IMPULS Lesson Study Immersion Program
2012
Overview Report

March 2013

"IMPULS Lesson Study Immersion Program 2012
Overview Report"
January 2013

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1 Preface

Project IMPLUS is a newly established project funded by the Ministry of Education, Culture, Sports, Science & Technology of Japan since 2011. The Project is housed in the Mathematics Education Department of Tokyo Gakugei University, Tokyo, Japan. The director of the project is Professor Toshiakira FUJII, and the project members include all the faculty members of the mathematics education department—Professors Koichi NAKAMURA, Shinya OHTA, and Keiichi NISHIMURA. Dr. Akihiko TAKAHASHI of DePaul University joined the project as a specially appointed professor. Ms. Naoko MATSUDA (KATSUMATA) also joined the project as a project staff member.

The purpose of the project is two-fold. First, as an international center of Lesson Study in mathematics, Tokyo Gakugei University and its network of laboratory schools will help teacher professionals from throughout the region learn about lesson study and will thereby prepare them to create lesson study systems in their own countries for long-term, independent educational improvement in mathematics teaching. Second, the project will conduct several research projects examining the mechanism of Japanese lesson study in order to maximize its impact on the schools in Japan.

Under these main purpose, we are working for ;

1) ***Research*** on Japanese Lesson Study to come up with ideas for establishing innovative teacher education systems for long-term, independent educational improvement in teaching mathematics.
2) ***Professional development*** to disseminate ideas for establishing innovative teacher education systems for long-term, independent educational improvement in mathematics teaching. Workshops and institutes would examine how to implement ideas for Lesson Study and innovative ideas for professional development in various schools with different systems and cultural back ground in order to prepare them to create in their own countries' systems for long-term, independent educational improvement in teaching mathematics.
3) Facilitate opportunities for researchers, administrators, and practicing school professionals throughout the region to ***exchange their ideas*** to improve their education systems for teaching mathematics.

The IMPULS lesson study immersion program was designed to give mathematics education researchers and practitioners from outside Japan an opportunity to examine authentic Japanese Lesson Study in mathematics classrooms. The major purpose of this program is for us to receive feedback on the strengths and weaknesses of Japanese Lesson Study and to discuss how to improve mathematics teacher professional development programs. To accomplish this, we invited leaders of mathematics education to immerse themselves in authentic Japanese lesson study, especially school-based lesson study, and to observe mathematics research lessons in elementary and lower secondary grades.

The program was held in Tokyo and Yamanashi in Japan from June 25, 2012 to July 5, 2012. In total 42 mathematics educators (36 form U.S., 2 from U.K, 2 from Australia and 2 from Singapore) including Phil Daro (CCSS co-chair), dean of Mills college, mathematics education professors, principals of school and so on. Project IMPULS invited 17 participants and others joined funded by Toyota foundation or funded by themselves applied through Global Education Resources (GER).

Two of IMPULS overseas support committee, Dr. Makoto Yoshida (President of GER and Director of Center for Lesson Study in William Paterson University) and Dr. Tad Watanabe (Professor of Mathematics Education at Kennesaw State University) interpret lesson and post lesson discussion observed. Almost all lesson plans were translated by Dr. Tad Watanabe and distributed before observation. And two of external evaluation committee, Dr. Catherine Lewis(Senior research scientist, Mills College) and Dr. Rebecca Perry (Senior Research Associate, Mills College) gave us useful feedback with objective evaluation of program. We would like to take this opportunity to thank all of our overseas support and evaluation committee, cooperative schools which kindly welcomed our visiting and all concerned professionals for their hard work.

2 Contents of Program

This program is designed for deeper understanding of Japanese school-based lesson study and it consist of these contents below.

1) Basic lecture on Japanese mathematics lesson and lesson study (1 day)
2) Observation of research lesson and post lesson discussion (7 classes)
3) Discussion among participants, Q/A and review session

Detailed schedule is shown as below.

Date	AM	PM
24, June	Arrival day	
25, June	Opening Ceremony Workshop "Mathematics teaching and learning in Japan, and lesson study" Welcome dinner party	
26, June	Preparation for school visit	School Visit (1) Funabashi Public Elementary School Observation School-Based Lesson Study
27, June	Move to Yamanashi by bus	School Visit (2) University of Yamanashi Model Junior High School Observation preparatory nation-wide Lesson Study
28, June	Courtesy call for local board of education Cultural trip to Takeda shrine to see Sangaku	School Visit (3) Oshihara Public Elementary School *School lunch Observation School-Based Lesson Study
29, June	Move to Tokyo by bus Reflective discussion and Q/A (in the bus)	School visit (4) Koganei Elementary School(TGU Affiliated school) Observation School-Based Lesson Study
30, June	Special seminar for " Common Core State Standards in the U.S. and Its Implementation: Potential of Lesson Study " -Common Core State Standards in the U.S. why and what is next? By Phil Daro -Implementation of the Common Core State Standards in the U.S.: What Might Lesson Study Offer? By Catherine Lewis	
1, July	Free	

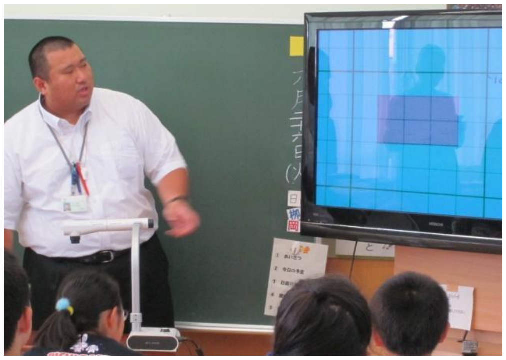
2, July	*Reflective Seminar* *Preparation for school visit*	*School Visit (5)* *Koganei Junior High School(TGU Affiliated school)* *Observation Special Lesson Study*
3, July	*Preparation for school visit*	*School Visit (6)* *Public Junior or Senior High School in Tokyo* *Observation Special Lesson Study*
4, July	*Reflective Seminar* *on Japanese mathematics instruction, strengths and weaknesses*	*School Visit (7)* *Hashido Public Elementary School* *Observation School-Based Lesson Study* *Farewell dinner party*
5, July	*Departure day*	

Participants made 7 group to make observation report for each research lesson.

Lesson Observation (1)

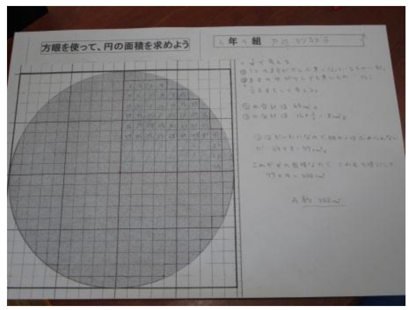
Research Lesson Observation Form (Use photos to document each section)

Names: Wanty, Connie, Tracy, Ruth, Miranda and Angeline
School: Funabashi Elementary School Grade: 6 Date: 26 June 2012

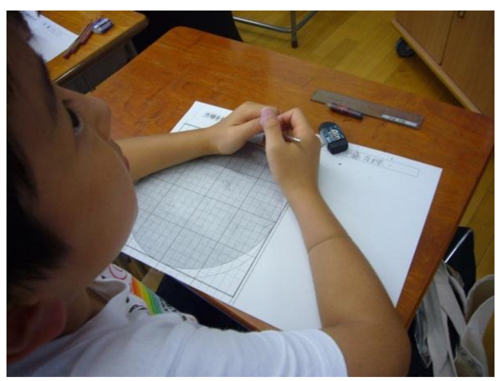
Start & End time	Lesson Phase	Notes
	1.a)Introduction	**Strategies to build interest or connect to prior knowledge** **-Exact posing of the problem, including visuals**
		Teacher started with posing the question “What is area? What is area about?” A student responded (S18?)“Area is about the space” (amount of space?)
	1.b)Posing the task	Teacher said “Yes” – he then showed a picture of a rectangle and asked students “What is the area of this rectangle?”  A student responded that 6 cm^2. The teacher said “Yes” and counted the unit squares “one, two, three, four, five, six”. The teacher also made a comment that this was already done in previous grades.  Teacher started by asking students “What is this shape?” Student identified that the shape is a circle.

		Teacher then introduced the problem of the day by asking students to find the area of the circle using the grid. He pasted the problem on the  Teacher handed in the worksheet to the students to find the area of circle as a challenge of today's lesson. He also mentioned to the students to "Try to tackle the problem; it is okay if you could not finish it but try to think about finding the area of the circle". Students were given 12 minutes to solve the problem individually and reminded to write their name on the worksheet.  He did not ask the class whether there is anyone who did not understand the problem of the day.
	2. Independent problem solving	The problem solving activities started as individual work. Students were given 12 minutes to work individually and then were asked to discuss in small groups to explain their ideas to others and find one method that is the best method to calculate the area of the circle. Below are samples of S30's solutions during individual problem solving. During the individual problem solving, teacher walked around the classroom with the classroom chart to identify students' strategies in preparing for the whole class discussion.

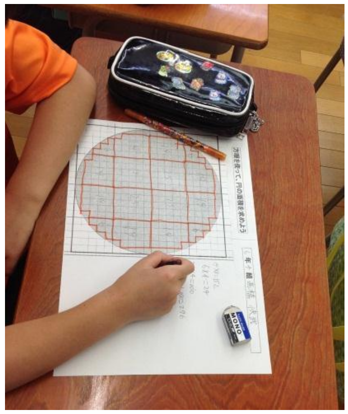




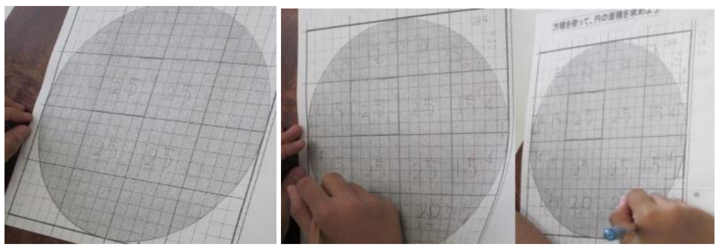
S27 started with counting the squares in the first quadrant and then she counted those squares that are partially shaded and divided by 2 which is 8 cm^2. She used a red pen to demark the quadrant that she was looking at. She added 8 cm^2 to 69 cm^2 and multiply by 4. The circle is made up of 4 quadrants.



S6 wrote down some words and erased them away. In his diagram are the labels “10cm” and “20cm”. Was he the one observed by Tad to have written down the correct formula for calculating the area of the circle but who erased his calculations shortly after?

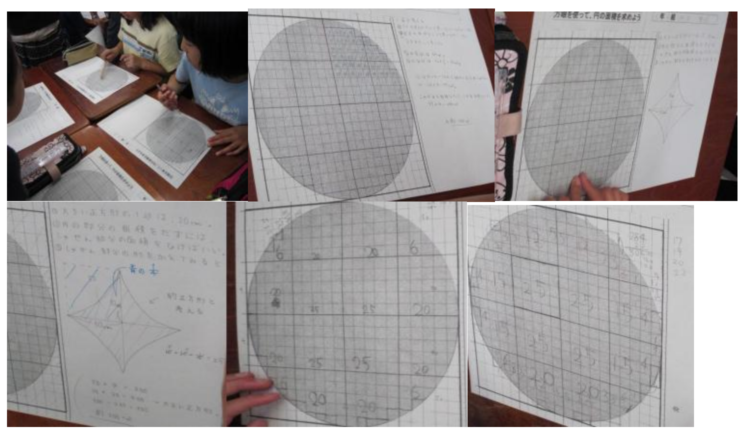


S18 identified the internal boundary and used the area of internal boundary as an approximation for the area of the circle.



S30 started her solutions by identifying the area of middle section of the circle, i.e., 4 of 5x5 squares; she noted the areas of each of these sections as 25. She then calculated the number of complete shaded unit squares as 15, 4, and 1 to the right and left of the middle squares, 3, 12, and 1 on top of the middle squares and 1, 4, 15 on the left middle squares by grouping some of the partially shaded squares. She then identified two partially shaded squares whose combined area was approximately close to one unit square. The following photos captured the progress of S30's group during an individual problem solving.

When 12 minutes was up, the teacher asked "Who have found the area?" and asked students to put their hand up if they have found the area. Some students put their hands up. Students were then asked to form a group of 4 students with their closest table. The instruction given was by the teacher was "Explain your thinking- explore different ideas, try to figure out one idea to find the area of a circle".

	Group Work	Observation of group discussion involving S30, S 29, S 28 and S 27. S27 started by explaining her strategy. Her strategy was counting the number of fully shaded unit squares of a quarter of the circle from 1 to 42 then for the next 3 rows she added multiples of 9 unit squares (42, 51, 60, 69). She then counted 16 “incomplete” unit squares and multiplied them by a half to get 8 and add 69+8 = 77 which was then multiplied by 4 to get the area of 308 for the whole circle. S28 was the second presenter; she has not found an area of the circle but explained her strategy which was based on identifying the area of a square/rhombus inside of the circle. After hearing S27’s presentation and her comments, S28 added some notes in her worksheet. S29 was the third presented, her strategy was to count the number of squares inside the circle followed by S30 who shared a similar strategy by S29 which grouped some “partially” shaded area inside the circles to approximate a unit square. An observer came around and added on to S28 thoughts and she added on to her initial thoughts. 
	3. Presentation of students’ thinking, class discussion	Whole class discussion started with teacher invited S17 to share her solution. She used counting an area of partially shaded area which was less than half as zero and partially shaded area that was larger than half as one. Her answer: area of the circle was 320 cm^2 was recorded on the blackboard by the teacher. S15 was invited as the second presenter and her strategy was to count the area of unit squares inside the circle (on the border line) and found the area of the circle as 326 cm^2. S12 was invited as the third presenter and she started with finding the area of the largest square and counted the area of the unshaded unit squares to subtract from the area of the square. She found the area of 400 – 88 = 312 cm^2. S27 was invited as the fourth presenter and her strategy was to count the number of fully shaded unit squares of a quarter of the circle from 1 to 42 then for the next 3 rows she added multiples of 9 unit squares (42, 51, 60, 69). She then counted 16 “incomplete” unit squares and multiplied them by a half to get 8 and add 69+8 = 77 which was then multiplied by 4 to get the area of 308 for the whole circle.

		
	Summary/ consolidation of knowledge	Teacher noted that students showed different ways of calculating an area of the circle using the unit squares. Teacher noticed that some students have not found the answer and asked who have not found an answer but close to solving the problem. Some students put up their hands. Teacher then asked "What is the source of difficulties in calculating the area?" S18 responded that "It was difficult to calculate the area because of the shape". Teacher ended the lesson by noting that these solutions are approximation and next lesson they will learn how to calculate an area of the circle more accurately.

What new insights did you gain about mathematics or pedagogy from debriefing and group discussion of the lesson?

Debriefing and group discussion of the lesson helped the teacher to identify the strength and areas for improvement based on colleagues observations of evidence in the classroom. Teachers worked in small groups based on the grade levels and focused their feedback on three aspects:

What kinds of instructional ideas were incorporated?

Students had opportunities to discuss their ideas in small groups and to communicate their ideas so this was a good point. However, the goo d points from the students were not highlighted on the board when they presented their strategies. Teacher only recorded the final answer on the board. It was important that teacher summarized students' ways to calculate the area of the circle. For instance middle group teachers pointed out that teacher needs to support students to contrast and compare ideas so students can discuss in depth and further the more effective ideas.

- Time for students to discuss their ideas

Enough time for students to study the problem.

Students might work in groups but they only listened to each other and reported their ideas but the discussion to improve their methods were not strongly featured during the discussion. This is an area that needs to be improved.

Students presented their ideas but teachers who observed were not sure whether other students really followed the explanations and understood the different ways of calculating areas.

Post lesson discussion was very insightful because teacher received feedback from the observers about areas for improvement and they helped the teacher to identify critical component of the lesson that needed to be improved. Other teachers also provided extra insights on students' ways of thinking which might not be observed in detail by the teacher.

What new insights did you gain about how administrators can support teachers to do lesson study?

Collaborative effort and support from other teachers, principals and experts in Lesson study are very important. In the post-lesson discussion, it was evident that the school provided a strong support for the teacher to conduct the research lesson and invited his colleagues to participate by observing the lesson and gave him insightful and constructive feedback. The support from external consultant/expert was very critical as this external provided teacher with a constructive advice on mathematical content knowledge to move forward. Prof Akihiko analysed various textbook approaches in teaching the topic to help teacher see the need to pay attention to the key ideas that students need to see that the area of the circle is about 3 times the area of 10x10 squares. This can be done first by dividing the square into 4 and knowing that the area of a circle will be more than 2 the area of the squares but less than 4 times the area of the squares. Prof Akihiko highlighted to teacher the importance of "neriage" by understanding students' background to help students to think and make connections in order to help students' ideas to develop. Teacher was advised to improve of "kyozoikenkyu" to deepen his understanding of students' thinking. By giving an example of how to analyse and compare the textbooks, teacher could see this point clearly as one of the ways to improve his practice.

Lesson Observation (2)

Research Lesson Observation Form (Use photos to document each section)

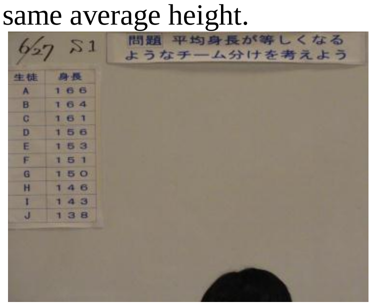
What are the primary lesson goals?

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

<table>
<tr><th>Start &End Time</th><th>Lesson Phase</th><th>Notes</th></tr>
<tr><td>2:09 – 2:15</td><td>1. Introduction, Posing Task</td><td>
Teacher states that this is a special lesson and they will be working on a worksheet. Passes out worksheet and reads the problem. Teacher posts the problem on the board.

[Problem] There are 10 new members on our school basketball team. In order to evaluate the skill levels of these new members, Mr. Sakuragi who is the faculty sponsor of the team decided to have them play intra-squad games by creating two teams, X and Y. Mr. Sakuragi figured out 3 different ways to create two teams so that the average heights are the same, and they played 3 intra-squad games. How did Mr. Sakuragi made these teams. Here are the heights of the players.
Name A B C D E F G H I J
Height (cm) 166 164 161 156 153 151 150 146 143 138

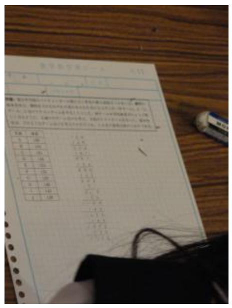
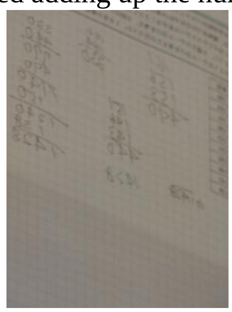
Teacher asks students if they understand the problem and reads it again emphasizing that the teams have to have the same average height.



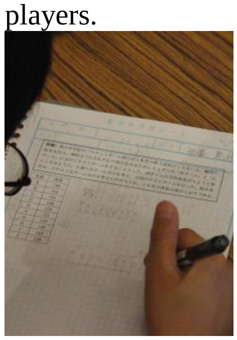
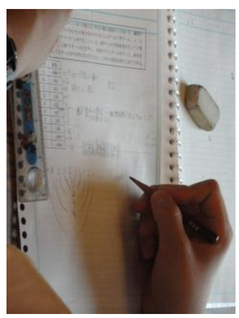
</td></tr>
<tr><td>2:15 – 2:28</td><td>2. Independent Problem-Solving</td><td>Teacher tells students that they will work in pairs except for one group in the back of the class that will work in a group of three</td></tr>
</table>



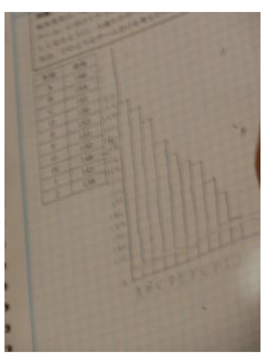
Some students started adding up the numbers to find the average.

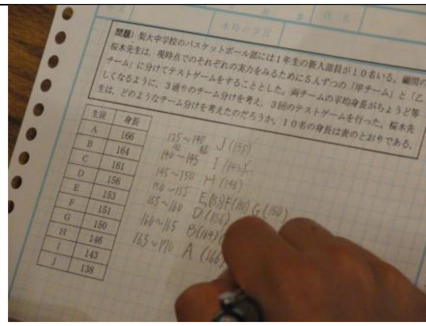
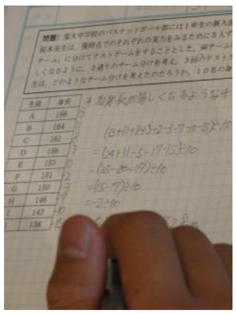
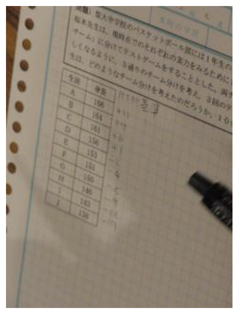
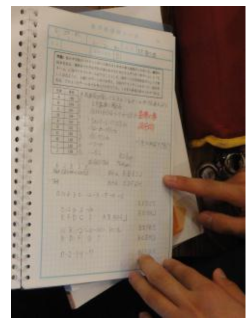
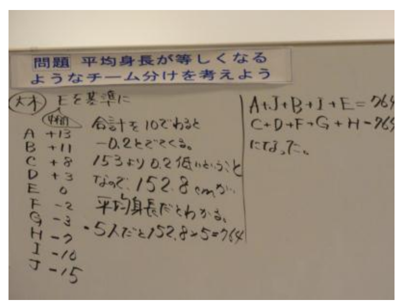
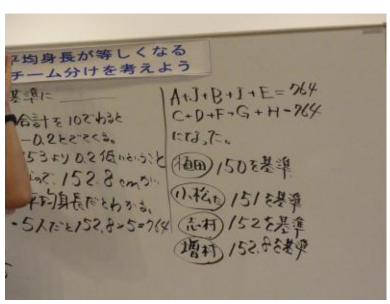
Some students started forming teams by systematically distributing tall and short players.

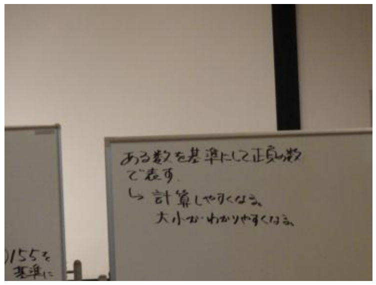
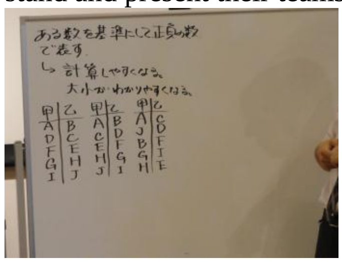
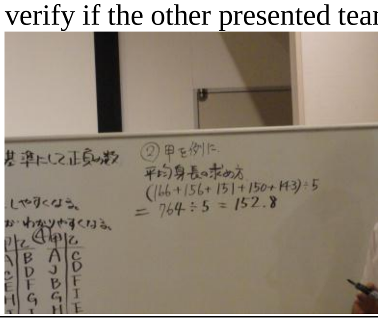
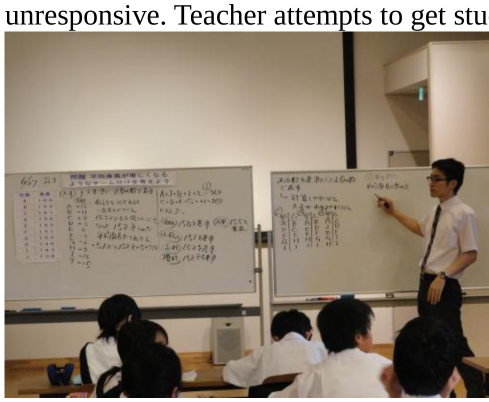



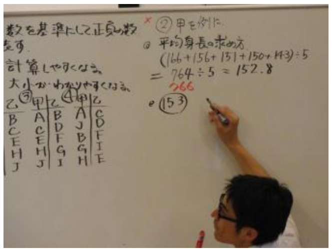
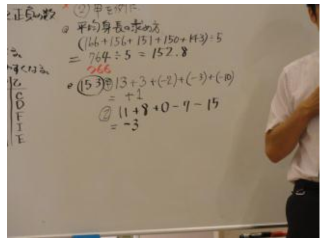
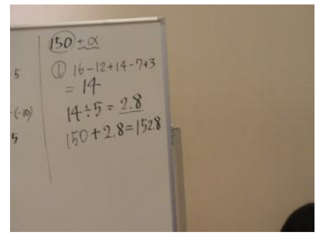
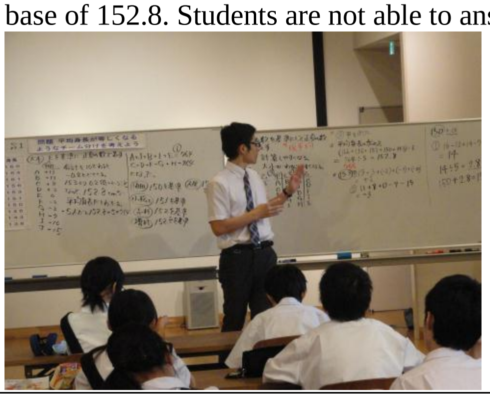
One student created a bar graph to analyze the range of heights and distribute students accordingly.



One student distributed the players within a range of heights.

		 And some students used the "tentative average" method.    
2:29 - 2:38	**3.Presentation of Students' Thinking, Class Discussion**	Teacher calls on first student to present. The student stands and reads his work as the teacher writes it on the board. Student used the "tentative average" method starting with a base of 153 and finding the average variation to be -0.2 and the average to be 152.8. the student then created teams.  
2:38 – 2:40		Teacher asks if any other students used bases and what bases they used. Teachers writes the bases down with the students names. 

<table>
<tr><td>2:40 –
2:42</td><td></td><td>Teacher asks why we might want to use bases instead of just finding the average in the normal way. A student volunteers that using bases makes the calculations easier because the numbers are smaller. Teacher writes that statement on the board.

</td></tr>
<tr><td>2:42 –
2:45</td><td></td><td>Teacher asks students what other arrangements of 3 teams did they find? Students stand and present their teams and teacher writes them down.

</td></tr>
<tr><td>2:45 –
2:47</td><td></td><td>Teacher demonstrates that one team does have an average height of 152.8. Teacher them asks students to use “tentative average” and positive and negative numbers to verify if the other presented teams have the same average height of 152.8

</td></tr>
<tr><td>2:47 –
3:00</td><td></td><td>Teacher brings class back together to ask if they were able to use “tentative average” and positive and negative numbers to find average height. Students are unresponsive. Teacher attempts to get students to respond but is unsuccessful.
</td></tr>
</table>

3:00 – 3:05		Using various questions and asking students if they are ok numerous times and after receiving incorrect responses, teacher decides to demonstrate how to use “tentative average” and positive and negative numbers to find the average height of 152.8 starting with a bas of 150. Tells students that they need to find the average deviation from the base by first finding the sums of the deviations from the base by using positive and negative numbers. Then they can divide that sum by 5 to get the average and then add the positive or negative average deviation to the base (150 + 2.8 = 152.8.)   
3:05 – 3:10	**4.Summary /Consolidation of Knowledge**	Teacher summarizes lesson using the board to go over the presented work and by asking more questions. Asks what would be the sum of the deviations if we used a base of 152.8. Students are not able to answer. Lesson ends 

Lesson Observation (3)

Summary of debriefing session for Lesson 3
Mental subtraction

Teacher who is the head of the research team introduced the speakers. She explained that they had chosen mathematics to focus on this year after doing many subjects at once. Their goal was to have more meaningful discussions.

Teacher Mr. Koji Koike spoke first.

Main points he made:
He knew the vision of the vertical algorithm was in their minds. He wanted them to understand that they could manipulate the numbers. Do calculations other than the algorithm is a hard habit to change.
QUESTION: WHY DO THEY TEACH THE ALGORITHM BEFORE TEACHING DIFFERENT WAYS OF SUBTRACTING?
Wanted them to divide and decompose numbers.

He used 3 different numbers. He believed that the first child chose 53 rather than 89 or 68 because he wanted to challenge himself. Teachers encourage children to challenge themselves.

He discussed the "on the spot" argument he had with himself about whether or not to discuss the 3-8 becoming 8-3 issue. He wanted to ask the child to explain this, but ultimately decided to cut this discussion short. Not a wrong idea, but he didn't want to emphasize it.

The last one, explaining addition and subtraction strategies. When are they used in everyday life. He commented that looking at the journals he found many interesting ideas.

He repeated that he wanted the children to focus on other ways of computing subtraction.

Comments on his remarks and teaching:

The students were mumbling explanations. He responded to them. One child in particular responded to every step. Why did he decide to emphasize the split strategy; 53-3=50-5=45-20=25.

Someone else commented on this as well.

Mr. Koike responded that he wanted this strategy to occur.

The principal commented that with the algorithm we want children to find the answer. Is this good or wrong? Think about that issue.

Teacher commented that students are often satisfied with one way to do it. Would like them to have other ideas.

Questions about children's responses:

Children's responses guided the lesson. Watching a student that had difficulty sharing ideas in public. Allowed her to share her idea based on the algorithm. Looking for things that triggered student ideas shift.

Question about Ray who did the split system (described above).

That child came up with several ideas yesterday. Also used this strategy yesterday.

There was a question about a child who got an incorrect number.

Mr. Koike said that on her own she has difficulty. She had ideas, but couldn't quite do it.

There was a child who could generate her own ideas.

Mr. Koike mentioned that he adjusts the lesson when he teachers it, but that one child couldn't quite finish what she was asked to do.

Observing the class as a whole, student listening to the teacher at the beginning were struggling to come up with different strategies. There were a couple of instances when students were really baffled. They made use of the posters in the back of the room. Shows how he helps students to use what they learned previously.

The teacher went around the room and monitored the students. Case by case, changes how he listens to the students and gives them follow-up.

One teacher said we have to improve our observation and help students.

The 2nd teacher in the classroom commented: She wanted to help the students. Words on the white board used to explain. Worked with one student to help her and then she was able to explain. She expressed the need to be clearer about what she can do in the classroom.

Mr. Koike said he asked her to be with the one child who is not good a focusing for a long time. She wanted to give her time to report. This gave her confidence. The 2nd teacher was very supportive of that child.

Summary of these comments:
Be clear and discuss the role of the 2nd teacher. Explore and consider this point..

2nd teacher: team teaching. I was involved for three years as a 2nd teacher. I wasn't sure about the position in the classroom. Often just stuck with the children not doing well, to build their confidence, helped students do work more carefully. Consulted with the teacher. Took 3 years to do this well. Share through my experience.

Professor: commented on 53-28, that the textbook showed 53-26. Did you pick this one on purpose?

Another question about the shopping situation. (72-48). You might have coins like 50 and would do 50-58. Students might not follow the logic.

Box—what if? 53-____ try out different #s.
How students' thinking changed and using this in everyday situations m=is more meaningful.

Mr. Koike: In the book it's 53-26. I chose 53-28 because it's closer to make it a rounded number. Didn't think they would do this. Some regret about this. As for the shopping scenario, debated which way to go. Wanted to use 28. Might have been easier to manipulate 53. It's harder to manipulate the subtrahend than the minuend. I will think more carefully.

Now some of us asked questions:

Courtney: Why did you direct them to one method?

Mr. Koike: they didn't have too many. I wanted the students to practice this one to see if they understood it.

Nick: what about more days on addition mental arithmetic? Would it have been better to learn subtraction?

