening arefully o that they can identify here heir own ideas belong to.<br><br>5.2 Students are trying to think about the rationale behind grouping. | ○ Display only the number of groups students made.<br><br>5.2 Encourage students to think about what rationale will give you a particular number of groups. |

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| Whole Class Discussion | ○ Students will share their rationales for making groups.<br><br>6. Let's have the sharing time. Please place a green card with the idea that matches yours, and place a yellow card on ideas ou re ot ure about.<br><br>Number of groups<br>1<br>2<br>3<br>4<br>5<br>6<br>Rationale<br>• lengths f ides<br>• relationship of sides<br>• angles | I ade roups f **** and ####. I looked t hether or not ---- is/are ••••.<br><br>6.1 I made groups of {a, g} and {b, c, d, e, f, h, i}.<br>You can make 2 groups if you check if there is a right angle.<br><br>6.2 I made groups of {c} and {a, b, d, e, f, g, h, i}.<br>I grouped them by whether or not there are parallel sides.<br><br>6.3 I made groups of {e, i}, {a, b, d, f, g, h} and {c}.<br>You can make 3 groups if you check if there is 1, 2 or no pair of parallel sides.<br><br>6.4 I made groups of {c}, {e, i}, {a, b, f}, and {d, g, h}.<br>I oked t he ides of equal lengths.<br><br>6.6 I made groups of {a}, {b, f}, {c}, {d, h}, {e, i}, and {g}.<br>I oked or arallel sides, perpendicular sides, and sides of the same length. | 6.1 Students are sorting quadrilaterals based on right angles.<br><br>6.2 Students are sorting sort quadrilaterals based on the presence of parallel sides.<br><br>6.3 Students are sorting quadrilaterals based on the presence and the number of parallel sides.<br><br>6.4 Students are sorting quadrilaterals by whether or not opposite sides are equal lengths.<br><br>6.5 Students are sorting quadrilaterals based on the presence and the number of equal sides.<br><br>6.6 Students are sorting quadrilaterals based on the presence of parallel sides, perpendicular sides, and lengths of sides. | ○ In rder o facilitate discussion with own positions, use name cards.<br>• same idea green<br>• not sure yellow<br><br>When all yellow cards are removed, we can conclude that all students understood the method of sorting.<br><br>○ Instruct tudents to use tools such as set squares and protractors and their previously learned kills o guide their sorting, not just based on the appearances of the figures.<br>○ Instead f tudents explaining their own ideas, let other students think about others methods.<br>○ Students who came up with the idea should try to give hints to other students. |
| Summary of the lesson | ○ Announce what we will be studying in the next lesson.<br>In he ext sson, e ill look ore arefully t he grouping method based on parallel sides in quadrilaterals. | ○ Students will listen to what they will be learning he next lesson. | ○ Did students understand what they will be learning he next lesson? | ○ Using the completed blackboard, bring students' attention to the methods that used parallel sides. |

June 28

# Grade 3, Class No. 2, Mathematics Lesson Plan

**Instructor:** Yuji hikawa
**School:** Showa machi Oshihara Elementary School

1. **Name of Unit:** "Let's think about division" (Division with remainders)

2. **About the Unit:**

Up to w, tudents ave arned ivision" a w nd f calculation. There re two different meanings of division—partitive division and quotative division. Students have learned both meanings of division, as ll how o press them in problem situations using math sentences. Since division is the nverse operation of multiplication, the answers to division calculations can be obtained by utilizing the multiplication table of the number in the ivisor.

In this unit, students will learn about division with remainders for cases in hich the multiplication table is used once. The content of this unit includes he meaning of division with remainders, calculation methods, what remainders are, and how to deal with remainders.

When the size of quotient is maximized, the remainder is what s ft from subtracting a product of a divisor and the quotient from a dividend. The division problems that students learned previously in which the remainder was always 0 are actually pecial cases. In der to help students understand this point, division with remainders needs to be integrated with division without remainders that tudents learned eviously as ne ncept. It is important to build on what tudents learned before to help them to learn to do division calculations with remainders. Moreover, students should solve problems involving oth partitive and uotative ivision situations in the same way they did before with division without remainders.

Through the nstruction f e ssons in his unit, I would like to help students to understand the meaning of remainder as ll expand their view of remainders. First, at e ntroduction f e unit, I will se a task at involves division without a remainder, similar to what students have learned before, and elp them to understand that division ncludes the cases that are th divisible and not divisible. For the introduction, a partitive division problem situation will be used. By dividing objects into several groups equally, udents ill earn that what s ft rough his process the remainder. By utilizing this problem situation, I would like to lead the students to learn how to calculate division with remainders. If we use a quotative division problem situation it is much easier to understand that at s ft s e remainder. However, by using the partitive division problem situation students can think about the act at depending on the objects they are dealing with there are object where they can still divide the remainders and objects here they an't ivide the remainders. This view of looking at remainders can be thought s connection o division with decimal numbers and fractions. After the introduction, I would like to use a quotative division problem situation and help students to understand the relationship between the size of the remainder and the divisor. Moreover, I would like to help students to understand how to utilize multiplication and addition to check division calculations by ing several examples.

Next, in he second half of the unit, which ncludes is sson, I would ike o help students to understand different ways to treat remainders that are different from what tudents arned n e irst alf f e unit (e.g. round up remainders and add 1 to a quotient or round down remainders and find a quotient). By inking about ow to teat remainders differently in situations that match the story problems students are ked to solve, I would like to help them to xpand eir understanding of remainders. At the end of the unit, students will apply is understanding by solving story problems that involve he size of the remainder. For example, "We are going to make basketball teams named by fruits. Students e lined up and ivided into three teams—the apple team, the range am and the banana team—by assigning students into ch group in the following order: apple, range, and banana. What oup will the 5th student in he ine elong o?" I would like to student to understand hat y thinking about e patterns involved in the problem and utilizing ivision ith remainders, this problem situation can be solved efficiently. In addition, I would like to tudents to see the merit of doing so.

## 3. Relationship to the School Based Professional Development:

(1) About the ability to think in logical steps with foresight and to express

This years' theme of the School-based professional development at Oshihara Elementary School is "Elementary career education[1] that fosters students' base for independency: Development of instruction that raises students' ability to think in logical steps with foresight, and to express." At this school, we have been trying to clarify the relationship between the career education and mathematics education, and wrestle with raising the abilities that students need for career advancement through mathematical instruction. The reasons for doing this at our school is because we see the concrete connections between the goals of career education and the goals of mathematics education in the following four points: "creative thinking," "autonomy and independence," "awareness of value of self," and "relationship between self and society."

The sub-theme, "Development of instruction that raises students' ability to think in logical steps with foresight and to express," is elaborated in the goals of mathematics education [Chapter 1, Section 1, Number 1(3) in the Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grades 1 6)]. In general, the process of earning in mathematics involves posing problems and solving problems. This process involves students using their prior knowledge and kills, asping e problem situation, thinking about how to solve the problem, utilizing easoning, and making inferences. The mathematics curriculum has strong coherence between e content hat students learn. Furthermore, the basis needed for thinking is relatively clear. Therefore, the subject is appropriate for developing students' ability to think in logical steps with foresight.

From the results form last year's professional development, at this school, we think of "the ability to think in logical steps with foresight and to express" in the following manner:

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[1] (Notes added by translator) Career education was introduced formally to the schools in 2008 by the Ministry of Education, Culture, Sports, Science and Technology (MEXT). The goal of the career education is to foster each student's view or willingness to work and become aware of the responsibilities of their profession. According to the MEXT, "Career education is important in educating children in their views of career and work, and in cultivating the ability to proactively select and decide career paths. For that purpose, MEXT is promoting systematic career education applicable to each school stage through experience in the workplace and so on."

Abilities to think inductively, analogically and deductively.
In addition:
**• Ability to explore clues to solve problems**
**• Have a basis to solve problems, organize own thinking, and be able to express them**
**• Reflect on and organize own thinking**

In this class, I have observed the following points among the status of students' learning on these three highlighted abilities.

About 70% of the class consists of students who can solve problems on their own and show the process in writing. When they have to discuss problem situations or other students' ideas about half of the students often mumble something that they noticed or have their own opinion. In addition, many students actively communicate their ideas and thoughts in the class. On the other hand, they are not so good at clarifying the commonalities and differences between their own ideas and their friends' ideas, and having the will and patience to try to understand what they don't get. These issues may be part of the developmental stage of 3rd grade students, however, something needs to be done to improve.

Up to now, students have learned to speak in the class using phrases such as, "The reason is …," "as same as …," and "different from …." In addition, they have learned to take notes in order to organize their thoughts such as using headings like "problem," "my idea," "friends' ideas," "summary," and "reflection." Students have been developing these skills for two months since the beginning of the school year in April. Lastly, students have been practicing sharing the thoughts that they mumble and describing own as well as other students' thinking. Through this the students are developing their attitudes for listening and the ability to respond to other students' ideas and thoughts.

Based on these students' state of learning, I would like to show an improvement in students' "ability to think in logical steps with foresight and to express" through the following five learning steps.

1.) Grasping goals/tasks
2.) Having foresight for solving a problem
3.) Solving a problem on their own
4.) Discussing methods to solve a problem in class as a whole
5.) Summarizing learning

(2) About "the ability to think in logical steps with foresight and to express" in this lesson:

1.) Evidence of "the ability to think in logical steps with foresight and to express"
I am hoping that students can show the following evidence of these abilities:

A. Students are able to find clues for solving problems

- In the case of division with remainder, students are able to think by using the partitive and quotative meanings of division that they learned previously through problem situations.
- **By utilizing the meanings of division that students learned previously, they try to express the problem situation with a math sentence.**
- Students think about a method to check the result of a division calculation by utilizing previously learned knowledge, such as division calculation is the reverse operation of multiplication calculation, and methods for checking the results of addition and subtraction calculations.

B. Students solve problems based on what they learned and are able to organize and express their own thinking.
   - In the case of division with remainders, students are able to express how they can utilize previously learned knowledge in problem solving with math sentences, words, diagrams, and pictures.
   - Students are able to express remainders (what part remains, if it is divisible or not divisible) using diagrams and pictures
   - Students are able to express why they need to carefully examine how to deal with remainders in order to come up with an appropriate answer for a specific problem situation.

C. Students who reflect and organize their own thoughts
   - Students are able to recognize that calculations utilizing multiplication tables are efficient for calculations of division with remainders, and utilize this method to solve problems.
   - **Students compare their own ideas and their friends' ideas to find similarities and differences.**
   - **Students recognize the merits of their friends' ideas and utilize them to solve problems.**
   - **Students record their own progression of learning in their reflections in their notebooks.**

2.) Teacher's role and support for raising students' "ability to think in logical steps with foresight and to express."

A. Teacher provides *hatsumon* that help students to have foresight for solving problems.
   The steps involve "grasping goals/tasks" and "having foresight for solving a problem." The teacher asks *hatsumon* such as, "I wonder if we can use something we learned before," "What should we use to think about the problem?", "What are we finding?", and "What do we know and what do we need to find out?" that help students to be able to have foresight for solving problems. By asking these *hatsumon*, the teacher listens and picks up students' mumblings and helps to connect them to understanding the goals/tasks of the lesson. By facilitating students sharing of their thinking in the class, help each student to have foresight for solving problems and engage them in problem solving.

B. The teacher asks about the basis of students' thinking and provides opportunities for them to express their thinking.
   The steps involve "students solving a problem on their own" and "discussing methods to solve a problem in class as a whole." The teacher asks students, "Why did they think that way?" to bring out there bases for their thinking. As the case may be, the teacher helps students to focus on a few ideas and provides them with opportunities to organize and express their own thinking in their notebooks in many different ways and for them to discuss what they notice and think about.

C. The teacher provides opportunities for students to exchange and examine ideas and creates *bansho* (board writing) that helps students to see the flow of the lesson and how ideas are connected to build understanding.
   Steps involve "students discussing methods to solve a problem in class as a whole" and "summarizing learning." The teacher helps students to be exposed to many different ideas and asks questions that lead to the essence of the learning of the content. The teacher asks students about the merits and expandability of each idea,

the similarities and differences among ideas, and the generalizability and consistency of ideas. As the case may be, help students to clarify the ideas that are easier for the students to understand such as, “a method that helps to solve a problem efficiently” and “a method that is easier for everybody to understand.” Also, the teacher will facilitate students’ thinking so they can move on to better ideas as well as deepen and utilize the ideas. In order to help students to think this way, the teacher will think about *bansho* that shows the flow of the lesson and how students’ ideas and thinking are incorporated to develop understanding of the content.

## 4. Objectives of the Unit:

| Mathematics and Moral Education | Career Education |
|---|---|
| ○ Help students to understand division with remainders, deepen their understanding of meaning of division, and utilize them.<br>• Students think about the meaning of and how to calculate division with remainders based on division without remainders, the relationship between division and multiplication that they have learned before, and manipulation of concrete materials. [Interest, motivation, and disposition]<br>• Students are able to explain the meaning of and how to calculate division with remainders (multiplication tables can be utilized to find answers) using concrete materials, diagrams, and math sentences.<br>• Students are able to pay attention to remainders and think about the structure of problems [Mathematical reasoning]<br>• Students are able to calculate division with remainders and check the answers. [Skills and Procedure]<br>• Students are able to understand the meaning of remainders and the relationship of the size of the remainder and the divisor, nd ow o calculate division with remainders. [Knowledge and understanding] | <Ability to take a part><br>○ Students try and are able to understand each other’s ideas [Formation of human relationship and ability to be a part of community]<br><br><Ability to find and resolve issues><br>○ Students are able to come up with strategies and solve problems/tasks by clarifying the basis for solving the problem by utilizing what they have learned previously [Ability to cope with problems/tasks]<br><br><Ability to turn into reality><br>○ Students are able to choose a more efficient way from many different ideas and utilize it to solve problems/tasks. [Ability for career planning] |
| <Moral Education><br>1 (2) Once one decides to do something, persevere to complete the task. | |

## 5. Unit plan (Total of 10 lessons):

| Sub Unit [# of lessons] | Lesson | Main Activities | Evaluation Points of This Lesson | Career Education |
|---|---|---|---|---|
| 1 [6] | 1 & 2 | • Think about problem situations that are divisible and not divisible based on a fair share (partitive) division problem situation.<br>• Understand remainders and think about problem situations where remainders can be still be divided and not be able to be divided. | Students think about division with remainders based on the division they have learned previously. [Interest, motivation, and disposition]<br>Students are able to understand how to calculate division that is not divisible and about remainders. [Knowledge and understanding]<br>Students think about problem situations that involve remainders that can be still divided and cannot be divided based on a partitive division problem situation. | Students try and are able to understand each other's ideas [Formation of human relationship and ability to be a part of community] |
| | 3 | • Think about a problem dealing with a quotative division situation and compare division that is divisible and division is not divisible. | Students are able to explain that both partitive and quotative non divisible division problem situations can be express as division, the calculation process can be thought as divisible division that they have learned. [Mathematical reasoning] | Students are able to solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks] |
| | 4 | • Investigate the relationship between the size of remainder and divisor. | Students are able to investigate the relationship between the size of remainder and divisor and understand this relationship. [Mathematical reasoning] | Students are able to solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks] |
| | 5 | • Think about how to check calculation of division with remainders. | Students are able to think about how to check the result of division with remainders based on understanding that division is the inverse operation of multiplication and knowledge of how to check addition and subtraction answers. [Mathematical reasoning]<br>Students are able to understand how to check calculation results of division with remainders. [Knowledge and understanding] | Students try and are able to understand each other's ideas [Formation of human relationship and ability to be a part of community] |
| | 6 | • Practice and become fluent on calculations of division with remainders and the method for checking the calculation. | Students are able to check the results of division calculation with remainders. [Skills and procedures] | Students are able to solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks] |
| 2 [3] | **1 [This Lesson]** | • Think about how to treat remainders appropriately according to the problem situation (round up the | Students are able to think about and explain how to treat remainders appropriately according to the problem situation (round up the reminders and | Students try and are able to understand each other's ideas [Formation of human |

| | | | | |
|---|---|---|---|---|
| | | reminders and add 1 to the quotient) | add 1 to the quotient) [Mathematical reasoning] | relationship and ability to be a part of community] |
| | 2 | • Think about how to treat remainders appropriately according to the problem situation (round the reminders down, keep the quotient as is) | Students are able to think and explain how to treat remainders appropriately according to the problem situation (round the reminders down, keep the quotient as is) [Mathematical reasoning] | Students are able to come up with strategies and solve problems/tasks by clarifying the basis for solving by utilizing what they have learned previously [Ability to cope with problems/tasks] |
| | 3 | • Solve application problems that utilize division with remainder efficiently. | Students are able to think about the meaning of finding remainders in a problem situation. [Mathematical reasoning] | Students are able to choose more efficient ways from many different ideas and utilize them to solve problems/tasks. [Ability for career planning] |
| 3 [1] | 1 | • Solve problems to summarize learning | Students are able to solve problems utilizing what they have learned in the unit. [Skills and procedure]<br>Students are able to develop deep understanding of the content they learned. [Knowledge and understanding] | |

### 6. Learning in this lesson:

(1) Goals of this esson:

Students are able hink bout and xplain how to treat remainders appropriately according to the problem situation (round up the reminders and add e quotient).