Mr. K: limited time, this is what was allocated to our unit. Maybe it can be carried into the next lesson. We examine different ideas; figure out what are good questions to ask.

Michelle: Discourse moves: three different moves: 1. Asking students to explain each other's reasoning. 2. Using someone else's method. 3. And children saying "he didn't use 28". How do you think about it and make these possible?

Mr. K: I always tell the students there's no wrong thinking. If you are not listening, what you say often isn't good. In small group discussions in other subject areas asked children to describe how this person is thinking. Praise them—it's a very good idea.

Tom: Students write down interesting ideas and then erase it. How do you think about that phenomenon (problem)?

Mr. K: I ask students not to erase whatever you started to write. Emphasize they should underline it (*Did he mean put a line through it?)* But there are always students who erase. I will mention that they shouldn't do it, again.

Summarizer: Move on to the final comment

Final commentator:

Textbook problem was 53-16. He waited for children's ideas. Getting children to come up with a variety of ideas is difficult. The algorithm is a tedious beginning but students keep using it. Began with the algorithm. This was about mental calculations, so the implication is that they did the algorithm in their heads (reasoning about what's hard). Need to refocus the task, to rethink about the nature of the calculation

Order of words important in describing the mental arithmetic process: first, then, finally. Need to discuss this with students explicitly.

The student notebook.

3-8 8-3. Not simply reversing the operation; 5 is just a number.

53-28
50-20=30 8-3=5
30-5 = 25

The student was really thinking, not just inverting. The teacher really valued the children's ideas.

Second final commentator:
Todays lessons and interaction between the teacher and the students. The teacher looked carefully at each student's ideas. They were being asked to make word explanation with mathematical explanation.

Tried to be with the students until they all understood. Maybe needed a more concrete situation. People nodding—meant they understood.
Then they stop nodding—does this mean they are not listening?

The teacher focused on who is listening and others who are not. Formal steps—may not understand. Look at students carefully to adjust our expectations. We can learn from Koike.

What image do children have in their head about this problem? Abacus? Charts? Coins? Writing notebook—go back and compare mental calculations, talk to each other. Enjoyment. The merit of writing. The attitude toward mental calculation. They can estimate.

Doing the problems with us: 83-15. Many different ideas. Some are very complicated. Using friendly numbers.

Talks about starting in the 10's column. Easy calculation might sometimes be complicated.

Lesson note—looking at student work, choosing which ideas to emphasize based on student work. Wrote in words—when it's in a math statement can really follow it. *(HE can really follow it—not so sure about the children!)*

Children write down what's on the board.

Professor comments:
Purpose of the lesson
Teachers aware of the value. Doing something good unconsciously. Many lesson study discussions focus on pedagogical strategies. Important, but have to talk about the content. Focus on kyosai genku. Focused discussion time on the content focus.

Value of this lesson today
Mental computation
Developing and enriching a number sense—major focus of the lesson.
From school experience and everyday experiences (refers to Brazilian candy sellers and everyday mathematics, differentiated from school mathematics.) Number sense *in context* is really good. How can you bring number sense from everyday situations into more formal situations. Maybe he should have used larger numbers to demonstrate the tediousness of using the algorithm and paper and pencil. Focus on the property of subtraction.

Picture of overlapping rectangles. No matter what the lengths the difference stays the same.

Look at the notebooks during the lesson. The length of the notes shows how much children have learned.

Lesson Observation (4)

Research Lesson Observation Form (Use photos to document each section)

What are the primary lesson goals?
From the lesson plan: “Students will deepen their understanding of characteristics and properties of cubes by examining and understanding the reason why seven edges must be cut in order to open a cube into a net.”
The teacher was following the school-wide goal of deepening understanding.

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
First lesson of this unit on cubes and cuboids based on prior work with solid figures. The students had discovered that there are 11 nets to make a cube in a lesson a few weeks prior to this one.

Start &End Time	**Lesson Phase**	**Notes**
2:00	**1. Introduction, Posing Task**	T hands out two small paper cubes made out of graph paper with taped edges to each student. T holds up a cube and attaches a net to the board with a magnet. Seemingly to activate students’ prior knowledge and create interest, T asks students, “what is this?,” solicits their opinion, and then confirms that it is net of a cube. He instructs them to keep it on the table. [picture 3 Sonny]  T: “We’re going to cut along the edges to make a net. Can you imagine what we’ll be doing? The question is how many edges to cut. You can look, but do not touch. How many do you think?” [Students guess 6, 7 edges.] T: “Take 1 cube and cut it up to verify how many edges to cut.” [Students: 5; “depends on which type of net.”]
2:07	2A. **Independent Problem-Solving**	Each student has their own cube, but they are working in pairs to focus together on one cube first.  T reports that one student has said that they need to make sure the squares stay together in one piece. T asks students to present their work. S1 presents cross shape T: “Did you see what she showed? To make a net that looks like a cross, cut 7 edges. What happens with a different net? What about the others?”

2:08	**3A.Presentation of Students' Thinking, Class Discussion**	3 additional students are asked to present their nets, and each report that 7 cuts are required to make the net. With each solution, T draws the net on the board and confirms that there are 7 cuts needed for each net. 
2:14	**2B. Independent Problem-Solving**	T: "A lot of the students have seven edges. Do you agree with that? Are there any other nets?" T asks students to use their second cube to "think about a different one [net] and cut it open to make the net. S: "I can't keep track of where we cut." T: Do you see the markers on your table? Use these to keep track of where you cut." Students' methods of getting the net varied: some student used colors; one student was numbering the edges. At this point, when students said they only had 2 cubes, the teacher told them to use their cube and visualize the other nets in their minds. This may have been an attempt to encourage more abstract thinking.  [Students' journals show that many of them have copied down all the nets the teacher posted. We hear that some students can only find the cross shape.]
2:17	**3B. Presentation of Students' Thinking, Class Discussion**	T: "This is what Junta-kun made. She also had to cut 7 edges." [Show Junta's solution.] Almost all of the students had their hands up. T calls on 7 more students until complete complement of 11 nets is posted. Each time, T confirms that 7 cuts are required. 
2:25	2C**. Independent Problem-Solving**	T: "With each one, we had to cut 7 edges. Is that a coincidence? We need to prove it. That's hard, but there must be a reason. Let's investigate. I can't give you any more cubes, so look at the ones you have. If you fold them up, you get a cube. Use what you have and think about it…. Using those 2 [cubes], see if you can figure out why. We know you have to cut 7." T writes hatsumon on board: "Let's explain why if we cut 7 edges, we make the net of a cube." [Some students report that they already know.] "If you already figured it out, write in our notebook so others can figure out the 

<table>
<tr><td></td><td></td><td>explanation.”
T walking the room with clipboard, observing and writing down notes.</td></tr>
<tr><td>2:32</td><td>3C.Presentation of Students’ Thinking, Class Discussion</td><td>


T: “It is time to discuss. Who hasn’t said anything yet?” Calls on S.
S. [Offers solution.]
T writes on board: “In all nets the number of sides is 14…”
T: “Hold on, what do you mean by that?’
S: “There are 14 sides around all these nets. In order to make a cube from a net for each side of a cube, 2 sides come together to make the edge. Each time I cut an edge, I use 2 sides. So if I write the expression, 14 ÷ 2, therefore you have to cut 7 edges.
T writes this out in words on board.
T asks if students understand, repeats this response, then asks again if students understand.
T: “Show me the drawings you have.” He then draws what she has drawn on board, showing the net and the cube with cuts in red.
T: “I’m going to try to make it the same.”
[Asks her clarifying questions about her drawing, like “then you have some numbers here…] T draws some and then has student draw some.
S: [Referring to drawing] “…The red one. If you cut the edge, you make two sides.”
T: “You have to look at the (other) students to see if they understand.”

T: “Who has a different idea?”
S: A cube has 12 edges altogether. When you look at a net, there are 5 places where the faces are kept together. Those are edge of a cube. So the number of edges cut is 12 – 5 = 7.

T writes this idea on board. Some students report they had the same idea. T asks if anyone wants to add anything.</td></tr>
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		After the girl came up and drew out her idea, there was a boy who provided the 12-7=5 response  T: “OK if I ask you a question? When the student said that there are 5 places and I checked different ones, is that a coincidence or is there a reason behind it? What about the 5? Do you understand the question?” S: “I checked all the nets I had drawn before. All the nets had 5 edges where they come together.” T: “We don’t want the net to be like this (left separated squares). To keep them together, there must be 5 places. Why?” S: “If you think the opposite, if I cut 6 edges, there are always 7 that have to be connected.” [Other students drawing on nets and cubes, and drawing in their journals to indicate their thinking] T: “His explanation shows that 7 edges need to be cut. But if you look at this explanation, is the 5 a coincidence?” S: “We don’t want to separate them. If I cut this edge one will be separated. If I cut 6 edges, there is always a point that is connected.” T: [Writes S’s idea.] “Let’s think about this part here. ___ said earlier that places where the faces come together are 5. From a cube… when you cut a cube open during the process the shape you get… If you think about the shapes and cut just 6 edges you can’t open a cube to get a net. In order to keep the faces together, you have to have 5 places that stay together. T: “So let’s stop thinking about the 5. It’s already time. __ said if you look at this cross, there are 5 places where they were all together. Did you check all the ones where you don’t get a net?” S: “I checked all the nets. Eleven.” T: “She checked all the nets. The question I ask you is, are there always 5 edges? We haven’t quite figured out why yet why it is 5, maybe that is just the way it is. Maybe that is for the future. Let’s stop here for today.”
2:50	**4.Summary /Consolidation of Knowledge**	T: “What we did today is talk about how many edges we need to cut to make a net. Think about what we did. Please write a journal entry and leave the notes here. Write you name on the nets and leave them here as well. You have until 2:52.” [Students quickly write journal entries.]

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- Blue and yellow notes as an interesting process for organizing the debriefing comments. Blue: Class observations; Yellow: Questions/ big issues to discuss, based on what you saw with students
- Mechanics of the lesson – e.g., whole group work vs. small group work – less important than content.
- Lesson study and post-lesson commentary is about the nudge to get the teacher to the next level in teaching. E.g., the idea of lesson mechanics or thinking? There were two groups of students: a) students wanted to find nets and b) students who wanted to find the reason for making 7 cuts. The teacher chose to focus on students who wanted to find nets and took the lesson in that direction, while trying to find places where he could connect these two ideas. By getting out all 11 nets, he wasn't able to get them to where he wanted to get them to deductive reasoning. Maybe the decision to focus on his idea of following the students to get all 11 nets was not the right one. The teacher was able to reinforce what they learned about the cube, and help them think more about the properties of a cube, the idea of 7 cuts is mundane ("where is the math"). Math is in the student thinking process.
- Teachers were concerned about whether the lesson as taught provided the opportunity for students to tell their story (i.e., articulate their thinking. There was discussion of students' incomplete opinions/ ideas. Encouraging students to express incomplete opinions is a good thing – something the teacher consciously thought about. "Incomplete opinion [idea]" isn't good terminology.
- How much experience do students have to have (e.g., the process of creating a net from a cube rather than studying how many nets form a cube) before a teacher introduces inductive/deductive reasoning?
- Teacher reported he wasn't helping students listen to each other's ideas –an area for improvement.
- We wonder about the significance or importance of storytelling. On one hand, there was the story about the student who had his own answer, then drew a dotted line and copied the answer from the board. The participant in the discussion said that the student should have told the story about how he (?) got from his own answer to the one that he copied. (There is a photo of this paper.) This seems useful. On the other hand, the final commentator (and Prof. Takahashi) seemed to be saying that storytelling was not a useful goal because it was too general and not sequential—you can have understanding and storytelling or storytelling and understanding.
- There were several key decision points that the teacher made during the lesson in response to the students. Teachers in the debriefing process recognized these decision points.
- In terms of board work, the teacher took a lot of time putting up the eleven nets on the board. He drew them fairly rapidly rather than asking the students to put them on the board, which saved time. Still this process took a significant amount of time.
- It was mentioned that it may have been too complex a lesson. This would depend on the goals and also the students' prior knowledge and grasp of nets. It was mentioned that maybe the teacher's expectations for how the students can express themselves were too high for this time of year. The question was raised about how to address the students who did not seem to understand the content or mathematics of the lesson.

What new insights did you gain about how administrators can support teachers to do lesson study?

- If you don't have good knowledge in the content area, you tend to focus on mechanics/procedural decisions. This would be a place for leaders to focus the discussion during the lesson study. Not necessarily that the administrator needs to have the content knowledge, but should ensure there is a content expert. Administrator should be present at the observation and debrief of the lesson.
- One important role for administrators is to clarify the connections between the school-wide goal and the research themes.
- There was a tension between following students' interests (making all 11 nets) and getting to the specific goal or focus of the lesson in the 45-minute period (teaching inductive/deductive reasoning.) How might the group have helped the teacher resolve this tension? This same tension is reflected in the tension of the goals—do you focus on helping students to work together, engage with the problem, present ideas, etc. or on the mathematical task and how do you do both? How should this be represented in the lesson plan and how can the teacher use the lesson plan to make strategic decisions.
- The teacher said, I would like to provide appropriate support for students to state their ideas even if they are not complete. This is an entry point for administrator (or collegial) support.

How does this lesson contribute to our understanding of high-impact practices?

- "Can you see?" [Teacher increases the size of the writing on the board.]
- Teacher writes the name (and probably not coincidentally also the purpose) of the lesson on the board: "Cube, again" [with date].
- At beginning of class, holding up a net, the teacher asks students, "what is this?" to activate prior knowledge and get their interest, solicits their opinion, and then confirms that it is net of a cube (he doesn't tell them).
- He asks them to not touch it and keep it on the table, and does not let them cut. He tells them they'll cut along the edges to make a net and asks them to imagine what they are to do (prior to actually doing it).
- "I can't hear you" to encourage students to speak with mathematical authority.
- [When calling on students to speak up in class] "are there no other girls?" [Very conscious of/ intentional about who he is calling on to share; he is conscious of calling on new students and also calling on a balance of boys and girls.]
- He used a strategy to get the students to think more deeply about whether there was logical reason for the observation that there were always 5 uncut edges. He asked, is this a coincidence?
- At the end when he ran out of time, he seemed to suggest that they would keep puzzling over this in the future.
- Balance of teacher intervention/public sharing and giving students time to work on their own. How to encourage greater participation, to time the lesson so students work out their own ideas before copying down their peer's ideas from the board. When students are interrupted during the independent problem solving period, they may stop their own thinking and copy other people's ideas. The remaining question is how best to balance the need to share and express ideas and to allow time and encourage independent thinking.
- The teacher mentioned that he was paying attention to the students' concentration levels. In the post-lesson discussion, the teacher said, "I should have spent more time on clarifying

(the problem), instead I spent more time paying attention to their concentration level." One high impact practice might be for the teacher to follow the students' understanding as she or he makes strategic decisions during the lesson.

- To make incomplete understandings complete, it's important for students to know what is missing. (This was part of the discussion that even if it is an incomplete idea, it is important to write it out.) Following this was a discussion about how to do that, e.g., group work vs. individual vs. whole class work. The teacher said that the term incomplete understanding is not a good term (or one he use with students. He said there are no mistakes, only misunderstandings and explained the importance of establishing a classroom where students can express all of their ideas even if they are "incomplete." He also mentioned the importance of understanding why the students did the problems in particular ways (which may seem incomplete or incorrect.)
- Teacher said he wants students to use their voice develop the task, think about it on their own, don't want to lead the students.
- Board was used effectively - it was clear and organized. It displayed the goal, the student ideas, the student thinking, summary of their strategies. (see picture). It was easy for students to see the flow of the lesson.
- He had 2 cubes available for students. The second one was used to push student thinking. "Don't touch" to get them imagining folds and nets.
- T walking around the room, looking at student work, and ordering student strategies. He may also have been questioning them what they were doing (but we don't really know because of translation).
- Intentional decision-making based on what student's offer.

Other observations

- It isn't clear who gets to decide on the purpose of the lesson.
- The team may have had a revised lesson plan that we did not have.
- One participant asked if it was too much for the students to pay attention to each other's solutions and think for themselves.

Lesson Observation (5)

July 3rd Observation: Construction of Bisectors of Angles
Grade 7 (20 Girls, 20 Boys), Mr. Kouichi Kabasawa

As reported by: Michelle Cirillo, Kelly Edenfield, Erik Moll, Joshua Rosen, Phil Tucher

What are the primary lesson goals?
Goals of the unit were explicitly provided, but there were no goals of the lesson provided explicitly beyond the topic: "Set of points that are equidistant from a pair of given lines (Construction of bisectors of Angles)" and the Research Theme: "Designing lessons that enhance the quality of mathematical activities." ." [Note: Enhancing the quality of mathematical activities we later learned meant to focus on the mathematical work – similar to Standards for Mathematical Practice from the Common Core State Standards – developed throughout the lesson.]

Unit Goals included:

- *Students will be able to construct bisectors of angles using points of symmetry.*
- *Students will be able to explain the steps of construction indicating the center of the circle, the radius, and the two points through which a straight line passes.*
- *Students will deepen their understanding about thinking behind each method and about bisectors of angles through examination of various ways of construction.*

Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?

This lesson appears in the last half of the unit. The students have previously used compasses and the properties of circles to construct hexagons, points equidistant from a given point (circle), points equidistant from two given points (perpendicular bisector), points that are equidistant from three given points (perpendicular bisector through one of the points), and points equidistant from a given line (parallel lines, copying angles, perpendicular lines). They have also summarized their work on these basic constructions prior to today's lesson. After today's lesson, they will construct points equidistant from three given lines, tangents, perpendiculars through a point on a line, other constructions, and transforming figures. The progression of the lessons is thoughtful; the students progress though constructions as more considerations are needed. They construct objects that are equidistant from one point, then two points, then three points. At this point, they move to lines; they construct objects equidistant from a single line, two given lines, and then three given lines. There is also a review day during the lessons and opportunities to engage with more complex constructions at the end of the unit.

This is topic seven of ten where the topics include:

1. *Construction of regular hexagons*
2. *Set of points that are equidistant from* a given point.
3. *Set of points that are equidistant from* two given points *(perpendicular bisector)*
4. *Set of points that are equidistant from* three given points.
5. *Set of points that are equidistant from* a given line *(construction of parallel lines,*

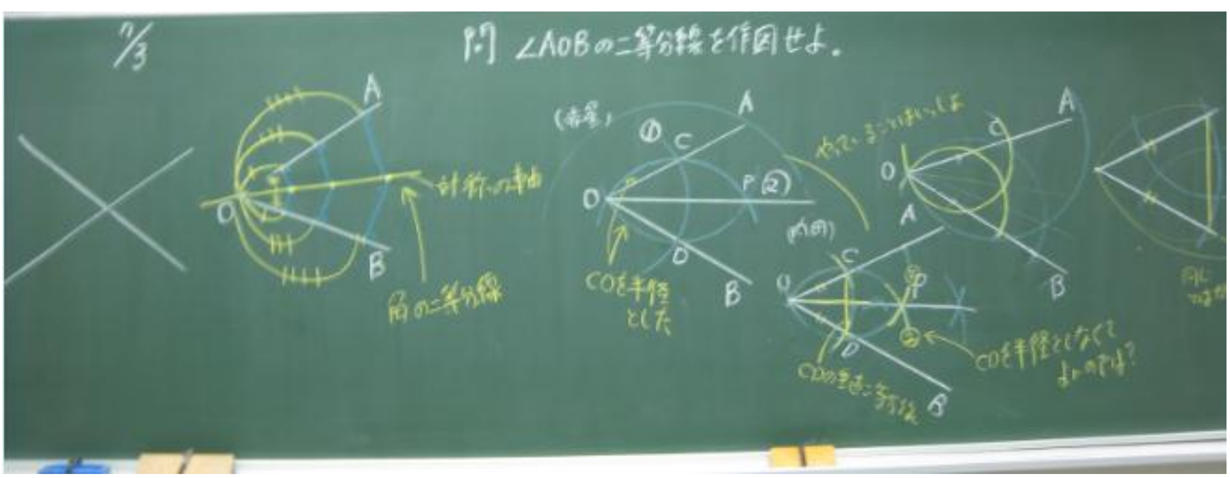
transformation of angles, construction of perpendicular lines)

6. *Consolidation of basic construction (1) (Basic construction, organizing the terms)*
7. ***Set of points that are equidistant from* a pair of given lines *(construction of bisectors of angles)***
8. *Set of points that are equidistant from the three given lines; construction of perpendicular line that passes through a point on the line; Construction of tangents*
9. *Various constructions*
10. *Transformations of figures*

<u>Summary of the Lesson</u>

This 58-minute lesson began with the formal greeting: students standing, greeting the teacher, and welcoming the lesson with a bow. The teacher, Mr. Kabasawa, took 13 minutes to introduce and motivate the lesson. The introduction began with a quick reminder of what constructions were done in the unit prior to today. Next, Mr. Kabasawa said that yesterday they discussed what the set of points that was equidistant from two lines might look like. They talked about two possible conditions: the two lines could be intersecting or not (i.e., parallel). Today they were going to focus only on the set of points equidistant from the rays of angle AOB. Mr. Kabasawa then asked his students what the set of points equidistant from rays OA and OB would look like. A student came up and sketched an angle bisector. Mr. Kabasawa finished off introducing the task by asking the students to construct the bisector of an angle AOB. Students worked on the task for 10 minutes before Mr. Kabasawa called for a discussion. During the 30-minute discussion, four student strategies were discussed. Toward the end of the 30 minutes, there was discussion about whether or not two of the strategies were actually the same. Mr. Kabasawa closed the lesson by saying that they would continue discussing these strategies and others.

The final board at the end of the lesson looked like this:



<table>
<tr><th>Start &End Time</th><th>Lesson Phase</th><th>Notes</th></tr>
<tr><td></td><td>1. Introduction, Posing Task</td><td>-Strategies to build interest or connect to prior knowledge [about 13 minutes]

2:20 Lesson begins after the formal bow.

Mr. Kabasawa reminded the students that during the last lesson they summarized the first part of the unit. He asked the students what they discussed. The students recalled previous activities such as drawing points equidistant from two and then three points.

Next, Mr. Kabasawa asked the students what he said they would be discussing today. Students remembered that the discussion would be about the set of points that were equidistant from two lines. Students volunteered that the two lines could be either parallel or not parallel.

Mr. Kabasawa drew a picture.

Then Mr. Kabasawa said, “So instead of thinking about these two cases, we’re going to just be looking at this part,” and he pointed to the angle.

He drew ∠ AOB, and he told the students that they would be looking at the set of points that were equidistant from “these two lines”. Mr. Kabasawa reminded students that they already discussed what the set of points equidistant from two segments looked like. He asked, “Can somebody come up and draw that?”

A female student drew a line that approximated the angle bisector.

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They discuss if he meant a point or a line since he said it was a collection of points. Mr. Kabasawa asked, “So the line is a set of points equidistant and that’s what you were thinking?” She said yes. Next Mr. Kabasawa asked, “Can somebody draw segments equidistant from these two segments? How do we know these points are equidistant from the two segments?” He then asked if someone could come to the board and draw them.

Mr. Kabasawa asked, “Can you explain what you did?”

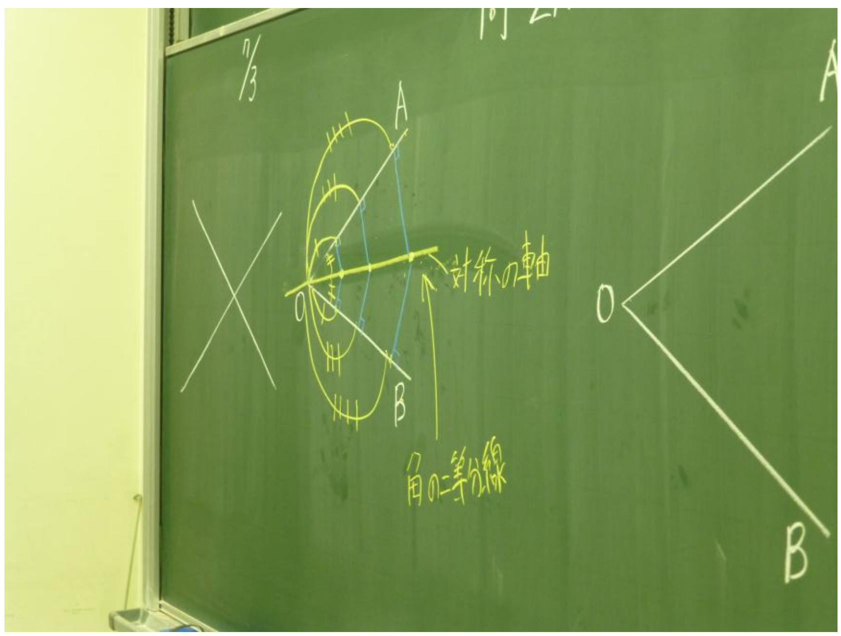
The student spoke about drawing points so that the line segments were perpendicular . The student said that the distance was equidistant from the two sides. Mr. Kabasawa checks to see if the students agreed. Mr. Kabasawa then asked the students if what they knew about the bisecting line.

2:27 A student said that the line was a collection of points that's equidistant from those two sides and as a result all of the angles are "the same."

Mr. Kabasawa asked, "Anybody else thought the same way? Can you raise your hands?"

A few students raised their hands.

Mr. Kabasawa then explained, "This line actually has a name. It's called the bisector of an angle." He then said that the points were equidistant from segments OA and OB. He asked, "But can you say anything more about this? What else can we say about this line? What do you notice about this line here?"



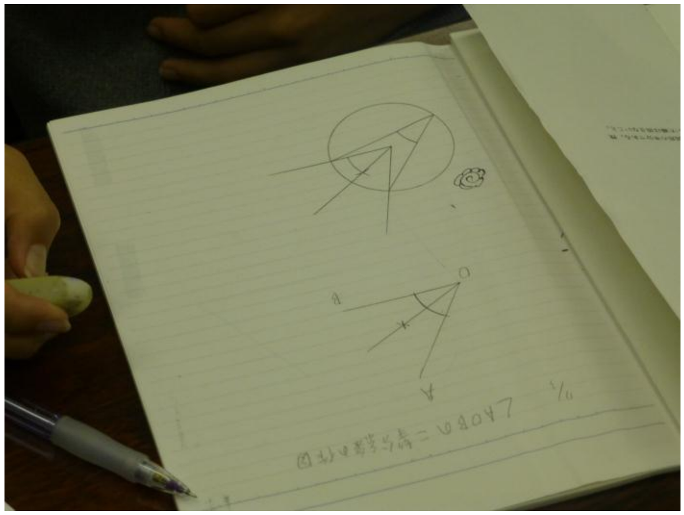
A student answered that it was an axis of reflection. After asking the student what he meant, the teacher said that it was an axis of reflective symmetry.

[This brief exchange about naming the axis of symmetry, and then subsequently defining the day's problem as constructing the bisector of the angle, became an important decision point in the lesson, one that the external commentator would ultimately argue was a serious misstep that launched students on an investigation that was considerably more procedural and mathematically less significant than it otherwise would have been, had the focus been on shape, on symmetry, and on perpendicular bisectors.]

		2:31 Mr. Kabasawa said, “Okay so today I’d like you to think about ways to bisect this angle. So first I’d like you to just draw an angle.” He then asked, “Can it be a right angle?,” and “Is this okay?” He drew a straight angle to have students consider the type of angle that they would draw. He asked, “Any other angles?” Responding to something a student said, Mr. Kabasawa asked, “The one that’s bigger than 90 degrees, like this one?,” and he drew a straight angle. Next Mr. Kabasawa said, “These angles, these angles are okay, but it might be easier to think about ways to do this construction if you draw an angle that’s less than 90 degrees. It might be necessary later to go back and look at other angles, but today just look at angles less than 90 degrees.” 2:33 Mr. Kabasawa wrote on the board, “Construct the bisector of an angle AOB.” He told them that after they found a way to construct an angle bisector, they should look at other ways. He reminded them that they talked about different kinds of constructions. After asking students if there were any questions, he told them, “Okay, please get started.” As students got started, Mr. Kabasawa asked if any student is having trouble with the compass or need a compass.
	2**. Independent Problem-Solving**	**-Individual, pairs, group, or combination of strategies?** **[about 10 minutes]** Students begin to look for individual strategies at 2:33. The following are student strategies described by our translator, Tad: * This student has three arcs inside an angle. * The third student from front looks like he has construction that looks like Method V. 

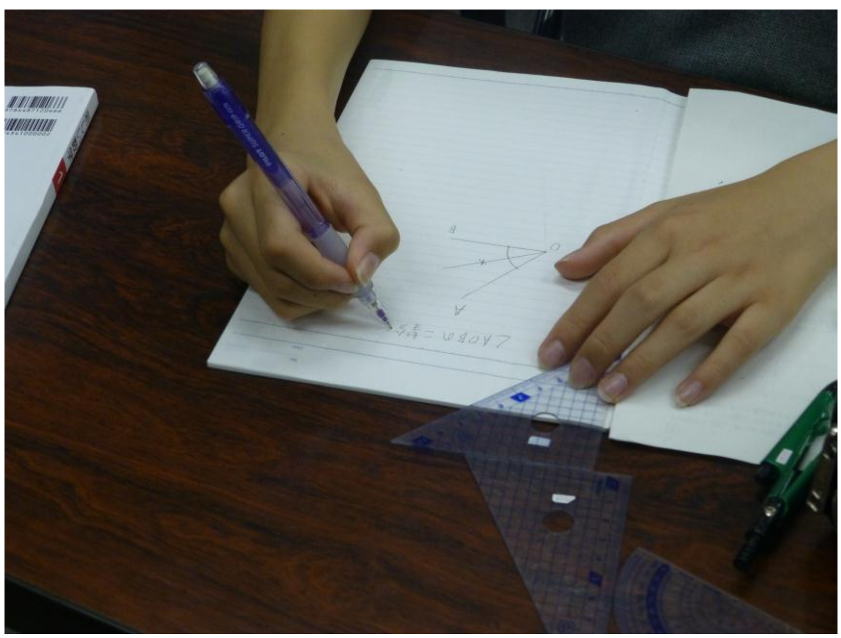
* The girl second from the front has an arc from the vertex of the angle and then connected two points to make isosceles triangle but seems stuck at this point.

2:37 A boy in front drew the bisector using Method II and drew another angle in his notebook but hasn't done anything else yet.

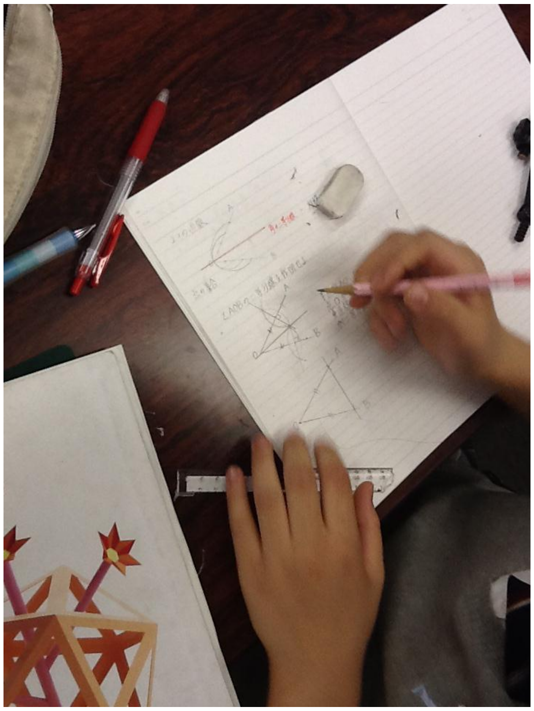


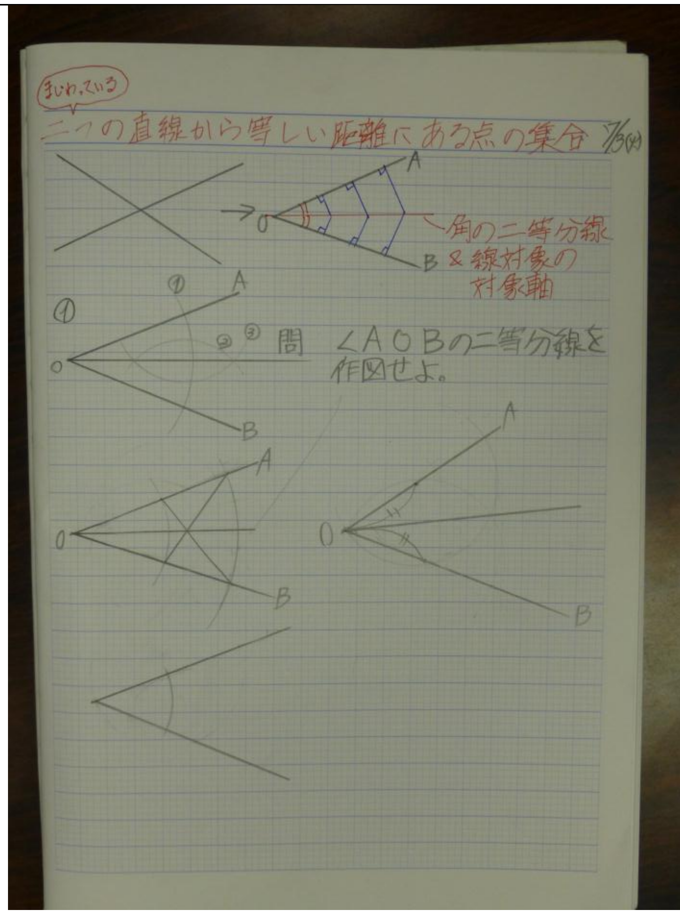
2:38 The girl second from the front was just measuring angle using protractor and said it was 36 degrees.

2:39 The girl with the jacket on has Method II.



The boy next to her, his drawing looks more like Method V, but he used distance from B_1 to the radius. It looks somewhere between Method II and V.

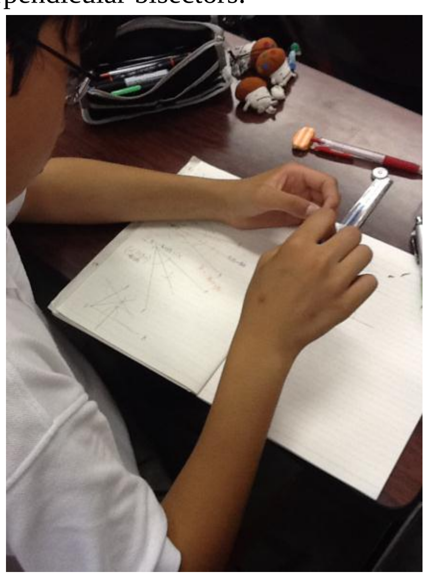
		2:40 A boy second from front, his method is Method V drawn with an isosceles triangle.  This girl with the gray vest did something like Method V and used B_1 to O as the radius. 2:41 This boy third from the front has Method V. He had written down the steps in addition to doing the actual construction. 2:42 The first girl from the front, she has taken two points that are equidistant from AB and then constructed the perpendicular. "I'm not sure if that method is on here." 2:43 Some boys in back have Method V using B_1 and O as the radius. This boy on my left has done three methods and is working on a fourth. He used Method II and then drew some perpendicular lines but they "are somewhat random." The third one is Method V using O and B_1. He drew an isosceles triangle. ---

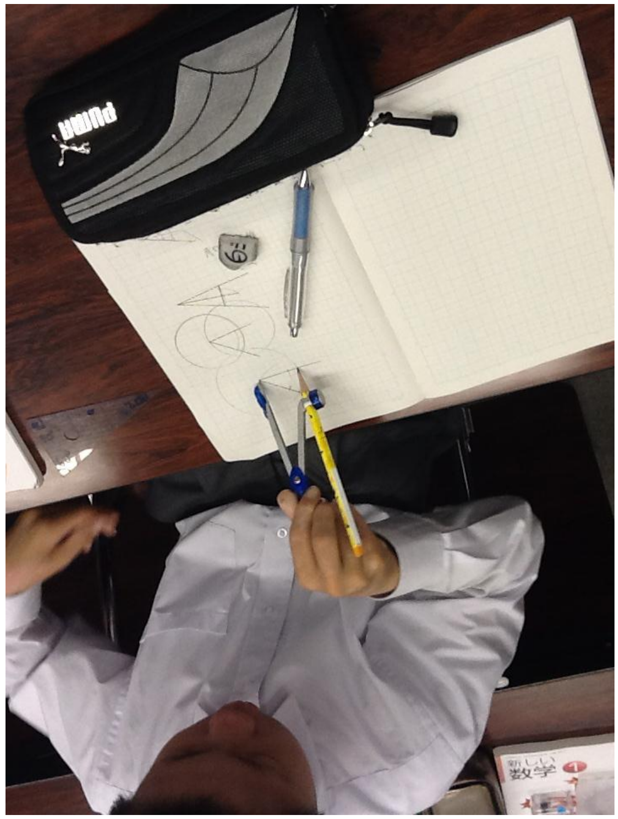


Here are some strategies observed by the team.

Phil's observations:

This one shows Method II and then an inaccurate strategy based off of perpendicular bisectors:

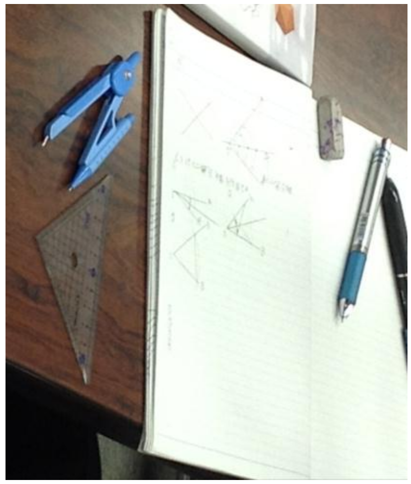
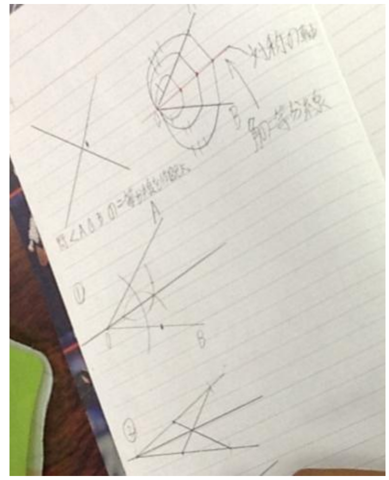
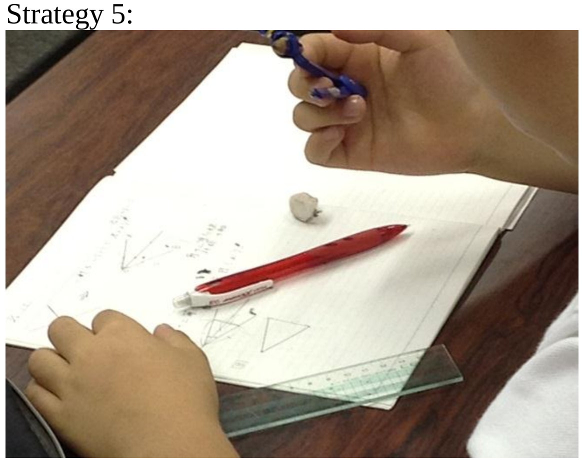


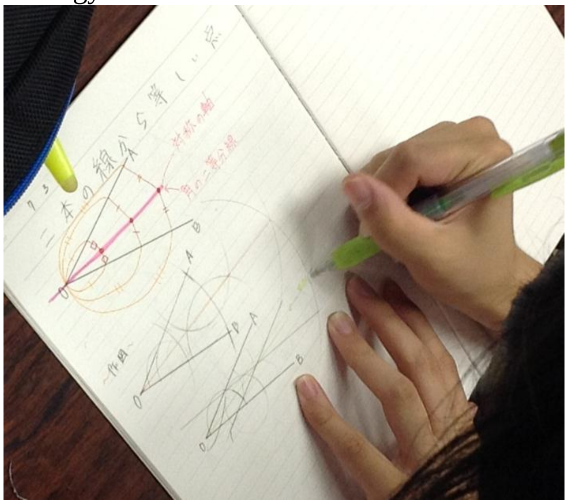
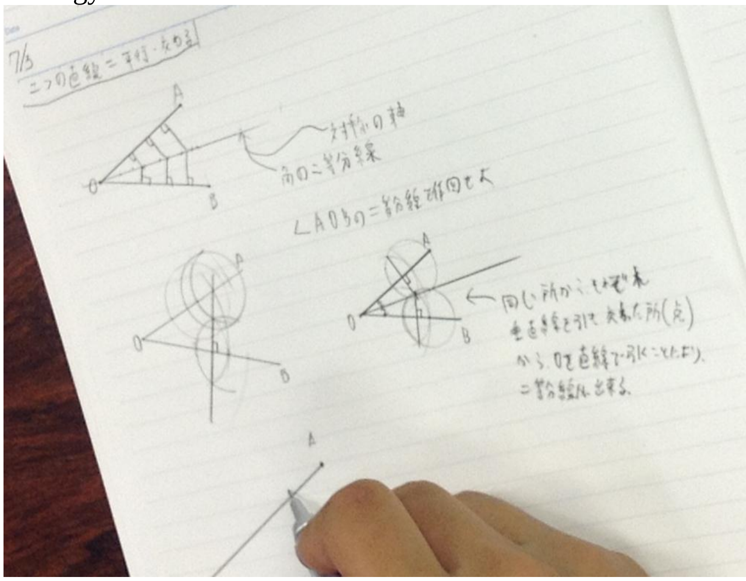
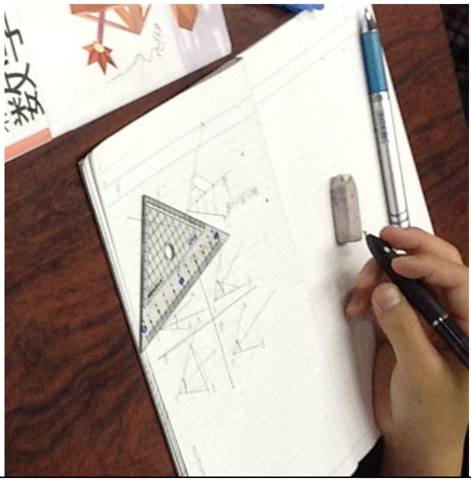


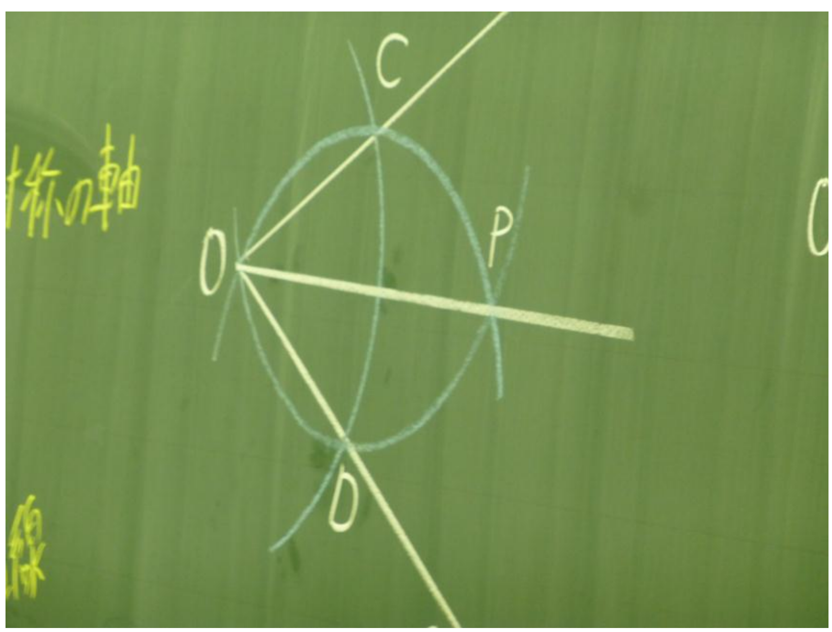
Kelly: I attempted to take pictures of every student’s notebook on the left side of the room, considered when facing the board. Based on the pictures, the following counts of construction strategies were determined. Note that many students had completed more than one possible construction.
Strategy 1: 4 students
Strategy 2: 8 students
Strategy 3: 0 students (If students did this approach, I may have listed it under Strategy 7.)
Strategy 4: 0 students
Strategy 5: 4 students
Strategy 6: 1 student
Strategy 7: 4 students
Other: 2 students (incorrect strategies such as drawing in a bisector with no construction action and using concentric circles)

Note: It is possible that some of the strategies were not categorized correctly. A number of the students did not clearly mark their constructions or the pencil was so light, it was difficult to see.

Strategy 1:

		  Strategy 2:  Strategy 5: 

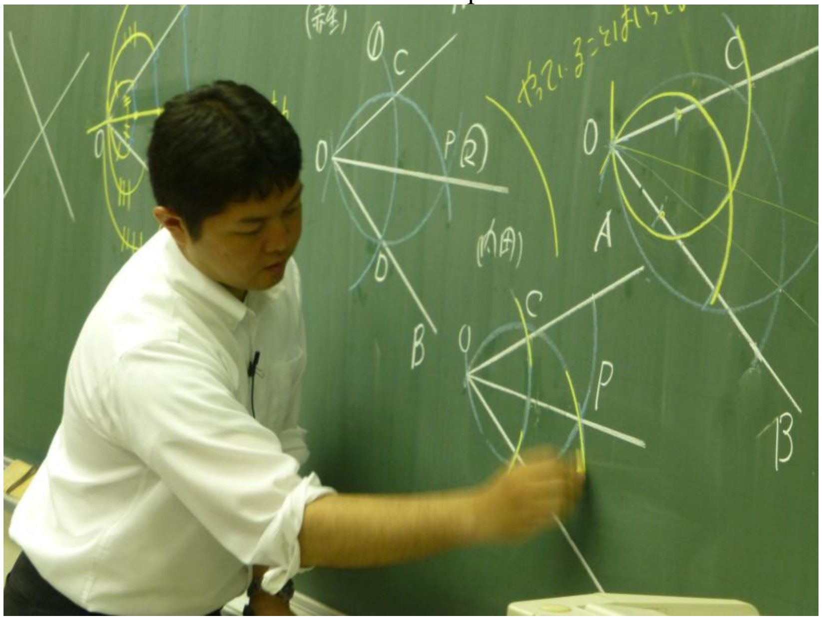
		Strategy 6:  Strategy 7:   Group of strategies:

	3.Presentation of Students' Thinking, Class Discussion	**Student Thinking / Visuals / Peer Responses /Teacher Responses [about 30 minutes]** 2:45 Mr. Kabasawa said, "Okay I think most of you have at least one way so let's think about these methods together. I don't think we can talk about all the methods today. I have a worksheet here for all the methods. As your friends [show] their methods, please try them on this sheet. I have plenty of angles here, so even if you make a mistake it's okay." 2:46 After Mr. Kabasawa asked students if they all have a worksheet, he asked a female student if she could please stand up and tell him about her method. He said that he was going to do the construction as she explained to him how to do it. The student explained her method. ***Strategy 1 (Female Student 1)*** Female Student 1: So first open the compass from O, centered at O and draw a line. Then from the point of intersection with OA using the same radius draw an arc this way. The student continued to describe something like this: "Draw a line that would go that point. If you do that, it will be the axis of symmetry." (Student did not name points of intersection; instead, the teacher added the labels.) Mr. Kabasawa asked, "How many of you used the same methods?" Many students raised their hands for Method II. (When polled, about 11 students on the left side of the room agreed that they had used a method like the first strategy provided. My counts from looking at their notebooks did not bear this out, but they could have done additional work after I passed their desks.) 

		Mr. Kabasawa noted that quite a few students used that method. One student said that it was theirs was a little different. They centered at C and D so the lengths were different. Mr. Kabasawa then called for other methods. ***Strategy 2 (Female Student 2)*** 2:51 Female Student 2: I did it a little bit different. I think we can omit a little bit of the steps than FS1 said. Well it's one less line. The student approaches the board, but Mr. Kabasawa said, "No, I want you to explain what you did." She asks, "Can I come up?" He said that she should explain it to him. After Mr. Kabasawa began a construction, FS2 said, no, that's not what I meant. She said, "Just open the compass at random. Put the chalk at O. Then put the compass down and draw an arc. So where the arc and OB intersect. So the point of intersection, no, it's not OB, it's OA. So this time put the center at C and match the other side to point O. Is that what I did? Can we change?" No, that's okay. That's okay. No, that doesn't work. Maybe I just lied. Okay that's not going to work (laughing)."
		Strategy 3(Male Student 1) Mr. Kabasawa then called on a student with his hand raised. The student said, "I think I understand what Miss M was trying to do. So if you start out with what Miss M said to do initially….put the center of the compass on any place on OA and then just draw on any point….move that other center of the compass onto side OB without changing the radius, the other way around…just keep the chalk at O then move the center onto side OB and then draw a circle centered at that point. Then were the two circles intersected, connect that to point O. 

Mr. Kabasawa asked the students if they thought his drawing reflected what the student meant. Mr. Kabasawa then asked students if they had any comments on the method, and asked one of the girls who if the method just described by the previous student was what she meant to explain.

2:57 FS 2 said, “So the first arc is not there” to which Mr. Kabasawa said, “ So in Miss M’s method there is no step one.”



Student: Well it’s still just as many things to do.
2:58 Mr. Kabasawa: Is that okay? Can we do this?
Student: We’re just copying the lengths.
Mr. Kabasawa: Who thinks it’s not okay?
Student: Well maybe we shouldn’t do it too often.
Mr. Kabasawa: So we took the random radius right? You just put the chalk at O, right?
Student: Inaudible.

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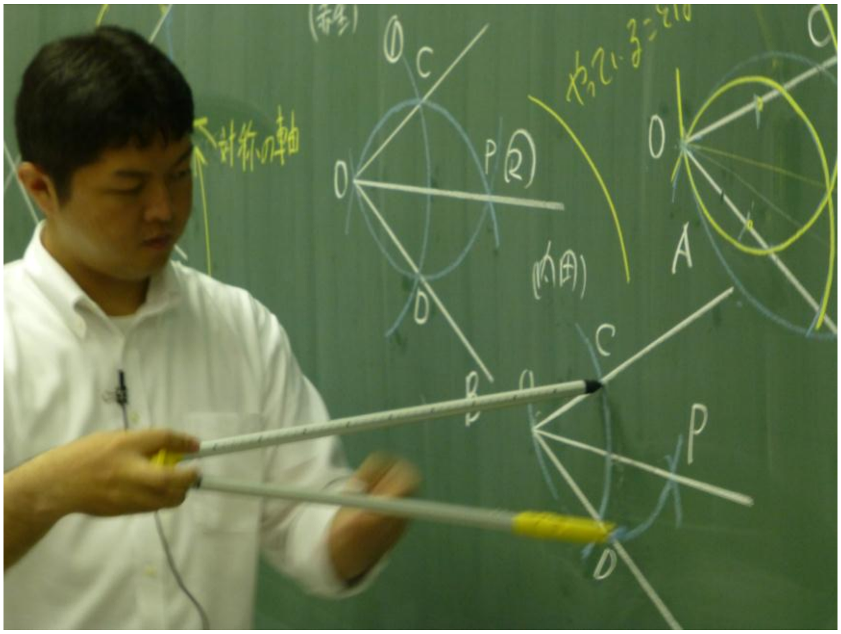
Mr. Kabasawa: So if we put the chalk here first, what is it doing?



Student: It’s like the needle.
Mr. Kabasawa: So when we put the chalk here, it’s acting like the center.
Student: But it’s not the center of the circle.
Mr. Kabasawa: But we are treating it like the center.
Student: Just use it as the center and then [inaudible].
Mr. Kabasawa: So it’s okay to measure this right?

Mr. Kabasawa: Doing like this, it’s like using the chalk as the center so it’s really like using the radius and marking the same distance on the size of angle. He didn’t want to draw the first line so it’s okay. So we don’t use all the chalk. I noticed some of you doing the same thing, but please don’t use the writing part as the center…Okay let’s look at another method.

Method 4 (Male Student 2)

3:02 Mr. Kabasawa drew a new angle. Male Student 2 said that he omitted something from the first one. He just made little marks at the intersections. MS2 said, “Just draw a little thing and do the same thing with the points of intersection.” Mr. Kabasawa commented that this part was the same method as the first one. He also asks students about it not just being reducing the steps.
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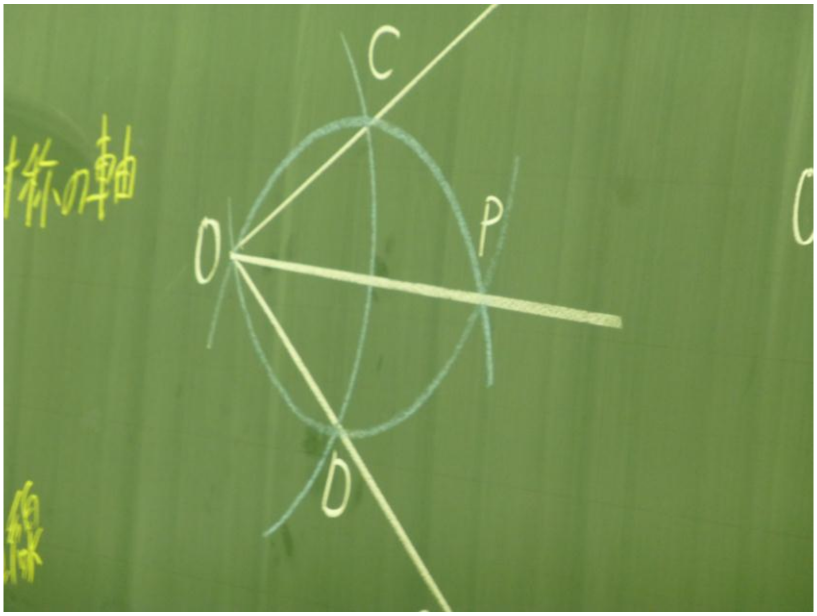


MS [not sure if same one who presented] said, “It’s not number of steps, instead of drawing a lot of parts, just to make it easier to see.
Mr. Kabasawa: Somebody else wanted to say something about this method? Some people are saying these two methods are the same. Are they the same?
FS1: He’s not drawing as much as I drew, but what he did is exactly what I did, so they’re the same. So if you say the same is this what you mean?

They continue discussing.

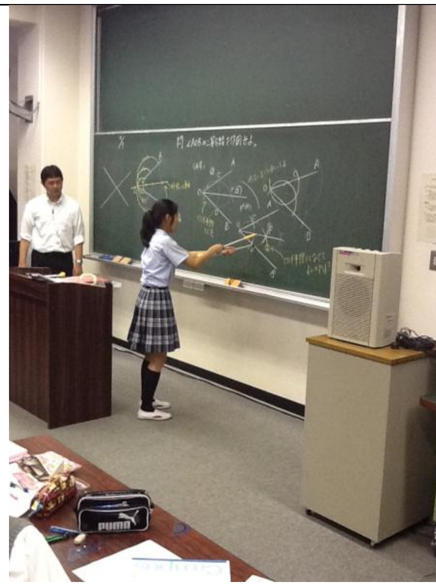
Student: “At C if you just keep the compass going and extend it like that and do the same thing at the center at D, so at D then you do the same thing, then you get the same picture as Miss O. So these two are the same.”

Mr. Kabasawa: So what Mr. W drew, I drew in yellow, but if you extend them then they’re the same.

They discuss what is similar and different about the two methods. A student makes a suggestion, and Mr. Kabasawa said, “Okay so come up and show us.”

Comparison of Strategies

A female student came up to draw, but she struggles with the compass.



FS: So if we made the radius longer, that point of intersection would be on the same line. Even if we made the radius shorter, like this and this, the range of pictures expands. So if we do it that way, Mr. W and Miss O's methods are not the same.

Mr. Kabasawa: What were you thinking Miss O?

Miss O: I didn't necessarily think about using the same radius. If we used the same one, it would be easier to go back later, But I think we can change the radius.

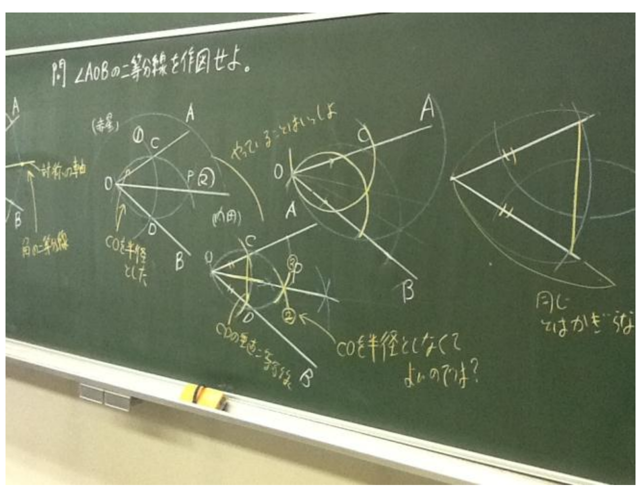
Mr. Kabasawa: Mr. W what were you thinking?

Mr. W: I wasn't thinking about the radius either. I was thinking about less complication.

Student: If you draw the line then draw the arcs, they intersect. [translation issue]

Mr. Kabasawa: I think I missed what you said.

Student: So if you set the radius, then they will intersect with the angle bisector.



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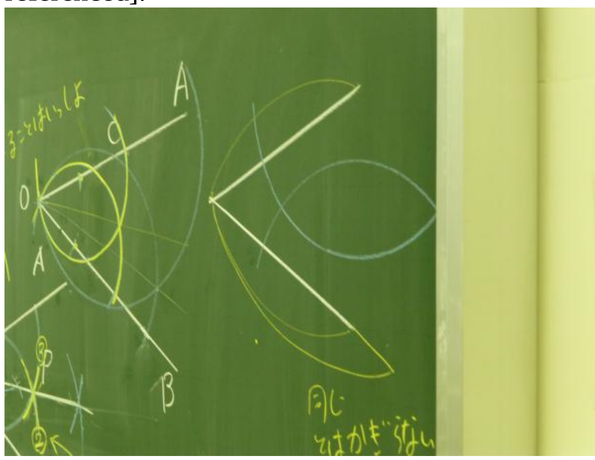
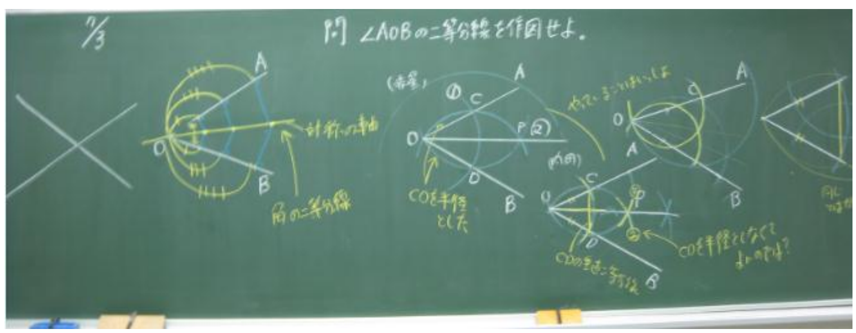
<td>3:14 Mr. Kabasawa continued the discussion for a few minutes asking about changing the radius. He then said that the time is up, but he invited one more student who had something to add to share her thinking.

FS: I think we can do it easier. I think we could use A.
Mr. Kabasawa: Where is A?

The student explained her thinking, and Mr. Kabasawa said that she was thinking about something that could overcome this [not sure what this referenced].


A student answered, “If we draw C and D…, then we have a triangle there. That triangle is an isosceles triangle, then the line in the middle is the angle bisector or the isosceles triangle is the reflective symmetry so that would be the line of symmetry.

Mr. Kabasawa wrote something on the board about the perpendicular bisector of BD. They talked about that strategy before closing the lesson.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<td>4.Summary /Consolidation of Knowledge</td>
<td>Strategies to support consolidation, e.g., blackboard writing, class discussion, math journals. [5 minutes]

About 5 minutes before class ended, the teacher tried to end the class by saying that they were out of time, but one more student wanted to speak so the teacher allowed the student to speak.</td>
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		Okay, so we are already get over the time. We'll think about these methods the next time and more the next time. Okay so you think these three methods are the same or different? St: The same. 3:20 Okay, so we think these are the same. We'll think more about them the next time.

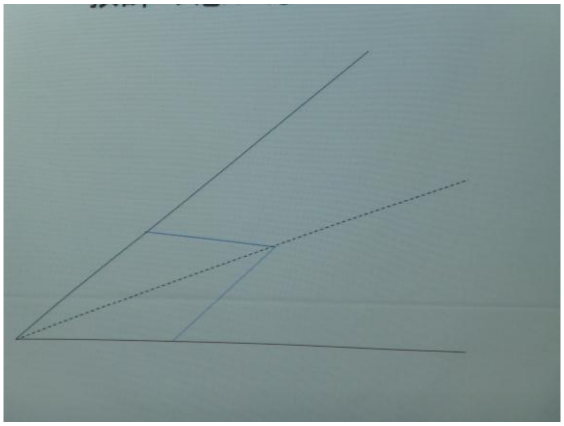
1. **What new insights did you gain about mathematics or pedagogy from *the debriefing and group discussion* of the lesson?**

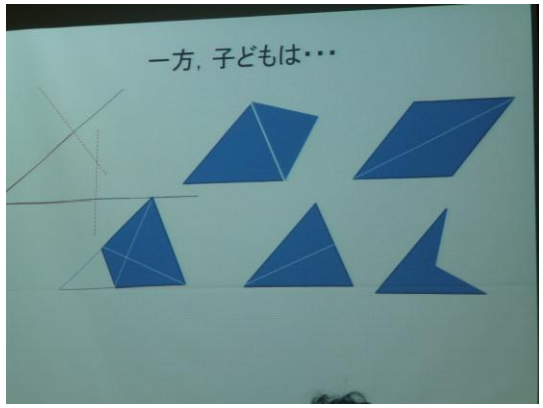
Michelle: I was intrigued by Mr. Kabasawa's decision to have the students describe their strategy while he constructed them on the board. We have no way to know what his intention was, but one consequence of that was that students were then forced to use verbal language to present their thinking. This could support their acquisition and use of mathematical language.

Kelly: It was interesting to me that the students seemed already familiar with the properties of triangles and quadrilaterals, properties that might help them explain why the constructions were appropriate (e.g., in an isosceles triangle, the perpendicular bisector is also the angle bisector of the vertex angle). In Georgia, we take an opposite approach. The students learn the constructions and later they learn, formally, the properties of the triangles and quadrilaterals. It appears that there is a greater emphasis on informal reasoning and proof in the lower grades, with the learning trajectory clearly pointing towards doing the proving when the students are more developmentally ready. We could take notes from this idea – it's okay to introduce topics in early grades but to put off formal proof until more developmentally-appropriate grade levels. On a constructive criticism note, it occurred to me that invoking the idea of equidistant from two lines would have lead naturally to Strategy 7, so it might have made sense to begin the discussion with that strategy. That might have placated the participant who asked how the constructions explored related to the opening of the lesson.