(2) Date and time: June 28, 2013 (Friday), 1:50 p.m. to 2:35 p.m. (5th period)

(3) Place: Showa City Oshihara Elementary School, Grade 3, Class 2

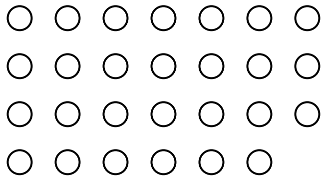
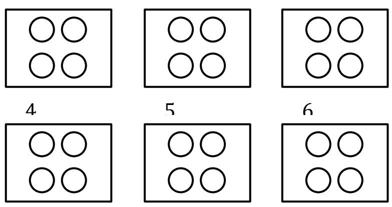
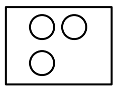
(4) Intention instruction:

Students have learned the meaning of remainders and how to calculate division with remainders up to he previous lesson. In this esson, udents ill earn how to treat remainders (round up the reminders and add 1 to the quotient) that s different than what they have learned. Because of this reason, I am going to provide a problem situation hat is close o he udents' ily ife. Through thinking about e task of his esson, I would like to students to notice that n some problem situations remainders can be treated differently and thought as a group that is added to the quotient.

Through several problem solving activities up to the previous lesson, tudents had o consciously hink about ituations where they ed find a quotient and remainder. Because of this reason, students might come up with a wrong answer. Based n a problem situation that involves "all the students need to ride," help students to think about how to treat the remainder. Through learning from this lesson, elp tudents xpand eir view of remainders and foster their ability to find an appropriate swer that corresponds with a problem situation.

(5) Flow of the lesson:

| Process | Learning activities and content | Instructional points to remember | Evaluation |
|---|---|---|---|
| Grasp the task and have foresight for solving the task<br>5 min. | 1. Grasp the task<br>(1) Understand the task for this lesson.<br>Grade 3, Class 2 is going to an amusement park. 27 students are divided into groups to ride boats that can hold 4 passengers each. How many boats do we need if everyone rides in the boats?<br>1.) Think about a math sentence for this problem and check it as whole class.<br>• 27 ÷ 4 | • Write the problem on the board and place the picture.<br>• By facilitating the conversation among students, come up with a math sentence. |  |
| Solve problem on their own<br>5min. | 2. Solve the problem on their own<br>a. 27 ÷ 4 = 6 R3 6 boats<br>b. 27 ÷ 4 = 6 R3<br>6 + 3 = 9 9 boats<br>c. 27 ÷ 4 = 6 R3<br>6 + 1 = 7 7 boats<br>Because we want all the students to be able to ride the boats, we need to add 1 more boat so the remainder of 3 students can ride in a boat. | • For students who are having difficulty calculating, ask them to look at their notes they took up to the last lesson, and utilize them to think about it.<br>• Use a seating chart to check students' problem solving progress and provide appropriate supports. | |
| Discuss solution methods as a whole class<br>25 min. | 3. Discuss solution methods as a whole class<br>(1) Ask students to present math sentence and answer.<br>27 ÷ 4 = 6 R3<br>a. 6 boats<br>b. 9 boats<br>c. 7 boats<br>(2) Students will explain their own thinking for various different answers that are obtained from the same math sentence using their own expression (e.g., using diagrams, words, math sentences, etc.<br>1.) Compare between one's own idea and friends' ideas and summarize their own idea one more time. | • Walk around the classroom and grasp students different solution methods<br>• Write down students' ideas that were presented on the board.<br>• Ask students why there are different answers even though they started with the same math sentence, and help students become conscious about the differences of thinking.<br>• If it is necessary, show students'' iagrams nd pictures that they wrote in their notebooks using the digital board. | Students are able to think about and explain how to treat remainders (round up the reminders and add 1 to the quotient) according to the problem situation. [Mathematical reasoning] (Notebook, presentation) |

| | | | |
|---|---|---|---|
| | Examples:<br><br>< Array Diagram ><br><br><br>< Picture of boats ><br><br><br><br><br>(Remainder of 3 students will be riding a boat)<br><br>2) Discuss each method as a whole class<br>a. We have 6 boats, 3 students cannot ride.<br>b. I think those 3 students were asked to get on to each boat (one person per one boat), ut the units for students and boats are different so we cannot add them together.<br>c. If we put the remainder of 3 students on a boat and make the total number of boats 7 boats, all the students can ride.<br><br>3) Students learn how to treat remainder according to the problem situation<br>• In order for all the students to ride in the boats, e eed to add 1 boat so the remainder of 3 students can ride. o he otal umber f boats is 7 boats.<br><br>(3) Students summarize what they have understood from today's task.<br>• In order for all the students to ride the boats, add 1 boat needed for the remainder of 3 students to the number of boats obtained by the calculation. | • Use a student's wrong answer for the class discussion. y aying attention to the commonalities and differences of the presented methods, help student to think about how to treat the remainder.<br><br>• Connect diagrams/pictures and math sentences. Help students understand the meaning of 6 + 1 = 7 and confirm that if we increase the quotient 1 more, all the students can ride the boats.<br><br>• Ask students to write in their notebooks with their own words. | |

| | | | |
|---|---|---|---|
| Summary<br>10 min. | 4. Summarize the lesson.<br>(1) Students solve an application problem that is similar to the problem they worked on.<br>1.) "26 students are divided into groups to ride boats that can hold 4 passengers each. How many boats do we need if everyone rides in the boats?"<br>• 26 ÷ 4 = 6 R 2<br>6 + 1 = 7 7 boats<br>(2) Write reflection in the notebook.<br>• "Depending on what the problem is asking to find, I understand that there is a case where you need to change the remainder to 1 and add to the answer. "<br>• "I was not thinking deeply about what I need to do with the remainder at the beginning, but in order to let all the students to ride the boats it is important to think about what to do with the remainder."<br>• "I listened to ____'s (a friend's) idea (or looked at a friend's diagram/picture) and I understand that we need to add 1, that is for the remainder, to the answer. " | | |

(6) Evaluation:

- Students are able hink bout and xplain how to treat remainders appropriately according to the problem situation (round up the reminders and add e quotient). [Mathematical reasoning]

**[References]**

1.新しい算数３上　教師用指導書　指導編　(2010) pp.98 110　東京書籍
2.新しい算数３上　教師用指導書　研究編　(2010) pp.198 211　東京書籍
3.文部科学省　(2010)「小学校学習指導要領解説　算数編」　東洋館出版
4.中村享史　(2008)「数学的な思考力・表現力を伸ばす算数授業」(2010) pp.105 113　明治図書
5.杉山吉茂　(2012)「確かな算数教育をもとめて」　東洋館出版

＊Translator's Note

· References 1 & 2 above refers to the textbook. An English translation of this textbook series may be purchased from Global Education Resources (www.globaledresources.com).

· Reference 3 is from *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grades 1 - 6)*, inistry f ducation ocument hat rovides etailed xplanations f he ourse f Study. It is available online at
http://e archive.criced.tsukuba.ac.jp/data/doc/pdf/2010/08/201008054956.pdf

· Other references are from books that are available only in Japanese.

Grade 2 Mathematics Lesson Plan

Multiplication (1): I can figure it out without counting them all!

Teacher: Daisuke Tsunoda
Learners: 33 Students from Classroom 1
Lesson Location: Aogiri Hall
Discussion Location: Aogiri Hall

## I. About the unit

In rade , tudents ngaged everal ctivities hat re oundational or he tudy f multiplication. For example, as counting activities, students counted by making groups like in 2's, 5's and 10's. They experienced the benefits of counting by groups such as the ease to insure each item is counted once and only once.

In ddition, tudents ave lso ngaged the activity of partitioning 12 items into equal groups, which is foundational for the study of division. With the exploration, "Let's make buildings," students worked on the problem: We are going to make a building using 12 color tiles. Pretend each □ is a room and make different buildings." The picture hown n he ight is a part of the blackboard from that lesson. Student comments that reflected the multiplicative nature of the problem situation included, "kept on going by 2's" and "4 tiles side by side then another one just like it underneath." We have tried to connect those statements to diagrams and repeated addition expressions.

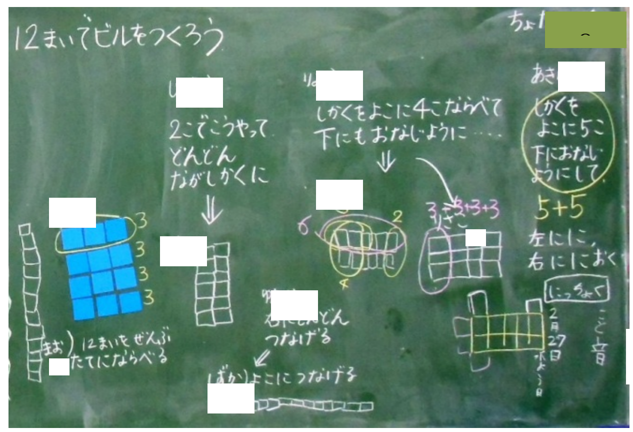


Fig. 1 A part of the blackboard from "Let's make buildings."

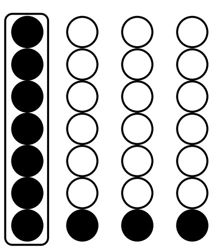


Fig 2 7 × 4

The most important goal of this unit is for students to realize the usefulness of multiplication. A usefulness of multiplication is we can determine the total number of objects without counting all of them once we identify equal grouping. For example, suppose there are 28 balls arranged as shown on the left. If e ee roups f alls, e an etermine the total number of balls by simply counting those that are shaded in black.

Students who have not yet learned ultiplication o ot now his articular sefulness f multiplication, that is, we can determine the total number without counting all objects. Therefore, in the beginning of the unit, we plan to include counting activities in which students can make groups of 2, 5, or 10 so that they can use the strategy shown in Figure 2. By making groups of 2, 5, or 10, students can determine the total number by counting “how many in a group” (group size) and “how many groups” (number of groups). We want students to experience the benefit of making use of equal groups. Even though students have yet to learn the basic multiplication facts, if they make equal groups of 2, 5, or 10, they can determine the total number easily by using their prior knowledge. However, they cannot determine the total number if they make equal groups of different sizes. Therefore, what needs to happen with those students who understand the benefit of making equal groups is for them to become aware that if we learn the basic multiplication acts, e an etermine he otal umber easily by making any size equal groups. That awareness will become a bridge to the study of the basic multiplication facts. Moreover, they should be able to experience the benefit of making equal groups in their everyday situations as well as in their study of mathematics.

We will also discuss the properties of multiplication such as the commutative property and the distributive property as well as the relationship between the multiplier and the product. Using these patterns, students will construct the multiplication table on their own. Moreover, we intend to deepen students’ understanding by incorporating activities in which students will represent and explain their ideas sing xpressions/equations, ictures, iagrams, nd ords. We want to develop students who can re construct the multiplication table on their own by making use of the patterns of multiplication, not students who simply memorized the multiplication table.

## II Goals of the Unit

○ Students will understand the meaning of multiplication and be able to use it.

[Interest, agerness, nd ttitude]

- Students will recognize the usefulness of multiplication and try to use multiplication to determine the total number of objects.

[Mathematical Way of Thinking]

- Based on the idea of repeated addition, the relationship between the multiplier and the product, the commutative property and other patterns, students can think about and explain ways to construct the multiplication table.

[Mathematical Skills]

- Students will be able to represent situations in which multiplication may be used by using pictures, diagrams, words, or expressions/equations.
- Students will construct the multiplication tables for 5’s, 2’s, 3’s and 4’s, and be able to calculate accurately.

[Knowledge and Understanding]

- Students will know the multiplication table and situations in which multiplication may be used, and they understand the meaning of multiplication.
- Students will understand the properties of multiplication (the relationship between the multiplier and the product, commutative property, distributive property).

**III Relationship between the research theme and this unit**

1. About characters and abilities necessary for learning toward harmonious living

Mathematics is a discipline which is taught systematically. Therefore, it is important that each student develops his or her own questions. Students can then recognize each other's strength from "differences" in their ideas and experience "understanding." We consider this to be the learning toward harmonious living. Moreover, the engine of this process is "questions" students develop. Therefore, we want to help students develop the following creative reasoning ability:

- Ability to think independently and develop own questions in problem solving situations,
- Ability to compare and contrast own idea with those of others in problem solving situations.

In his nit, e tend o rganize he ssons ased n he roperties f ultiplication during the lesson, the phrase, "patterns of multiplication," will be used). We want students to enjoy constructing the multiplication table on their own by using the relationship between the multiplier and the product, the commutative property, or the distributive property, instead of focusing on the memorization of the table. Therefore, we hope to see students with eagerness to find patterns of multiplication themselves. In addition, during the comparing and discussing stage of a lesson, we hope to see students who display their desire to make sense of patterns discovered by other students. The questions relevant to this unit include "What patterns are there?" "Is there another pattern?" and "Which pattern will make the calculation simpler?" Whatever the situation is, we want students to explain he attern f ultiplication hey iscovered. s hey olve heir uestions, e ant tudents o use various ways of representations to support their explanations.

2. Dispositions for learning toward harmonious living

In eaching athematics, I aim for a lesson in which students can develop a series of questions by carefully developing problems. Problems should arise from everyday situations, and by putting them on the mathematical stage, students can develop their own "questions." My goal is for students to experience "understanding" by thinking about their questions and comparing and contrasting their ideas ith ther tudents' eas onsciously nd tentionally. or uch ssons, ssential hat students have the disposition to generate and solve their own questions. For students to generate and solve their own questions, it is necessary that they can think logically. And, to think logically, students must be able to express their ideas clearly.

This unit, from the beginning, will be organized based on students' prior learning. The unit will begin with students realizing the benefit of repeated addition. Then, when students realize the expressions/equations for repeated addition are rather long, we will introduce multiplication expressions/equations. Finally, students will know that if they learn the multiplication table, they can determine the total number of objects without using repeated addition even if we have groups other than 2's, 5's or 10's. The construction of the multiplication table with begins with the 5's facts. As we examine the 5's facts, we will help students notice the patterns in the numerals, which is a characteristic of multiplication, and the relationship to the numerals on clock faces so that they can construct the table for the 5's facts on their own. Once students can construct the table for the 5's facts, students should be able to identify patterns in the 2's facts to construct the table on their own as well.

If he ocus f he nit imply emorizing he multiplication table, it will be difficult for students to experience "understanding." However, by organizing the unit in which students will construct the table on their own, they can feel "Even if I forget the multiplication table, I can find the answer on my own." In this unit, the focus of assessment is students' disposition "to construct the multiplication table using the patterns of multiplication." The assessment question will be, "How will you find the answer if you forgot the answer to 4 × 7?"