Phil: The recommendation from Prof Nishimura in the post-lesson discussion was to rework the written goal of the lesson slightly from focusing on the angle bisector to working to find the line of symmetry (i.e. the collection of points equidistant from the two rays that make up the angle). The mathematics that surfaced during the lesson was criticized during the post-lesson discussion as insufficiently narrow and procedural in nature.

If students and the teacher are to do mathematics together (Level 3 teaching) then the decisions at every point of the lesson need to be made in the service of this goal. During the lesson, the teacher worked so hard to let students lead him through their thinking. However there were a few key turning points in the lesson that resulted in the conversation going where it did. The posing of the problem was one such important moment, where perhaps not introducing the angle bisector but rather focusing on symmetry and shape would have led to a richer exploration of the axis of symmetry. Here are some of the shapes that might have been presented:





The students had defined the problem to be to find the line of symmetry, when immediately after the teacher gave that line a name: angle bisector, and stated that this was the question central to the lesson. In retrospect, this may have been an significant determinant in the mathematical discussion that followed individual student work time.

2. **What new insights did you gain about how administrators can support teachers to do lesson study?**

Michelle and Kelly did not have comments here.
Phil: This research lesson may end up being one that has significant impact on the research team. Following the post-lesson debrief, the research team stayed after for about an hour of further discussion, with Profs Fuji and Akihashi, I believe. Phil Daro asked Professor Fuji the topic of discussion and the response was that it was, “very serious.” The conversation seemed serious. My understanding, third hand, was that the research lesson (and debrief?) were not up to the expected level of quality that the university partners would expect.

So, consistent with what we saw in several public lessons, we are seeing that there are multiple audiences and purposes in any particular lesson study. And in this case, the administrators presumably play an important role in creating a place for conversation to occur – and then follow up to happen – so that the relationship between university partner and faculty stays vibrant and satisfactory.

3. **How does this lesson contribute to our understanding of high-impact practices? (See below)**

(a) Helping students make their mathematical thinking visible and compare it to classmates’ thinking;

Phil: This might be an example where we can say there’s more to learn from what we didn’t see during the lesson than from what we did. We didn't see the vast majority of students with solid solutions; we didn't see an abundance of alternate methods; we didn't see many students participating in the mathematical discussion (only 5-6, by some of our counts). However, the teacher was a relentless listener to students' thinking, gently prodding there thinking by saying: 'is this what you mean?" as he moved the compass and chalk at the board. His insistence that students explain their thinking from their seats served as a nice counter to what we often do in US classrooms, and did in fact, increase the precision of students' directions, as well as a keep a blackboard organized as the teacher wanted.

(b) Anticipating students' responses and using them to strategically plan in advance for the discussion;

Michelle: At the PLD, Mr. Kabasawa discussed his anticipated strategies. He said that he expected additional strategies to surface, for example, the use of parallel lines, but that's not what happened. Because he had anticipated multiple strategies in advance, Mr. Kabasawa seemed ready to deal with the students' strategies that surfaced. He said that what he does in the next class will be quite important because he did not cover the point of conclusion that he had wanted to. He seemed as though he might be surprised that a bigger variety of strategies was not used. Mr. Kabasawa also mentioned in the PLD that he had anticipated that students would draw a set of points rather than a line when he asked them what the set of points equidistant from two lines would look like. One might also wonder if he was out of touch with the potential strategies that students might use since students did not use some of the more complex methods that he anticipated. In addition, it did not seem that Mr. Kabasawa had selected and sequenced student presentations so that multiple *different* strategies surfaced during the discussion.

Phil: as described above, the student response analysis may have been undermined with the focus on angle bisectors when students were more ready to explore symmetry within the possible shapes.

(c) Helping students link representations (words, pictures, diagrams, mathematical expressions);

Kelly: The teacher clearly wanted his students to be able to communicate their ideas about constructions, their diagrams, into words. He was insistent on the students explaining the steps to their constructions in clear detail so that he could recreate the construction on the board. Although this strategy might be said to be for the speaking student's benefit, it was also for the benefit of the students who had not come up with the particular construction. All students were to be recreating the constructions on their own paper. The best way for them to accomplish this was for the fellow student whose construction it was to clearly communicate the steps to the construction to the teacher, and thereby, to the class.

Also, at the beginning of the lesson, the teacher drew attention to the words "equidistant from two lines" and required students to determine what that meant in terms of a diagram. A huge component of this lesson seemed to be linking the diagram and language representations.

(d) Planning for the development of mathematical concepts over multiple lessons, units, and grades;

Kelly: The overview of the unit plan shows a clear development of the skills needed to prepare students for construction of angle bisectors. The students have had significant practice with using compasses in this course and the constructions in which they have previously engaged have increased in complexity each lesson. See above for more on the progression of the unit and the place of the lessons. This unit is situated within a larger picture of constructions that began in grade 3 and continues into the upper secondary courses.

In grade 3, students begin laying the foundation for this lesson. They discuss the definition of a circle and begin using a compass to construct circles and measure distances. They also investigate triangles using circles and construct isosceles and equilateral triangles using compass and straightedge. In grade 5, the students construct congruent triangles using a compass and straightedge. Then they determine the minimum criteria needed to draw congruent triangles: (1) corresponding sides with included angles congruent, (2) corresponding angles with included sides congruent, or (3) corresponding sides all congruent. The focus in grade 5 is on informal understanding, not proof. Proving the triangle congruency theorem and other facts about triangles and other shapes is not taught until grade 8. However, as we see in this lesson, students should know sufficient properties about triangles, quadrilaterals, and circles to intuitively reason through why a construction might work.

It would seem that the students should have a robust understanding of the merits of compass constructions and the power of the circle to construct "equidistant" situations. In the "development" section of the lesson plan is a statement that the students should be able to say that the correct construction methods are correct because they all use congruent triangles (or other figures).

As the final commentator mentioned, this lesson may help lay the foundation for topics in the upper secondary school, namely construction of the conic sections. The commentator also mentioned that the teacher chose an arrangement of topics that deviated from the text's arrangement and that arrangement was not well-thought out or appropriate.

(e) Using blackboard and journals to promote student meta-cognition, reflection, and integration of mathematical ideas;

It may have helped if all students had to take a pass at writing up their constructions. Simply stating the procedure for the construction might have truncated the "are they different?" conversation – more appropriately. The board drawings represent the detailed discussions that a few students had during the lesson, but not the mathematical diversity of ideas that the lesson was supposed to bring forth.

(f) Encouraging students' sense of commitment, interest, and capacity to solve challenging mathematics problems.

Lesson Observation (6)

Research Lesson Observation Form

Lesson: Calculation of Expressions with Square Roots
School: Sengen Lower Secondar School
Grade: 9
Date: July 2, 2012
Names: Belinda Edwards, Tom McDougal, Courtney Ortega, Elizabeth Torres, Colleen Vale, Geoffrey Wake

What are the primary lesson goals?
Students will discover and understands ways to multiply square roots.
Where is the lesson located within the unit (in relation to previously studied topics and ideas to be studied in the future)?
Lesson 1 in a unit of 8 lessons

Start &End Time	**Lesson Phase**	**Notes**
1:30 – 1:46	**1. Introduction, Posing Task**	**[Task] What is the expression to calculate the area of the rectangle with length $\sqrt{2}$ and width $\sqrt{5}$?** T: You are a little bit excited today. Please open your note books. This is the first lesson in July. We're going to do calculation with square roots. BB T writes the heading – Multiplication and Divisions including the square roots. T: We've been looking at square roots and how big they are. Starting today we're going to look at the calculation. BB T writes the goal of the lesson: What happens with the product of square roots? T: Think about ways to find the product. Here's what I want you to think about. BB – pastes the problem on the board

		 T: What’s the area of a rectangle with width root 2 and length root 5? (posts text description of problem) [A student reads the problem aloud. All students write the topic and goal in their notebooks.] T: Let’s make sure we understand. Can you develop the image of this problem? The width is longer right? (Boy toward the back draws rectangle in the air.) (T draws rectangle with labeled lengths.) I’m going to ask many of you to answer. T: So you all know how to calculate the area, so write down an expression for this. S: √2 x √5 T (writes on board) √2 x √5. So what happens to the result of the calculation? T: Can you make prediction of what would happen? S(C3): √10 √10 — but I’m not sure. T: We’re just predicting. He thought maybe √10. T asks the class to indicate their thinking: Is it root 10? Raise your hand. (About half the class raised their hand.) Is it not root 10? (No hands are raised.) T: Why do you say it’s root 10? S(C3): Because it’s 2 x 5 T: Root 2 is not 2 right? Root 5 is not 5 right? So let’s think about this more. 2 and 5 are inside. T: What if we had different numbers inside? What about √4 x √9 ? (writes) S1: 6 — S2: 2 x 3 T records: √4 x √9 = 2 x 3 = 6 = √_ T: Will you use the square root symbol with 6? Think carefully. Can you express 6 with the radical symbol? T writes on board: √36 (did a student suggest anything?) T: So 4x9 = 36, √4x√9 =√(4x9), but √2 & √5 are different. What do we call these numbers? Irrational. It looks like we can multiply √2 and √5.

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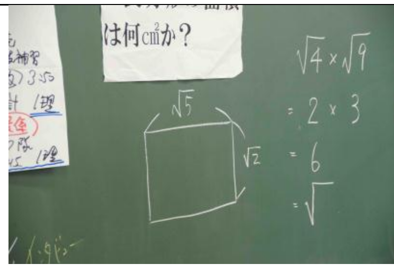
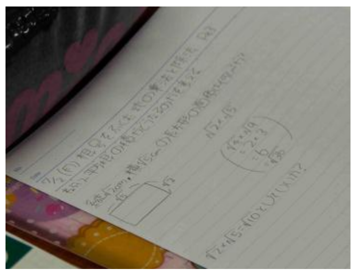
<td>

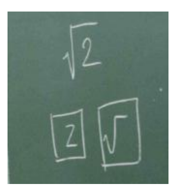
We are going to verify more carefully. √2 times √5 equals √10 Is that really the case? √2 and √5 are different types of numbers than 2 and 3

I'm going to pass out the calculators. Please use the calculators to verify. I'm not going to give any details, but explain why it is okay or not okay to say √2x√5=√10.

Ss have copied board into their notebooks.

</td>
</tr>
<tr>
<td>1:46-
1:49

1:49-
2:00p
m</td>
<td>2.
Independent Problem-Solving</td>
<td>The teacher asks the students to make sure they understand and can develop an image of the problem. "You know how to calculate area? Write down the expression and determine the area of the rectangle with length $\sqrt{2}$ and width$\sqrt{5}$".

The teacher asks the students to think about the results of this calculation. "Can you make a prediction of what will happen?"
Student replies, "square root of 10. I'm not sure, but…"
Teacher: "Is it okay to say $\sqrt{2}$ x $\sqrt{5}$ = $\sqrt{10}$?
Let's verify formally. Use the calculator to check to see if $\sqrt{2} \times \sqrt{5} = \sqrt{10}$.
The teacher hands out calculators and reminds students how to enter √ into the calculator using the correct key strokes. Initially many students had forgotten.

Teacher roams between desks. The teacher urges, "Just write down what you did in your notebooks."</td>
</tr>
</table>

Students begin to work independently at their desks, using the calculator to verify the product of the square roots. Some students begin using the calculator to find the values of $\sqrt{2}$, $\sqrt{5}$ individually and then take the product and compare it to the calculator's value of $\sqrt{10}$.



As the students work independently at their desks,—several students erase their work. It appeared that some students were uncertain placing a question mark after the equals sign. A few were looking through their notebook. Many are up out of their seats to consult with a neighbor. Many are turning around to check answers or share calculator results.

T to a student (A3): How do we enter square roots? So what did you calculate? What are you comparing it to? What are you trying to get?
S (A3): Oh the $\sqrt{10}$. I understand.

Another S: Do we have to use it in the usual way?
T: Just write it down in your notebook.

2 girls kept calculating $\sqrt{2}$ times $\sqrt{5}$ then $\sqrt{10}$ to verify if they get the same number.
Another student is writing decimal approximations of all the numbers.

T to a student: Where did you get these numbers?
S shows the T her calculator.
T: I'm going to ask you to write on the blackboard in a few minutes.

S (A3 to A2): How am I going to explain why it works? What should I say?
S (A2) rewrote her journal to say the same thing as S (A3).



<table>
<tr>
<td>2:00-
2:20</td>
<td>3.Presentation of Students' Thinking, Class Discussion</td>
<td>
The teacher brings the class back together and asks for a student to present their solution at the board.
The students writes $\sqrt{2} \times \sqrt{5} =$ and then writes the decimal equivalent from the calculator 3.1622777 and then writes, $= \sqrt{10}$. The student explains that she squared $\sqrt{2} \times \sqrt{5}$ and then used the calculator to find $\sqrt{10}$
 
The teacher restates the values for $\sqrt{2} = 1.41\ and\ \sqrt{5} = 2.4$ and their product is 10.
The teacher explains: $\sqrt{2}$ is not the same as 2. If we use the calculator we can see this. What is this called? The teacher answers her own question—approximate value. The teacher explains, "you are using the approximate value. We don't really know…they are approximate values".
Teacher explains, "The calculator does rounding so we don't know the exact value. We need to check it for truth".
Teacher writes the equation $(\sqrt{2} \times \sqrt{5})^2 = (\sqrt{2} \times \sqrt{5}) \times (\sqrt{2} \times \sqrt{5})$ on the board and begins to work through the steps on the board.

Teacher: "Square the left side by rearranging the right side".
$$\begin{aligned}(\sqrt{2} \times \sqrt{5})^2 &= (\sqrt{2} \times \sqrt{5}) \times (\sqrt{2} \times \sqrt{5}) \\ &= (\sqrt{2} \times \sqrt{2} \times \sqrt{5} \times \sqrt{5}) \\ &= (\sqrt{2})^2 \times (\sqrt{5})^2 \\ &= 2 \times 5 \\ &= 10\end{aligned}$$
Teacher: "What happens when you square a square root?"
"We didn't use any approximations here so we get the exact amount.

BB: $A^2 = 10$

T: If the answer after squaring this is 10, then what did it look like before?

BB: A=$\pm\sqrt{10}$

T: Is this negative or positive? How do you know?

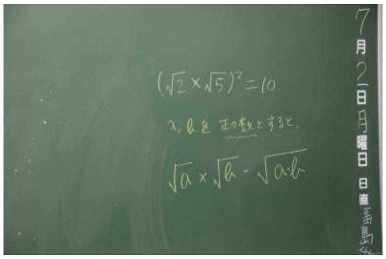
S: It is positive because you multiplied it twice.
</td>
</tr>
</table>

T: Let's go back to $\sqrt{2} \times \sqrt{5} = \sqrt{10}$. "Those of you who think that this is right, raise your hand."
S: Almost everyone's hand goes up.

BB: $\sqrt{3} \times \sqrt{5} = \sqrt{15}$
T: Is this right?

Student sitting in the middle right shakes his head yes and says, "We did the same thing, $\sqrt{2} \times \sqrt{5}$ must be $\sqrt{15}$"

BB: $\sqrt{a} \times \sqrt{b}$.
T: $\sqrt{a}$ is a positive number, and $\sqrt{b}$ is a positive number. Therefore, these first ones are positive numbers. They are inside of the square roots.



Two girls sitting in the back left stare at the board. They both copy down the notes exactly how they are on the board. One student writes $\sqrt{ab}$ (?)
Teacher writes the following questions on the board:

1) $\sqrt{3} \times \sqrt{7}$
2) $\sqrt{5} \times (-\sqrt{6})$
3) $\sqrt{2} \times \sqrt{8}$

Girl sitting in the back left, switches from writing in a pencil to a pen. Then she proceeds to copy the teacher's problems from the board. She completely answers all the questions in less than one minute. After she is done, she turns around and shows her answers to her friend. They discuss, then the friend copies the answer to the last question.

		   Teacher begins to write the answers on the board as she orally explains them in brief. T: I’m going to ask you to share your results. Look at the inside of the radical signs. It’s really just the product. S: √21 T: Be careful with the next one. If you have a positive times a negative, what’s your result? Pay attention to your sign. What’ a positive times a negative? S: Negative. - √30 [bell rings – students start packing up] T: Last one S: √16 T: Remember it’s a rational number. You can take it out of the radical sign
2:20	**4.Summary /Consolidation of Knowledge**	Teacher reminds students again that when you are calculating the product of square roots, you can just multiply the numbers inside the radical sign and the answer to the problem is the square root of the product. She tells students that in the next lesson they will work on division.

What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

The debriefing and group discussion of the lesson was well organized. The teachers were asked to organize their feedback using post-it notes. They were asked to provide their positive feedback on blue notes, negative feedback on pink notes, and improvement measures on yellow notes. This method clearly provides the classroom teacher with feedback that she can refer to as she reflects on her lesson. The lead teacher indicated that there would be six factors based on the three categorizations: 1. Understanding and development, 2. Lesson skills, 3. Instruction and Assessment, 4. Leadership, 5. Passion/commitment/sensitivity, 6. Understanding the students. I believe that one of the goals of the school is to produce teacher leaders. With this in mind, I think it’s critically important that the team leader be fair and honest when providing feedback to the teacher and the expectations of each teacher on the team should be high. When the teacher was asked, “why did you select $\sqrt{2}, \sqrt{5}$ when introducing the concept of the product of the two irrational numbers”? The teacher replied, “Well those were the numbers in the textbook”. I think the team leader should have probed her more so that she states her answer in terms of student understanding and development since that was one of the six factors resulting from the categorization of feedback.
The role of the teacher leader is important. The leader had to provide commentary for the teacher

while connecting what was written on the post-it notes to improving the overall lesson. I don't think the team leader probed her to the point that she believed that she could have done a better job. I think it's very important that we provide positive feedback to teachers; but the feedback needs to be constructive and convincing. Constructive feedback should make the teacher somewhat uncomfortable with their teaching to the point where he/she begins to immediately reflect on their teaching as it relates to student development and understanding or, more specifically, the six factors discussed in the debriefing.

What new insights did you gain about how administrators can support teachers to do lesson study?
Administrators can participate in the post lesson discussion by organizing the feedback.

How does this lesson contribute to our understanding of high-impact practices?
The students did not participate very much in this lesson. Students were able to make their mathematical thinking visible during the independent work session and presentation of solutions. The teacher was able to anticipate students' responses and provided probing questions for the students. Students were able to link mathematical representations to area.

Lesson Observation (7)

Division with Remainders
Planning Team: Honobe, Koh; Seki, Satoe; Arashi, Genshu
Hashido School
July 4, 2012

Observation Team: Veronica Chavez, Debbie Brown, Lorelei Nadel, Josh Lerner, Lisa Lam

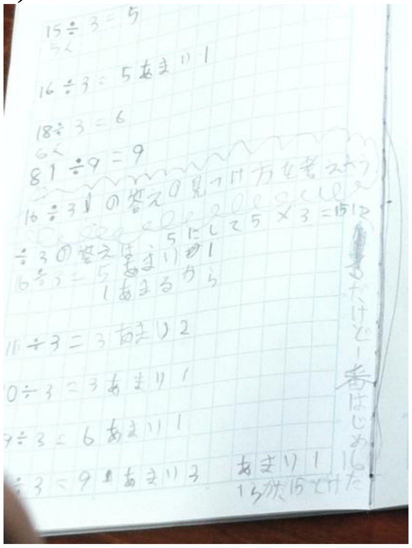
What are the primary lesson goals?

- Students will think about ways to find answers for division situations with remainders and explain their methods in their notebooks.
- Students will understand that they can also use the basic multiplication facts even when there are remainders.

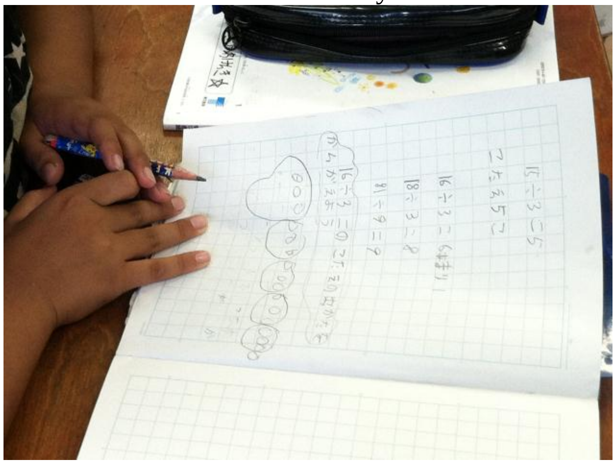
Where is the lesson located within the unit?

0	Understand and be able to explain how to find answers for division without remainders (review of prior learning)
1	Calculate division with remainders using concrete materials (counters)
2	Think about ways to find the answers for division with remainders and explain them in notebooks. Understand that the answers for division with remainders can be found by using the basic multiplication facts.
3	Understand relationship between divisor and remainders.
4	Understand the division operation can be applied to partitive (fair sharing) situations.
5	Understand how to check the results of calculation for division with remainders.
6	Practice calculations.
7, 8	Understand problem situations and write and calculate the appropriate expressions.
9, 10	Application, consolidation

Start and End Time	Lesson Phases
8 min	Introduction, Posing Task The teacher started by building on prior knowledge from the previous day. In yesterday's lesson, they used 14 divided 3 and 20 divided by 3. After giving a variety of expressions, the teacher picked 16 divided by 3 and instructed students to use any method to find the answer. Before allowing the students to begin, the teacher explained that they would not be using counters today, and elicited responses for how the students could solve without using counters. Writing numbers and drawing pictures were given as possible methods. The students began their work.
9 min	Independent Problem-Solving The following are solution methods we saw in student notebooks during independent problem-solving.

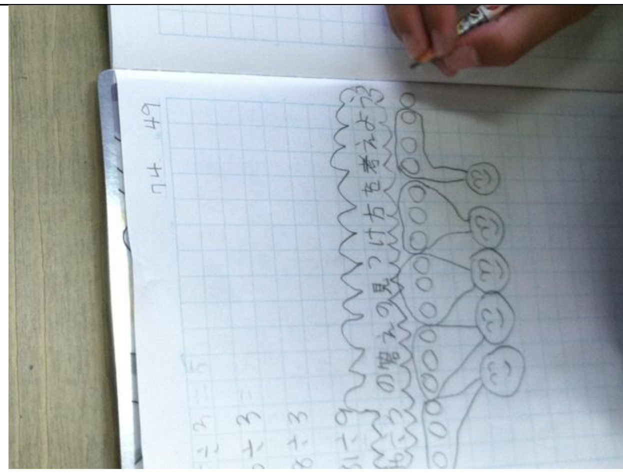
<table>
<tr>
<td></td>
<td>a) 16 ÷ 3 = 5 remainder 1


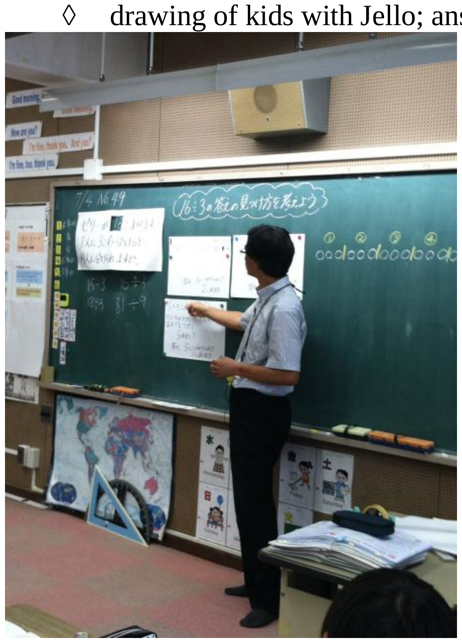
b) 3 x ☐ = 16

c) Draw groups of circles (or other images) and find how many groups of circles there were and how many circles were left over.

d) A line of squares, broken up into groups of 3, with one square left over.

e) Break circles into groups of 3 in different formations.

f) Happy faces with groups of three attached to each face.</td>
</tr>
</table>

<table>
<tr><td></td><td></td></tr>
<tr><td>7 min [presentation of student thinking]</td><td>Presentation of Student Thinking, Class Discussion

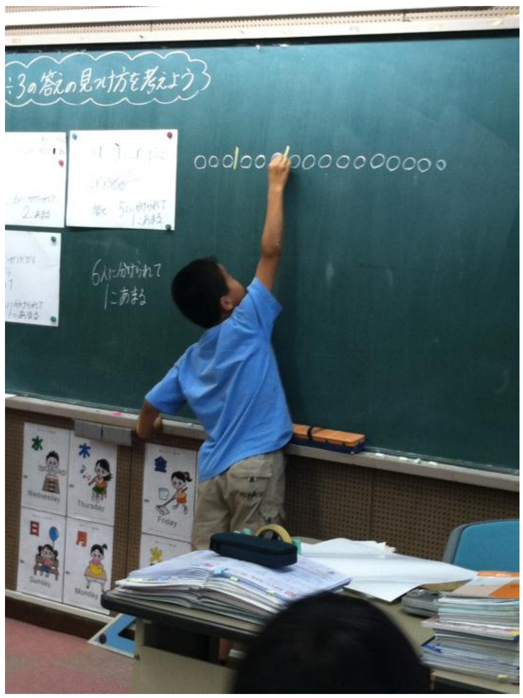
The teacher put up three students’ solution methods, which he had selected during independent work time. The three solution methods he included were:
◊ 3 x 5 = 15, 15 + 1 = 16; answer is 5, remainder 1
◊ 3 x ☐ = 16; answer is 6, remainder 2
◊ drawing of kids with Jello; answer is 5 kids, remainder 1
</td></tr>
<tr><td>2 min [pair share]</td><td>The teacher points to “6 remainder 2” and “5 remainder 1” and says: “These students have two different answers. Which one is correct?”

A student explains that “5 remainder 1” is correct because 3 x 5 is 15, and plus 1 is 16.</td></tr>
<tr><td>24 min [continued discussion, practice]</td><td>The teacher continued to ask if students agree. He instructed students to talk with their neighbors about which could be right.</td></tr>
</table>

In one group, a student explains why 16 remainder 2 is incorrect: “6 x 3 is 18. If 2 are left over, then that means you need 20 altogether.”

When the group came back together, the teacher asked which is correct. A student volunteered his opinion that both are wrong. He thought the answer was 6 remainder 1. However, this answer was not taken up by the class for discussion.

The teacher asked students to raise their hands if they drew pictures. Most students raised their hands. The teacher then made the assumption that if students draw pictures they will get the right answer. One student came up to the board and showed how to draw 16 circles and divide into 5 groups plus 1 left over.



The teacher then asked kids to find the answer 17 ÷ 3 *without* drawing pictures, since drawing will not always be useful.

When back together as a class, a student said he used the “3 facts” to find the answer. The teacher asked how many found the answer using the “3 facts”. He started writing each 3 fact (starting with 3 x 1) and the students called out that he had to keep going to get to the fact 3 x 5. He also wrote 3 x 6 = 18 and put an “X” next to the equation showing it was too far for this situation.

The teacher asked, “How is 16 ÷ 3 similar to 17 ÷ 3?” A student explained that both used 3 x 5 but the former has a remainder of 1 and the latter has a remainder of 2.

The teacher used these expressions and the ones from the beginning of the lesson (15 ÷ 3, 16 ÷ 3, 18 ÷ 3, and 81 ÷ 9) to define and differentiate between “divisible” and “indivisible”. He then gave additional examples and students individually wrote them in the two categories.

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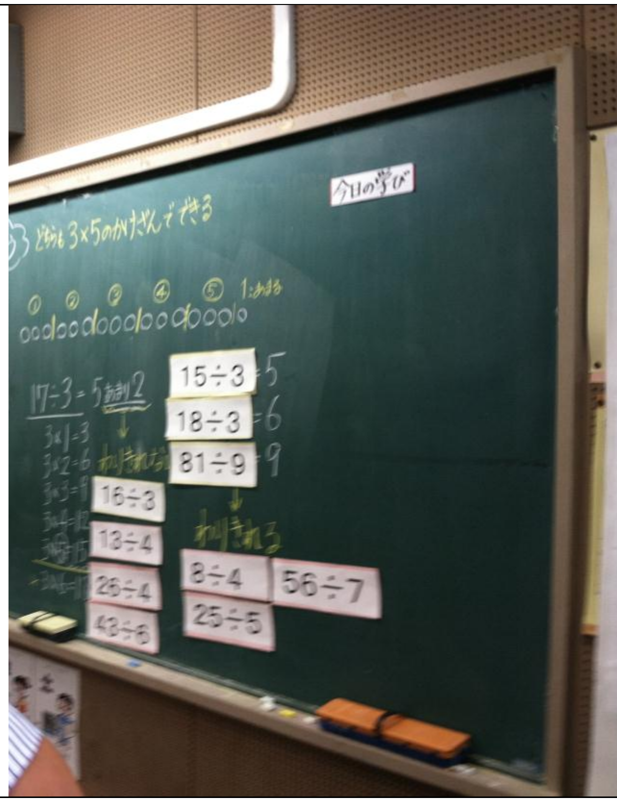
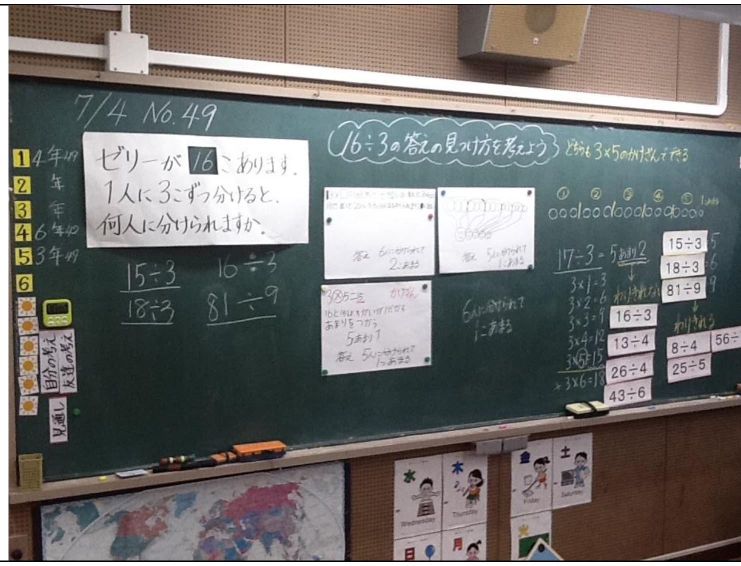
<tr><td></td><td>
</td></tr>
<tr><td></td><td>Summary, Consolidation of Knowledge

After checking each answer with the class, the teacher summed up the learning for the day by reviewing how they found remainders using multiplication facts. He reiterated the differences between divisible and indivisible.

The teacher then told students to write about the “main point” of the learning in their journals. Two students shared their journal entries.

Below is the blackboard at the end of the lesson.</td></tr>
</table>



What new insights did you gain about mathematics or pedagogy from the debriefing and group discussion of the lesson?