**IV. Unit plan Total f 4 ssons: ssons hown elow 6 dditional ssons)**

Sub Unit 1 Multiplication

| # | Goals | Learning Activities | Evaluation |
|---|---|---|---|
| 1 | Students will be able to grasp numbers as "how many in one group" and "how many groups," and try to explain heir eas. **(Today's lesson)** | Make buildings using o tiles, hen think about simple ways to determine the total number of blocks. | Students are trying to use equal groups. (Interest, Eagerness, and Attitude)<br><br>Students can determine the total number of tiles by making use of equal groups and explain their ideas. Mathematical Way of Thinking)<br>[Disposition to build on their prior learning.] |
| 2 | Students will learn that multiplication expressions/ equations will be more concise than repeated addition expressions/expressions. | Based on the repeated addition expressions/equations used to determine the total number of tiles, evelop ultiplication expressions/equations. | Students understand that repeated addition expressions/equations can be written as multiplication expressions/equations. (Knowledge and understanding) |
| 3 | Students will grasp "how many in roup" group ize) nd "how many groups."<br><br>Represent various situations using multiplication expressions/equations. | From pictures, students will grasp "how many in 1 group" and "how many groups."<br><br>From various pictures showing equal groups, students write multiplication expressions/ equations to represent them. | Students can grasp "how many in 1 group" in equal group situations and represent the situations using multiplication expressions/equations. (Mathematical Skills) |

| | | | |
|---|---|---|---|
| 4 & 5 | Students will translate multiplication expressions into concrete situations, then determine the total number by repeated addition. | Students will represent multiplication expressions using counters.<br><br>Students will determine the products by using repeated addition. | Students can represent situations in which multiplication is appropriate calculation using counters and multiplication expressions/ equations. (Mathematical Skills) |
| 6 | Students will understand the meaning of "times as much." | Students will understand that the length ade p f wo cm segments is "2 times as much as 3 cm."<br><br>Students will understand that 3 × 2 is used to determine 2 times as much as 3 cm. | Students understand the meaning of "times as much," and they know multiplication can be used to find the amount so many times as much of the given amount. (Knowledge and Understanding) |
| 7 | Students will find objects in their surroundings that can be represented using multiplication expressions/equations. | Students will identify situations in their surroundings that can be represented using multiplication expressions/equations. | Students can identify situations in their surroundings in which multiplication can be used and explain them using words and expressions/ equations. (Mathematical Way of Thinking) [Disposition to build on their prior learning.] [Disposition to use manipulation of concrete objects and diagrams/ pictures to represent and communicate their ideas to others.] |
| 8 | Summary of the sub unit. | Students will solve the assessment problem. | Students can solve problems using what they have learned. (Knowledge and Understanding) |

Sub Unit 2 Multiplication tables for 5's, 2's, 3's, and 4's (omitted)

**V. Today's lesson**

(1) Date Saturday, June 29, 2013 (9:00 – 9:45)

(2) Location University of Yamanashi Model Elementary School Aogiri Hall

(3) Goal of the lesson

- Students will be able to grasp numbers as "how many in one group" and "how many groups," and try to explain their ideas.

(4) Rationale of the lesson

This lesson is the introduction of multiplication. The goal is for students to realize the usefulness of multiplication. I would like to communicate to the students the usefulness of multiplication, that is, if we know how many in one group and how many groups, we can determine the total number without counting them all.

The lesson will open by reminding students about a Grade 1 lesson, "Let's make buildings." In that lesson, students explored the following task: We are going to make a building using 12 color tiles. Pretend each □ is a room and make different buildings. Once students remembered that we used expressions like 4 + 4 + 4 or 6 + 6 to represent the buildings, today's task will be presented.

Today's task is "Let's make buildings with ○○ tiles." Each group will receive 30 color tiles and they will make different buildings with them. However, students will not be told how many tiles there are. Students can make buildings such as 5 × 6, 3 × 10, or 2 × 15 as shown in Figure 3 without knowing the total number of tiles. Some might make buildings such as those shown in Figure 4. It ill e difficult to represent the arrangements like those in Figure 4 using multiplication, but we will utilize these as non examples of repeated addition situations.

Fig. 3 Anticipated buildings (1)

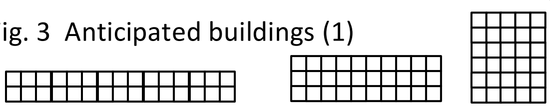
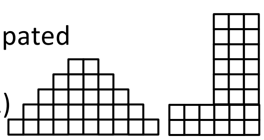
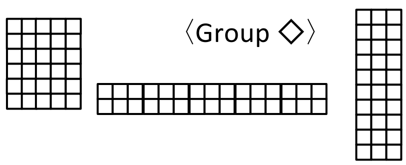


Fig 4 Anticipated buildings (2)



After students make buildings, a follow up task will be given: Let's think about ways to describe your buildings to other groups without actually howing hat ou ade. Each group will discuss how they might describe their buildings. Verbal explanations like the following will give others a good idea of the buildings: "It is a 5 story building and there are 6 rooms in each floor," or "We kept building up 3 rooms at a time." At that point, a new question about the total number of tiles will be posed to the students: "How many color tiles did Group * use?" As we discuss this question, I will try to help students attend to "how many in one group" and "how many groups." By translating verbal explanations into diagrams, or connecting verbal descriptions with mathematical expressions such as "we can use the expression 5 + 5 + 5 + 5 + 5 + 5 to show a 6 story building with 5 rooms on each floor," students will realize that the total number of tiles can be determined without counting them all if we know "how many in one group" and "how many groups." Then, we will introduce the multiplication expression: "5 + 5 + 5 + 5 + 5 + 5 can be written as 5 × 6." I will have students represent other buildings using multiplication expressions, such as 2 × 15 or 3 × 10. It is hoped that students will realize that buildings can be represented by multiplication expressions much more simply than repeated addition expressions.

(5) Flow of the lesson

| Min | Content and Tasks | Instructional onsiderations/Relationship o he research theme |
|---|---|---|
| 7 | 1 Introduction<br>Let's Make buildings.<br>〈Group ○〉 〈Group △〉<br>〈Group ◇〉<br> | ・Students will be told that each group have different numbers of tiles. They are not supposed to count the number of tiles. There are 8 groups in this class.<br>(Each group actually will receive 30 tiles.)<br>・Make sure students understand that all tiles must be used, and adjacent tiles must share a side completely.<br>・Each group should cover up the building so that others cannot see it. |
| 3 | 2 Posing the task<br>Let's think about ways to describe your buildings to other groups without actually showing what you made. | |
| 10 | 3 Individual problem solving<br>〈For Group ○〉<br>C: There are 5 rooms in one floor, and it is story building.<br>〈For Group ◇〉<br>C: It is a 2 story building. There are 15 floors in one floor.<br>〈For Group△〉<br>C: It is tall, and there are 10 groups of 3. | ・During the individual problem solving time, each group will discuss ways to describe the building.<br>・Have students think about a concise way to describe their buildings to others.<br>・They are not supposed to include the total number of tiles in their description.<br>・Post each group's description on the blackboard. |

| | | |
|---|---|---|
| 15 | 4 Comparing and discussing solutions<br><br>How many tiles did each group use?<br><br>C: Let's use diagrams.<br><br>C: Maybe they used the same number of tiles as we did.<br><br>C: If we switch the dimensions, it will be the same as ours.<br><br>C: The description of Group △ was easy to understand.<br><br>C: For group ○, he umber f iles s 0 because 5 + 5 + 5 + 5 + 5 + 5.<br><br>C: It's easier to figure out 5 + 5 + 5 + 5 + 5 + 5 than 6 + 6 + 6 + 6 + 6.<br><br>C: It's easier to count groups of 5's or 10's.<br><br>C: We can tell the total number of tiles easily without counting them all if we have equal groups.<br><br>C: It's too tedious to do 2 + 2 + … + 2. | ・Tell students to record the reasons they knew the shapes of the buildings other groups made during the comparing and discussing solutions stage. As they do so, tell them to use words, pictures, and mathematical representations like expressions.<br><br>・If ny tudent bserves ommonality etween his/her own group's building and another group's building, ask the student to share the observation.<br><br>・Ask students about the total number of tiles so that a repeated addition expression may be brought up.<br><br>・Guide students so that they realize that the total number of tiles can be determined easily without counting them all if we make equal groups, particularly groups of 5 or 10.<br><br>・ Communicate to the students the usefulness of expressions by incorporating the activities to translate from words to expressions or from diagrams to expressions.<br><br>During the process of<br>"learning oward armonious living"<br><br>◎ Abilities to nurture<br>・Ability to investigate the commonalities and differences between their own buildings and those of other groups'.<br>・Ability to think about more efficient ways of counting.<br>◎ Desired students responses<br>・Students can determine the number of tiles by considering equal groups.<br>・Students can translate other's ideas into different representations.<br>・ Students realize that the total number can be determined based by reflecting on various representations.<br>◎ Strategy<br>・Carefully design the learning tasks.<br>◎ Support<br>・Have students repeat other students' ideas.<br>・Encourage students to use mathematical representations used by others. |

| | | |
|---|---|---|
| 10 | 5 Summary of the lesson<br><br>・Students will know that repeated addition expressions can be written as multiplication expressions.<br><br>５＋５＋５＋５＋５＋５＝５×６<br><br>C: We can write multiplication expressions for other repeated addition expressions.<br><br>・Write a journal entry.<br><br>C: It's asy o ount e ake roups of 5 or 10.<br><br>C: It's impler o epresent ith multiplication expressions. | ・Tell students that we will discuss other groups' ideas in he ext sson. |

(6) Assessment points

1. Was the "ability to nurture" today's lesson focused in alignment with the dispositions for learning oward armonious living?
2. Was today's lesson (its organization, the choice of tasks, instructional approach, etc.) effective to nurture characters and abilities necessary for learning toward harmonious living?
3. Through today's lesson, id tudents generate ew eas rom omparing nd ontrasting own ideas with others'"? What were some of the new ideas?
4. Were students appropriately assessed and given appropriate support?

(7) References

1 藤井 斉亮・飯高 茂 ほか 40 名(2011) 「あたらしいさんすう 1」 東京書籍

Translator's Note: This refers to a textbook series. An English translation of this textbook series may be purchased from Global Education Resources (www.globaledresources.com).

June 29
Lesson 2R

University of Yamanashi Model Elementary School

Grade 3 Mathematics Lesson Plan

Division with Remainders: Utilizing Remainders

Teacher: Sayuri Kasai
Learners: 31 Students from Classroom 1
Lesson Location: Aogiri Hall
Discussion Location: Aogiri Hall

## I. About the Unit

A 2012 report by the National Institute of Educational Policy Research examined the 4 year trends on students' erformance n the National Assessment of Academic Ability[i] from 2007 to 2010. One of the areas where successful results have been achieved is the calculation of the four arithmetic operations with whole numbers, decimal numbers and fractions. The average success rate on questions on whole number multiplication and division was 89.3%. The report concludes that mastery of computational fluency is generally good. One of the challenges that still need to be addressed is "understanding the meaning of multiplication and division." This suggests that students can calculate fluently but unable to apply calculation appropriately because they do not understand the meaning of operations well enough. This unit is organized to address this challenge by examining the meaning of remainders and developing the disposition to utilize the idea of remainders in daily lives.

Division is introduced in Grade 3. Mathematically, division is the inverse operation of multiplication. Thus, we can define this way: if b × q = a (or q × b = a), then q = a ÷ b. However, when division is introduced in Grade 3, we study division as a separate operation because we teach calculation through manipulation in concrete situations. For example, we learn that the following 2 situations can both be represented as "12 ÷ 3 = 4": if we give 3 items to each person, 12 items can be shared among 4 people (quotitive division), and if we share 12 items equally among 3 people, each person will receive 4 items (partitive division). The meaning of division studied in this unit remains unchanged. Whether we are calculating division without remainder like 12 ÷ 3 or division with remainders like 14 ÷ 3, the process of making groups of 3 or making 3 equal groups remain the same.

When calculating for quotients, we related division as the inverse operation of multiplication. By connecting the process of making equal groups to the basic multiplication facts, we found the quotient, q, by using the multiplication facts to find the number that satisfies b × q = a (or q × b = a). For division examined in this unit, there is no natural number for q. Generally, division is dealt without considering remainders by using the relationship studied in Grade 5, a ÷ b = a/b (a and b are integers nd ). However, the idea of division with remainders is supported by the following theorem: For any two integers a and b, there is a unique pair of integers q and r such that a = b × q + r and 0 ≤ r < b. Thus, by applying this theorem to a = 14 and b = 3, we know that there is a unique pair of integers, 4 and 2 so that 14 = 3 × 4 + 2. We represent this relationship by "14 ÷ 3 = 4 Rem. 2." Thus, the meaning of 14 ÷ 3 is to find the greatest integer, □, o hat □ is ss han 4, and the remainder.

In eaching his nit, e ant o elp students concretely and visually grasp the numbers corresponding to “group size” and “number of groups,” and distinguish them from “remainder” through activities of making equal groups. At the same time, we will incorporate activities in which students will eason nd xplain heir eas y onnecting anipulation f oncrete bjects ith mathematical expressions and equations.

Furthermore, division we have previously studied (without remainder) can be considered as the special case of the new division (with remainders) where remainder is 0. By integrating their previous knowledge into the new knowledge, it is hoped that students can solidify their understanding of division.

We will also incorporate problems that will require students to interpret the meaning of the remainder. In some problem situations, we cannot have a remainder while in other situations, we can simply ignore remainders. Students will be required to examine the results of the calculation in the context of the original problem situations and derive the appropriate answers to the problems.

## II Goals of the Unit

Students will understand division with remainders. They will also deepen their understanding of division and be able to use division.

- Building on the prior knowledge of division without remainder, students will grasp the meaning of division with remainders and ways to calculate by making use of the relationship between multiplication and division operations and the processes of making equal groups. (Interest, agerness, nd Attitude)
- Students can grasp division by integrating division without remainder and division with remainder. They can represent the meaning of division and the ways of calculation by using concrete objects, diagrams and/or mathematical expressions and equations. (Mathematical Way of Thinking)
- Students can calculate division with remainder – they can determine the quotients and the remainders. (Mathematical Skills)
- Students will deepen their understanding of division by knowing the meaning of the remainder and the relationship between the remainder and the divisor. (Knowledge and Understanding)

## III Relationship between the research theme and this unit

1. About characters and abilities necessary for learning oward armonious living

Mathematics is a discipline hich aught ystematically. Therefore, it is important that each student develops his or her own questions. Students can then recognize each other’s strength from “differences” in their ideas and experience “understanding.” We consider this to be the learning oward armonious living. oreover, he ngine f his rocess questions” tudents develop. Therefore, we want to help students develop the following creative reasoning ability:

- Ability to think independently and develop own questions in roblem olving ituations,
- Ability to compare and contrast own idea with those of others in problem solving situations.

In his nit, tudents hould evelop uestions uch s he ollowing s e rogress hrough the unit. The question at the beginning f he nit , Can e ind he nswer sing ivision e have already learned?" The questions that should arise as students manipulate concrete objects and think about the meaning and methods of division calculation are, "Can we represent the process of manipulation using an equations?" and "Can we represent division with remainders using an equation?" As we explore ways to utilize division in our everyday situations, students should ask questions such as "Can we use division?" and "How can we make use of remainders?"

2. Dispositions for learning toward harmonious living

In eaching athematics, I aim or sson hich tudents an evelop eries f questions by carefully developing problems. Problems should arise from everyday situations, and by putting them on the mathematical stage, students can develop their own "questions." My goal is for students to experience "understanding" by thinking about their questions and comparing and contrasting their ideas with other students' ideas consciously and intentionally. For such lessons, it is ssential hat tudents ave he isposition o enerate nd olve heir wn uestions. or students to generate and solve their own questions, it is necessary that they can think logically. And, to think logically, students must be able to express their ideas clearly.

The emphases in this unit are understanding the meaning of division and appropriately using division to solve problems. Therefore, students will engage in activities of making equal groups and relating manipulation of concrete objects to diagrams and expressions/equations as they did with division without remainder. By engaging in these activities repeatedly, students can understand the meaning of "dividing." Moreover, instead of just calculating rocedurally, e ill corporate problems in which students must interpret the results of calculations. Through those problems, students will develop the habit of examining the meaning of remainders.

As a way to assess students' creative reasoning ability, we will make use of students' notebooks. During this academic year, we have been exploring ways to create notebooks that will clearly reflect the format of teaching through problem solving, "individual problem solving → comparing and discussing solutions → reflection." Students have learned to not only recording their own ideas developed during the individual problem solving time but also recording their classmates' ideas shared during the comparing and discussing solutions stage of the lesson. Moreover, as students record their own and classmates' ideas, they try to record reasons and explanations for an idea using words, pictures, diagrams, and expressions/equations. Finally, by writing a journal entry, students can reflect and organize their reasoning so that they can make use of their own learning in future problem solving. By examining "individual problem solving," "comparing and discussing solutions" and "journal" components of students' notebooks, we can infer hat tudents ere hinking at each stage and how their thinking evolved during the lesson. By assessing how students thinking evolved, we want to examine how well students' creating reasoning ability has developed.

**IV. Unit plan 10 ssons)**

| # | Goals | Learning Activities | Evaluation |
|---|---|---|---|
| (1) Division with remainders (5 lessons) | | | |
| 1 | Students will understand how to calculate division with remainder in the case of single digit divisor and single digit quotient. | • Students will think about ways to calculate 14 ÷ 3. | • Building on the prior knowledge of division without remainder, students are trying to figure out ways to calculate division with remainders. (Interest, agerness, and Attitude)<br>• Students can think about ways to calculate division with remainders based on their prior knowledge of division without remainder and explain using concrete objects, diagrams and/or equation. (Mathematical Way of Thinking)<br>[Disposition to build on their prior learning.]<br>[Disposition to use manipulation of concrete objects and diagrams to represent and communicate their ideas o thers.] |
| 2 | | • Students will learn that 14 ÷ 3 = 4 Rem. 2.<br>• Students will understand the meaning of remainders. | |
| 3 | Students will understand the relationship between the divisor and the remainder. | • Students will investigate the relationship between the divisor and the remainder in the calculations of the form, ☐ ÷ 4. | • Students understand that the remainder will be less than the divisor. (Knowledge and understanding) |
| 4 | Students will understand division with remainder in the case of partitive division. | • After understanding the problem, students know that 16 ÷ 3 is the appropriate calculation and think about ways to calculate. | • Students can think about ways to calculate the quotient and the remainder of partitive division based on their prior knowledge of partitive division without remainder |

| | | | |
|---|---|---|---|
| | | | and explain using concrete objects, diagrams and/or equation. (Mathematical Way of Thinking)<br>[Disposition to build on their prior learning.]<br>[Disposition to use manipulation of concrete objects and diagrams to represent and communicate their ideas o thers.] |
| 5 | Students will understand how to check their calculation in the case of division with remainders. | • Students will think about ways to check their calculation in the case of division with remainders. | • Students understand ways to check their calculation in the case of division with remainders. |
| (2) Problems that requires students to reason about remainders (3 lessons) | | | |
| 1 | Students will deepen their understanding of how to interpret emainders. | • After understanding the problem, students know that 23 ÷ 4 is appropriate and calculate.<br>• Discuss whether or not the answer should be 5 because the results of the calculation is em. .<br>• Summarize that the answer will be Quotient + 1. | • Students understand how to process quotients and remainders appropriately based on problem contexts. (Knowledge and understanding) |
| 2 | | • After understanding the problem, students know that 30 ÷ 4 is appropriate and calculate.<br>• Although the remainder will be 2, discuss whether the answer should be the Quotient or Quotient + 1. | |
| 3 | | • After understanding the problem, students know that 32 ÷ 5 is appropriate and calculate.<br>• Although the results of the calculation is 6 Rem. 2, discuss what should be the answer based on the problem context of making groups. | |

| (3) Summary of the unit (2 lessons) | | | |
|---|---|---|---|
| 1 | Students solve problems by focusing on remainders. (Today's lesson) | • Determine the answer by listing all possibilities.<br>• After understanding the problem, students will determine the appropriate calculation and calculate.<br>• Determine the answer to the problem using the remainder. | • Students can identify patterns from problem contexts and explain their ideas using diagrams and expressions/ equations.<br>[Disposition to build on their prior learning.]<br>[Disposition to use manipulation of concrete objects, expressions/equations and diagrams to represent and to compare and contrast their own ideas with those of others.] |
| 2 | Students will create and solve problems using what they learned his nit. | • Students will create their own problems.<br>• Students will share and solve each other's problems. | Students can solve problems using what they learned in this unit. (Mathematical Skills) |

**V. Today's lesson**

(1) Date Saturday, June 29, 2013 (10:00 – 10:45)

(2) Location University of Yamanashi Model Elementary School Aogiri Hall

(3) Goal of the lesson

- Students will recognize that problems may be solved by focusing on the remainders.

(4) Rationale of the lesson

The problem used in today's lesson does not involve key words/phrases that are often associated with division such as "share," "how many times," or "how many groups." Students must identify atterns rom he roblem situation and represent the identified patterns in a table and mathematical expressions with addition, multiplication and division in order to solve the problem.

In ur lass, very ednesday esignated s he ay e lay ogether s lass. he games used on those days are chosen by the member of the Recreation Committee. The problem situation involves creating a circular chart for those games similar to the one that show various classroom duties that must be rotated. Suppose there are four different ames, dodge ee," "keidro," "dodge ball," and "kohri," nd e lay ne ame eek. e lay dodge ee" n eek , "keidro" in week 2, "dodge ball" in week 3 and "kohri" in week 4. Then, in week 5, we go back to "dodge bee," in week 6, we play "keidro," in week 7, we play "dodge ball," and so on. The question will be to determine which game we will be playing in week 26. The goal is to represent the situation mathematically and find the answer.

The mathematical representation we expect in today's lesson is expressions/equations. One way to find the answer is to list the games we play through week 26 one by one. This method might be a comfortable one for some students because they know that the correct answer will be found for sure. However, s he umber f eeks ecomes arge, t s edious ethod. t hat oint, they will be asked to think about "methods that can be accurate yet simple even when numbers become large." It is hoped that students will notice that the numbers (for a particular game) increases y , nd ome ay ry o se he ultiplication able or 's o ind he nswer.

If tudents ake able, likely hat hey ill otice hat or he ourth ame, kohri," he week numbers will be 4, 8, 12, …, that is, it matches the multiplication table for the 4's facts. Then, they will note that 4 × 6 = 24. Therefore, the game for week 26 should be the 2nd game from "kohri," or "keidro." When students make this observation, they will be encouraged to represent the idea "the 2nd from 24" using mathematical expressions/equations, in particular, 26 = 4 × 6 + 2. It is hoped that students will realize that this equation is the same form as the one we used to check the answers for division calculations. It oped hat his bservation will lead to another "question, " "Can we use division (or the remainder)?"

The week numbers for the 4th game, "kohri," ill e hose umbers hat an e venly divided by 4. Thus, the week numbers for the 1st game, "dodge bee," will have the remainder of 1 when divided by 4. Similarly, the week numbers for the 2nd game, "keidro," will have the remainder of 2 while the week numbers for the 3rd game, "dodge ball,"will have the remainder of 3. 26 ÷ 4 = 6

Rem. 2. Therefore, the game with the week numbers with the remainder of 2 is "keidro." Students should be able to experience the benefit that by focusing on the remainder, they can determine accurately which game will be played in which week no matter what the week number may be. This solution method leads to the idea of generalizability since it can be used "for any number."

In rder o enerate he eries f questions," tudents ill e iven ime o listen o nd make sense of their classmates' ideas. It is nticipated hat ariety f trategies ill e resented during the comparing and discussing solutions stage of the lesson. By having students explain other students' ideas or discussing unique features of their solution strategies, they might notice the differences between their own solution methods and other approaches and identify advantages of other strategies. Moreover, they may be able to incorporate those advantages to their own strategies in the future.

As a way to reflect on lessons, students have been keeping journals. Hopefully, there will be entries that reflect students' recognition of the usefulness of mathematical representations such as "By listing the numbers (in a table), I was able to find the answer," or "I learned that we can find he answer by calculation instead of listing all the numbers." I also hope to see some entries that might suggest the disposition to use mathematics in dealing with everyday problems such as "I understood that the remainders can be useful to solve problems."

(5) Flow of the lesson

| Min | Content and Tasks | Instructional onsiderations/Relationship o the research theme |
|---|---|---|
| 5<br><br><br><br><br><br><br><br><br><br><br><br><br><br>10 | 1 Understanding the problem<br>・ Notice the structure of the Recreation chart.<br>What game will we be playing in week ☐?<br>〇 We play kohri in weeks 8 and 12.<br>〇 The week numbers for kohri are the multiplication table for 4's.<br><br>Let's think about ways to find out which game we play in week 26.<br><br>2. Individual problem solving<br><br>(a) List all week numbers up to 22.<br>dodge bee 1 5 9 13 17 21 25<br>keidro 2 6 10 14 18 22 26<br>dodge ball 3 7 11 15 19 23<br>kohri 4 8 12 16 20 24<br>We will be playing keidro in eek 6.<br><br>(b) Use addition.<br>From the table, I see that numbers for each game is increasing by 4.<br>dodge bee 1 + 4 + 4 + 4 + 4 + 4 + 4 = 25<br>keidro 2 + 4 + 4 + 4 + 4 + 4 + 4 = 26<br>We get 22 with keidro.<br><br>(c) Use multiplication.<br>From the table, I see that the numbers for kohri are the same as the multiplication table for 4. The closest fact to 26 is × 6 = 24.<br>Since 26 is the 2nd from 24, he ame ust e "keidro."<br>If e epresent his n quation, ill e:<br>4 × 6 + 2 = 26. | ・ Display the circular Recreation chart.<br><br>・ By displaying how 4 games are rotated from week to week, help students notice the pattern that a cycle of 4 games is completed in 4 weeks.<br>・ By discussing the week numbers for kohri (4, 8, 12, ...), help students realize that they might be able to use patterns to solve the problem.<br><br>During the process of "learning oward harmonious living"<br>◎ Abilities to nurture<br>・Ability to reason using what they have learned previously.<br>・Ability to express their ideas by making connections among manipulation, expressions/ equations, and diagrams.<br>◎ Desired students responses<br>・Students will list all numbers to find the answer.<br>・Students will write a mathematical expression/ equation from the table and find the answer.<br>・Students will interpret others' ideas from diagrams or expressions/equations.<br>・Students will recognize a new solution strategy after listening to others' ideas.<br>◎ Strategy<br>・Carefully design the learning tasks.<br>・"Questions"<br>"Can we find the answer more quickly and accurately?"<br>"Can we use division?"<br>"Do the numbers for keidro always have the remainder of 2?"<br>◎ Support<br>・Help students think about ways to find the answer.<br>・Organize the blackboard writing so that students can identify expressions/equations from a table.<br>・Give students opportunities to interpret others' ideas. |

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| 25 | (d) Use remainders.<br>26 ÷ 4 = 6 Rem. 2<br>dodge bee 5 ÷ 4 = 1 Rem. 1<br>keidro 6 ÷ 4 = 1 Rem. 2<br>dodge ball 7 ÷ 4 = 1 Rem. 3<br>kohri 8 ÷ 4 = 1 (Rem. 0)<br><br>Of the four games, "remainder 2" will be keidro.<br><br>3. Comparing and discussing the solutions.<br><br>・Make a table showing all numbers up to 22 and find the answer. (a)<br>○ Week 26 will be keidro.<br><br>・ Discuss the solution that used multiplication?<br>"Is there a quicker and accurate way to find the answer?"<br>4 × 6 + 2 = 26 (c)<br>○ + 2 means the 2nd from that game. So, it is eidro.<br>○ If we can use multiplication, we should be able to use division, too.<br>○ This equation looks like what we used to check the answers for division calculations.<br><br>・ Discuss the solution that used remainders.<br>"Can we use division to find the answer?"<br>26 ÷ 4 = 6 Rem. 2 (d)<br>○ The numbers for kohri can be divided evenly. The remainder of 2 means it will be the 2nd from kohri. That will be keidro.<br>○ Do the numbers for keidro always have the remainder of 2?<br><br>・ Verify if the remainder method can be used with other week numbers.<br><br>"Do the numbers for keidro always have the remainder of 2?"<br>22 ÷ 4 = 45 Rem. 2 18 ÷ 4 = 4 Rem. 2<br>14 ÷ 4 = 3 Rem. 2 10 ÷ 4 = 2 Rem. 2<br>6 ÷ 4 = 1 Rem. 2 2 ÷ 4 = 0 Rem. 2<br><br>○ The numbers for keidro always have the remainder of 2.<br>○ Which game will have numbers with the | ・ Start with idea (a).<br><br>・ Listen for the comment, "I think we can find the answer without listing all of the numbers." Lead to the "question," "Is there a quicker and accurate way to find the answer?"<br><br>・ Discuss the interpretations of the equation, 4 × 6 + 2 = 26, nd elp tudents realize that this equation is in the same form as the equations used to check the answers for division calculations.<br><br>・ By verifying the remainder method will work for keidro in weeks other than week 26, ave tudents xperience he relationship, "keidro = remainder 2.".<br><br>・ By focusing on the remainders for other games, help students appreciate the usefulness of using division. |

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| | remainder of 1?<br>◯ If we use remainders, we can find the answer quickly even if the number becomes large. | |
| 5 | 4 Reflecting on the lesson<br>◯ We can figure out which game we will be playing if we make a table.<br>◯ We could use calculation to find the answer instead of writing all the numbers.<br>◯ I arned or he irst ime hat emainders can be useful to solve problems. | • From journal entries assess students' learning nd he ffectiveness f he lesson. |

(6) Assessment points

1. Was the "ability to nurture" today's lesson ocused lignment ith he ispositions or learning oward armonious living?
2. Was today's lesson (its organization, the choice of tasks, instructional approach, etc.) effective to nurture characters and abilities necessary for learning toward harmonious living?
3. Through today's lesson, did students "generate new ideas from comparing and contrasting own ideas with others'"? What were some of the new ideas?
4. Were students appropriately assessed and given appropriate support?

(7) References

（１） 国立教育政策研究所(2012)
「全国学力・学習状況調査の４年間の調査結果から今後の取り組みが期待される内容のまとめ　～児童生徒への学習指導の改善・充実に向けて～　小学校編」

Translator's Note: This is the report written by the National Institute of Educational Policy Research. It is vailable only apanese) rom http://www.nier.go.jp/4nenmatome/.

（２） 文部科学省（平成 14 年８月）
「個に応じた指導に関する指導資料　～発展的な学習や補充的な学習の推進～（小学校算数編）」
（３） 中村享史（1991）国土社
「算数　考える力をのばす教材」

[i] Since FY2007, the Ministry of Education, Culture, Sports, Science and Technology (MEXT) has carried out the National Assessment of Academic Ability in mathematics and Japanese for students in the sixth year of elementary school and the third year of lower secondary school. According to the National Institute of Educational Policy Research, "the assessment seeks to ascertain and analyze the academic abilities and learning atterns f choolchildren hroughout apan nd o vestigate he utcomes of educational policies and programs, identify issues requiring attention, and achieve improvements therein."

# Grade 3 Mathematics Lesson Plan

Monday, July 1, 2013, Period 5
Matsuzawa Elementary School
Grade 3, Classroom 1
Teacher: Sachiko Kawabata

1. Unit: Division

2. Goals of the unit

- Students will understand the meaning of division and be able to use it.
- Students will be able to represent division situations in expressions/equations and interpret division expressions/equations.

3. Evaluation standards

| Interest, Eagerness, and Attitude | Mathematical Way of Thinking | Mathematical Skills | Knowledge and understanding |
|---|---|---|---|
| • Students are thinking about the meaning and ways of division calculations by relating them to multiplication and subtraction. | • Students are thinking about ways to calculate division problems with 1-digit divisors and 2-digit quotients. | • Students can fluently calculate division with 1-digit divisors and 1-digit quotients. | • Students understand the meaning of division for both partitive and quotative cases.<br>• Students undertand how division relates to multiplication and subtraction. |
| • Students are interested in interpreting and representing situations using division expressions/ equations, and they are trying to represent various situations in expressions/ equations. | • Students can think about division situations using tools such as concrete objects and diagrams, and they can represent them using division expressions/ equations.<br>• Students can relate division expressions/ equations to concrete situations. | • Students can represent division situations using expressions/ equations, and they can interpret division expressions/ equations. | • Students understand the relationships among quantities in situations where division is appropriate by interpreting or representing with expressions/ equations. |

4. About mathematics in the unit

(1) Partitive and quotative division

Most of situations in which division is used can be divided into the following two cases.