- ◊ The teachers discussed the difference between using counters and drawing circles to represent counters. Some teachers said they thoughts drawings of circles are a transition between concrete and abstract. Professor Fuji emphasized that there is very little difference between the two. Members of our team had never considered these differences.
- ◊ We appreciated Professor Fuji's question of why students were given counters in Lesson 1 and then told they could not use them in Lesson 2. The teacher deliberately did not allow students to use the counters so that they would use multiplication facts and division expressions to represent the situation. We think this was too fast a transition for the students.
- ◊ We also wondered why the teacher did not pursue a discussion about one student's incorrect response of "6 remainder 2". We agreed that it was a strange decision to put this up as one of the first few student responses even when the teacher knew he did not want to engage the class in a discussion of its reasoning.
- ◊ Professor Fuji said it was important for the students not just to think of division as the inverse of multiplication, but also as additive subtraction. We found this interesting.

What new insights did you gain about how administrators can support teachers to do lesson study?

- ◊ We liked the system of three teachers teaching the same lesson at the same time. In this way, the teachers can communicate to each other about how the lesson went in each of their classrooms.
- ◊ We thought it was helpful that the post-lesson discussion was concise and well moderated. This can help teachers feel less overwhelmed and more motivated to participate in lesson study.

How does this lesson contribute to our understanding of high-impact practices?

- ◊ The teacher had students raise hands to engage in repeated whole-group checks for understanding.
- ◊ The teacher asked higher-order, open-ended questions and used high-impact learning tasks:
  - o "What's the difference between…?"
  - o "These two can't both be right. Which one is right and why? Turn to your neighbor and discuss."
  - o "Categorize these expressions as *divisible* and *indivisible*.
  - o "In your journals, write down what the important idea of learning today."
- ◊ The teacher collected the journals to get an immediate read on students' understanding.
- ◊ From the post-lesson discussion, we learned that the teacher missed a valuable opportunity when he did not take up the "6 remainder 2" response for discussion. Like in *kendo*, only by "stepping closer" can a teacher truly understand and clarify a student's thinking.

3 Reflection Journals

Invited IMPLUS participants were requested to write a reflective journal about mathematics teaching and learning in Japan and Japanese lesson study.

IMPULS Lesson Study Immersion Program Reflective Journal
Andrew Friesema
Dr. Jorge Prieto Math and Science Academy, IL, U.S.

The IMPULS Lesson Study Immersion Program was an incredible experience. The time spent observing lessons and post lesson discussions in Japan has given me so much to think about as I begin another school year here in Chicago.

First, there was the overall impression of the culture of school in Japan. The first thing that I encountered in the first school that we visited were the smiles of the children, followed by the energy, noise, and joy of teachers and students working, teaching, and learning together. I found this atmosphere to be typical of the learning environment in the other schools that we visited as well.

I was impressed by the professional approach taken by teachers and other educational leaders to analyzing the various aspects of the lessons that I observed. It was exciting for me to see examples of the philosophy of student directed instruction put into action through careful teacher questioning and facilitation of the learning experience to ensure that student thinking remained at the forefront of the lesson.

Along with my general impression of the typical learning environment in schools in Japan, I was also fascinated by specific instructional strategies that I saw in most of the lessons that we observed. Most of the lessons that we observed started with a problem that was posed to the entire class. The question was designed to be open enough to allow an entry point to the problem for students of various developmental levels of mathematical understanding. While students worked on this one problem, the teacher circulated the room and took notes on the problem solving strategies of various students. This one problem which may have taken around 10 minutes for students to solve became the basis of the discussion and the mathematics that occurred for the duration of the 45 minute lesson. The teacher used the notes that they had taken while the students had been working on the problem to make decisions on how they would facilitate the comparison and discussion of mathematics that followed. It was fascinating to watch how effective teachers would call on students in a purposeful way to share solutions, ask clarifying questions, record ideas on the board, and keep student thinking, student ideas, and student mathematics as the focal point of the lesson. The focal point of an effective lesson was not the teacher's ideas and the teacher's thinking, but rather the mathematics and the ideas of the ***student***.

Although I am familiar with using problem solving to teach mathematics, it was very inspiring to me to see lessons in which the majority of the lesson was not spent in computation or in answer getting, but rather in a comparison of strategies, a discussion of mathematics, and in students justifying to themselves and to each other the validity of the strategies used in the course of the problem solving. To see a lesson in which the first 10 minutes of the class was spent problem solving, and the remaining time was spent comparing and discussing solutions was very different from the typical math lesson I have observed in Chicago in which the majority of the class time is spent engaged in problem solving or answer getting, with comparison and discussion of strategies tacked on to the end of the lesson, rather than serving as the focal point.

The lesson component that I found the most interesting in the lessons that we observed was the relationship between the teacher's recording of information on the board, the student note taking, and how they came together to provide a framework for the mathematics done by the students. The class begins with the teacher posing the problem on the board, and the students writing the problem in their notebook. Students then record their work and reflect on their ideas in their notebook. The teacher is able to see the thinking of the students reflected in the notebooks, and use these notes to take notes of their own as to how they will have students share out their thinking in a way that builds the story of the mathematics that they did collectively as a class. As the teacher calls on students to share their ideas, the teacher records important points on the board, which models to students what are some of the ideas of their classmates that they should also be recording in their notes. Throughout the course of the lesson, the teacher records the comparison and discussion of the students' mathematics while the students are recording similar notes in their notebooks. Together, the board work and the students' note books serve as an artifact of the mathematics that the class did as a whole. The routine of the note taking and the careful facilitation of the comparison and discussion that comprises the majority of the lesson helps to ensure that for the majority of the students, their notebook can serve as a reference for future mathematical learning.

I was fortunate to have several fellow members of the IMPULS Lesson Study Immersion Program come to my school for a teaching through problem solving conference a couple of weeks after the trip to Japan. With my experiences watching Japanese lessons still fresh in my mind, I was able to watch a teacher teach a group of 15 of my schools' incoming 4th graders using an English translation of a Japanese math text, using the Japanese lesson structure. It was exciting to watch over the course of just 5 lessons how my students enjoyed and improved in their ability to justify their problem-solving strategies, refer to classmates' ideas, communicate their thinking, and do mathematics above and beyond simple computation and answer getting. To have the opportunity to watch how my students reacted in such a positive way to the instructional strategies that I had observed in Japan as a part of the IMPULS Lesson Study Immersion Program made the impact of the experience even more powerful.

As I begin my third year of using lesson study at my school and reflect on my experiences in Japan this summer, it is very exciting to see lesson study grow from our initial team of three teachers, to a tool that was used by several grade level teams to address grade level teaching and learning problems, to this year as we attempt to use lesson study as a tool to address the school wide teaching and learning problem of using student note taking and teacher board work to address the Common Core State Standards for Mathematical Practice. I am very excited to share my experiences in Japan as a part of the IMPULS Lesson Study Immersion Program with my colleagues as we address this challenge together.

Reflections on my first trip to Japanese Schools

Barton Dassinger
Principal, Chavez Elementary Multicultural Academy, IL, U.S.

My reflection includes thoughts on Japanese schooling in general as well as some specifics of lesson study professional development that I observed.

In General

The IMPULS trip was my first time to visit Japan (hopefully not my last) and many aspects of the schools we visited intrigued me. In particular, the students' access to equal buildings, instruction, and teachers across seven different schools in different areas of Japan captivated me because it contrasts sharply with my experience as a principal in Chicago.

Each of the schools we visited had structural similarities -- swimming pools, classroom windows that provided natural light, a large area for recreation, open spaces. Additionally, classrooms seemed to be more or less the same size across schools. Schools in Chicago vary dramatically in regard to this basic aspect of schooling. I oversee two buildings blocks apart from one another that are considerably different.

Student in the schools we visited also had greater equality of access to math curriculum and instruction than what we have in Chicago. All of the lessons we observed in Japan had a similar lesson plan structure and a similar constructivist / problem-solving approach. The math textbooks in use may not have been identical, but their differences were slight compared to what is found in Chicago classrooms. For example, schools located less than one kilometer away from where I work use math textbooks and teaching methods completely different from what is used at my school (in the same district). Further, there is no required or standard lesson plan format for teachers to use. Consequently, the math instruction in Chicago can vary widely from school to school and teacher to teacher.

My conversations with Japanese educators on the trip led me to believe that schools (and children) in Japan have greater equality with regards to their teachers. I learned that Japanese teachers and administrators do not usually stay at one school for over six years and their rate of pay is approximately the same throughout the country. Teachers generally change schools a few times during their career. This helps ensure that teachers develop professionally and that schools have teachers with a variety of experiences. Again, this is very different from my experience in Chicago where principals are responsible for hiring all staff members – from teachers to secretaries. Some schools may be able to recruit great teachers (better pay, better working conditions, better neighborhood) while other schools are forced to hire teachers unable to get jobs at the most desired schools. This results in Chicago Public Schools students having unequal access to great teachers.

Student equality of access to education is very different in Chicago because there are very different school buildings, principals are responsible for all hiring and teacher development, and local schools have autonomy in determining how they instruct students (textbooks and methodologies). I have always appreciated having the freedom make major decision for my school; however, my experience in Japan has allowed me to see how sacrificing this local freedom could positively benefit the larger school system by giving students greater equality to educational access.

Lesson Study

I have implemented lesson study for four years (with the generous support of Dr. Takahashi) as the principal of two different schools in Chicago. In both schools, I have aimed to increase teacher interest and the number of teachers actively participating in Lesson Study. With this in mind, I observed with particular interest how the structure and organization of the post-lesson discussion might contribute to my goals.

Five of the seven schools we visited used a structure and organization similar to what I have learned from Dr. Takahashi. This includes a room arrangement where the lesson study teacher, moderator, and expert commentator sit at a table facing other teachers and observers. The flow of the discussion is as follows: introductions, the lesson study teacher and / or team speak to provide summary and reflections; teacher observers then speak to provide comments or questions to the teaching teacher or team; finally, a final commentator summarizes the comments of others and provides an expert analysis. During the discussion the moderator guides the direction of the discussion and keeps time.

Two of the seven schools we visited used a somewhat different structure to their post-lesson discussion. At Funabashi E.S., teacher teams divided into groups and wrote their notes onto large sheets of paper for about ten minutes before beginning the discussion. Then, one representative from each of the teacher teams spoke first during the discussion. At Sengen Junior High School, observers wrote observations on three different colored post-it notes corresponding to three different categories (yellow = improvements, pink = problems, and blue = good). Observers then worked together to organize those notes on a board where all could see them. The moderator and final commentator were able to use these notes to facilitate discussion and analysis.

I observed greater participation among the observers at Funabashi and Sengen and incorporated aspects of their post-lesson discussion into the most recent research lesson at my school (12/05/2012) in order to ensure all teachers actively reflected on the lesson. Before beginning the post-lesson discussion, I asked teams of teachers to reflect on the lesson using large sheets of paper (like Funabashi); they did this in the style of Sengen, transferring their personal observation notes onto post-it notes and sticking those to a large sheet of paper their team shared. Teacher observers then categorized their post-it notes into three categories – similar to the categories in Sengen. When I moderated the post-lesson discussion, a representative from each team commented on the lesson. Using the techniques I learned in Sengen and Funabashi increased the number of teachers participating in the post-lesson discussion and teachers commented that they enjoyed this more than previous post-lesson discussions.

I am grateful to have been able to observe seven different lesson study lessons and learn specific ways to improve the lesson study experience for my teachers in Chicago.

Overall comments on the trip

I am extraordinarily grateful to project IMPULS and all those responsible for funding the project. It was the most beneficial professional development of my life. Within a short period of time, I have been able to incorporate specific aspects of Japanese lesson study at my school and improve the mathematics instruction for my students. Thank you!

Lesson Study Reflection
Belinda Edwards
Assistant Professor of Mathematics Education at Kennesaw State University, U.S.

The purpose of my participation in IMPULS Lesson Study professional development program was to learn about how Lesson Study can be implemented effectively in American secondary mathematics classrooms and in teacher education programs from those who practice Lesson Study on a daily bases—Japanese teachers and teacher educators. In the past year, I collaborated with a group of secondary mathematics teachers using aspects of Japanese lesson study and found that there are many challenges at the secondary level—lack of common planning, large class sizes to name a few. Before participating in IMPULS professional development, most of what I knew about Lesson Study was through conference presentations and readings. During my time observing mathematics classrooms and teaching in Japan, I was able to clear up misconceptions/misunderstandings about Lesson Study. A gained a better understanding of how effective teachers are developed. I learned that it takes a great deal of time to develop effective mathematics teachers.

I now have a better understanding of the 3 levels of teaching and embrace the idea that a pre-service teacher can be an effective teacher at level 2; that is, a teacher who can explain reasoning). A Level 3 teacher is someone who has many years of teaching experience—10 years, perhaps. Many of the teachers I observed in Japan could be classified as level 3 teachers. In the US, we consider teachers who have 3 years of teaching experience to be eligible to mentor pre-service teachers during their field experience. When I think about how long it takes to develop an effective teacher in Japan, 3 years doesn't appear to be long enough to develop a teacher who has deep knowledge of the content, knowledge for teaching, and expertise in teaching. Clearly, the teaching levels have me rethinking what kind of teacher makes a good mentor or collaborating teacher for pre-service teachers.

The principle form of teacher development in Japan is Lesson Study and it is an inherently collaborative process. I was impressed with the notion that school is a place where both students and teachers learn. Because Japanese teachers hold this belief, I believe they're more open to trying to understand how improving their teaching improves student learning. Teachers work together to design, implement and observe a lesson aimed at meeting a communal goal for student learning. After observing a number of lessons and lesson debriefing sessions, it's apparent to me that the broader knowledge base than that of one teacher creates increased opportunity for a lesson to provide a quality learning experience for students. Teachers continually interact about effective teaching methods and develop shared understanding of how to improve students' learning. A primary focus of lesson study is the improvement of student learning. One aspect of lesson study that sustains this focus is its careful attention to anticipating students' responses. As I reviewed the lesson plans and observed the lessons, anticipating students' responses, monitoring their solutions, and sequencing student solutions clearly played an important role in the successful progression of the lesson.

My experience observing Japanese lessons confirms my belief that when designing a lesson, teachers must consider what knowledge students are likely to bring, what strategies students may use, and how students' knowledge connects to the various mathematical concepts. Observation and reflection focused on students' thinking and knowledge was evident during the lessons and lesson debriefings. Focusing on student thinking during observation and lesson debriefing provides opportunities for teachers to develop and understanding of how students think about and learn from the mathematical activities and tasks they engage in while working together to determine what makes certain learning experiences effective for students.

An important aspect of teaching includes teaching through problem solving as opposed to teaching problem solving. Japanese teachers present the problem to students and students solve the problem. Most US teachers demonstrate a procedure and assign similar problems for students to work. This method of teaching does not provide students with the opportunity to engage in problem solving. In the Japanese classrooms I observed, students are introduced to a mathematics problem placed in context. The students provide multiple solution approaches which leads to rich classroom discussions. The teacher is not the one who is solving the problems. When developing a problem solving lesson, Japanese teachers consider the student, the curriculum, and the problem. But the lesson is not effective without the teacher facilitating a discussion of the problem which will help students develop and understanding of mathematical concepts and skills. But in order to do this, teachers need knowledge of the concept. In each lesson that I observed, the teacher was able to facilitate a mathematical discussion that seemed to help students progress toward an understanding of the concepts or at least learn something new. This entire process is an approach (Neriage) to problem solving that I plan to spend discussing with pre-service and in-service teachers. Another unfamiliar term at the time of my arrival in Japan, "Kyozaikenkyu", involves knowing both the curriculum and content very well. This is an important aspect of effective teaching that every pre-service and in-service teacher needs to spend time perfecting.

The flow of the lesson in each of the lessons I observed appeared to be effortless. I was impressed with the way in which teachers documented the flow of the lesson using something as simple as a blackboard. The blackboard simply told the story of the lesson. The blackboard included student work and teacher notes. At the end of the lesson, students are asked to journal about the mathematics in the lesson. As they journal, they are able to reflect on the lesson by looking back at their notes or at the problem solutions written on the blackboard. On a number of occasions during the teacher's facilitation of discussion the teacher asked the student to describe their solution as opposed to writing their solution on the board. I realized that allowing students to verbally provide the teacher with steps involved in solving a problem can be equally effective as asking the student to the board to write down the steps. In doing so, students have the opportunity to communicate their mathematical understanding and thinking.

One of the most important aspects of lesson study is that it focuses on promoting teacher self-reflection. As opposed to asking students to examine their teaching and student learning, lesson study simply asks teachers to plan a lesson, teach the lesson, refine the lesson with the expectation that this process will lead to reflection. When teachers reflect on their teaching, it provides opportunities for them to improve their teaching and student learning.

IMPULS REFLECTION

Colleen Vale
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Firstly I'd like to thank the teachers and Principals of the schools that we visited during the IMPULS tour for opening their doors and inviting us into their schools and classrooms in order for us to learn more about Japanese Lesson Study, Japanese mathematics curriculum and Japanese school culture. I'd also like to thank the IMPULS team who made our visit so easy and enjoyable.

There are many aspects of Japanese Lesson Study (JLS) that are relevant to the Australian, and especially the Victorian school context, and a number that I'd like to incorporate into my work as a teacher educator and education researcher and in my projects with schools and teachers. These aspects relate to the whole school approach to improvement and ongoing professional learning of Japanese Lesson Study, the focus on developing teachers' mathematical content knowledge along with their mathematics pedagogical content knowledge, their approach to planning for teaching and attending to students' thinking and learning.

One of the things that I hadn't realised about JLS was the way in which it is a whole school approach not just to mathematics teaching and learning but to teaching and learning in all aspects of the school curriculum. Each school identifies a theme that is the object of school improvement and teacher professional development. The themes at each of the primary schools we visited were:

- Nurture students who express own thoughts and deepen each other's understanding (through *neriage* in mathematics lessons);
- Raising students' ability to think coherently by anticipating and to represent their ideas;
- Nurturing students who can thing on their own, express their ideas, and learn from each other (developing mathematical ways of thinking); and
- Mathematics learning that nurtures students who can use what they have learned – through activities to express own thinking.

Each of these themes focuses on students' thinking and their capacity to express this thinking. The documentation of the secondary mathematics research lessons we observed did not refer to a whole school research theme. The JLS district model involving novice teachers from three secondary schools in one district worked on developing mathematical instructional leadership among the team and their theme was "planning materials from which students can feel satisfaction and fulfilment." I'd like to know more about how these themes are developed and how they relate to education policy and programs in the regions or in partnership with universities.

The membership and organisation of the school research teams appeared to vary between the schools we visited and this was evident by the different structures for the post-research lesson observation discussions. Also the degree of collaboration throughout the JLS appeared to vary from school to school. A number of the lesson plans appeared to have been documented by the teacher of the research lesson so the nature and extent of collaboration during the Kyozaikenkyu phase of curriculum, mathematics and pedagogical research and preparation was not clearly evident.

In the lesson plan the research teams (or research lesson teachers) demonstrate their knowledge of the mathematics curriculum and the scope and sequence of students' learning that relate to the content of the unit. Some research teams also included detailed analysis of their students' prior learning, noting both strengths and weaknesses or misconceptions of students' understanding relevant to the proposed unit and lesson. The more detailed plans from the primary schools also related the unit and the lesson to the school's theme for improvement. Each of these plans included goals for the unit which aligned to the main components of Japanese mathematics curriculum: interest, eagerness and attitude; mathematical way of thinking; mathematical skill; and knowledge and understanding. The team also documents the way in which it will evaluate students' learning in each lesson. This type of detail is not evident in the planning documents of teachers that I have worked with on various school improvement projects in Victorian schools.

Along with goals for the lesson and a documentation of the planned flow of the lesson, most research teams/teachers included anticipated student responses in the documentation of the flow of the lesson. This practice provided the clearest evidence of teachers' mathematical content knowledge (MCK) and/or knowledge of their students. While I have insisted that my pre-service teachers plan and document the statements and questions that they will use when implementing the lesson I have not previously required that they also document anticipated students' responses. I will be doing this in future.

The most impressive aspect of Japanese teachers' teaching approach observed in all classrooms was the teacher's capacity to orchestrate a discussion of students' solutions and mathematical thinking and to record students' ideas and thinking on the blackboard. This follows from careful anticipation of student responses and from knowing their students as well as from careful monitoring of students during the independent student work time in the lesson. Almost all teachers included the students' name on the board alongside their ideas. These practices demonstrated respectful relationships between the students and teacher and the valuing of students' ideas whether or not they were correct or the most efficient. Sometimes during the post-lesson discussion the teacher observers or the independent observer questioned the selection and probing of particular student ideas or the direction of the discussion with regard to students' mathematics learning and the lesson objective. While this critique may be well-founded I thought that, in general, these teachers demonstrated the way in which students' ideas can be connected and their understanding scaffolded and enriched through the discussion. If I have one criticism, it is that boys were more often asked to report than girls.

In Victoria, teachers of mathematics are urged to use student-centred teaching approaches, model mathematical thinking, guide students' inquiry and thinking and to use 'explicit' teaching. Unfortunately many Victorian teachers interpret this to mean selecting tasks for groups of students organised by ability along with the use of a transmission model of instruction. Very often students report their findings at the end of lessons in a 'show and tell' format without any expectation that they explain and justify their thinking or that others will learn from their thinking. Orchestration of discussion is a practice that I think is very much needed in Victorian mathematics primary and secondary classrooms.

I was amazed by the number of teachers participating in the observation of the research lesson and the post-lesson discussion in the primary schools. I had not anticipated that all the teachers at one school would be involved. This practice must certainly support teachers to know the curriculum and learn mathematics and to develop their horizon mathematics knowledge in particular. The post-lesson discussions were organised differently at different sites, especially with respect to who chaired the discussion and the way in which teachers were organised to observe, reflect on and comment on the lesson. The post-lesson discussion appeared to be more meaningful when the observers had a particular objective and when student work samples and/or the blackboard summary could be viewed during the discussion. The external expert commentator appears to make a significant contribution to the post-lesson discussion and to teachers' thinking about their knowledge and work. These comments mostly focussed on the mathematical ideas and alternate pathways student thinking and discussion might take as well as on teachers' preparation and attention to curriculum and textbook resources. I'd like to know more about what teachers learn during the post-lesson discussion and what changes they make to their practice as a result of participating and contributing.

Research in Victorian schools that is driving school reform and improvement has identified a whole-school approach and teachers working collaboratively in professional learning teams as necessary elements of an effective school. In the last 5 – 10 years Victorian state schools have been develop Annual Action Plans and establishing collaborative professional learning teams. Professional learning is increasingly school-based addressing the needs identified by each school in their Action Plans. Improved teaching and learning depends on the quality of instructional leadership and the structural support for schools provided by the centre and districts and at the school-level by leadership. I think that JLS is a viable model for sustained school improvement in Victorian schools especially where instructional leadership is strong and the time and space is provided for professional learning. Certainly there are many cultural differences between Australian and Japanese schools but I don't perceive these differences should impede the implementation of JLS in Victorian schools, though these cultural differences will inevitably result in a different practice of JLS.

My participation in IMPULS is impacting on my practice as a teacher educator and researcher in a number of ways:

- A research project involving three Victorian primary schools: *Implementing structured problem-solving mathematics lessons through Lesson Study*. In this project teachers from three schools will conduct two research lesson cycles in 6 months. We have used one of the Lesson Plans from IMPULS as a model for teachers for the first cycle.
- A second research project involving four Victorian primary schools and one Canadian school: *Primary school teachers' and students' perceptions and understanding of mathematical reasoning*. This project includes demonstration lessons as a means of collecting data from teachers and students. We have adapted a Japanese Lesson Study protocol for the pre-lesson briefing, lesson observation and post-lesson discussion for this study.
- I have used two elements from "Do I have a window or an aisle seat?" lesson in my teaching of second year pre-service teachers. We have been studying students understanding of operations including division with remainders. They have viewed the video of this lesson and completed the problem solving task documented as the prior lesson.
- I propose to discuss elements of Japanese Lesson Study, especially the orchestration of class discussion in a keynote address that I will deliver at the Annual Conference of the Mathematics Association of Victoria (MAV) in December this year.

Thanks again to everyone in the IMPULS team for a wonderful thought provoking experience.

Teacher professional development through lesson study
- some thoughts drawing on Cultural Historical Activity Theory-

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I have used the opportunity of reflecting on the lesson study process to consider this using Cultural Historical Activity Theory (CHAT) which has its roots in the soviet school of social psychology. This may seem a somewhat academic approach to take but I have been working with colleagues in the UK to consider professional development from this perspective and to explore if the theory can be used as a useful tool to assist with the design of professional development for teachers of science and mathematics. Importantly the theory provides some ideas with which to unpack the activity of the different 'communities of practice' involved in lesson study. I therefore want to use some of these ideas to help me analyse what I have seen of lesson study, consider what insights that provides, and then attempt to identify implications for the lesson study groups working in my own context of mathematics education in England.

CHAT views the activity of communities as the joint production of the actions of individuals directed towards a shared goal. The theory has been developed through a number of generations building on Vygotsky's initial ideas of how our individual actions are mediated by artefacts or tools (including ideas and language). These ideas were expanded by Leont'ev to take into account the activity of communities drawing our attention to how an individual's actions are socially constructed and mediated by rules (both implicit and explicit) and the division of labour between members of the community. Further, in third generation activity theory, the interaction of two or more activity systems has been considered (notably by Engestrom) leading to notions of boundary objects (artefacts that have 'currency' in each system) and boundary crossing by individuals who move between systems

In mathematics classrooms, in schools, the object of activity of the community, that is the class, is the learning of mathematics with teacher and pupils working together with distinct roles and ways of operating governed by implicit and explicit rules (the social norms of the classroom) that are culturally and historically evolved. Lesson study provides a new activity system(s) in which the teacher has a new role to play alongside colleagues including the 'outside expert' or 'knowledgeable other'. It is important to note that this new activity system has an object of activity that is not the same as that of the mathematics class, it is expanded beyond that. Not only is the learning of mathematics fundamental to the activity of the lesson study group, but also the learning of the group/community in mathematics education as a whole is important: the agreed research agenda helps form the goal of the group (although from our observations of post-lesson discussions this was not always at the forefront and discussions tended to focus to a large extent on activity that had occurred more generally in the immediately lesson). The members of the lesson study group work to new rules and norms and with a division of labour which is to some extent negotiable but is based on cultural rules and expectations (for example, Higher Education participants have a status that reflects that of their specific expertise). The research lesson teacher's individual professional learning and that of all members of the lesson study group lies not only in their experiences of the lesson study activities but also as they cross boundaries between classroom and lesson study meetings and in their interaction with boundary objects that have different intentions and meanings in the different communities in which they operate. The learning of members of the group is not restricted to lesson study events: for example for the teachers it is mutually recursive between on-going experiences in their classrooms on a day-to-day basis, the research lessons and lesson study discussions (as well as elsewhere). The professional learning of the group's members, and the group as a whole, results from the totality of these experiences.

Two important issues come to mind taking a CHAT view of lesson study: these focus on (i) shared ownership/understanding of the object of the community's activity and (ii) lesson plans and text books as boundary objects.

(i) In my experiences of observing post- lesson discussions in Japan, these quite rightly focused on the recent lesson as an event, however, often without much if any reflection of the on-going process towards the long term goals of the group and development of individuals and the group as a whole towards these. CHAT itself can be used as a tool by a community to explore important aspects of its behaviour and development towards its goals (although the group does not necessarily have to use such tools to consider its aims, objectives, purposes and ways of working towards these). Expansive learning of the group/community can take place as these important aspects of their activity are explored and reconceptualised by the members of the group themselves. In implementing lesson study, therefore, it seems important that the group considers how they might achieve such development and how they will facilitate this in their meetings (for example, this may be a responsibility of the facilitator/convener of the group who builds this into an agenda for the post-lesson discussion or planning meeting).

This raises issues of division of labour and community in the lesson study group.
In the two activity systems of classroom and lesson study group the division of labour and how hierarchical, and in this regard how rigid or flexible this might be provides an important aspect to consider. It seems important to bear in mind here there are cultural and historical expectations to which one should be sensitive.

In the classroom the socio-mathematical norms we observed appeared highly developed (as in classrooms everywhere): students knew exactly what was expected of them and how they were to 'do' mathematics and to learn. Some observations that seem important are:
(a) getting the 'correct' answer and way of working are important to students even though the lessons are 'problem' driven and mathematical exploration is considered fundamental to them. The process of solution and underlying thinking did not appear as valued by the students as getting the correct answer. We observed students working towards a solution but erasing their work if the teacher provided an alternative (and therefore validated as the accepted way of thinking and working).
(b) peer-to-peer collaboration appeared difficult to stimulate. Students worked almost exclusively on their own and the teachers did not use any specific pedagogies that deliberately prompt collaborative work (such as think-pair-share, card sorting and so on). However, students did engage with the thinking of others in whole-class discussions that the teacher conducted. It was noticeable that students were following the reasoning of others.

Equally in the lesson study community, roles and responsibilities provided a seemingly rigid organising structure. Ways of working of the different groups we observed following research lessons had some aspects in common although these were never exactly the same. However, for example, the teacher of the lesson always had an early opportunity to give their initial reflections but on the other hand the group facilitator appeared to take a more or less pro-active role in the different groups.

Given the adaptation required to implement lesson study in England, and without an established form for this, of particular importance, therefore, are the roles and responsibilities of the group members. Who is being brought together and the roles, responsibilities and status that they both bring with them and will adopt and develop as the lesson study group develops are important factors. With the important goal of developing teaching and classroom activity that better supports student learning in classrooms how can each member of the group be facilitated so that their contributions are valued and respected? How can joint ownership of the work of the community (for example, for the research lesson) be ensured? It seems that this needs to be planned for from the outset. Further discussion of lesson study in the Discussion Group at ICME explored the role of the 'knowledgeable other', for example. Can their input be more proactive than mainly being to provide a summarising input at the end of the research lesson? Should they be part of the planning and development phases of the lesson study cycle?

Tellingly in the final post-lesson discussion Professor Fujii suggested that to move to the next level of effective learning in the classroom teachers and pupils need to move to a position where they are developing mathematics together. This seems a somewhat prescient comment: perhaps in the learning of the lesson study community the same might be said to be necessary. In other words for the lesson study community to move to the next level, that is to move to a situation in which the professional learning of mathematics education is taking place effectively the community needs to be developing mathematics education *together*. It appears here with the lesson study group, as in the classroom, there exists the potential for a greater cohesion in the community's learning.

(ii) The lesson plan (for both the research lesson and the sequence of lessons of which this is a part) is a boundary object having purpose in both activity systems within which the teachers participate: those of the classroom and the lesson study group. As such the lesson plan has meaning in each but importantly serves different purposes. In the classroom it provides the intended plan of action of the teacher so in that sense it may be considered to have a very utilitarian function (in some ways providing a potential script as it anticipates students' responses to the activities they will undertake). However, it also encapsulates the teachers' (or in some cases teacher's?) understanding of curriculum and pedagogy of an area of mathematics education in general and for the school class in particular. Depending on the manner in which the lesson study group has been operating, this therefore shares and makes public their jointly constructed vision in these important aspects. These underpinning understandings appear to provide an important starting point for the critique of the lesson by the community in the post-lesson discussion - including for the knowledgeable-other asked to prompt thinking at the end of the discussion. Consequently for the lesson study group the lesson plan not only signals the intended actions for the teacher but it also provides a publically visible underlying rationale for the lesson within a developing sequence of knowledge. In my experience of the IMPULS post-research lesson discussions it seems that perhaps inevitably the teacher(s) focused heavily on the development of the lesson plan for teaching action and only to a lesser extent the wider rationale for knowledge and lesson development. Indeed in the teachers' desire to ensure the lesson was good they often scripted the lesson leaving little room for manoeuvre and perhaps losing sight of important principles in the sequencing of mathematics that should inform their lesson (and student knowledge) development. On the other hand the scripting ensure very detailed thinking by teachers about subject knowledge and how this is likely to be developed by students.