In one case, division is used to determine how many of one quantity equals the second quantity. This type of division is called quotative division. In the other, division is used to determine how many in one group when a quantity is made into so many equal groups. This type of division is called quatotive division.

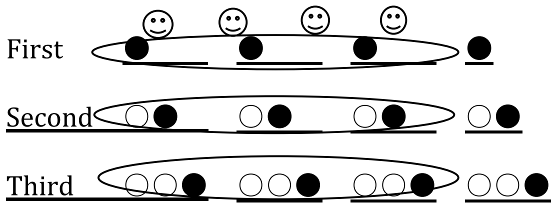
The idea of partitive division, "sharing equally among ○ people," is common in students' everyday situations, and many students understand how to distribute items equally through their experiences. However, it is difficult to represent those situations in diagrams, and diagrams are often not easy to interpret. Therefore, it is necessary to provide careful explanations and to have students manipulate concrete objects so that they understand how the amount was equally grouped.

The idea of quotative division, "if each person receives ○ items, how many people can share the given amount," is not as commonly encountered in everyday life. Therefore, it is anticipated that not many students have everyday experience of this type. However, the idea of sharing by giving each person ○ items is easier to explain with diagrams. Moreover, when situations are represented with concrete objects, the process of "removing ○ items" repeatedly may be easier to grasp visually.

Comparing partitive and quotative division, it may be better to teach quotative division first because the concrete representation/manipulation is easier to understand. Moreover, the action involved matches the idea of "removing."[1] Moreover, if we start with quotative division, we can explain partitive division situations using the repeated subtraction idea by focusing on "the group of items when one item is given to ○ people." (See Figure 1)

Figure 1 (Partitive division situation)

4 people are sharing 12 candies. How many candies will each person get?




Each time, 4 candies are removed.

(12 − 4 − 4 − 4 = 0)

(2) Relationships between multiplication and division

[1] The kanji character used for the formal term, division, is the same character as the kanji character used for "to remove."

Division can be considered as the inverse operation of multiplication. It is possible for students to clearly understand the meaning of partitive and quotative division by representing division situations using multiplication equations and noticing whether we are determining the multiplicand or the multiplier. (See Figures 2 and 3)

In this unit, it is important for students to understand the meaning of division equations by relating them to corresponding multiplication equations by using words and diagrams. For example, students should understand that "the divisor is the multiplicand or the multiplier of the corresponding multiplication equation."

The distinction between division to find the group size and division to find the number of groups is also important in students' future learning. For example, in Grade 5 discussion of "division of decimal numbers" and Grade 6 discussion of "division of fractions," this distinction is critical as students determine the appropriate expression based on the word problems or their representations on number lines.

Figure 2

partitive…division to find group size

12 ÷ 4 = 3

(Total) ÷ (# of groups) = (group size)

↓

□ × 4 = 12

(group size) × (# of groups) = (Total)

quotative…division to find number of groups

12 ÷ 4 = 3

(Total) ÷ (group size) = (# of groups)

↓

4 × □ = 12

(group size) × (# of groups) = (Total)

Figure 3

Understanding of "division as the inverse of multiplication" → In A × B = C, if either A or B is unknown, it can be determined by C ÷ B or C ÷ A.

group size × # of groups = Total

A × B = C

? = C ÷ B (Partitive)

? = C ÷ A (Quotative)

(3) Teaching of this unit

As stated in (1), we felt that introducing division using the quotative situations may be easier for students. However, if we treat partitive division as the same operation as quotative division, the distinction between "division to find group size" and "division to find the number of groups" discussed in (2) may become unclear.

Therefore, in this research lesson, we decided to focus on making the distinction between partitive and quoatitive division based on the relationship between division and multiplication. We will discuss partitive division first, then quotative division.

5. Students' current state

There are many students who eagerly tackle mathematics lessons and try to reason independently in problem solving. On the other hand, there is a wide range in the levels of understanding and mastery of basic ideas. There are some students who require individual support during mathematics lessons.

In a recent class survey, students were asked "When do you feel the enjoyment in a math lesson?" Large percents of the students agreed with the following statements: "When I score well on a test" (94%), "When I understand something I didn't understand before" (69%), and "When teacher or friends understood my idea" (69%). On the other hand, much fewer percents of the students agreed with the following statements: "When I am writing my ideas in notebook," (29 %), "When I am sharing my idea," (31 %), "When I'm doing calculation," (34 %), and "When I am working on application problems" (34 %). From this survey, we can see that the students in this class enjoy and feel satisfaction with mathematics when they understand something or do well on a test. Moreover, although some students do not feel good about writing or sharing their own ideas, they feel good when their ideas are understood by the teacher or their classmates.

The results of the readiness test for the unit, there are several students who have not completely mastered the basic multiplication facts. In particular, 7 × 6, 6 × 7, 7 × 4, 4 × 7, 6 × 8, and 8 × 6 were missed by several students. In addition, there are some students who do not completely understand the meaning of multiplication. Those students wrote multiplication expressions/equations with the multiplicand the multiplier reversed. They also drew pictures of 3 × 4 for the expression 4 × 3.[2]

Therefore, in this unit, the meaning of "divide" will be carefully taught by utilizing concrete materials and drawings. In addition, as we have been emphasizing throughout this year, we will incorporate activities in which students will represent their own ideas using words, expressions/equations, or diagrams and explaining their ideas to their classmates. Through those activities, we want students to realize the enjoyment in reasoning. We will also pay close attention to the understanding problem stage of the lesson so that students can tackle problem solving with clear strategies. We will also make sure to provide appropriate individual support during the independent problem solving time and secure the sufficient amount of time for whole class discussion. Through these efforts we want all students to experience success.

6. Instructional strategies

[2] In Japan, the multiplicand is written first, which is the reverse of the order indicated in the Common Core State Standards. Therefore, 4 × 3 in Japan means 3 groups of 4.

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| Through writing division problems and comparing those problems, students will understand that there are two types of division: division to find group size (partitive division), and division to find the number of groups (quotative division). |

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| The relationship between multiplication and division will be carefully taught. |

○ Ideas for unit structure

Today's lesson was set up after the meaning and calculation of partitive division (2 hours) and the meaning and calculation of quotative division (2 hours). We use the same numbers in both partitive and quotative division problems discussed in lessons 1 through 4 so that comparison of diagrams and equations will be easier.

○ Manipulation of concrete materials and representations using diagrams

While teaching the meaning of division, we will incorporate manipulation of concrete objects so that students can develop the solid image of "equal sharing" and "making groups of equal size." In addition, we hope to deepen students' understanding by representing manipulation of concrete objects in diagrams or expressions/equations.

○ Making students conscious of the differences of two types of division through problem writing

In today's lesson, we will show a picture of 8 strawberries, and students will be asked to create problems for 8 ÷ 2. As we discuss students' problems, we will compare and contrast "division to find group size" and "division to find the number of groups" so that students can think about the differences of the meaning and unknown quantities in these types of division.

In addition, we want to help students realize that the quotients can be determined by using multiplication, and the difference is what quantity is to be found.

Finally, we will ask students to write problems for 10 ÷ 2 so that we can assess whether or not students understand the distinction between partitive and quotative division.

7. Scope and sequence of relevant topics

**Grade 2** → **Grade 3** → **Grade 4**

**Grade 2**

- Meaning of multiplication
- Multiplication table

[10. Multiplication 1]

[11. Multiplication 2]

[12. Multiplication 3]

**Grade 3**

- Multiplication with 0, properties
- Multiplication of multiples of 10 and 100
- Multiplication algorithm, mental calculation

[2. Multiplication]

↓ [5. Multiplication algorithm]

- Meaning of division
- Division that is the inverse of the basic multiplication, division with 1 and 0
- Division with 1-digit divisors and 2-digit quotients.

↓

- Division with remainders

↓ [7. Division with remainders]

- (2-, 3-digit) × (2-digit)

[14. Multiplication of 2-digit numbers]

**Grade 4**

- Division algorithm
- Dividend = divisor × quotient + remainder
- (1- ~ 3-digit) ÷ (1-digit)

[2. Division]

↓ [5. Division: 1-digit divisors]

- Division by 2-digit numbers
- Properties of multiplication and

↓ [7. Division: 2-digit divisors]

- (decimal) ÷ (whole)
- (whole) ÷ (whole) = (decimal)

[16 Decimal Multiplication and Division]

**Grade 5**

[5. Decimal Division]

**Grade 6**

[4. Division of Fractions]

In Grade 2, students first learned about the meaning of multiplication and constructed the basic multiplication table. Then, they tried to master the basic multiplication. By the end of Grade 2, about 90% of students were able to recite the basic multiplication table fluently. As we studied the multiplication algorithm in Grade 3, we checked students' mastery level of the basic facts, and some students have forgotten or confused some facts. In addition, we learned that some students were having difficulty with division in Grade 4 because their mastery of the basic multiplication facts was unsatisfactory.

In this unit, we want students to be able to write appropriate expressions/equations by relating multiplication and division and making clear the type of the unknown quantity. What students study in this unit will relate to other topics studied in later grades.

In Grade 4 unit of "Division: 1-digit divisors," there is a task titled, "What calculation do we need?". In that task, students will examine word problems and represent them on numbers lines to identify how quantities are related and whether the appropriate calculations are multiplication or division. Students must determine which of the quantities in the equation, [group size] × [# of groups] = [Total], needs to be found. In Grade 5 unit of "Decimal Division" and Grade 6 unit of "Division of Fractions," the reasoning such as "because division is inverse of multiplication, we can do ÷ decimal or ÷ fraction," or "since we are finding group size, the calculation is partitive division" play an important role.

8. Unit and evaluation plan

| Sub-Unit | Lesson # | Content | Evaluation Standards | | | |
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| | | | Interest, Eagerness, and Attitude | Mathematical Way of Thinking | Mathematical Skills | Knowledge and understanding |
| Division | 1 | • Discuss the situation in which "12 pieces of chocolate are shared among 4 people."<br>• Understand the meaning of partitive division as the process of determining the number of pieces for one person when "12 pieces of chocolate are equally shared among 4 people" through manipulation of concrete objects.<br>• Understand how to represent situations using division expressions/equations. | • Students are trying to figure out the meaning of partitive division by representing the problem situations using concrete objects or diagrams. | • Students are reasoning about division situations by representing the problem situations using concrete objects or diagrams. | | |

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| | 2 | • Think about ways to find the quotient for 24 ÷ 6 in a partitive situation.<br>• Think about the relationship between the quotient and the multiplicand (☐ × 6 = 24).<br>• Find the quotients using the basic multiplication facts. | | • Students can relate partitive division situations with multiplication. Through diagrams and manipulation of concrete materials, students can think about ways to find the quotients using the basic multiplication facts. | | • Students understand how division relate to multiplication and subtraction. |
| | 3 | • Understand the meaning of quotative division as the process of finding the number of people who will get cookies when "each person receives 4 cookies and there are 12 cookies" through manipulation of concrete objects or representing problem situations in diagrams.<br>• Understand how to represent situations using division expressions/equations. | | • Students are thinking about division situations using diagrams and concrete objects. | | • Students understand that quotative situations can also be represented by division expressions. |

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| | 4 | • Think about ways to find the quotient for 24 ÷ 6 in a quotative situation.<br><br>• Think about the relationship between the quotient and the multiplicand (☐ × 6 = 24).<br><br>• Find the quotients using the basic multiplication facts. | | • Relating division situations to multiplication, students can explain how quotative divisions can be calculated by using the basic multiplication facts. | | • Students understand that quotative situations can also be represented by division expressions. |
| | 5 | • While looking at pictures of 8 strawberries, make problems for which 8 ÷ 2 is the appropriate calculation.<br><br>• Think about and share the differences between two types of problems (partitive and quotative).<br><br>• Summarize the two types of division.<br><br>• Make two types of division problems for which 10 ÷ 2 is appropriate calculation. | | • Students are thinking about the differences of the meaning of the two types of division problems. | | • Students understand the differences of the two types of division. |

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| | 6 | • Write division word problems and solve each other's problems. | | | • Students can represent division situations using expressions, and they can interpret division expressions. | • Students understand the meaning of division in both partitive and quotative cases. |
| | 7 | • Practice division calculations.<br>• Create division books. | • Students are trying to identify situations in their daily life where division may be used. They are enjoying the activity of making division books. | | • Students calculate the quotients when both the divisors and the quotients are 1-digit numbers. | |
| Division with 0 & 1 | 8 | • Think about the expressions and answers for sharing 12, 4, and 0 cookies among 4 people equally.<br>• Summarize division problems with the quotients of 1 and 0. | | • Students can make sense of division with the quotients of 1 or 0 based on their previous learning. They can think about ways to calculate such division. | | |

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| Using patterns of calculation | 9 | • Think about ways to calculate 36 ÷ 3 using properties and rules of calculations.<br>• Look for patterns among expressions with the constant multiplier or divisor. | | • Students are thinking about ways to find the quotient using diagrams or the relationship between multiplication and division. | | |
| | 10 | • Think about ways to calculate 80 ÷ 4.<br>• Learn about the division algorithm. | | • Students are thinking about ways to find the answers by drawing diagrams or using what they have previously learned about multiplication and division. | | |
| | 11 | • Apply division in everyday situations and think about problems. | | • Students can connect division equations to concrete situations. | • Students can write and calculate division expressions based on diagrams. | |

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| Review | 12 | • Deepen students' understanding. | | | • Students can fluently calculate division with 1-digit divisors and 1-digit quotients. | |
| Exercises | 13 | • Re-affirm what students learned in the unit. | | | | • Students understand the relationships of quantities in situations in which division is used. |

9 Goal of the lesson

- By comparing and contrasting partitive and quotative division problems, students will understand that there are two types of division: division to find the group size and division to find the number of groups.

10 Flow of the lesson (Lesson # 5/13)

| | ■ Learning Activity<br>(T) *Hatsumon* (question)<br>(C) Anticipated responses | ⦿ Points of consideration<br>□ Evaluation<br>❖ Teaching strategy |
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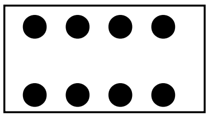
| Posing the problem/Plan strategies | ■ Understanding the problem and planning solution strategies<br>T1 Today, we will be making division word problems.<br><br>T2 There are 8 strawberries. We want to share these strawberries so that it will match the expression, 8 ÷ 2. How can we share these strawberries?<br><br><br>C1 2 people share strawberries.<br>C2 Put 8 strawberries on 2 plates.<br>C3 Make groups of 2 with 8 strawberries.<br><br>T4 OK, let's write word problems that can be solved by 8 ÷ 2. Draw a picture or diagram for the problem situation. Also, write an equation and the answer, too. | ⦿ By showing a picture of strawberries, help students understand the problem situation more easily.<br><br>⦿ By discussing ways to share and some key words to be included in problems, help students develop solution strategies.<br><br>⦿ To make it easier to compare and contrast problems, clearly mark spaces for the problem, diagram, equation, and answer on the worksheet. |

<table>
<tr>
<td rowspan="2">Individual Problem Solving</td>
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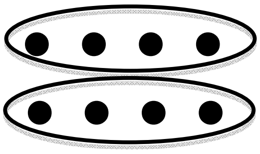
A: Division to find the group size (partitive division)

2 people are sharing 8 strawberries. How many strawberries does each person get?

(^o^)
(^o^)



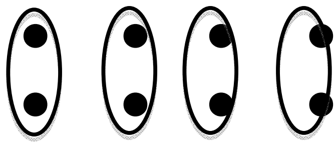
Equation 8 ÷ 2 = 4

Answer 4 strawberries

B: Division to find the number of groups (quotative division)

We are going to give 2 strawberries to each person. If there are 8 strawberries, how many people will get strawberries?

(^o^) (^o^) (^o^) (^o^)



Equation 8 ÷ 2 = 4

Answer 4 people

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⦿ Make sure each students will write one problem.