In a similar way, but a little less obviously close to the lesson, text books provide more than tasks for students: they encapsulate what is known regarding effective sequencing of knowledge for learning and connections within and across school mathematics. They act as an important archive of knowledge for mathematics education: it is not clear how explicit this sequencing is made to new generations of teachers as they enter the profession. However, it seems clear that contrary to text books in England, their sequencing of knowledge and use of tasks to engage students in appropriate activity that result in what might be thought to be guided discovery of key mathematical concepts, Japanese textbooks capture the essence of what educators over many years have learned in relation to knowledge development.
The lesson study group needs to think very carefully about the sequencing of knowledge, how it develops and interconnects: this seemed particularly pertinent in the lesson "bisecting an angle" when the expert summariser Dr Nishimura critically questioned how the students were asked to consider the construction of the angle-bisector.

In this lesson students considered how to construct an angle bisector using pencil compasses and straight edge. As the photograph here (figure 1) suggests this was not particularly successful as students struggled to consider how this could be done. Dr Nishimura suggested that the lesson did not successfully build on knowledge that had been developed earlier, that is on ideas of constructing sets of points that are equidistant from geometrical objects (see slide 1 below). Rather than seeking standard construction procedures he suggested prompting students to build understanding on prior knowledge (see slides 2 and 3 below) - perhaps dynamic geometry tools might have better facilitated this.

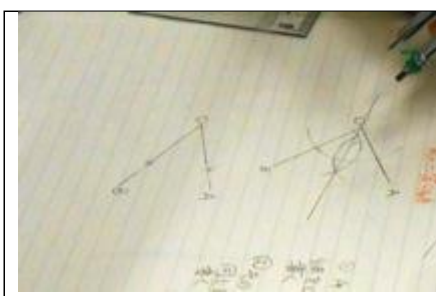
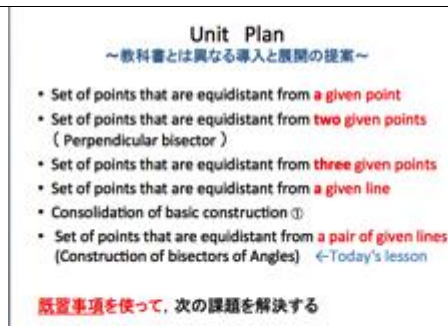
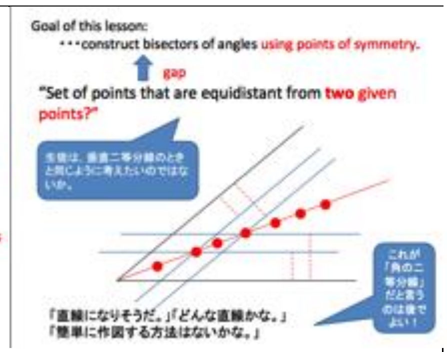


Figure 1: student explores how to construct angle bisector



Slide 1: unit plan for research lesson sequence



Slide 2: considering the set of points equidistant from two given lines

In summary these initial reflections drawing on something of a CHAT analysis (with a light-touch here) suggest some key aspects of lesson study which we might wish to consider as we adopt and adapt lesson study to achieve our own goals working within our specific culturally and historically developed systems. In particular as we generate new activity systems within which teachers and others will take part in lesson study there is much we can learn from the Japanese experience. We cannot adopt in their entirety the systems and ways of working that have been developed over many years in Japan, rather they provide some very helpful insights into how we might adapt these so that lesson study might work in our cultural setting. My initial skirmishes with lesson study and the need to help facilitate a new community of lesson study in England suggest that there are some important aspects of boundaries, boundary crossing and division of labour to be considered from the outset if we are to ensure our joint enterprise can best draw on, recognise and value the strengths of individual contributors. It is important to think carefully about this and plan accordingly so that the totality of the joint enterprise is more effective than the sum of its parts.

REFLECTION

Joshua Rosen
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I would like to begin by expressing my profound gratitude to IMPULS for the opportunity to experience this rare and unforgettable experience. It is unusual to have the opportunity to exchange ideas with teachers from around the world, and to experience an authentic immersion into a foreign culture and its education system. I am eager to maintain the contacts that I have made through the trip, and to expand my Lesson Study work to other communities outside the New York area.

I left Japan impressed by many things, of which the education system is one element. I now realize that the emphasis placed on education in Japan is a reflection of the prevailing culture that places value on many things. I noticed the care, value, and respect in many corners of Japan: The man who spent 20 minutes sharpening the knife that I bought atTsukiji market, the woman who wrapped my present at Daimaru with angular precision, the chef preparing my ramen with obvious care, and the teacher thinking about Bansho in the planning of a lesson to help the students understand the content. I find Japan such a fascinating place, as I see traditions from hundreds of years being enacted in seemingly innocuous everyday activities. I observed an obvious pride in one's work, and an effort to do things the right way, even if that required more time and effort. Education is perhaps the most profound reflection of a nation's culture and society, and I left inspired by the passion and devotion of the educators.

One of my strongest memories was having lunch with the students of Oshihara Elementary School. I was struck by how welcoming they were and how proud of the school they felt. I satamazed as the students buzzed around, preparing lunch and serving us, while the teacher mostly facilitated the process. The students have a great deal of freedom and responsibility at the same time.

Three significant themes emerged as I observed the lessons and post-lesson discussions in the Japanese schools: Collaboration, Research, and Reflection. I will comment on each of these ideas in the following paragraphs.

1) In the US, we use the expression "team player" to describe a worker who is willing to collaborate or even make sacrifices for the betterment of the team. I imagine this expression doesn't exist in Japanese in the same way, as that appears to be a cultural norm. I was particularly struck by the way in which all of the teachers at the school jumped up to assist in any way needed, whether it be to hand out papers, set up a room with tables, hand out snacks to a visitor, or videotape a lesson. It is clear that all of the teachers are there to educate all of the students. This collaborative feel is evident in the students behavior.

There is a palpable feeling that the schools belong to the students, and there is evident pride in this on the part of the children. One sees the collaboration in some of the structures, including the set-up of the teachers' room. The desks' location inside the teachers' room allows for constant discussion and reflection. This is in stark contrast to the typical set-up in the US, which seems to foster teachers' isolation and individualism. If asked, teachers in the US will discuss lessons and ideas, but it isn't structural in nature.
The collaboration is there in the efforts to improve teaching and learning. Lesson Study and the

intense planning that is required fosters the sharing of curricular ideas and teaching practices. There appears to be a constant striving to perfect that which is imperfect. There is a glory in the group accomplishment. Lesson Study is a vehicle to genuine collaboration.

At the same time that collaboration is fostered, a sense of student independence is an obvious value in Japanese classrooms. The students are expected to solve problems on their own and persevere through difficult tasks. They are given time to contemplate the problem situation and to develop a plan. They are given time to struggle with the mathematics, which is a necessary component of learning. I feel that in the US, we intervene far too quickly to offer assistance to the students before they have genuinely attempted the problem. I saw an example of this during the 6th grade area of a circle lesson. I chose to observe one girl working for the duration of the lesson. She sat for quite some time (although it probably was about 10 minutes) struggling (I mean this in a positive sense) with the task until she developed a very unique strategy. The teacher didn't intervene to show her what to do; she was expected to solve the problem using mathematics that she has learned. The teachers engage in Kikanshido, a kind of teaching in which they walk between desks offering hints but not showing the students what to do. Perhaps the most powerful thing that I observed in this respect was many students simply sitting, flipping their pencils in their hands, thinking. They were just thinking about different ideas. They didn't necessarily know what path to take, but were given ample time to consider different options. We must give our students this time to just engage in the simple act of thinking. How do we accomplish this task of fostering collaboration and independence simultaneously?

2) Research: In many of the final comments after the lessons, the importance of a thorough Kyozaikenkyu was emphasized. It is critical to thoroughly analyze the curricular materials to see how different topics can be treated. The teachers in Japan carefully look at the unit plan, and systematically structure the lessons so that the units are coherent. They look at the learning that takes place before and after the grade that is being taught to see how the lesson fits in the coherent whole. I am profoundly impressed by the level of detail and precision in the lesson plans. It is apparent that the teachers discuss each element of the task carefully, try the problems in order to anticipate possible student solutions, and think about what questions to ask in order to elicit the responses that will lead to maximized learning. In my work with teachers in the US, I intend to use the lesson plan as a vehicle for developing these skills of analysis and in improving the teacher's capacity to plan generally. The anticipating of student responses is an underdeveloped idea in the US, in large part because we don't necessarily value the student thinking and alternative ideas in the way Japanese teachers do.

The observation of student thinking during the lessons has a feel of authentic research. I noticed how carefully the teachers were observing the students' work and thinking. This observation allows for a richer post-lesson discussion and a clearer sense of what the students have learned. This observation is an example of data collection in the natural habitat of the students. Lesson Study affords us the opportunity to have extra pairs of eyes in the classroom to see different things that are happening in the classroom that the teacher can't see. We can get a much sharper sense of the effectiveness of our teaching through the comments of the observers in our classroom. In a sense, this is the intersection of collaboration and research.

3) Reflection: I was thoroughly amazed by the level of reflection embedded in the Japanese

teachers' practice. At times, it almost felt as though the lesson was taught so that there would be a excuse to have a reflective discussion. I was astounded by the length of the post-lesson discussion after the cube cutting 5th grade lesson, as well as the focus and stamina that the teachers displayed. I will never forget the teachers, after a 1 hour discussion about angle bisectors (including a high level final comment) standing in front of the blackboard for another hour analyzing the lesson. This kind of deep reflection about the successes and weaknesses of a lesson is a necessary component of successful teaching. As I look to improve and broaden our implementation of Lesson Study at my school, I found it very useful to see various forms of LS practice. After the 9th Grade lesson about operations with square roots, the teachers engaged in an activity with post-its in which they recorded strengths of the lesson, areas to improve, and ways to improve the lesson. I think this reflection before the actual post-lesson discussion enables the discussion to go more smoothly, as the participants have had the opportunity to formulate their thoughts more coherently. Generally speaking, the teachers are given ample opportunities (as the students are in the classrooms they are teaching) to reflect on their practice in a constructive way to improve student learning.

I return to my school with not merely new ideas to implement but a new mind-set about teaching and learning. Before this trip, I felt as though I was imagining a form of professional development that I could only conceptualize through books and articles. After the trip, I have seen and experienced Lesson Study in its most authentic state. I was pleased to see that LS is not a prescribed set of steps to follow, but can be implemented differently from school to school and depending on whether the LS is school based or district based. It is a practice that is clearly grounded within Japanese culture, and fits perfectly within the cultural norms. There is no reason, however, that teachers in the US and around the world can't practice Lesson Study with fidelity. I am extraordinarily grateful for the opportunity to be immersed in teaching and learning in Japan, and to meet the children and their teachers.

Project IMPULS Reflection
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To reflect on my Lesson Study Immersion experience, I chose to consider two major questions: (1) what did I learn about Japanese mathematics instruction and lesson study and (2) what are the main ideas I will take away from the experience? I have organized my reflection into comments about Japanese education and culture, in general, and comments about lesson study, in particular.

Japanese Culture and Education

The Japanese education system and the philosophy behind mathematics teaching are grounded in the work of Americans. John Dewey, an American philosopher whose work has been generally rejected in the United States, was a major influence on Japanese educational philosophy. Progressive education focuses on the whole child – the intellectual and the social. We saw evidence of this movement throughout our visits in the schools: students took responsibility for cleaning their schools, they served and cleared lunch for each other, and the schools were often arranged into open classrooms. Howson, Keitel, and Kilpatrick (1982) stated that we cannot apply an innovation from one country and expect the same results; there are too many other factors to consider. What I find intriguing, however, is that the Japanese took a philosophy from America and have been quite successful. What might have happened in the US had we collectively embraced Dewey's ideas? Would the result have been the same? I doubt it. The Japanese culture appears to value education and respect teachers more than the American culture.

In 1980, the U.S. National Council of Teachers of Mathematics published *An Agenda for Action*, a report that made a variety of recommendations for the improvement of mathematics teaching and learning. The first recommendation was for problem solving to be the focus of school mathematics. The view of problem solving taken by the Japanese reflects George Polya's view of problem solving as an art; teachers must teach children how to think and process of problem solving is one way to accomplish such a goal. The National Council of Teachers of Mathematics elaborated on the value of teaching through problem solving: "A problem centered approach… uses interesting and well-selected problems to launch mathematical lessons and engage students" (2000, p. 182). We saw this approach in each classroom we visited. In each lesson, there was a major mathematical question, usually one that required students to develop strategies, test their strategies, compare strategies, and come to a closure that answered the main mathematical question.

Throughout the problem solving process that transpired over the course of a 45-minute class session, the teachers typically kept a record of the conjectures, reasoning, and conclusions made by the class. The practice of keeping track of the class interactions is called *neriage*. (See below for an example of a class board.) This practice makes student thinking visible and encourages students to construct arguments and critique the reasoning of others, one of the new Standards for Mathematical Practice embedded in the United States Common Core State Standards for Mathematics. Although the potential for critiquing each other's reasoning was present, the teachers did not often capitalize on such opportunities. Like U.S. teachers, the observed teachers might have been concerned about the time required for students to engage with other's ideas. Most of the teachers used traditional chalk boards to display student thinking; however, one teacher used a document camera. This use of technology appeared innovative, yet it did not allow students to compare the various strategies. As document cameras and interactive white boards – placed directly over the chalk/dry erase boards in classrooms – become more and more popular in the United States, I wonder at U.S. teachers' logistical abilities to display a variety of student solutions.

The most striking thing I have learned is that the teachers in Japan struggle with the same issues as U.S. teachers. In discussing teaching, too often the focus is on surface pedagogical issues than on the substantial mathematics in the lesson. Also, like in the United States, this lack of focus on the mathematics might be somewhat exacerbated by alternative certification programs that focus on general pedagogy rather than on deep understanding of mathematics.

As I have worked with teachers from different grade levels, I have noticed that teachers in grades K-8 are often more willing to engage students in problem solving and investigative activities. In Japan, we were told that the Japanese way of teaching is based on Polya's ideas of teaching through problem solving. Furthermore, in Lesson Study, we would see many instances of teaching through problem solving. However, lesson study only occurs in grades 1-9 in Japan. When I asked why this was, I was told that upper secondary teachers in Japan are under pressure to prepare their students for the college entrance exams and so often focus on teaching skills rather than investigation, conjecture, and verification. The college entrance exams are not as high-stakes in the US, yet it often seems that U.S. high school teachers teach for skill mastery when teaching through problem solving would serve U.S. students better in their college and careers than skill mastery.

Japanese Lesson Study

Lesson study, on the surface, is similar to Smith's (2001) Reflective Teaching Cycles. In both systems, teachers work collaboratively to improve the teaching and learning of mathematics. Teachers plan well-thought out lessons, teach the lesson – and others observe, and then discuss the lessons. In Lesson Study, the lessons generally should be planned in collaboration and the post-lesson discussion praises and critiques the entire group who wrote the lesson, not just the teacher who implemented the lesson. This team mentality might make Lesson Study a more enjoyable system than what many presently experience – one teacher plans a lesson, gives it to others, but the teachers do not discuss how the lesson went and all take individual, personal responsibility for implementation of the lesson.

Lesson study can be a very powerful avenue for teachers' learning about mathematics and about pedagogy. However, like all things, successful lesson study is contingent on keeping the purpose of the study at the forefront. If the purpose is to learn more mathematics, the participants must engage in meaningful *kyozai kenkyu*, study and exploration of instructional materials, and a thoughtful post-lesson discussion. However, like in the US, it did not appear that the teachers always planned lessons together, thus how they translated the intended curriculum (the national Course of Study) into the planned curriculum (lesson plan) was often based on one teacher's knowledge and experiences, not a collective knowledge, which might lead to more effective lessons.

What is convenient about planning lessons in Japan is that the intended curriculum is shared by all teachers; that is, the national Course of Study and corresponding Teaching Guides outline what students should learn in each topic of study. This shared understanding of the mathematics that students should learn increases the likelihood that the planned and implemented curricula will be similar for all students. In the US, we are moving in such a direction with the Common Core State Standards for Mathematics. For the first time in recent history, U.S. educators can engage in conversations across states about what students should know and be able to do and how best to teach students these ideas. Schools are still under the jurisdiction of the states (prefectures); however, the move to the Common Core allows for smoother transitions in our transient student population and for greater collaboration among teachers and teacher educators.

We observed a number of variations to the structure of Lesson Study. A number of particular variations or activities resonated particularly strong with me. These are activities I hope to engage teachers with in the upcoming year:

- Planning really should be collaborative. Teachers should work together to discuss the mathematics of the lesson, research the treatment of the topic in a variety of textbooks and resources, consider how students have learned the topic in the past, consider students' prior knowledge and future needs, plan how to orchestrate a discussion and what solution strategies should be highlighted or brought up if not conjectured by students.
- In one school, three teachers simultaneously taught the same lesson and were observed. All three implementations were discussed after the lesson, aided by transcripts of the classes and pictures of the chalk boards from the end of class. This style of Lesson Study would be conducive to the mindset and limitations existing in U.S. classrooms. Numerous curriculum directors have recently told me that they cannot excuse their teachers from the classroom for professional development; Lesson Study activities in which teachers can plan

together but are not required to miss their own classes to gain the benefit of knowing and being able to discuss what transpired in other's classes can prove a value form of professional development for U.S. teachers.

- All teachers involved in the planning and implementation of the lessons should be involved in the post-lesson discussion. All participants should reflect on what occurred in the class(es) before an official discussion begins. This can take place by recording the mathematical positives and mathematical negatives from the lesson and posting on a community board/poster. Pedagogical pros and cons could also be noted, but the focus of the discussion should be on the mathematics. Finally, all should be encouraged to write "what if", "what now", etc., type questions for the group to consider. Once each participant has been given sufficient private think time, the discussion can commence.

I am thankful for my opportunity to travel to Japan to observe Lesson Study in action. It is more complex and more varied than I previously thought. I look forward to developing a plan to use Lesson Study with teachers in my daily work.

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Math Teachers' Professionalisation through Lesson Study 2012
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In my current organisation, the Academy of Singapore Teachers (AST), we are advocating a catalyst and an enabler for professionalism. We encourage teachers to take greater ownership of their professional development and bring forth stronger teacher-leadership. The vision and mission of the academy established in 2010 is "to be the leading academy for professional excellence in education" and "building a teacher-led culture of professional excellence centred on the holistic development of the child" to transform Singapore's teaching service in the next few years. Parallel to the Japanese system, raising the teaching quality is also paramount, strengthening their professional development and fostering a deep sense of professionalism was clearly seen throughout my 10 days of observations in Tokyo. Lesson study was held with upmost importance in all 7 schools that I visited. School Leaders gave great support and time was set aside for teachers to discuss in-depth the mathematical content, knowledge, concepts and skills in teaching and learning. In the schools, teachers were expected to observe one lesson of their peers every month. Teachers would consolidate all learning and write a reflection report on what had been learnt and these would be discussed at the end of each year.

In the process of doing lesson study, the teachers, whether they are beginning or experienced teachers, have the opportunity to deepen their pedagogical content knowledge (PCK) and subject matter knowledge (SMK) learning alongside the 'Knowledgeable others (KO)'. In this case, the KOs were the Professors from the universities or retired teachers. The teachers in school went through rigorous learning, discussion and sharing, not just mathematics alone but also learning together as a community with regards to mathematics education. Together with the KO, they agreed on the research theme which helped the group to focus on the year's goal. The group worked together based on norms; though this was not verbally mentioned. I had the opportunity to observe this unspoken level of trust and respect for one another. I supposed this was due largely to the Japanese culture and expectations that had been developed in them. Japanese teachers grew, learnt from mistakes and graciously accepted feedback from their peers.

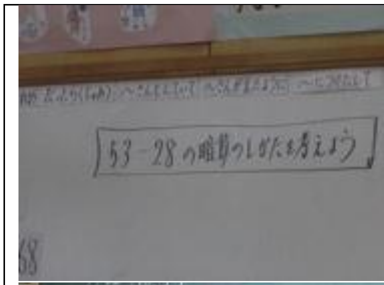
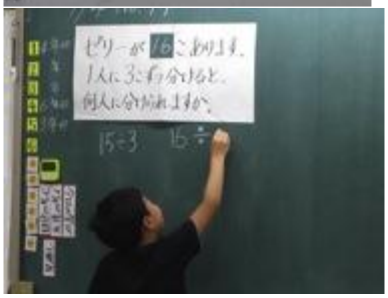
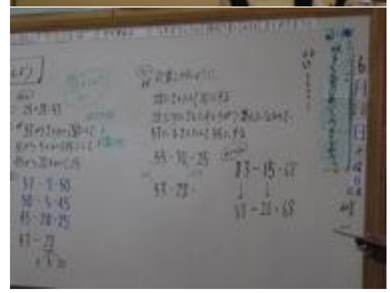
All Japanese teachers were given the opportunities to teach all levels throughout their years of service, likewise for lesson observations. This was to ensure that teachers had a good grasp of the PCK and SMK and developed to be an effective teacher. Having that knowledge enabled the teacher to build on students' prior knowledge in their teaching and learning and to take that into consideration in the design of their lessons. Professional learning did not just benefit the group of teachers carrying out the research; it reached out to the entire school. The research team engaged in different and multiple levels of interactions. They gained their knowledge through interactions with teachers within and from another district, professors and other communities. This was over and above their on-going daily classroom experiences.

Though I was not part of the research teams in the planning of the 7 lessons, it was necessary for me as an observer to study and analyse the thinking behind the lesson planning. The lesson plan encapsulated the teachers' knowledge of the curriculum, pedagogy and content. It was jointly planned and discussed, elaborating in detail the intent of the research and approach. The lesson plan worked as a blue print to help observer anticipates students' response, look out for key observable learning behaviours and teachers' facilitation discourse in drawing out the essence of the learning in the lesson. All the lesson plans were aligned to four evaluation criteria; interest, eagerness and attitude, mathematical way of thinking, mathematical skill and knowledge and understanding. This was similar to our Singapore Mathematical Framework. This was clearly seen in the enactment of the lesson in the classroom and brought up frequently in the post-lesson discussion.

Through research discussions and reflections, Japanese teachers acquired knowledge of curriculum, assessment, instructional strategies and students' understanding. The classroom teacher used questioning techniques to probe and clarify the students' thoughts and understanding in his teaching. Students' confidently presented their ideas or solutions to the class. They demonstrated metacognitive skills and self regulated their thinking by gathering information and feedback from classroom teacher and peers. This was supported by Vygotsky's zone of proximal development (ZPD). In the collaborative dialogue with teacher and peers, the students internalised the information and used it to guide their performance. This allowed the students to clarify thoughts and reorganize their own facts. In the midst of the interaction and engagement in reasoning, communicating their conjectures and verifying among themselves, students made assumptions and generalised what they had discovered. Solutions were shared by different students and discussed in depth to understand the concepts behind it. Classroom teacher would occasionally identify misconceptions and errors, getting students to make sense out of their own reasoning and thinking. They also found patterns in their solutions and developed their own mathematical concepts and skills.

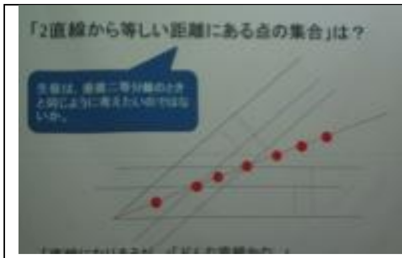
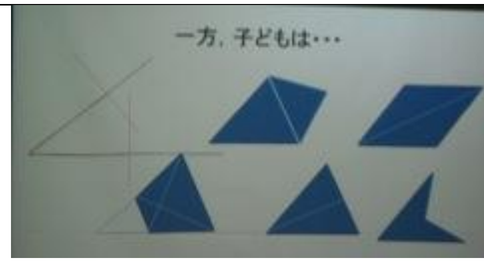
The post-lesson discussion provided a platform for the Japanese teachers to continue their professional development by engaging teacher observers as reflective practitioners. The objectives of making the lesson public was to encourage the teachers to reflect on their own teaching practice and further improved their teaching based on their reflections. The strength of the lesson study was centred in carefully selecting appropriate problem, extensive classroom discussion (Neriage) and emphasis on blackboard practice (Bansho). The teacher would present, summarise and consolidate students' discussions and thoughts sequentially on the blackboard. This was a common practice and a good presentation on the blackboard provided insights to the flow of the lesson, the mathematical concepts and strategies being extensively discussed and contributed by students.



 	 	 
Slide 1 Select appropriate problem, students to investigate	Slide 2 Neriage	Slide 3 Bansho

The research team or the teacher would present how they sequenced the teaching and the knowledge of the lesson which displayed their rich knowledge of PCK and SMK in the designing of the lesson. They also discussed deeply how students would have developed the concepts. Surprisingly, some lessons were deemed unsatisfactory, as the classroom teacher failed to help students made connections to the key aspects of the lesson, though teachers were able to orchestrate constructive teacher-students discourse in the classroom and the extensive discussion in the lesson.

The comment given by teacher observers very often mentioned that the teacher overlooked the mathematical concepts in their planning and teaching, and failed to tap on students' prior knowledge. It was brought up, on several occasions, the importance of 'Kyozaikenkyu'. Studying and comparing more than one textbook and instructional material in the context helped students to think about and understand the concepts they were going to learn. It was pointed out by the professor that in the delivery of the lessons, teacher needs to make connections to what they had learnt previously (Slide 1) and what was going to be learnt (Slide 2) as shown below. It was impressive to see how intellectually the KO picked out key mathematical concepts and skills that needed further development. KO also highlighted that in planning a lesson teacher needs to identify the key concept that was to be taught to the students. In the classroom discussions teacher should draw out students' learning and guide them to extend their learning beyond the concept supposedly to be learnt. KO emphasised that if lesson was sequenced well to develop student's ability, teachers would expect students to mathematize beautifully and make connections.

 Slide 1 Consider the set of points on the line of symmetry, equidistant from the given 2 lines	 Slide 2 The symmetry and the shape – lead to rich exploration of axis of symmetry

In summary, the whole process of lesson study provided a platform for teachers to grow continuously and be effective classroom teachers by learning with, learning from and learning on behalf of others. Professionally learning with a community of practitioners, enhanced the PCK and SMK of the teachers and provided insights of teaching and learning in a new dimension. There were some key aspects which I might want to seed in my various networked and workshops. Particularly, I would like to start with the Lesson Study network by having extensive discussions about teaching a particular key concept in a lesson and reflecting on teaching and learning to deepen teachers' PCK and SMK.

Acknowledgement:
I would like to thank all Professors, IMPULS project members, AST, School Leaders, teachers and students from the 7 schools in giving me this opportunity to experience Japanese Lesson Study and culture, making this trip enriching and possible.

Reflection – IMPULS Lesson Study Immersion Program 2012
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During the period of June 25 – July 4, 2012 I visited seven different schools in Tokyo and Kofu, Japan where I observed mathematics research lessons and the post lesson discussions for those lessons. As part of the program I was asked to provide this reflection paper focusing on one significant finding from my observations and experiences.
I have chosen to focus on nine components of a math lesson that, to a certain extent, were observable in each of the seven research lessons.

The nine components are:

1) Lesson set up – teacher carefully describes the goal of the lesson and/or the problem the students will attempt to solve
2) Lesson is built around 1,2 or 3 carefully selected problems or applications
3) Hatsumon – key question(s) during the lesson
4) Time for student individual or small group work
5) Teacher walking the room during student individual or small group work time to observe and take notes on student work and solution methods
6) Bansho – carefully planning what will be written or posted on the board and where it will be placed
7) Neriage – facilitating class discussion based on the thoughtful sequencing of student solutions to bring the students to understand the concept being taught
8) Summary of lesson
9) Journal writing in student notebook

I consider this finding to be significant because of the following reasons:

- I consider each of the nine components to be effective strategies for providing an engaging and effective lesson to students.
- The appearance of these nine components in seven different schools provides evidence of a consistent delivery of in service teacher education and ongoing teacher professional development in Japan regardless of where teachers are trained or where they teach.
- The appearance of these nine components in seven different schools provides evidence that Lesson Study provides an effective means for transmission of effective pedagogy and content knowledge between teachers through collaboration.

Although the nine components were evident in all seven lessons, the execution of the components was delivered with varying degrees of effectiveness. Even so, the fact that the components were present in each lesson demonstrates consistency in high level mathematics lesson planning in each of schools. With this high level planning in place and with the effective, ongoing professional development provided by Lesson Study, the teachers in these schools are in a position to improve each and every year no matter their current ability level.

The following table will provided the evidence of each of the nine components in each of the seven lessons. The evidence was gathered from my observation notes and pictures and from review of provided lesson plans.

Table: Evidence of nine components of effective mathematics instruction in seven different lessons

Lesson	Lesson Set Up	Lesson Problem or Application	Hatsumon	Student individual or small group work	Teacher observation of student independent or small group work	Bansho	Neriage	Summary	Journal writing in notebook
Grade 6 Area of Various Shapes Funabashi Elementary School	Teacher reviews how to find area of rectangles using graph paper where each square represents 1 sq. cm	Teacher provides student with a circle drawn on graph paper and asks students to find the area of the circle	Try to find approximate area of circles using graph paper	Students work independently for 10 minutes and then share their findings with other students for 10 minutes	Teacher walking the room and recording observations on clipboard	Teacher uses chalkboard as well as document camera to present student work. Teacher organizes the chalkboard so that similar solutions are grouped together	Teacher facilitates discussion around two solution methods that resulted in similar answers	Teacher uses work posted on chalk board to summarize learning content	Teacher asks students to reflect on what they learned today and write their impressions in their journal
Grade 7 Calculation of arithmetic mean Yamanashi University Lower Secondary School	Teacher explains that this is a special lesson, not part of regular curriculum. Reads problem and posts it on the board	Distribute 10 basketball players into two teams that have the same average hieght	To make teams with equal average height, is there a way to make the calculation simpler?	Students work in pairs for 10 minutes	Teacher walking the room looking for 4 particular solutions as indicated on the lesson plan	As students orally present their solutions from their seats, teacher records the work on the board	Teacher attempt to lead the students to see that by using a base number a tentative average the numbers become smaller and the calculations become easier. And that by using integers, the	Teacher uses work on the board to re-iterate how using tentative average can make the calculations simpler.	Teacher asks the students to reflect on their learning by writing in their journal

							positive and negative numbers cancel each other out		
Grade 3 Mental calculation of the differences of two 2 digit numbers Oshihara Elementary School	Teacher discusses the merits of mental calculation in everyday situations. Refers to yesterday's lesson – "it was hard but I learned by listening to other students	53 – 28 68 – 28 89 - 28	Let's think about ways to mentally calculate 53 - 28	Students asked to write down their ideas for mentally calculating the problem in their notebook – 12 minutes	Teacher observing all student work and taking notes	Teacher recording student ideas from left to right on blackboard	Teacher leads students to understand that it is ok to split 28 into 20, 5, and 3 to make the mental calculation possible. Leads class through other methods of mental calculation	Uses blackboard to summarize all the different solution methods	Asks students what should we make the title for our journal entry. Students decide on Mental subtraction
Grade 5 Deepening student's understanding of the characteristics and properties of cubes	Teacher uses a cube and a few different nets for demonstration of task. Teacher provides cubes to students to cut.	Write in your notebook why you think we need to cut 7 edges to open a cube.	How many edges of a cube do we need to cut to open it to be a net?	Gives students 7 minutes to write down their thoughts	Teacher observing student work looking for 3 particular solutions as indicated in lesson plan	Teacher organizes blackboard so that all three solutions are posted	Through student presentation and class discussion, teacher leads students to understand the three solution methods	Teacher uses work on the board to summarize the lesson	Teacher asks student to reflect on their learning by writing in their journal
Grade 9 Calculation of Expressions with square roots Sengen Junior High	Draws rectangle on the board with side lengths to demonstrate the task	What is the area of a rectangle with sides square root of 2 and square root of 5	Can we say that the square root of 2 x the square root of 5 = the square root of 10	Students work individually and in pairs to verify that the square root of 2 x the square root of 5 = the square root of	Teacher walks around the room to observe student work. Also giving hints to students who are stuck.	Teacher records all the steps of the calculation on the blackboard	Uses student solutions to lead the class to generalize that the square root of a x the square root of b = the	Teacher uses blackboard to summarize the lesson and reiterate the generalization	Students write lesson reflection in their journal

				10 by using a calculator			square root of ab		
Grade 7 Constructing bisectors of angles using points of symmetry Koganei Junior High School	Teacher draws an angle on the board and asks a student to come up and draw a point equidistant to the two lines that form the angle	What can you say about a set of points that are equidistant from two given lines? Construct the bisector of an angle.	Segments connecting points that are symmetric around the axis of symmetry will intersect on the axis of symmetry	Teacher asks students to work independently to find as many different ways to construct an angle bisector as they can	Teacher observing student work looking for 7 specific methods to construct an angle bisector	Teacher has students come to the board to record their solution methods	Teacher uses student solutions and class discussion to lead the students to understand that segments connecting points that are symmetric around the axis of symmetry will intersect on the axis of symmetry	Teacher organizes solutions on the board to summarize the lesson	Teacher asks students to write what they thought about today's lesson
Grade 3 Division situation with remainders	Teacher uses several division expressions and asks students if they can figure out which ones will give us remainders	16 ÷ 3	How is this problem different that the others?	Teacher asks students to think about ways to find the answer to the problem and write down their methods in their notebooks	Teacher observes students independent work and takes notes	Teacher uses the black board to write down the expressions, equations and diagrams that the students used to solve the problem	Teacher uses student work and class discussion to lead students to understand that calculation for division with remainders can be carried out using the same method for division without remainders	Teacher reminds students of the days task and uses work on the board to summarize the lesson	Teacher encourages students to write the details of how they thought in their journal

Each of the seven lessons had individual strengths and weaknesses due to the plan of the lesson and the ability of the teacher. And, each of the teachers demonstrated varying degrees of skill in executing the nine common components, especially Neriage and Bansho.