❖ Set up a "Hint Time" for students who are having trouble. Remind them the sharing strategies discussed earlier in the lesson and different division word problems we have studied previously so that they might be able to write their own word problems.<br>
❖ For those students who completed the task, encourage them to write a second word problem. Also, ask them how they found the answer and write their steps down.

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■ Compare and contrast 2 problems

T5 Let's look at the problems Students A and B. Which of these two problems is the next problem look like?

C8 It looks like Problem A.<br>
T6 Why did you think so?<br>
C9 Because they both "share equally between 2 people."<br>
C10 If you look at the diagrams, they both have made groups of 2.

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⦿ Select several partitive and quotative problems and have students compare and contrast.

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|---|---|---|
| | T7 Now, let's look at the problems you wrote. Is your problem like Problem A or Problem B? Please put your name plate under the problem.<br><br>T8 What is similar between the two problems, A and B? What is different? Let's share what you noticed.<br><br>C11 They both have 8 ÷ 2.<br>C12 The answers are both 4.<br>C13 Even though they both calculate 8 ÷ 2, the "2" in A is "2 people," and in B, it is "2 strawberries." So, it means something different.<br>C14 If you look at the diagram, the way ● are circled are different. In A, there are groups of 4, and in B, there are groups of 2.<br>C15 The answers are both 4, but A's answer is 4 "strawberries" and B's answer is 4 "people." So, the units are different.<br><br>T9 So, why are the answers, "4 strawberries" and "4 people," different even though the problems have the same equation?<br><br>C16 Problems A and B are asking for different things. In A, "how many does each person get," and in B, "how many people will get strawberries."<br><br>T10 OK, so the quotients are both 4, but the meanings are different. How did you find the answer, 4?<br><br>C17 I used multiplication.<br>C18 2 × 4.<br><br>T11 Do A and B both use 2 × 4?<br><br>C19 For A, it's 4 × 2. In multiplication, "(group size) × (number of groups) = (total)." In A, the group size is 4 (4 strawberries each) and the number of group is 2 (peole). | ⦿ By having students identify their own problems with A or B, help students attend to the problem situations.<br><br>□ Mathematical Way of Thinking<br>Students can compare 2 problems and think about the difference in the meaning of the types of division.<br><br>❖ Have students think about the difference between "sharing between 2 people" and "making groups of 2" from the diagrams and word problems.<br><br>❖ Encourage students to think about ways to explain the differences of the problem so that others can understand it easily.<br><br>⦿ If the relationship of multiplication and division or the meaning of multiplication, "(group size) × (number of groups) = (total)" are not raised by students, ask questions to help students think about these ideas. |

C20 For B, it is 2 × 4, because the group size is 2 (2 strawberries each) and there are 4 (people).

T12 OK, let's look at Problems A and B once again.

| A (division to find how many in one) | B (division to find how many groups) |
|---|---|
| 8 ÷ 2 = [   ] | 8 ÷ 2 = [   ] |
| [   ] × 2 = 8 | 2 × [   ] = 8 |
| group size / # of groups / Total | group size / # of groups / Total |

From the same expression 8 ÷ 2, we could write two different types of division word problems. In Problem A, what is the division calculation finding?

⦿ For Problems A and B, use the labels students invent to make it easier to distinguish the two types.

C** Problem A is finding "how many in one."

T** How about B?

C** Problem B is finding "how many groups."

T** OK, please write a journal on what you learned today.

So, there are 2 different types of division problems, aren't there?

| If ○ × Δ = □, we can use division to find |
|---|

| | | |
|---|---|---|
| | ○ (group size), OR<br>Δ (number of groups).<br><br>■ Solve the application problem<br>T13 OK, now let's write word problems for which we use 10 ÷ 2 to solve. This time, please write two types of problems. You don't have to use strawberries for things to be shared. Use your own ideas. For each of the two types of the problems, write your problem, the equation, and the answer. If you finish, also draw diagrams. | ⊙ By using the application problem, evaluate students' understanding.<br>(Compare what students wrote initially and what they write here to capture changes in students' thinking.)<br><br>□ Knowledge and understanding<br>Students understand the difference of the meaning of the two types of division problems. |

11 Evaluation

- Do students understand there are two types of division based on the meaning of multiplication, "(group size) × (number of groups) = (total)": division to find the "group size," and division to find the "number of groups."

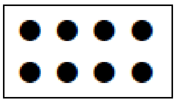
12 Board writing plan

7／1

There are 8 strawberries

Way to share that matches

**8 ÷ 2**



・2 people share 8 strawberries

・Make groups of 2 strawberries with 8 strawberries.

Goal

Let's write word problems that can be solved by 8 ÷ 2. Write an equation and solve the problems.

**Problem A**

**Problem B**

…
8÷2=[]
[]×2=8
…

…
8÷2=[]
2×[]=8
…

What is different

**Different**

•4 strawberries and 4 people

•Shared between 2 people, or give 2 strawberries to each

•(diagram) groups of 4 or 4 grous

•(what we have to find)
How many for 1 person? OR
How many people will get strawberries?

**Summary**

Division may be used
* to find "how many in 1," OR
* to find how many groups.

# Grade 8, Mathematics Lesson Plan

1. Date & Time: 6th period, Tuesday, July 2, 2013
2. Theme: Explaining with Algebraic Expressions
3. Instructor: Tokyo Gakugei University Affiliated Koganei Junior High School, Hideyuki Kawamura
4. Class: Tokyo Gakugei University Koganei Junior High School, Grade 8, Class D (40 Students)
5. Place: Educational Technology room
6. Name of the Unit: Calculations with Algebraic Expressions
7. About the Theme of This Lesson

The instructional material that I will be providing to the students in this lesson requires the students to describe a statement that is a revers statement of usual statements for determining numbers if they are multiples or not.[1] The objective for dealing with this material is providing an opportunity for the students to have experiences for generalizing and specializing by interpreting transformation process of algebraic expressions and grasping the meaning of them clearly.

About the process of utilizing algebraic expressions, Miwa (1896) describes it using the following diagram (see figure 1).

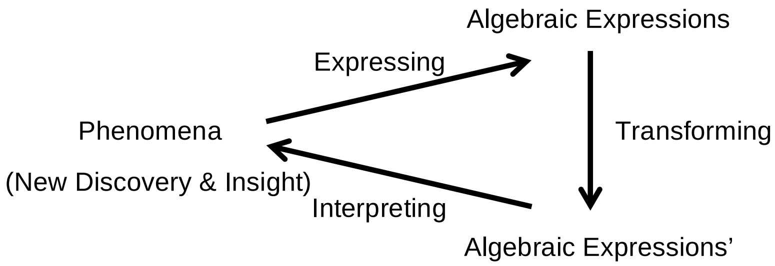



Figure 1: Diagram for Use of Algebraic Expressions (Miwa, 1996)

The diagram shows, when a algebraic expression is utilized, the processes of "expressing," "transforming" and "interpreting" take in the place. In the process of interpreting the algebraic expressions, generalization and specialization can be conducted by reexamining and grasping the meaning of the algebraic expressions. (Miwa, 2001) In other word, we can think that the act of interpreting an algebra expression facilitates an opportunity to create new mathematics. I thought if students could experience this process through a lesson, they would use algebraic expressions more actively and try to interpret them more willingly.

In the article written by Miwa (2001), an example of a process of interpreting algebraic expression for generalization and specialization is discussed by providing an example regarding how to distinguish multiples.

The discussion of the transformation process of the algebraic expression for distinguishing multiples of 9 (If $a + b$ of the 2-digit numbers $10a + b$ are multiples of 9, the numbers are multiple of 9.) describes that $10a + b$

---

[1] Translator's note: Usually the statement for determining a 2-digit number is a multiple of 9 or not is written as "A 2-digit number $10a + b$ is a multiple of 9 when $a + b$ is a multiple of 9." So the reverse statement that the instructor explaining here is "If a 2-digit number $10a + b$ is a multiple of 9, $a + b$ is also a multiple of 9."

can be split into the part shows the multiples of 9 (9*a*) and the remainder part (*a* + *b*). By generalizing this idea, students could create methods for distinguishing multiples of other numbers.

In this lesson, I decided to ask students to think about the reverse of this problem that is “If a 2-digt number is a multiple of 9, (a number in the tens place) + (a number in the ones place) is also a multiple of 9.” I have two reasons for setting up the problem like this way.

The first reason is when the algebraic expression $10a + b = 9n$ was established, because of the goal of the transformation of the algebra expression is $a + b = 9(n - a)$, it might be easier for the students to recognize the process of subtracting 9*a* from the both side of the equation.

The second reason is in the case of distinguishing if a number is the multiple 9 or not, students need to explain a reason why a number is a multiple of 9 or not by determining if the sum of the numbers in each place is a multiple of 9 or not. Therefore, many students would try to examine if the statement is valid or not by substituting the algebra expression with actual numbers that are the multiple of 9. If students examine the statement this way, students might proof this problem using a wrong reasoning that is because the original number is a multiple of 9 so the statement is valid. If this is the case, I thought it might be easier for the students think about the explanation if we make the supposition to “the 2-digit number is the multiple of 9.” In other word, I thought the statement can be reversed and provide it to the students to work on. In this way, it might be easier for the students to explain and understand the logical story of generalizing idea by interpreting the algebraic expressions.

## ８. Goal of the Lesson

・ Students generalize the statement by interpreting the process of explanation using algebraic expressions and grasp the mechanism of the expression.

＜References＞

三輪辰郎 (1996). 文字式の指導：序説，筑波数学教育研究 15，pp.1-14.
三輪辰郎 (2001). 文字式の指導に関する重要な諸問題，筑波数学教育研究 20，pp.23-38.

9. Flow of the Lesson

| Learning Process | Instruction | Anticipated Student Reactions | ・Instructional Pints to Remember<br>〇Evaluation Viewpoint |
|---|---|---|---|
| 1.<br>Introduction | ・The last lesson I asked you to explain "If a 2-digt number is a multiple of 9, (a digit in the tens place) + (a digit in the ones place) is also a multiple of 9." Do you remember how you explained about that?<br><br>・Is there any part of the statement that you might want to change? | ・ If we assign the numbers in the tens place as *a* and in the ones place as *b*, the original number can be expressed as $10a + b$. This number is a multiple of 9 so:<br>$10a + b = 9n$<br>$9a + a + b = 9n$<br>$a + b = 9n - 9a$<br>$a + b = 9(n - a)$<br>From this algebraic equation, (a digit in the tens place) + (a digit in the ones place) is a multiple of 9.<br><br>・ 2-digit number → Increase the number of digits.<br>・ Multiple of 9 → Multiple of other numbers. | 〇 Students are able to understand the explanation with algebraic expressions that they worked on in the previous lesson.<br><br>・Ask the students to show all the steps involve for transformation of algebraic equations.<br><br>〇 Students are eager to think about what part of the statement can be change. |

| | | | |
|---|---|---|---|
| 2.<br>Expansion | ・If we change the digits to 3-digit, what should we do to the part, (a digit in the tens place) + (a digit in the ones place)?<br>・If we change it to (a digit in the hundred place) + (digits less than and equal to the tens place), how should we change the algebraic equation.<br><br>・If the statement says multiple of 7 instead of multiple of 9, what do we need to do?<br>"If a 2-digt number is a <u>multiple of 7</u>, (a digit in the hundred place) + (digits less than and equal to the tens place) is also a multiple of 9."<br>Is there any part of the statement you need to change?<br><br>・Let's think about the statement "If a 2-digt number is a multiple of 7, (a digit in the hundred place) + (digits less than and equal to the tens place) is also a multiple of 7." | ・(a digit in the hundreds place) + (a digit in the tens place) + (a digit in the ones place), (a digit in the hundreds place) + (digits less than and equal to the tens place), (digits higher than and equal to the tens place) + (a digit in the ones place).<br>・If we assign the numbers in the hundreds place as *a* and in less than and equal to the tens place as *b*:<br>100*a* + b = 9n<br>99*a* + *a* + *b* = 9*n*<br>*a* + *b* = 9*n* – 99*a*<br>*a* + *b* = 9 (*n* – 11*a*)<br><br>・Multiple of 9 → multiple of 7 | ○ Students are thinking enthusiastically about the part that they could change in the statement.<br><br>・Make sure to write the statements side by side so that the students could understand clearly about what were changed and what were not changes. |

| | | | |
|---|---|---|---|
| | • What kind of ideas did you use to do the transformation of the algebraic equation? | • If we assign the numbers in the hundreds place as *a* and in less than and equal to the tens place as *b*: $100a + b = 7n$ $98a + 2a + b = 7n$ $2a + b = 7n - 98a$ $2a + b = 7(n - 14a)$ <br>• The method (a digit in the hundreds place) + (digits lesson than and equal to the tens place) does not work well. <br>• We need to change it to (a digit in the hundreds place) × 2 + (digits less than and equal to the tens place). <br>• For the multiples of 9, 100 is split into 99 and 1. <br>• 99 is a multiple of 9 so it that we can factor 9 out. 1 is what is left. | ○ By recalling and utilizing how the algebraic equation was transformed in the case of multiple of 9, students are trying to think about the case of multiple of 7. <br>• By interpreting the explanations using algebra equations carefully, help students understand the necessity of transformation of the equation to reach to the conclusion of this problem solving. <br>• Help students to become conscious about they are thinking about the case of multiple of 7 based on the case they worked on multiple of 9. |
| 3. Summary | What are the commonalities of these three transformations of algebraic equations? | • Leaving *b* at the left side of equal sign as it is. <br>• Splitting 10 and 100 into the number that is the multiple of 9 or 7 and the number that is left. <br>• Transform the right side of the equations into something like 7×(…) or 9×(…). | • By asking the students identify the commonality among the algebraic equations, help students to pay attention to the part of the structure of the algebraic equations that have not changed although the statements were changed. |

# Grade 6 Mathematics Lesson Plan

## Who is the fastest?（Speed）

Wed. July 3, 2013, 5th eriod
Megurol ku Sugekari Elementary School Grade 6 (Class 1) 35 Students Instructor: Koko orita

**Research Theme: "I did it! I understand it!" Designing lessons that students become absorbed.**
**Devising instructions that care about students' questions and provide experiences for students to enjoy thinking and expressing.**

## １． Name of the Unit: Speed

## ２． Goals of the Unit

○ Students are able to understand the meaning of speed, how to express it, and how to find it.
○ Students are able to understand the relationship of three quantities: speed, time, and distance.

## ３． Evaluation of the Unit

| | |
|---|---|
| Interest, Motivation, & Disposition | Students are applying idea of per unit quantities when they are finding speed. Also they are eager to apply speed in their study and daily lives. |
| Mathematical Reasoning | Students are utilizing idea of per unit quantities when they are finding speed. |
| Skills & Procedures | Students are able to find speed, distance, and time, based on the idea of per unit quantities. |
| Knowledge & Understanding | Students are able to understand how to express speed that is based on the idea of per unit quantities. |

## ４． Structure of the Unit

### （１） Goals

The main goals of the unit is that the student to understand speed is expressed using the distance traveled in a unit mount of time, and them to be able to compare speeds and finding distance or time based on the study of "per unit quantity" that students learned in grade 5.

In the sub unit entitled "1. Speed," a sprint race, which is familiar situation to the students, will be used to introduce this topic. Comparisons of speed in the cases of "different travel distance in the same period of time" and "different travel time for the same distance " are easier to carry out, owever, he ase ith "the different travel distances in different period of time" is very difficult to determine. Therefore, students need to think about how they can compare speed in this case by thinking about making "time" or "distance" the same. Through this exploration of thinking, help students understand the meaning of speed as the distance traveled in a unit amount of time. In addition, the students will learn various ways to express speed (e.g., distance per hour, istance er minute, distance per second) and the relationship among these expressed speeds. Finally, the students will learn about how to solve problems that require them to find distance or time.

In the sub unit entitled "2. Speed and graphs," students will be representing the relationship of time and travel distance of the cases of walking and bicycling in a table and on a graph, and discuss and interpret them.

**（２） Students' state of learning**

Many students in this class are serious about learning. They can think carefully and participate enthusiastically in the class. For example, they can hold their own ideas and opinions, explain ideas to their friends willingly, listen to friends' ideas while comparing them with their own ideas, and describe the similarities and differences of ideas that are presented. However, there is a large ability difference among the students. Some students have difficulty explaining their ideas logically in the class and understanding their friends' deas. If I am not careful, a lesson could be progressed with participation of only a handful of students.

Because of this reason, I would like to think about better ways to pose problems, se instructional materials, develop flow of lessons, nd support individual students to match their needs. I would like to apply these ideas to my lessons and help to bring out students' uestions nd nterests. Moreover, I would like to provide problem solving approach of instruction to the students and promote their active participation to the lessons. Lastly, I would like to support student learning carefully so that each student in the class could understand the importance of thinking on their own logically and expressing their ideas.

**（３） Students' bilities hat oster n he nit**

Speed is related with time and distance. It can be compared by travel distance per unit time or amount of time per unit distance. These ideas for comparing are as same as the idea of unit per quantity (crowdedness) that students have learned before.

Students might have some ideas about speed from their experiences in their daily lives intuitively. However, I will help the students to understand clearly that speed can be express with the relationship between time and distance through engaging them to a mathematical activity. Amount of time is a quantity that is not easily see or feel. This point is different from other kinds of quantities. Also, distance (length) can be obtained by the result of movement of an object in the continuous time. The travel distance per 1 second and the amount of time per 1m that students will learn in this unit are "an average travel distance" and "an average speed." In other words, "speed" is "an average speed" that is thought non constant speed as an ideal constant speed by theorizing it as "if object is moving as a constant speed."

By helping students to understand the difference between "average speed" and real speed, develop students' awareness that is they are learning about speed by applying the idea of per unit quantity that students have learned before. Clear understanding of speed in this unit becomes a base for future learning such as idea of acceleration speed, instant speed, movement, etc.

## ５． Relationship and Expansion of Content

【Grade 】

Multiplication and Division of Decimal Numbers

- (Decimal Number)×(Whole Number)
- (Decimal Number)÷(Whole Number)

↓

【Grade 】

Per Unit Quantity

- Per unit quantity
- Average

Multiplication of Decimal Numbers

- (Decimal Number)×(Whole Number)
- (Decimal Number)×(Decimal Number)

Division of Decimal Numbers

- (Decimal Number)÷(Whole Number)
- (Decimal Number)÷(Decimal Number)

Multiplication and Division of Fractions

- (Fraction)×(Whole Number)
- (Fraction)÷(Whole Number)

↓

【Grade 】

Multiplication of Fractions

- (Whole Number)×(Fraction)
- (Fraction)×(Fraction)

Division of Fractions

- (Whole Number)÷(Fraction)
- (Fraction)÷(Fraction)

Speed

- Meaning of speed, how to find it (per hour, per minute, per second)

**6. Unit Plan (A total of 8 lessons)**

| Sub-Unit | Lesson | Learning Activities | Evaluation View P | | | |
|---|---|---|---|---|---|---|
| | | | IMD | MR | SP | NU |
| 1. Speed | 1 | ● Think about speed in a sprint race.<br>● Think about what quantities are related to speed.<br>● Think about how to compare speed. | ◎ | ◎ | | |
| | 2 | ● Understand how to find speed.<br>● Understand distance per hour, distance per minute, and distance per second, and compare speed using formula for speed. | | ○ | | ◎ |
| | 3 | ● Understand the relationship among distance per hour, distance per minute, and distance per second; and learn how to find them. | | ○ | ◎ | ○ |
| | 4 | ● Investigate how many seconds a person takes to walk 50m, and find the speed for distance per hour, distance per minute, and distance per second. | ○ | | ◎ | |
| | 5 | ● Investigate how distance change when the time becomes twice or three times as much, and think about how to find distance.<br>● Think about how to find time when speed and distance are known from the math sentence for finding distance. | | ○ | ◎ | |
| 2. Speed & Graphs | 6 | ● Representing the relationship between time and distance using a table and a graph. | | | ◎ | |
| Practice | 7・8 | ● Check the understanding of content student have learned. | | | ◎ | ◎ |
| | | ● Find speed and distance from a graph. | | ◎ | | |

* The first lesson of the 8 lessons is this research lesson.

** IMS (Interest, Motivation, Disposition), MR (Mathematical Reasoning), SP (Skills & Procedures), NU (Knowledge & Understanding)

## 7. Instruction of This Lesson

### （１） Goals of this lesson

• Students are able to think about how to compare the speed of running and explain about it by paying attention to the two quantities involve, the distance a person run and the amount of time a person run.
• Students are able to recognize the merit for finding per unit quantity and utilizing it willingly.

### （２） Rational of this lesson

The task of this lesson is for the students to think about how to compare the running speed of two children when both the distance and the time they run are different. The comparison can be done mainly using the following four methods:

① Finding a common multiple of distance to compare
② Finding a common multiple of time to compare
③ Finding amount of time per 1m to compare
④ Finding distance per 1 second to compare

All these methods are correct because when times or distances of two children are made the same, the students can compare them. However, this lesson does not end at this point. The goal of the student discussion of this lesson is for them to search "more effective ideas."

By helping students to have viewpoints such as "concise," "clear," "accurate," and "general," uide student to search "more effective ideas." Then help them to understand the meaning of speed, that is "speed is expressed using the distance traveled in a unit amount of time" with their agreement.

### （3） Concrete measures for achieving the research theme

**① Wrestle with the problem and bring out students' questions --- (grasp)**

First, show the students only the three students' records of time of a sprint race (insufficient information), and ask them "who is the fastest?" The students might say, "the smallest value is the fastest." But at the same time, other students might say, "we can't tell who is the fastest because we don't know if they run the same distance." By pick up such students' voices, bring out idea such as to compare the running speed of these children, the distance (travel distance) need to be set the same. Then provide information of the distance of those children run and help them to recognize that speed of those children can be compared when either the distance or the time they run is made the same. However the problem the students dealing with in this lesson tells two children from the three children run differently both the distance and time, so it is not easy to compare the speed of the three children. At this instance, students would come up with a question, "when both the time and distance of running are different, how can we compare them?" In this lesson, using the question that are rose from the students and use it to establish a main problem, elieve t ould help the students to engage in autonomous learning through problem solving.

**② Through solving students' own question, help students to feel the joy of solving the problem on their own (pursue)**

In problem solving where students think about problem on their own, helps students to think about how to resolve the problem by utilizing the clue that is if either the distance or the time they run is made the same we can compare the speed of the children. For the students who are having difficulty coming up with a solution idea, provide a small group instruction. Discuss with them by providing hints in question format such as "Why can't we compare the speed of A and B?" and "I wonder if we could make one of them (time or distance)," and help them to find a clue to solve the problem. Support students in the way that they can feel joy of learning such as "I did it! I understand it!"

**③ Through activity involves interpreting friend's ideas, students answer to the question and deepen their thinking (deepen and rise)**

First, look at the math sentences of student solution methods that utilize common multiple to make the distances or the times the same. Then help the students to recognize that "when both distance and time are different, if one of them is made the same, they can be compared."

Next, bring up the math sentences of student solution methods that incorporate "per 1m" and "per 1 second" ideas, and discuss and interpret these ideas. This time also help the students to understand "when both the distance and time are different, if one of them is made the same, they can be compared." In addition, help students to recognize that "the former methods used idea of common multiple and the latter methods used idea of per unit quantity to make one of the quantities the same."

After all four methods are discussed, confirm that all the methods are in fact correct, and all of them used idea of "making one of the quantities the same." Then ask the students "If you need to find out the order of speed of the following six children what method do you want to use?" The common multiple method is easier to understand but the calculation is complicated and the method is not appropriate when the number of children that you need to compare increased. Thus, the discussion becomes focused on which one of the methods, "per 1m" and "per 1 second," is better. In the case of "per 1m," the smallest value obtained from the calculation is the fastest. In the case of "per 1 second" the largest value obtained from the calculation is the fastest. In our daily life, in general, "the distance run per 1 second" is used. By recognizing the strength of both methods and thinking about how these ideas are used in our daily life, help students to understand that "speed is expressed using the distance traveled in a unit amount of time."

**④ Through activities such as summarizing, utilizing, and expanding, bring out students' new questions. (Summarize, expand)**

By helping students to reflect on the process of learning of this lesson along with their thinking process, ask students to summarize what they have learned in their words by having viewpoints such as "I see!," "discovery," "question," and "challenge."

## 8. Flow of the Lesson (1/8)

| | Learning content (Main *hatumon* and students' anticipated responses) | ☆Measures to achieve the research theme<br>〇Support and points to remember ◎Evaluation |
|---|---|---|
| Grasp | | |

**1. Grasp the goal of the learning.**

Problem: Let's think about the order of speed of these 3 children.

| | Time (second) |
|---|---|
| A | 6 |
| B | 6 |
| C | 5 |

T Order the speed of a sprint race of 3 children from the fastest.
C C is the 1st place, and A and B are the 2nd place
C We can not tell only with this information.
T Why do you think so?
C Because we do not know how far they run.
C We can not compare the speed if we don't know the amount of time and the distance they run.

T The distance they run were like this.

| | Distance (m) | Time (Second) |
|---|---|---|
| A | 40 | 6 |
| B | 30 | 6 |
| C | 30 | 5 |

C C's time is short but the distance C run is also short so we can't tell if C is the fastest.
C But we can tell that C is faster than B.
T Is that true? Why can 〇〇 tell about it by just looking at it?
C Because B and C run the same distance but B is taking longer time to run.
C I think so, too.
T So we can say that C is faster than B.
C Yes, I can tell more. A is faster than B.
C A and B run the same time but A run longer distance, so we can say A is faster.
T Well, we have not compare A and B, who is faster?
C …

☆ Pose the problem with an insufficient information (providing only the time), and bring out students' questions.

〇When we think about distance in the context of a sprint race of the sports day, the use of distance is more common. So in this lesson, we use distance instead of travel distance to pose the problem.

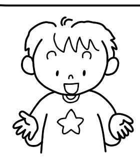
We cannot compare speed using only with the time.

〇Open the table and show the information of distance.

Both the distance and the time are different for A and B. What can we do?

〇By comparing 2 persons at a time, help all students to understand the main question of the lesson.

We might be able to compare them if I make one of them the same.



| | | |
|---|---|---|
| | T　Why can' t you tell about it?<br>C　Because both the distance and the time are different.<br>C　Well, we can compare them if we make them the same.<br>C　We can compare them if we make the same.<br>C　If we make one of them the same we can compare.<br>T　If you make the same…? Can you compare them?<br>C　Yes, we can! Yes, we can! | ◎Students are clear about the main question and trying to solve the problem enthusiastically. 【Interest, motivation, disposition】 |
| Pursue | **2. Comparing Speed of A and B by making one of them the same.**<br><br>⑤ Finding common multiple of distance to compare<br>The smallest common multiple of 30 and 40 →１２０<br>A　１２０÷４０＝３　　６×３＝１８<br>C　１２０÷３０＝４　　５×４＝２０<br>A is faster.<br>⑥ Finding common multiple of time to compare<br>The smallest common multiple of 6 and 5→３０<br>A　３０÷６＝５　　４０×５＝２００<br>C　３０÷５＝６　　３０×６＝１８０<br>A is faster.<br>③ Finding distance run per 1 second to compare<br>A　４０÷６＝６．６６６６６・・・<br>C　３０÷５＝６<br>A is faster.<br>④ Finding time per 1m to compare<br>A　６÷４０＝０．１５<br>C　５÷３０＝０．１６６６６６・・・<br>A is faster.<br>⑤　I don' t know | ◎ Students are able to come up with ideas to compare speeds by making one of time of distance the same and write their thinking in their notebooks.<br>【Mathematical Reasoning】<br><br>〇Ask students who are having difficulty to come to in front of the blackboard for small group discussion. By providing hints in question sentence format, help them to notice the method ③.<br><br>【Examples of hints】<br>・C can run 30m for 5 seconds. If he runs for 1 second, how many m can he run?<br>・How about the case of A?<br>・I wander what we can do to find distance a person can run in 1 second. |

| | | |
|---|---|---|
| Deepen & Rise | **3. Students present their solutions and understand them**<br>Show the math sentences of the method ①.<br>T　Look at ○○' s math sentences and think about what he/she was thinking.<br>C　I think ○○ used the least common multiple to make those distance the same.<br>C　If both of them run for 120m, A' s time would be 18 seconds and C' s time would be 20 seconds. So A is faster.<br>C　I used the common multiple but my method is a little different.<br>T　What is that mean? Who used the method with the common multiple but different from what presented.<br>C　Yes, I also used the method with the least common multiple. But I made the time the same.<br>T　Can you see the method of ○○s' on the blackboard?<br>C　Yes, that is the method ②.<br>C　If both A and B run for 30 seconds, A would run 200m and C would run 180m. A is faster because he/she would run farther.<br><br>Show the math sentences of the method ③.<br>T　Someone came up with these math sentences.<br>40÷6＝6．666・・・<br>30÷5＝6<br>Do you understand what this person was thinking? Talk with your partner and describe what you think.<br>T　If you your explanation was similar to your partner' s, please rise your hands.<br>T　If you think your partners' explanations were easy to understand, please rise your hands.<br>C　Yes.<br>T　Okay, _____. Please explain the meaning of these math sentences.<br>C　Yes, I think this person was thinking to find how far each person run in 1 second.<br>C　For 1 second, A run 6.66666… and C run 6m. So A is faster.<br>T　Why can you say A is faster? Does everybody understand?<br>C　Because the time is the same so we can compare with the distance.<br>T　What do you mean?<br>C　Because you can say someone is faster if the person could run farther in 1 second, so we can say A is faster.<br>T　I see. | ☆ Show only the math sentences and ask students to think about what the person was thinking. Facilitate an active discussion to promote students' participation.<br><br>☆ Ask students to talk with their partners in order to increase each individual student' s opportunity to explain their thinking.<br><br>○Ask students to write their solution ideas on the portable whiteboards.<br><br>◎Students are thinking about the solution methods by recalling how they have compared the crowdedness of rooms in grade 5. 【Mathematical reasoning】<br><br>☆ Ask students to talk with their partners in order to increase each individual student' s opportunity to explain their thinking.<br><br>☆By asking *hatumon* to students' responses, help each individual student to understand the method clearly. |

| Deepen & Rise | | |
|---|---|---|
| | C　I used the ides of per unit quantity.　But my math sentences are different.<br>T　What are your math sentences.<br>C　④６÷４０＝０．１５<br>　５÷３０＝０．１６６・・・・・<br>C　Wow, this one also uses per unit quantity.<br>T　Do you mean they are the same method?<br>C　Well, they are reversed.<br>T　They are reversed… Do you understand what is the meaning of reversed, everyone?<br>C　Yes, I understand. It is the reverse of the method ③.<br>T　What do you mean by the reverse od the method ③? Talk with your partner and describe what you think. If your partner does not understand it please give her/him a hint.<br>C　The method ③ is finding the distance run per 1 second to compare but the method ④ is finding the time took to run for 1m to compare.<br>C　Both methods use the idea of per unit quantity. But one uses 1 second as the unit quantity and the other one uses 1m as the unit quantity. So that is different.<br>C　When we think about the comparison with "per 1 second," we can say that the greater value means faster just like the method ③. On the other hand, when we think about the comparison with "per 1m," we can say that the smaller value means faster.<br>T　I see, we came up with 4 different methods. What is the similarity among these methods?<br>C　All methods use the idea of making either time or distance the same to compare.<br>T　The important idea is making either time or distance the same.　So which methods used the idea of making time the same?<br>C　The methods ② and ③.<br>T　How about the methods used the idea of making distance the same?<br>C　The methods ① and ④. | ☆ Listen to students' mumbles carefully and proceed the discussion according to student thinking and their pace of thought process.<br><br>☆By asking *hatumon* to students' responses, help each individual student to understand the method clearly. |

Deepen & Rise

T Okay, let' s say 3 more children run. So we want to decide the order of places of these 6 children. What method do you want to use to determine? Please place your magnet name cards to one of the methods you wan to use on the blackboard.

| | Distance (m) | Time (second) | Place |
|---|---|---|---|
| A | 40 | 6 | |
| B | 30 | 6 | |
| C | 30 | 5 | |
| D | 35 | 5.5 | |
| E | 45 | 6.5 | |
| F | 50 | 8 | |

T Please describe why you chose a particular method.
C I thought it is complicated to find the least common multiples for this.
C Distance÷Time can be used every time and easier to understand (method ③).
C Time÷Distance can be also used every time and easier to understand.
C But it is little difficult to say a person is faster when the value is smaller so even though the method ③ and ④ are similar, I chose the method ③.