However and in conclusion, the consistency in lesson design and in the content knowledge of teachers was significantly greater than what I have seen in U.S. mathematics instruction during the hundreds of lesson observations I have conducted as a Lesson Study practitioner, school academic coach and provider of teacher professional development.

Reflecting on the purpose and success of lesson Study Japan 2012

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For me, the aim behind lesson study is to identify areas of development for individual teachers and schools with the sole intention on improving these areas in order to raise the standards of teaching and learning.

Pre-Japan my experience of lesson study had either been in school or across the borough. The approach we had was always on a smaller scale. The maximum being 16 members of staff (at all levels from across the borough) involved who were split into groups of 4. Here, we all had a particular area of teaching and learning we were to report on. In house lesson study had involved cross curricula work either in pairs or triads.

The approach to lesson study must be delivered in a way to create a supportive, team working environment. Planning should be collaborative and feedback constructive with follow up meetings to plan ways to move forward. In Japan, most post lesson discussions created the supportive environment. Indeed there were many people from different areas of education present who were able to contribute their vast experience and advice. Perhaps at times, the feedback seemed to offer no suggestions of improvements, but instead critical solely towards the teacher. Those post lesson discussions I believed to be more successful in identifying areas to focus on were those that were smaller and structured to include directed activities which followed a carefully thought of agenda. When the group was too large comments given in the feedback were repeated which is inevitable when everyone is keen to share their ideas.

It was unclear how much pre-training for observing a lesson members of the lesson study had had. Some appeared to look for specific attributes of a lesson, where as others had a less focus driven approach. Would a lesson observation template have improved post lesson discussions with people offering comments on key areas?

When the question was raised as to what the follow up plan after the lesson observation was it appeared that varied between schools. It would be a shame if there were no opportunities for teachers to work on the areas identified and move forward with their classroom practices.

Initially it was the Mathematical content and delivery of the Japanese Mathematics curriculum that excited me about visiting Japan. Possibly more so than observing the processes of lesson study. From all lessons the comparison to the Maths curriculum in the UK was clear. Although the content was the same, at which point students first met it differed. In Japan, students are taught topics much earlier. Their approach to the teaching is for students to understand why a concept works. This is a key in developing good problem- solving students and ultimately good Mathematicians. Topics were mostly presented in context with opportunities for students to discover connections and try different approaches to reach a solution. Indeed these were the learning objectives to almost all lessons observed.

Although I was impressed by the content and learning objectives of each lesson the delivery and particularly the engagement was lacking. The introduction and setting of each task was almost identical in every lesson. Students worked by themselves with some collaboration at times. The activity of students sharing their ideas with their neighbour or the class was not planned enough to be fully utilised. It was unclear the purpose for the pair work. Due to the nature of the students they did listen and explain their ideas. However, a very small proportion used their neighbours' ideas to re-evaluate their own work and try a different approach. Perhaps paired discussion would of worked better if students were assigned roles or given prompt questions to ask each other to fully understand someone else's work. It was observed many times that during class discussions students continued to work on solving the problem instead of listening to the ideas being shared. When ideas were shared some teachers used this opportunity to refine students' mathematical language in their explanations. This worked when well planned open questions were asked. However, far too often questions were closed and did not lead to an open discussion building on students' thoughts. At times key misconceptions rose which were not explored to consolidate students understanding, achieving that all important lesson objective. With carefully scaffolded questioning I believe the teacher could have supported students better to understanding the reasoning being the solution.

Observing students practices in the classroom with a particular focus on their approach to their work, it was evident students had developed many good thinking skills. I often wondered how students were aware of how they learn. All but four to five students observed were resilient in their approach to their work. The safe and positive learning environment created learners who were not afraid to 'get stuck in'. Often students would made mistakes but this did not stop them attempting the problem again. Unfortunately not all students have this approach in the UK, often with students giving up after the first attempt or not even attempting to put some ideas down. There were many important thinking skills that were not so well developed. Most students did not learn from each other or able to distil their thoughts into coherent explanations which the teacher so often asked for.

During lessons all students worked on the same problem. Since the problem was open to being solved with many approaches it worked to an extent. There was no support offered to students who needed it. In the UK if teachers did not plan to support students of all abilities to make better than expected progress in their lesson it would be graded Unsatisfactory. With optional support available, students would be able to try a solution on their own first yet have prompt starting points when unsure. It was not only the lower ability students that appeared not to be supported but also the gifted Mathematicians. There was no stretch or challenge given to these students and at times they were the ones who drove the class discussions.

In the UK, teachers will spend a considerable amount of time on thinking how to achieve the lesson's objective. They would plan a range of activities to develop thinking skills and engage students. In Japan a range of tasks was not seen. With Japan's focus on the Mathematical content both countries can learn a lot from each other here.

There is a lot one can draw from both the lesson and post lesson discussions seen. The practice of lesson study appears to be well established and valued in Japan. This has taken time. In the UK in order to reach such a high standard our approach must be reflective with us constantly looking to refine and improve the process.

IMPULS Lesson Study Immersion Program 2012- My reflection

Siew Ling Connie NG

Curriculum, Teaching and Learning Academic Group, NIE, Singapore

This very well organized and enriching Lesson Study Immersion Program provided me with an understanding of the key elements of Lesson Study and its potential in developing student understanding and teacher competency. Besides the opportunity to observe authentic Lesson Study in mathematics classrooms and learn about Japanese school culture, the interaction with fellow participants of this immersion program also gave me insights on how Lesson Study is implemented in other countries.

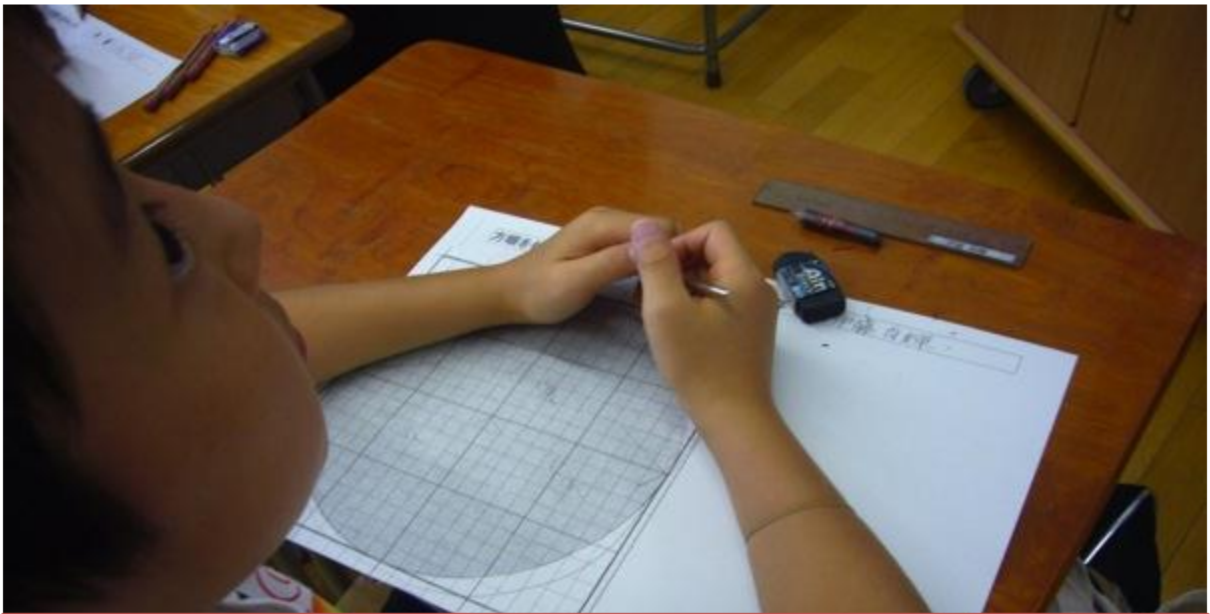
For this reflection, I will concentrate on sharing one important take-away from this immersion program.- the power of focusing on student understanding. While this may be a simple idea, I witnessed how the strong focus on student understanding throughout the cycle of lesson study (planning, lesson and post-lesson activities) enabled student learning to be achieved.

Being new to Lesson Study, I was initially not sure what to focus on during lesson observations. What should I look out for? Pedagogy? Classroom management? After Professor Akihiko's presentation on Lesson Study, I realized that the emphasis is on student understanding. I decided to focus on this during the first lesson observation of a Grade 6 Mathematics class at the Funabashi Elementary School. In this lesson, each student was given a grid paper and asked to use it to estimate the area of a circle. Students were given 12 minutes to work individually and then gather in small groups to explain their ideas to others and find the best method to calculate the area of the circle. Students from some groups were then selected to present their group's method to the whole class.

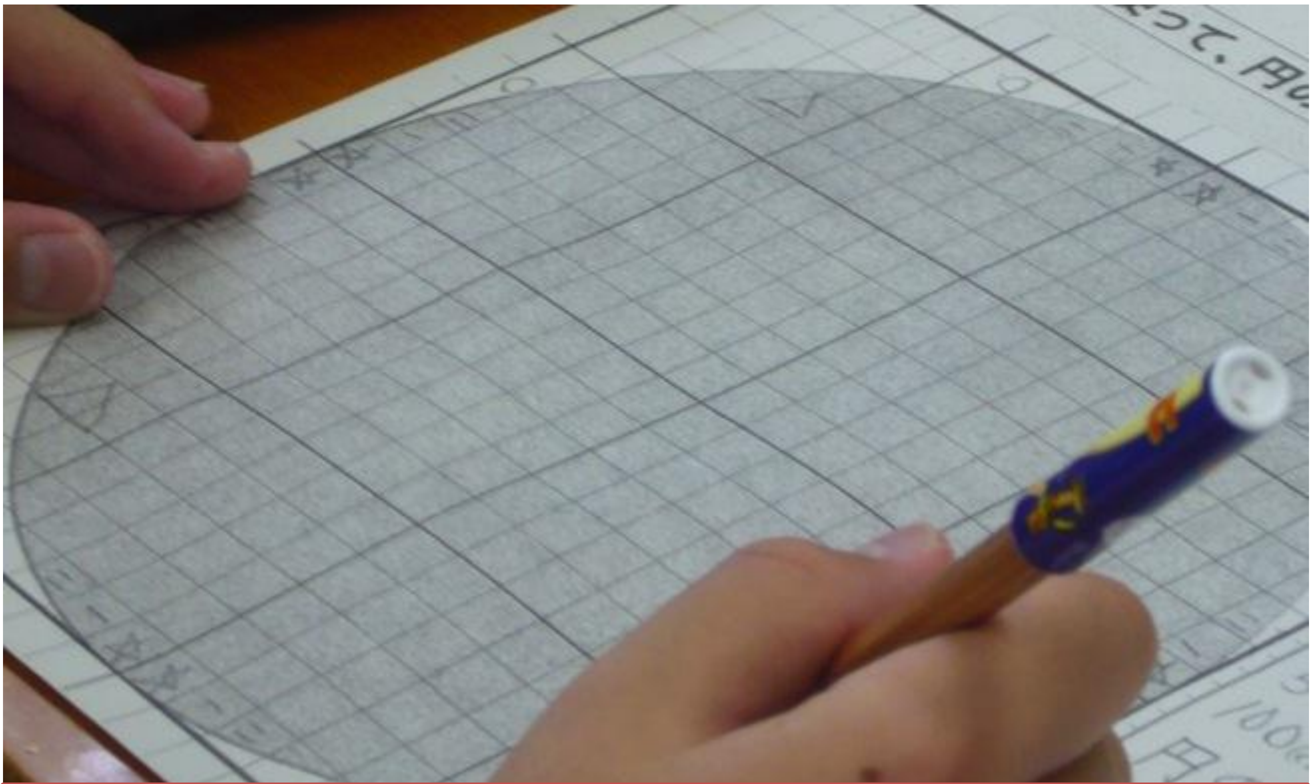
My observation of a few students revealed that the students used the following methods to calculate the area of the circle: (refer to the photos for more details):

1. Counting of individual squares (Students A and D)
2. (Possibly) making use of the radius to calculate the area. This student (Student B) might already know the formula for the calculating the area of circle
3. Pairing/Grouping the partial squares at the boundary to estimate the rounded regions of the circle (Student C)
4. Dividing the inner squares into smaller regions to calculate the regular portions of the circle (Student E)

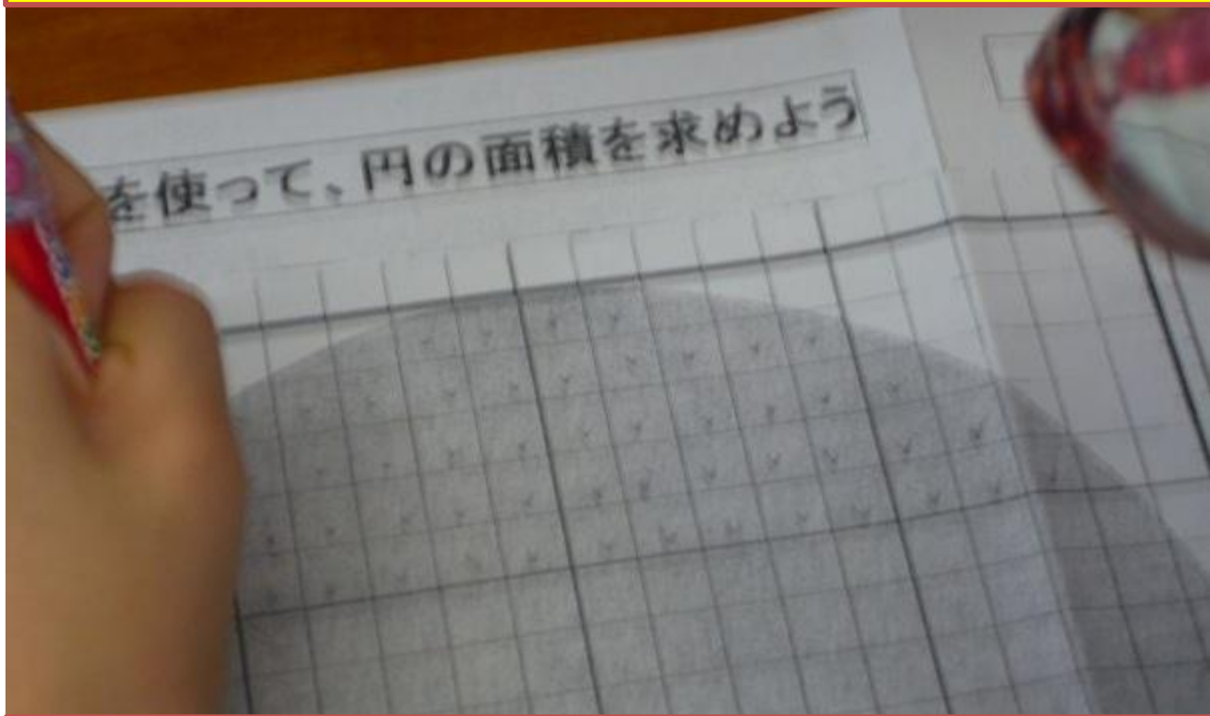
Student A outlining the boundary of the complete squares and counting them



Student B wrote down some words and erased them away. In his diagram are the labels “10cm” and “20cm”.



Student C marking some of the squares



Student D ticking the complete squares row by row.

Student E dividing the complete squares into smaller regions

It was obvious that some of the students were using the laborious method of ticking and counting the squares one by one. A number of the students who did it using this method did not complete counting in time to share their answers in the small group discussion. For students such as Student E who divided the grid paper into smaller regions, they were most likely applying their prior knowledge of calculating the areas of squares/rectangles.

A number of questions came to mind when I focused on observing student understanding. What prior knowledge did the students have? Did all of them already know the formula for the area of rectangles/squares? If so, why was it that only some applied it to calculate the regular segments of the area of the circle? Were these students more remindful that they had only limited time to complete the activity individually? Were they more aware of how mathematics formulae can be applied to daily life? Did those who counted the squares one by one not know the advantage of using a formula?

Perhaps I had more questions because I have not taught Mathematics before and was less familiar with the subject and students' understanding of the topic. However, the strong emphasis on collecting evidence of student understanding captivated my attention and helped me focus on observing the students rather than the teacher. Through observation of this and the other six lessons and my understanding of the post-lesson discussions, I noted how this focus on student understanding acted as a catalyst to encourage the teachers to collaboratively plan and identify the goals for the lessons. Besides pondering and deciding on the overarching goals of the lesson guided by resources such as textbooks, the teachers built in opportunities for students to make their thinking visible through the explanations which they had to write down in their worksheets and through sharing their answers in group discussions and/or whole class presentations. These allowed students' prior knowledge of the topic to be made visible and identified or confirmed. Sometimes, scaffolding by the teachers in the form of higher order questioning was needed to guide students to articulate their ideas.

To consolidate the learning points of the lesson, the teachers selected representatives from some groups to present their group's answer to the whole class .I was not sure how the teachers decide on which students to select but I thought that if the focus was on student understanding, a variety of various methods used by students could be presented for further discussion. For example, for Student B from the Funabashi Elementary School, he might already have known the formula for calculating the area of a circle. It would have been interesting to hear his explanation during the whole class presentation. Bearing in mind the focus on student learning, the criteria for selecting student work for whole-class presentations could be decided during the lesson planning stage as well. This will contribute to the consolidation of learning at the end of the lesson.

Although every class is different in the real world, I saw how the strong emphasis on student understanding guided teachers to look for evidence of student learning and improved their teaching. The ability to do so- to "see a lesson from the student's point of view" and develop "the eyes to see children" is "the most important goal of lesson study" (Lewis, 2004, p. 36) This understanding convinces me of the potential of Lesson Study in improving student learning and teacher. With this understanding and other learning points from this immersion program, I will propagate the practice of Lesson Study.

Thank you IMPULS team for putting together this enriching and very well-organised immersion program.

Reference

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Reflections on the Project IMPULS 2012 Lesson Study Immersion Program

Tom McDougal

Executive Director, Lesson Study Alliance, Chicago, U.S.

Let me begin by expressing my gratitude to Project IMPULS and its funders for making it possible for me to participate in this Lesson Study Immersion Program. I learned a lot from it and enjoyed myself thoroughly, and all of the logistics were extremely well handled.
Having attended a similar program 5 years ago, I came on this trip with slightly different goals and expectations than I might have had otherwise. I was less concerned with learning about what Japanese elementary schools are like and about how lesson study is done in Japan, and more interested in learning about nuances in Japanese lesson study, such as the different purposes served by lesson study; about how post-lesson discussions are handled; and about the kinds of points that knowledgeable others make in their final comments.

With the very first lesson, which focused on area of circles, I learned something about all of these. In the briefing before the lesson, we learned about the structure of school-based lesson study, the role of the steering committee and chairperson, and the importance of lesson study for helping teachers, in this case younger teachers especially, meet the challenges of implementing a new national curriculum. In his final comments following the lesson, Dr. Takahashi focused on the importance of conducting a thorough kyozaikenkyu, contrasting the approach taken by the textbook used at that school with the approach taken in another textbook. He explained to us later that he was addressing his comments to the steering committee. I also saw how a person responsible for final comments might take advantage of early access to the lesson plan to prepare slides illustrating some of the points he expects to want to make.

In the discussion of a later lesson, on mental subtraction, I was able to see an example of very effective moderating. The moderator kept the conversation focused by occasionally summarizing the points that have been brought up, and then either inviting additional remarks or explicitly shifting the conversation to a new topic.
It was interesting for me to see, especially against this one strong positive example, that some of the other discussions were not well moderated. This leads me to wonder: in Japan, how does one develop the skills to moderate a post-lesson discussion? Is there any explicit system, or is one expected to pick it up by example? I wonder whether ideas about effective moderating are widely shared, and, if not, whether there might be some way to facilitate the spread of this kind of expertise.

The middle school lesson we observed about the product of square roots involved an application of lesson study that I had not heard of before: the explicit development of teacher leadership. This application makes perfect sense, as it gives teachers an opportunity in a fairly small group to practice the role of moderator, to push their own teaching, and to develop their analytic abilities.

This program gave me plenty of opportunities to observe and think about effective teaching. In the lesson on mental calculation, the teacher began by having students think about several different problems and decide which one they thought might be more difficult to compute. He then had them actually calculate the correct answer prior to asking them to consider how they might calculate that answer mentally instead of using a paper and pencil algorithm. The first step helped get students thinking about the difficulty of computation, and what it is that makes one problem more difficult than another, which also helped prime them for coming up with a possible solution—i.e., to transform the problem into a simpler one, with the same answer. The second step eliminated the answer itself as a matter of concern, helping students focus on finding an efficient process for calculating it.

For one lesson we observed that did not go so well, on the construction of the angle bisector, the discussion we had with Dr. Takahashi later highlighted the importance of making sure that every lesson had something new in it, and keeping the goals of the lesson in mind. The lesson got mired down in a discussion of the specific steps a student had used, and whether those steps were the same or different than the steps another student used. Critically lacking was any discussion of why those steps actually produced the angle bisector. This issue was not highlighted during the post-lesson discussion, which I believe led to some sharp private critique after the discussion was over.

Although it wasn't an explicit goal of mine, I also gained some useful insights into student thinking. A prominent instance of this was the last lesson, on division with remainder. From the final comments given by Prof. Fujii, I came to appreciate how students might think of remainders in ways different than I do. For example, given 16 divided by 3, a student might think of this as 6 remainder 2, which can make sense depending on the context. If there were 18 children going on a carnival ride that has cars seating 3 children apiece, then clearly we need 6 cars and there will be 2 additional spaces. During the lesson, there was at least one student who had this “incorrect” answer, and professors Fujii and Takahashi argued that a valuable opportunity was missed by not discussing that answer.

I always treasured the discussions that we had on the bus after some of the lessons, led by Dr. Takahashi. These were enormously helpful for understanding the main points of the post lesson discussion and for solidifying my learning. I think this view was widely shared among the participants, and so if the program could be improved, I think it would be by formalizing this process so that it happened more regularly, either on the bus or the next morning.

One lesson in particular that I would've wished to discuss more was the lesson on multiplication of square roots. As Prof. Fujii remarked to me privately, this was a Level II lesson, in which most of the key ideas came from the teacher. Some of this might have been a result of student shyness beneath the eyes of 40 observers. But how might this lesson have been restructured to be more Level III?

Overall, this was a fascinating, enjoyable, and valuable experience. I am confident that my own practice as a facilitator of lesson study in the United States will benefit from what I have observed and learned, even though there is still clearly very much more for me to learn.

IMPULS Lesson Study Immersion Program Reflections

Tracy Sola

Lesson Study Co-Coordinator Silicon Valley Mathematics Initiative,

Mathematics Coach and Teacher Belmont Redwood Shores School District

I am profoundly grateful to the IMPULS Program for providing to me the opportunity to study lesson study work in Japan. The experience of participating in the 2012 IMPULS Lesson Study Immersion Program will be transformative to my teaching, coaching, and lesson study coordinating practice.

Silicon Valley Mathematics Initiative Lesson Study Coordination

Publishing Findings

As the co-coordinator of the Silicon Valley Mathematics Initiative's (SVMI) Lesson Study Project, I am responsible, along with my co-coordinating partner, for organizing lesson study teams across 23 districts to participate in a 5-month lesson study project each year. We establish a broad research theme and teams work on their research for the 5 months then submit a report on their findings. We write a report at the end of each cycle to the funders of the project. My report is usually a Word document without pictures; it is very business-like but also somewhat dry. Teacher reflections about insights gained are included but a unified research conclusion has not been included in our report in the past. In addition, we have been fairly liberal about how closely we have defined the course of individual teams' research. So long as our broad research theme (problem-solving, for example) is included in each team's research, we have been satisfied.

In Japan, I saw the research findings of lesson studies published in brochures to share with entities outside of the lesson study team. I love this product and think it would be a powerful addition to our project. We have always asked individual teams to share their findings throughout their district - at board meetings, parent group meetings, and with their greater community of local colleagues. I hope to expand this process of sharing to include a brochure for our project that integrates the findings of individual districts and reports the results of our research. We can then share the results of our research with colleagues throughout the many districts in our project, with other schools and districts outside of our project, and with our funders and other interested entities. In addition, I hope to provide tools for individual districts to create their own local brochures to share with their immediate communities.

Defining a Common Research Hypothesis

With a more focused goal of integrating the research findings of many groups and presenting project results, I realize the need to be more detailed in our expectations for each participating lesson study team. Related to the theme each year, SVMI will provide a focused research question and be very clear about the data that we need back from each team. This will help teams be more purposeful in their collection of data related to our collaborative's research topic, and help us to better report our research findings in ways that can inform best teaching practice across our collaborative and beyond.

In keeping with these goals, the Silicon Valley Mathematics Initiative's common research theme and goals for the 2012-13 cycle are:

Hypothesis:

Constructing more than one mathematical statement, and making logical connections among statements, will help students to construct viable arguments.

Questions to Explore, related to the hypothesis:

- ***How did students' statements help them to build mathematical arguments?***
- ***How viable were these arguments?***
- ***Are there teacher moves that support students to build more viable arguments?***
- ***Are there student moves that help students to help one another to build more viable arguments?***
- ***Were there any missed opportunities (comments made by students that could have been built upon by peers or the teacher to help students to build more viable arguments)?***
- ***What were these missed opportunities and what else could have happened at that point to make the most of that opportunity?***

Teaching and Coaching

Studying the Mathematics Deeply Before Teaching the Lesson

I have been extremely impressed with the extent to which Japanese teachers study the curriculum before teaching the lesson. Although I have been doing and supporting that to a good degree, I believe that the Japanese teachers do it much more deeply, evidenced by their very thorough referencing of the content in their discussions following the lesson. I am inspired to support my teams to more thoughtfully and thoroughly identify and study the landscape of the mathematics concepts involved in a lesson before considering the implications of teaching the concepts of the lesson to students

Considering the Lesson Within the Context of the Mathematics a Unit

Another aspect of Japanese lesson study that I plan to bring back to my work in the United States is the practice of studying lessons more cohesively within the context of a unit. Before my trip to Japan, my understanding of lesson study was that a single lesson is taught, and retaught, and improved upon over the course of teaching it to several different groups of students. It appears to me that, in Japan, a unit is studied as a whole and the "studied" lesson is considered within the context of a unit. Next, a lesson is taught, studied, and then a plan is formulated to move students on to the next lesson in a meaningful way, based on the findings of the lesson study. The difference is one between studying a single lesson and perfecting it versus studying a series of lessons and making sure that lessons earlier in the series purposefully inform the teaching of lessons that follow, so that reflections about students' experiences in one lesson guide the subsequent lessons. Moving away from looking at a lesson in isolation and toward looking at a series of lessons that adapt to meet students' learning needs will more closely match the daily decisions that teachers need to make to assure that instruction is cohesive and responsive within a unit.

Boardwork

The practice with which I am most fascinated, and which I am excited to experiment with as a teacher when I get home, is the practice by Japanese teachers of the use of the blackboard to document the story of the lesson concept as it builds throughout the lesson. The power of having the story in a sequential, cohesive format, available for students to reference throughout the lesson as they build meaning, is a very powerful learning strategy.

I see how the use of the blackboard in this way requires teachers to carefully consider the way in which they will develop a concept with students. I watched teachers in Japan methodically facilitate students to build the story of a lesson, and to build the story in a way that makes sense and grows as the concept is mastered. This careful record of the development of the lesson is brilliant and helps students to refer to ideas in the previous parts of the lesson as the grapple with ideas near the end of the lesson.

I can't wait to build proficiency with this method of ongoing documentation. I also think that it would be interesting to photograph the finished board product and post for student reference after the lesson has passed and the board erased. I could post on my classroom webpage for reference by both students and parents.

Collaboration and Networking with Educational Professionals From Around the World

Finally, I very much look forward to continued collaboration with this network of educators with whom I have been so very privileged to study for these two weeks. I highly value their varied perspectives and insights and hope to continue to work with all of them, both virtually and in person, to build a strong international network that utilizes and advances lesson study.

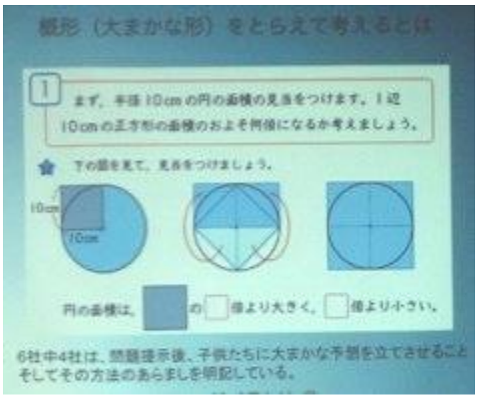
My learning journey on Japanese lesson study

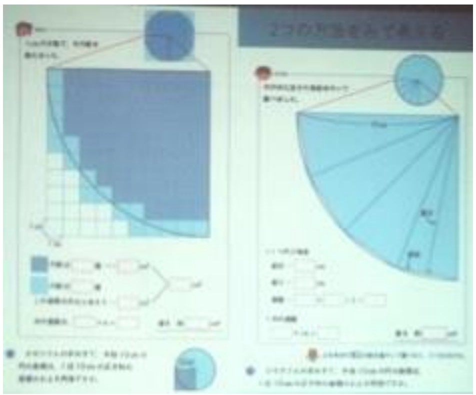
Wanty Widjaja

Deakin University Melbourne Australia

Being ‘immersed’ in the IMPULS project to learn about Japanese Lesson Study for 10 days was a rich and thought provoking experience to examine authentic Japanese Lesson Study. Observing research lessons and participating in pre- and post-lesson briefings prompted me to think about mathematics, elements of mathematics lessons such as anticipating and eliciting children’s strategies, orchestrating classroom discussions to help children progress to the next level of their mathematical thinking and documenting students’ strategies and the flow of the lesson on the board.