〇By asking students to place their magnet name cards, each person need to hold his/her own opinion.

〇It is not necessary for students to actuary do the calculation to find the order of places but ask students to just think about what method might be the best one to use. Through this discussion, help students to think about the idea of how to express speed such as, distance per second, distance per minute, and distant per hour.

**Deepen & Rise**

T For example, have you seen the writing showing the speed of the bullet trains? Which one is faster?

A

Super Komachi 300km per hour

B

Hayabusa 320km per hour

C Hayabusa is faster.
C I think the larger value (number) means faster.
C I wonder if "per hour" means how far the train can run in 1 hour.
T Both methods ③ and ④ can be used to compare the speed but when we show a larger value (number) for indicating faster speed, it is easier to imagine so in general, we show the speed of something using distance per time (e.g., 1 second, 1 hour).
T In the next lesson, let's compare the speed of these 6 children and other objects that use expression of speed such as distance per hour.

I have heard about the speed of the bullet trains. We also say the speed of cars something

**Summarize & Expand**

### 4． Summarize the learning

T Today, we have thought about ways to compare speed. When both distance and time are different, we learned that we can make one of them the same to compare. We came up with these four methods to compare. ① and ② use the idea of the least common multiple to make one of them the same. ③ and ④ use the idea of per unit quantity such as per 1 second and per 1m to compare. All the methods are similar because they are making something the same such as "distance a person run" and "time a person run."

T However, when you have many things to compare all at once, using ideas of "per unit quantity" is the best. In addition, in general, it is easier for people to have image of a larger value means faster. So the method ③ is usually used for expressing speed. However, both ideas using the least common multiple and per unit quantity can be used to solve this problem. So we can say that both methods are correct.

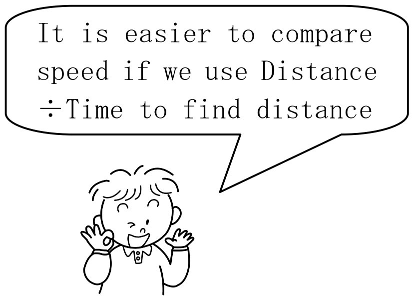
T At last, let' s write down the points for comparing speed and the easy ways to compare based on what we learned from today' s lesson.

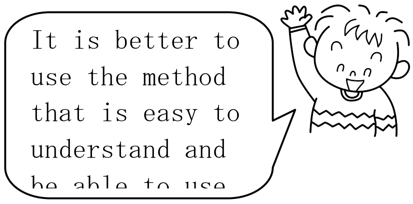
C "I See!" --- We can compare speed if we make time or distance the same.

C "Discovery" --- I think the idea of using the least common multiple is a good idea but when we compere many things all at once the method is not easy to do. So I would like to use the idea of distance per 1 second.

C "Question" --- We talked about "distance per hour" but for the case of the sprint race, I wonder id we can call it as "distance per second." Is there "distant per minute" also?

C "Challenge" I want to compare speed of cars, trains, airplanes. I also want to compare speed of many other things.




◎Students are thinking about new questions and try to solve problems willingly. 【Interest, motivation, disposition】

## 9. Blackboard Plan

Problem

**Let's think about the order of speed of these 3 children!**

① Make distance the same using common multiple to compare

The least common multiple of 30 and 40 → 120

A -- 120  40 = 3  6 x 3 = 18

(Shorter time means fast)

C -- 120  30 = 4  5 x 4 = 20

| | Distance (m) | Time (second) |
|---|---|---|
| A | 40 | 6 |
| B | 30 | 6 |
| C | 30 | 5 |

Easy to

A is faster.

② Make time the same using common multiple to compare

The least common multiple of 6 and 5 → 30

A -- 30  6 = 5  40 x 5 = 200

(Longer distance means fast)

C -- 30  5 = 6  30 x 6 = 180

| | Distance (m) | Time (second) |
|---|---|---|
| A | 40 | 6 |
| B | 30 | 6 |
| C | 30 | 5 |

Easy to

A is faster.

③ Find distance traveled in 1 second to compare

A -- 40  6 = 6.666…

(Longer distance means fast)

C -- 30  5 = 6

**Distance ÷ Time = Distance per 1 second**

| | Distance (m) | Time (second) |
|---|---|---|
| A | 40 | 6 |
| B | 30 | 6 |
| C | 30 | 5 |

Be able to use the numbers as they are for the

A is faster.

④ Find time took in 1m to compare

A - 6  40 = 0.15

(Shorter time means fast)

C -- 5  30 = 0.166666…

**Time ÷ Distance = Time per 1m**

| | Distance (m) | Time (second) |
|---|---|---|
| A | 40 | 6 |
| B | 30 | 6 |
| C | 30 | 5 |

Be able to use the numbers as they are for the

A is faster.

## １０. Viewpoints for looking at the lesson

① When the students are solving the problem on their own and discussing solution methods, did the teaching methods, such as providing hints with question sentence format, asking students to explain each other with partner, and asking *hatumon* to students' responses help the students to deepen their thinking as well as engaging them learning autonomously.

② Did the student discussion help to achieve the goal of discussion, "search more effective ideas."

③ Other points that observes noticed during the lesson. (e.g., How main goal/task was established during the lesson?).

## 2.

1. Please enter the unique participant ID number emailed to you.

## 3.

2.
Please describe your experiences with lesson study to date, including:

a. Number of years you have been involved in lesson study;

b. Content area (e.g., math, English/ language arts) of lessons you have experienced;

c. Number of times you have observed and participated in lesson study;

d. Whether these experiences were within your home country or in another country.

## 4.

3. What do you think are the strengths/ benefits of using lesson study in your local context(s) (e.g., district, school, university setting)?

## 5.

4. What do you think are the challenges to using lesson study in your local context(s)?

## 6.

**5. Please describe how your current organizational contexts use lesson study for educational improvement.**

## 7.

**6. Please describe how you hope to use lesson study for educational improvement in your current organizational contexts after this trip.**

## 8.

**7. Please select and rank in order of importance the five items from the previous question that you believe will be most professionally useful for you within the next year. Please remember to rank only 5 items.**

| | 1st Most Useful | 2nd Most Useful | 3rd Most Useful | 4th Most Useful | 5th Most Useful |
|---|---|---|---|---|---|
| a. Mathematics content | | | | | |
| b. How to build students' problem solving ability | | | | | |
| c. Evaluating a lesson on the basis of a written lesson plan | | | | | |
| d. How lesson study is conducted in another country | | | | | |
| e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.) | | | | | |
| f. Collecting data on student thinking to inform instruction | | | | | |
| g. Strategies for making students' thinking visible | | | | | |
| h. Analyzing/studying curriculum materials | | | | | |
| i. Ways to build connections among educators at multiple levels of the education system | | | | | |
| j. Anticipating student responses | | | | | |
| k. Writing a useful lesson plan | | | | | |
| l. Supporting participants to have powerful and effective lesson study experiences | | | | | |
| m. . Organizational/structural supports for lesson study | | | | | |
| n. Students' mathematical reasoning | | | | | |
| o. Differentiating/ offering support for struggling learners | | | | | |
| p. Cultural influences on mathematics teaching and learning | | | | | |
| q. Organizing a successful post-lesson debriefing session | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| r. A typical school day at a Japanese elementary school | | | | | |
| s. Developing mathematics units and lessons | | | | | |
| t. Strategies for working effectively in a lesson study group | | | | | |
| u. My own country's approaches to mathematics instruction | | | | | |
| v. Analyzing written student work/ responses | | | | | |
| w. Analyzing and interpreting verbal student comments | | | | | |
| x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards) | | | | | |
| y. How to build a classroom learning community | | | | | |

## 9.

**8. Four teachers were discussing the way they believe mathematics is learned by students. To their surprise, no two of them agreed on the principal way mathematics is learned, although each suggested that intellectual processes were necessary.**

**You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about the way mathematics is learned. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.**

**MARY: "To learn mathematics, students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."**
**SUSAN: "The most important thing is reasoning. If students can reason logically and can see how one mathematical idea relates to another, they will understand what is taught."**
**BARBARA: "The primary thought process in learning mathematics is memory. Once students have the facts and rules memorized, everything else falls into place."**
**DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures–right or wrong–and discover things for themselves, they will understand the mathematics and how it is used."**

**Please write about your view of how students learn mathematics.**

## 10.

**9. Four teachers were discussing the role of problem solving in students' learning of mathematics. To their surprise, no two of them agreed on the role of problem solving in mathematics learning.**

**You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about problem solving in mathematics. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.**

**MARY: "Problem solving is like any other skill in mathematics. Students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."**

**SUSAN: "The most important thing in problem solving is to develop logical reasoning. Problem solving helps students learn to reason logically and can see how one mathematical idea relates to another. Thus it helps them understand mathematics."**
**BARBARA: "Students should first master the prerequisite facts and skills of mathematics before they are assigned problem solving. Problem solving should emphasize the application of these facts and skills to real life situations."**
**DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures–right or wrong–and discover things for themselves, they will understand the mathematics and how it is used."**

**Please write about your view of the role of problem solving in students' learning of mathematics.**

## 11.

**10. Please indicate how well each of the following statements describes your current attitude. (Circle ONE for each statement.)**

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| a. I enjoy learning about mathematics. | | | | | | |
| b. I have learned a lot about student thinking by working with colleagues. | | | | | | |
| c. I have strong knowledge of the mathematical content taught at my grade level. | | | | | | |
| d. I have good opportunities to learn about the mathematics taught at different grade levels. | | | | | | |
| e. I think of myself as a researcher in the classroom. | | | | | | |
| f. I have learned a great deal about mathematics teaching from colleagues. | | | | | | |
| g. I am always curious about student thinking. | | | | | | |
| h. By trying a different teaching method, teachers can significantly affect a student's achievement. | | | | | | |
| i. I am interested in the mathematics taught at many grade levels. | | | | | | |
| j. I would like to learn more about the mathematical content taught at my grade level. | | | | | | |
| k. Working on mathematics tasks with colleagues is often unpleasant. | | | | | | |
| l. I find it useful to solve mathematics problems with colleagues. | | | | | | |
| m. Japanese mathematics teaching approaches are not likely to be useful outside of Japan. | | | | | | |

## 12.

**11. Please indicate your current position (Check ALL that apply.)**

## 13.

**12. How many years of teaching experience do you have?**

## 14.

**13. Please list any grades to which you have ever taught mathematics.**

## 15.

**14. Please add any comments or feedback you have about this survey.**

# 2013 Japan Trip - Post Survey

## 2.

**1. Please enter the unique participant ID number emailed to you.**

## 3.

**2. After the trip, what do you now think are the strengths/ benefits of using lesson study in your local context(s) (e.g., district, school, university setting)?**

## 4.

**3. What do you think are the challenges to using lesson study in your local context(s)?**

## 5.

**4. Please describe how you hope to use lesson study for educational improvement in your current organizational contexts after this trip.**

## 6.

**5. How much did you learn about each of the following during the immersion trip to Japan?**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| a. Mathematics content | | | | | |
| b. How to build students' problem solving | | | | | |
| c. Evaluating a lesson on the basis of a written lesson plan | | | | | |
| d. How lesson study is conducted in another country | | | | | |
| e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.) | | | | | |
| f. Collecting data on student thinking to inform instruction | | | | | |
| g. Strategies for making students' thinking visible | | | | | |
| h. Analyzing/studying curriculum materials | | | | | |
| i. Ways to build connections among educators at multiple levels of the education system | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| j. Anticipating student responses | | | | | |
| k. Writing a useful lesson plan | | | | | |
| l. Supporting participants to have powerful and effective lesson study experiences | | | | | |
| m. Organizational/structural supports for lesson study | | | | | |
| n. Students' mathematical reasoning | | | | | |
| o. Differentiating/ offering support for struggling learners | | | | | |
| p. Cultural influences on mathematics teaching and learning | | | | | |
| q. Organizing a successful post-lesson debriefing session | | | | | |
| r. A typical school day at a Japanese elementary school | | | | | |
| s. Developing mathematics units and lessons | | | | | |
| t. Strategies for working effectively in a lesson study group | | | | | |
| u. My own country's approaches to mathematics instruction | | | | | |
| v. Analyzing written student work/ responses | | | | | |
| w. Analyzing and interpreting verbal student comments | | | | | |
| x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards) | | | | | |
| y. How to build a classroom learning community | | | | | |

## 7.

**6. Please <u>select and rank in order of importance</u> the <u>five items</u> from the previous question that you believe will be most professionally useful for you within the next year. Please remember to rank <u>only 5 items</u>.**

| | 1st Most Useful | 2nd Most Useful | 3rd Most Useful | 4th Most Useful | 5th Most Useful |
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| | | | | | |
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| w. Analyzing and interpreting verbal student comments | | | | | |
| x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards) | | | | | |
| y. How to build a classroom learning community | | | | | |

**7. If you remember the items you chose before the trip, comment on any changes made to your responses after the trip. Why do you now consider some knowledge more or less useful than before the trip?**

## 8.

**8. Please review the following three problems, and provide your ratings again after the trip.**

**Four teachers were discussing the way they believe mathematics is learned by students. To their surprise, no two of them agreed on the principal way mathematics is learned, although each suggested that intellectual processes were necessary.**

**You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about the way mathematics is learned. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.**

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| c. I have strong knowledge of the mathematical content taught at my grade level. | | | | | | |
| d. I have good opportunities to learn about the mathematics taught at different grade levels. | | | | | | |
| e. I think of myself as a researcher in the classroom. | | | | | | |
| f. I have learned a great deal about mathematics teaching from colleagues. | | | | | | |
| g. I am always curious about student thinking. | | | | | | |
| h. By trying a different teaching method, teachers can significantly affect a student's achievement. | | | | | | |
| i. I am interested in the mathematics taught at many grade levels. | | | | | | |
| j. I would like to learn more about the mathematical content taught at my grade level. | | | | | | |
| | | | | | | |

| | | | | | | |
|---|---|---|---|---|---|---|
| k. Working on mathematics tasks with colleagues is often unpleasant. | | | | | | |
| l. I find it useful to solve mathematics problems with colleagues. | | | | | | |
| m. . Japanese mathematics teaching approaches are not likely to be useful outside of Japan. | | | | | | |

## 11.

**11. Please select the research lesson and post-lesson discussion that you feel was most professionally informative for you.**

**12. Please explain why you selected this lesson and post-lesson discussion. What about the lesson and post-lesson discussion was informative for you?**

## 12. Copy of

**13. Please select the research lesson and post-lesson discussion that you feel was least professionally informative for you.**

**14. Please explain why you selected this lesson and post-lesson discussion as the least professionally informative for you. What specifically was missing?**

## 13. New Page

**15. Please comment on the number of lessons you observed during the program.**

**16. Please indicate the degree to which you agree or disagree with the following statement: There were too many items on the itinerary, and as a result, the program felt too busy.**

**17. Other Comments:**

## 14.

**18. What changes to the trip itinerary might have helped to deepen your own learning about lesson study and mathematics teaching and learning?**

## 15. New Page

**19. Please indicate your level of satisfaction with the following aspects of the program:**

| | 1 | 2 | 3 | 4 | 5 | NA |
|---|---|---|---|---|---|---|
| Accommodations (Hotel Mets) | | | | | | |
| Meals (Hotel Mets) | | | | | | |
| Accommodations (Hotel Fuji) | | | | | | |
| Meals (Hotel Fuji) | | | | | | |
| Communication with program staff prior to arrival | | | | | | |
| Communication with program staff during the program | | | | | | |

**20. Other comments:**

## 16.

**21. Please add any comments or feedback you have about this survey.**