Lesson plans provide insights for observers into the teacher’s or the planning team’s knowledge of scope and sequence of the curriculum, of mathematics, of teaching resources and of their students’ mathematical development. Lesson plans have gone through several rounds of review process before they were made public for research lessons. However, as informed by Dr. Tad, some reviews focused on superficial features of the lesson plans such as format alignment. Clearly this sort of reviews is not helpful for teachers especially considering the fact that the lesson plan is ‘sealed’ and changes to the plan during the research lesson are not viewed in a positive way. The lesson plan was expected to be followed closely by the teacher during the public research lesson. Providing a better quality review of lesson plans would support teachers or the planning team in developing their capacity to plan. Lesson plan is a vehicle for teachers to develop and practice their *Kyouzai-kenkyu*. The importance of *Kyouzai-kenkyu* was highlighted many times during the post-lesson discussion. For instance, Dr. Akihiko pointed out that studying and comparing more than one textbooks would help the planning team to focus the lesson in helping students to make a connection between what they have learnt previously (i.e., area of 10×10 squares) and what students need to learn (i.e., the area of the circle is about 3 times the area of 10×10 squares).

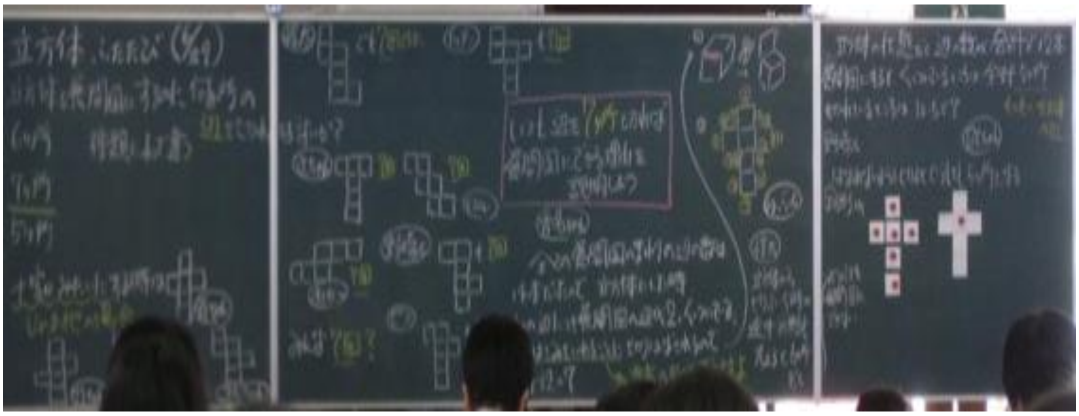




At the heart of Japanese Lesson Study is the public research lesson. It is a proving ground for teachers to test their plan and ideas as well as getting contributions from observers and knowledgeable-other. Although public research lessons are not meant to produce a perfect lesson, based on observing the post-lesson discussions, high expectations were set for public research lessons. In fact, it was challenging for me to get a good grasp of what was not ‘up to the standard’. In some lessons where students came up with different strategies and teachers showed a great skill of orchestrating productive mathematical discussion throughout the lessons. Yet these lessons were being evaluated as unsatisfactory. It was indeed impressive to see how efficient and effective the knowledgeable-other pinpoint critical issues surfaced during the research lessons such as key mathematical ideas that needed further development or lack of clarity in the problem statements.

During the IMPULS lesson study program, we were fortunate to observe school-based lesson study, district-based lesson study, and a professional development-based lesson study for training of novice teachers to become mathematics specialists. However, it was unclear how schools or teachers decide on the type of Lesson Study that they embark on. What are the factors that influence this decision? I will be interested to learn more about this. There were no differences in terms of the role that the observers played among various types of research lessons. However, there were different sets of protocols when teacher observers worked collaboratively to discuss their lesson observations prior to providing feedbacks during the post-lesson discussion. The most insightful post-lesson discussions in my personal opinion were when teachers and observers brought in students' work as evidence from the research lesson to provide feedback for the teachers. Students' work served as evidence when the observers and teachers reflected on the research lesson and discussed ways to improve the lesson. We also participated in a lengthy post-lesson discussion which seemed to be lacking of clear focus and directions. The lesson learnt from this experience was the critical role of moderator in ensuring a focused post-lesson discussion by summarising key ideas raised by observers.

Japanese Lesson Study engages teachers and observers as reflective practitioners and provides a platform for their ongoing professional development. The main intention of making the research lessons public is for teachers who taught the lesson to reflect on their own practice and to make improvement based on these reflections. It was pointed out several times that post-lesson discussions should focus more on the content and less on pedagogical strategies. Insights shared by the knowledge-other and teacher observers on mathematics and students' mathematical strategies during post-lesson discussions were really valuable to see the bigger picture. It would be valuable for the teacher and the planning team to have access to these insights from knowledgeable-other prior to the research lessons. It will be interesting to learn more on how the teacher and the planning team follow up on the feedback shared during the post-lesson discussion. However, the strengths of Japanese lesson study also are centred on teacher's pedagogical decisions in choosing the key questions to pose (*hatsumon*) and the recording and summarising of the lesson on blackboard (*bansho*). In some of the research lessons when these two elements were done well by teachers, students showed more engagement in the mathematical discussions because the mathematical ideas were clearly represented. A good blackboard presentation is like having a clear window glass and you can get a good look inside the mathematics lesson to learn the flow of the lesson and the mathematical ideas and strategies being discussed and students who contributed to these ideas.



All seven research lessons shared a common intention to engage students in exploring various strategies and to develop students' ability to communicate their mathematical ideas. Having only one or two main problems for the whole lesson allowed teachers and students the time to devote more time for in-depth discussion on various mathematical strategies. However, the whole-class discussions varied in their extent to engage students in mathematical thinking. In some research lessons, students' sharing of strategies were at the level of '*show and tell*' as there was no clear follow up discussion to engage other students in questioning the shared strategies. In other research lessons, the teachers carefully orchestrated the whole-class discussion by asking students to compare and contrast the various strategies. Students did 'beyond show and tell'. I recalled the episode when Kokei sensei asked one student to repeat the explanation several times so that other students could understand this explanation. In Japanese Lesson Study, the main purpose of whole class discussion is to help students in comparing various strategies in order to expand their repertoire of strategies (*Neriage*). Teachers and observers use the seating chart to carefully map students' strategies. Careful observation of students' strategies along with anticipating students' mathematical strategies prior to the lesson were critical for teachers in orchestrating productive mathematical discussions. Students' mathematical strategies are selected to be presented in a sequence so that students can engage in a fruitful discussion. However, recording students' strategies alone did not guarantee a successful '*neriage'*. Personally, I found that solving mathematical problems and anticipating students' strategies by examining the textbooks were critical in making sense of students' strategies and following the whole class discussion during the research lessons.

With colleagues at Deakin University, I am learning more about Japanese Lesson Study by working together with our teachers in Melbourne on our project to adapt Japanese Lesson Study. Currently we are at the beginning phase of planning our research lessons. The immersion program run by IMPULS has been very valuable for our own journey. I look forward to continue the conversation about Japanese Lesson Study and hope to share more about our journey at later stage.

Acknowledgement:
I would like to thank all sensei, students from the 7 schools and the IMPULS project staff for providing us with a rich learning experience on Japanese Lesson Study and Japanese cultural experience. Doumo arigatou gozaimashita.どうも有難う御座いました

4 External Evaluation of the Program

External evaluation was done by Dr. Rebecca Perry as below.

2012 Immersion Program Evaluation Report

Rebecca Perry, Mills College, Oakland, CA, USA

6 December, 2012

Background

In late June and early July 2012, Project IMPULS, in collaboration with Global Education Resources and the Mills College Lesson Study Group, organized a Japanese lesson study immersion program. Designed to familiarize participants with authentic Japanese lesson study, the ten-day program included visits to seven schools in Tokyo and Yamanashi prefectures to enable participants to experience mathematics lesson study in multiple contexts. In addition to lesson study events, the program began with a brief seminar on lesson study, included opportunities for participants to see local sights and experience Japanese culture, and provided time for reflection on learning from program activities.

The forty participants were from 4 countries (the United States, Great Britain, Australia, and Singapore) and 6 U.S. states, and had a range of familiarity with lesson study prior to the trip (18% of the participants had no prior lesson study experience and 35% of participants had 2 or more years of experience with lesson study). Most participants (73%) had no prior experience with mathematics lesson study, although may have had some exposure to lesson study in other subject areas. Only a small percentage of participants (15%) had any previous direct exposure to Japanese mathematics instruction, through seeing Japanese teachers teach in the U.S.

This report draws on multiple forms of data, including a participant survey administered before and after the program, observation notes collected during the trip, and participant reflections gathered during and after the trip. The report begins with a brief summary of findings and recommendations, followed by additional daily learning highlights. Participants' comments are used throughout the report to exemplify findings.

Executive Summary of Findings

The program offered an exceptional opportunity for participants to learn about many aspects of Japanese lesson study, mathematics education, and culture (e.g., see Figure 1 showing mean participant ratings of extent of learning about 25 program elements). The seven lessons demonstrated how Japanese lesson study can be adapted to a range of diverse organizational settings. The lessons also enabled participants to see how the common intentionality and focus on student thinking that exists across levels of the Japanese education system (despite variety in local contexts) supports excellence in mathematics education. The program logistics as well as the openness and friendliness of program organizers and staff were critical to the success of the program; participants felt taken care of as travelers in a foreign country and as learners with a broad range of experiences and questions.

Although statistical analysis shows that participants rated their actual learning (posttest rating) about program elements significantly lower than their anticipated learning (pretest rating) on 15 of 16 items[1] (see Figure 2), a review of qualitative data revealed quite nuanced learning about lesson study, mathematics, and instruction, as summarized below.

1. Learning about Lesson Study

Participants reported that the diversity of the schools included in the program itinerary enabled them to have a broader understanding of lesson study as a process centered around a live classroom lesson that is adaptable and useful across a range of organizational contexts. Two exemplar comments give a sense of how participants' ideas about lesson study were enhanced by their program experience:

> "I have learned that I was terribly naïve about Japanese lesson study, given that I had read quite a bit and seen videos of Japanese lesson study prior to coming on the trip. [This] makes me think there are some real difficulties in trying to describe elements of lesson study faithfully…"

> "There is more variation in the process of Lesson Study than we thought – there is room for variation and adaptation. I came here and filled out the survey with the intention of focusing on how lesson study is done in Japan. I have come to the realization that there are an indefinite number of components that need to be in place, but there is not one specific way to structure lesson study in your school. You have to make it work for your school, for teachers' buy in."

Additionally, the myth that lesson study is about creating a lesson from scratch and using the cycle to perfect the single lesson was also dispelled during the program. Rather than seeing lesson study as a single model lesson that teachers devote much time to, participants began to understand that "each lesson is more of a case study on how to teach more effectively" and the research lesson should be considered in relation to the mathematical unit. Another participant comment illustrates how this view of a lesson enables learning from the full lesson study cycle:

> "What was interesting was 1) that each lesson is based on and derived from a particular lesson in the textbook—none of this inventing lessons from scratch, which often happened when I was a classroom teacher; 2) the debriefing of the lesson is more important even than the lesson itself, although it seemed to me that the connections between good lessons and good discussions were evident—that is that a well-crafted lesson also engendered a better discussion; 3) however, an organized and focused discussion is much more interesting and useful than everyone simply sharing their reactions—that was an interesting aha! for me."

2. Learning about Mathematical Content and Coherence

The role of mathematical content in lesson study was emphasized in the program; i.e., that

[1] There were no significant differences between pretest and posttest ratings of learning on 9 additional items.

content knowledge is both a critical input to guide the lesson study process and an outcome of lesson study if the cycle is well-implemented. Participants reported gaining new appreciation for *kyouzai kenkyu* as an important beginning stage of lesson study. "Analyzing/ studying curriculum material" was one of only two program elements listed on the survey that participants considered much more professionally useful after the trip than before.[2] After the trip, several participants like the teacher below reported that *kyouzai kenkyu* would be incorporated more into their lesson study work at home:

> "I plan to continue working with my 3rd grade team on math lessons. I know that, after this trip, my focus will now be on kyozai-kenkyu and really doing the research on how the math goal is taught over multiple grade levels and what the students will need to know in the future."

Participants also learned about the central role of lesson plans for sharing mathematical ideas. By reviewing lesson plans in relation to the seven observed lessons, participants were shown lesson plans models with coherent information about mathematical ideas (especially prior and subsequent student learning) and how this information included in the lesson plan could guide student observations and interpretations (e.g., whether student misunderstandings may be caused by previous lessons or by what happened in the observed lesson).

Post-lesson discussions also played a particularly important role in helping participants understand the importance of content knowledge as described in the lesson plan and shown in the lesson. Although many participants registered surprise that some final commentators had prepared remarks ahead of time (a difference from their prior lesson study experiences), final commentators highlighted the trajectory of mathematical ideas and gave a rationale for why a certain topic was important to teach (e.g., the angle bisector lesson), or provided evidence on how a lesson implementation supported or failed to support students' achievement of mathematical goals (e.g., net of a cube lesson). Several participants commented on understanding how valuable support from content-knowledgeable outsiders was in helping to develop lesson ideas and support teacher learning, as two example comments illustrate:

> "I am coming back with many ideas for supporting my practice as a commentator and knowledgeable other. In particular, I am recognizing that it is particularly important to facilitate lesson study work in ways that brings it back to the scope and sequence of the curriculum. Most lesson study groups I've worked with spend too much time in the shallow end of mathematical discussions. I have been impressed how some commentators swim into the "deep end" with regards to the math content."

> "Having a good facilitator with content knowledge and skills to move/ steer the conversation one direction or another. Seeing a final commentator close the debrief is intriguing. I've never seen that before. Again having someone who is insightful w/ content background to bring all of the comments together and move/push the teacher/ school forward."

Despite participants' reporting learning from the post-lesson discussions, on average

[2] A non-parametric Friedman test showed that ranking of this survey item increased from 18th most useful on the pretest (mean ranking of 11.68) to 3rd most useful on the posttest (mean ranking of 15.43).

participants rated their learning about "organizing a successful post-lesson debriefing session" the lowest of 25 survey items (see Figure 1). The variety of different structures and the level of the mathematical comments made in the observed post-lesson discussions may have made the idea of trying to organize a "successful" post-lesson debriefing even more mystifying to participants.

Participants also reported that the program supported their own learning of mathematics. When asked on the survey about the extent to which they agreed with the statement "I have strong knowledge of the mathematical content taught at my grade level," participants' posttest ratings were significantly higher than their pretest ratings,[3] and learning about mathematics content was one of the ten most highly rated program elements (see Figure 1). For example, one participant reported:

> "I have learned more mathematics on this trip than in a lifetime. In elementary school we are facing resistance from teachers who are not interested in learning more content knowledge. When content emerges from the lesson and the analysis afterwards is tied into curriculum, that's powerful."

The coherence of the Japanese mathematics education system was also demonstrated through the program by the fact that teachers in the observed lessons were not working in isolation, but rather alongside and with support from school and district administrators, school leadership teams, college faculty, and even ministry of education officials. Several participants remarked on the value of including these diverse perspectives – to enhance learning from the lesson and to ensure that all levels of the education system are focused on student learning – and hoped to establish stronger connections to individuals at other levels of the system back in their home country.

3. Learning about the Teachers' Role (Content Knowledge/ Instructional Skill)

A third umbrella category of participant learning involved developing a strong(er) appreciation for Japanese teachers' knowledge and skill in orchestrating content-rich, problem-solving instruction. While participants' beliefs about problem-solving teaching and learning did not change substantially as a result of the program (most participants believed in the importance of student exploration and inquiry instruction before the program), some participants did begin to draw a distinction between teaching problem solving and teaching *through* problem solving, or began to feel that problem solving should not be taught as an intermittent stand-alone idea or a series of problem-solving steps. For example, one participant reported:

> "After the IMPULS program, I think problem solving should be included in every mathematics lesson so that students build their logical reasoning. Acquiring problem solving skills will be harder if it is only included once or twice a week in the lessons."

Bansho and *neriage* were new ideas for most participants, and the diversity of the lessons helped them understand the knowledge and skill needed to implement these ideas well and showed multiple examples of their use in practice. For example, the following comments illustrate some of participants' learning about these instructional ideas:

> "I was struck by the blackboard. If teacher is facilitating student thinking, the blackboard is

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[3] Pretest mean = 4.23, posttest mean = 4.87 on a 5-point Likert scale, $t(26)=2.849$, $p< .01$.

like a tableau. It's not erased so that it builds over the classroom and the students can see their thinking progressing... It's the whole story."

"After this trip and seeing the power of the board work and student note-taking in Japanese classrooms, I am planning to use lesson study to design, implement, observe, reflect, and revise how we use board work and note taking to do mathematics and specifically to address the Mathematical Standards for Practice outlined in the CCSS for Mathematics."

"…My other goal is to pursue discussion and clarification of student misconceptions instead of occasionally avoiding them. As I learned on the trip, the greatest student learning occurs when the teacher can "step closer" to the child and really see what his/her thinking is."

"I'm particularly interested in the pedagogical moves the teachers make during the actual teaching and how these moves ultimately have a significant impact on how the lesson plays out. The observation of student work by the teacher and then the ordering of calling on students and decisions about how to proceed – who to call on, where to spend time – sees particularly important…. I also have been thinking a lot about how to apply this to the ELA and SS work we're doing… Learning how to do discussion seems particularly important."

"I was struck by how the teachers choose who shares. The idea of going desk to desk. I'm used to getting everyone to share, but this is very strategic, almost surgical."

While participants learned about the central role of *kyouzai kenkyu* in the lesson study cycle, they also got view of the centrality of the teachers' role in developing and implementing a lesson plan that embodies the critical research and mathematical goals. The group saw and heard about how teachers drew on deep understanding of mathematics content to make decisions about instruction when interpreting the curriculum into a lesson plan. As one participant commented:

"Teachers allowed the flow to happen rather than intentionally thinking about how to set up the flow. And you can see flow on the blackboard. The teachers have to be responsive to the students and their thoughts. That's a real skill we would have to develop as American teachers. Deep mathematical thinking is the most salient point in lessons. That's something we miss in the US."

Participants also learned that anticipating student responses provided a foundation for the skillful teachers' use of student thinking during the lesson. In fact, "anticipating student responses" was the second program element on the survey that participants considered much more professionally useful after the trip than before.[4] Post-lesson discussions also pointed out how teachers could have done a better job of anticipating student thinking during the lesson.

In summary, evidence from the program suggests that the trip helped to clarify the qualities of lesson study, and supported participants to focus on and learn about key mathematical ideas and gain a broader perspective on how to support students' mathematical problem-solving and mathematics

[4] A Friedman test showed that ranking of this survey item increased from 20th most useful on the pretest (mean ranking of 11.24) to 6th most useful on the posttest (mean ranking of 14.82).

teaching and learning more generally.

Recommendations for Program Improvement

Overall, participants were very pleased with the program and offered few recommendations for improvement. The change most requested by participants and most obviously needed from this evaluator's point of view was more time in the agenda for discussion and reflection about the lessons, mathematics, and lesson study. More reflection time would have enabled participants to deepen their learning by drawing more on the diverse knowledge and perspectives of other participants and program organizers. Similarly, although participants appreciated the task of documenting a single research lesson to immerse themselves more fully in a lesson, many felt that they did not have sufficient time to complete this task to their satisfaction. Suggestions differed on how much additional time should have been allocated for reflection and whether making time for this should have reduced the number of school visits. (Most participants were satisfied with the number of school visits, as this was, after all, a lesson study *immersion* program.) Perhaps having each two days of school visits followed by a third day of reflection and pre-briefing on subsequent lessons might have been useful. The few comments below exemplify participants' feelings about reflection time:

> "Because our lessons were in the afternoon, I feel that we could have utilized our morning time together much more effectively. Since we didn't get a chance to engage in the post-lesson discussion at most sites, it would be nice to have a structured discussion the following morning. I liked that we chose a lesson to document, instead of trying to focus on all the lessons. This is a good model for future programs (even if there is no research project like the Mills research project) but it would be nice to have an hour set aside each morning for lesson study teams to meet, or organizations to meet together (like the Oakland team)..."

> "It would have been great to build in more time for group (and individual reflections on the lessons afterwards and time to really understand the mathematics and the place of the lesson in the larger Japanese curriculum before the lesson. There were simply too many competing agenda and it always seemed as though there wasn't enough time to process what we were learning and think deeply with each other and our Japanese colleagues."

> "I loved seeing the different classrooms, but we really needed more time to discuss the lessons before they were taught and then afterwards."

Participants' second most frequent recommendation involved the desire to see other aspects of lesson study, particularly *kyouzai kenkyu* and lesson planning.

> "I'd like to know more about Kuyozai Kenkyo. It would have been good to observe teachers engaged in this aspect lesson study. We were led to understand that this was a collaborative process and that each research lesson was developed by a team of teachers. However there did not seem to be much evidence of this team work. It seemed to me that in almost all cases the teacher of the Research Lesson wrote the lesson plan alone. Likewise it was less clear how instructional leadership operates during this phase of Lesson Study."

> "All in all I am very greateful to have witness the 7 lessons but I feel that I saw 7 different

versions of the same scene. To further my understanding of lesson study I would have loved to have seen some of the research team's planning sessions. I think it would have been extremely helpful to hear what they believed to be the anticipated students response, why that is and what were they going to do about it. Additionally It would be great to see a lesson and post-lesson debrief and then return to that classroom to see how and what the teacher does after the lesson. Does the teacher re-teach the lesson using the information that came up in the debrief or not? What happens after the lesson study. Being able to see the whole process from the planning stage to the lesson and post lesson to the next's day's lesson would have provided me with a fuller picture of what lesson study is all about and what it would look like at my site."

"i would've liked to see or heard about in depth how they as a research team planned a lesson together."

Other recommendations, offered only by one or two participants each, included having more rest time for jet-lagged participants, visiting other low-performing or more diverse schools, seeing lessons at a wider range of grade levels, organizing morning lesson observations (to help tired travelers), limiting program participation to a smaller group to facilitate movement around the classroom, and improving on the poor translation offered by the professional translators. A few participants would also have appreciated a more active role in the post-lesson discussions, as one participant noted:

"The biggest challenge was having to sit to 2 to 3 hour post lesson discussions with poor translation. As we all know passive learning is not a very effective way of learning. At the post-debriefs where we (the implus group) were asked to provide feedback were much more stimulating and informative. The ability to ask the presenting teacher or the research team some questions of what say throughout the lesson would have been helpful."

When considering future immersion trips, IMPULS may want to consider two additional suggestions. Given the positive reaction of the Oakland Unified School District team (including administrators and teachers at all levels) to this opportunity, IMPULS may want to consider organizing similar programs for other school district teams as a way to support simultaneous learning across a system about the requirements for successful lesson study. Program organizers could then work with the district team more closely along the way to help them think about how to translate their learning into practice back at home. One OUSD representative reported on the impact of this immersion experience for their participant team:

"This trip has enabled many more leaders from Oakland to get an in depth professional development into Lesson Study and how it shapes Japanese instruction and curriculum. I hope that mutual learning experience leads Oakland to include more teachers and administrators into the process in the upcoming years, and therefore, develops Lesson Study into a districtwide signature pedagogy that many teachers and sites utilize as we implement the Common Core."

Secondly, the translated lesson plans were very useful resources to support participants' understanding of lesson study, as were two translated or partially translated district/ school brochures. The "research promotion school of the Setagaya public school district (2010-2011 and 2011-2012)" was especially rich because it described several elements of the lesson study process in the district, including the research theme and rationale, the structure for the overall research within the district, lessons to aim for to enhance problem solving, the lesson flow and effective teacher questioning that might occur at various lesson stages, and the lesson and board writing plan, etc. More translated resources like this could facilitate participants' learning about lesson study in a different way, giving them something to study more carefully and use to explain lesson study to others at home.

Daily Learning Highlights

Day 1, June 25

Activities – Full-day seminar introducing lesson study, Japanese mathematics instruction and teacher preparation, and the program (goals and agenda); Q&A; evening reception

Through presentations, vivid descriptions, question and answer, and a series of video clips (Hase lesson), the first seminar day laid a foundation for participants to learn about lesson study generally, the specific activities and goals of each lesson study step, and Japanese instruction.

The diversity of participants (teachers and non-teachers, those with prior experience with Japanese instruction or lesson study and those without, etc.) became quickly apparent and this proved to be a program strength. While some observations this first day inevitably focused on Mr. Hase's instructional choices (e.g., student grouping, methods of differentiating), others participants were ready to discuss student solution methods shown in the video or their current understandings of mathematics and lesson study in relation to the information presented. "Slowing down" instruction on the Hase video supported participants to learn about elements of teaching through problem-solving style instruction.

A discussion about *neriage* exemplifies how this first day provided important foundational learning. After watching a video clip, a participant noted that Mr. Hase asked students "who wants to show their solution?" rather than choosing the student solution he wanted to present first. The participant asked, "Is it common to select the student who has the best solution first when a lot of students' hands go up?" Another participant also asked, "How common is it to begin with a misconception in order to discuss it? …I've gotten it into my head that starting with a misconception is useful." Dr. Takahashi suggested, and others agreed, that order of presenting student solutions should be decided on a case-by-case basis. Through discussion like this, participants new to lesson study could be attuned to the importance of carefully noting student solutions and ordering the presentation of solutions to support *neriage*.

Day 2, June 26

Activities – Continuing introductory seminar; 6th grade lesson at Funabashi Elementary School (Tokyo) on area of a circle

Participants' observations on this day focused on several elements of teaching through problem solving demonstrated – or not – in the lesson, including the ways the teacher elicited students' solutions and thinking, how he allowed time for students to productively struggle to solve the problem independently prior to group work, how he used the board to present students' solutions, and the importance of *neriage* to compare and contrast student solutions. Several participants commented on how little *neriage* occurred during this first lesson, helping them to realize how the absence of good *neriage* (and the instructional work of choosing student solutions prior to *neriage*) can cause the lesson to fall short of goals for students. Several participant comments about teaching and learning arose from this lesson example, as shown below:

> "Bancho [sic] → portrayed each students work on large, legal size paper and students expressed their rational, group work. However, students only summarized their method and board only depicted the answer, not key idea to later critique/ argue. In group, shared, but was a consensus achieved? Methods refined? New learning?"
>
> "It is important for the teacher to rotate through the class, gathering data to determine who understands the problem – and to strategically sequence the follow-up discussion."
>
> "I would like to practice/ develop a better sense of how to facilitate the conversation/ discussion at the end of the lesson."

Participants appreciated seeing the grade level teacher teams working together prior to the post-lesson discussion to organize their thoughts for the debriefing, but the heavy focus on pedagogy rather than student thinking during the discussion led some participants to wonder about what training teachers receive to take notes and observe lessons. Participants also asked about existing guidelines to structure post-lesson discussions, a question (i.e., how to organize post-lesson discussions) that arose several times.

Day 3, June 27

Activities – Travel to Yamanashi with Q&A; 7th grade lesson at Yamanashi University attached lab school on positive and negative numbers

During the morning, Dr. Takahashi summarized comments from the previous day's post-lesson discussion, elaborating on the mathematical lesson goals, ways the teacher failed to achieve his goals, and what might have been done differently by the teacher and steering committee to achieve a better lesson for students (e.g., studying other textbooks to see how the ideas on that topic are presented). This discussion also highlighted the varying role of knowledgeable others and the kinds of topics that are typically raised by school teachers versus a knowledgeable other. Dr. Takahashi informed the group that final commentators frequently offer thoughts about content, adding, "If you have good math ideas, pedagogical ideas can follow this."

One key idea that arose from the lesson and post-lesson discussion on this day was that each element of a lesson needs a strong rationale for its use. For example, although this lesson used a real world problem context (something which U.S. teachers frequently think will help to interest and engage students), students did not understand the problem well. One participant paraphrased learning from an observer's comment that the instructor should have given "space for students to think about the problem 'How might we decide to split the students into two teams?' Students might come up w/ idea of making avg. heights the same." Similarly, participants observed that although the lesson included group work, the purpose of having students work in groups was questioned during the post-lesson discussion. One participant wrote:

> "I like the idea of thinking about the goal of group work in the particular way that was

discussed: Is my goal to get multiple strategies to surface and discuss those different strategies or one efficient particular strategy?"

Reflections from this day also focused on elements of teaching through problem solving that could be observed during the lesson, such as the explicit example of *bansho* (where the instructor wrote across the board from left to right, using multiple colors) and how the lesson demonstrated the importance of thinking carefully about the order of presenting student solutions. Two participants commented on this idea of selecting students:

> "The most sophisticated idea should not be presented first – monitoring, selecting, and ordering presentations is <u>so</u> important."

> "I saw that it was important to probe and to think carefully about what to look for during student work time and to choose students to share in a thoughtful way that will steer the thinking of other students."

The mathematical idea of "tentative average" used in this lesson was new for many participants, and the lesson also led some participants to wonder about the characteristics of a "good" problem for a problem-solving style lesson or about where to place such open-ended problems within a unit.[5] Participants commented:

> "Although I have used the idea of tentative average instinctively in my own work, I have never used it explicitly to explore positive and negative integers before. I think this could potentially be a powerful tool and will be thinking about how to incorporate it into my own practice."

> "This unit helped me reconsider where to place lessons on application within the unit. I tend to want to 'hook' students at the beginning of a unit with a topic's application, and then couch the whole lesson in terms of that (i.e., +/-, temperature). In this case, there must have been many lessons dedicated to the mathematics/ number line, etc., which ideally allows students to be ready for applications such as this one."

Finally, the contrast between this lesson study experience and the one on the previous day left some participants with questions about the nature of lesson study. For example, participants wondered whether the focus of the post-lesson discussion should be about pedagogy or mathematics

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[5] In a survey item asked before and after the trip, participants were asked about the role of problem solving in students' learning of math. After the trip, participants were on average 8% less likely to agree with the statement that: "Students should first master the prerequisite facts and skills of mathematics before they are assigned problem solving. Problem solving should emphasize the application of these facts and skills to real life situations." This survey result is consistent with the following participant comment that problem-solving instruction may be advantageously located at different places within a given unit.

and the task, or whether the focus of lesson study is on improving one lesson at a time or improving teaching overall.

Day 4, June 28

Activities – Visit to Kofu's Takeda shrine; travel to Showa; Tour of "green" Oshihara Elementary School; lunch with students; 3rd grade lesson on mental calculation; evening banquet

Thirty-two percent of survey respondents nominated this lesson and post-lesson discussion as the most professionally useful for them. In the lesson, students were asked to consider the number sentence: __ - 28=___. They were given three numbers to choose from (89, 53, and 68), and had to decide which they wanted to put in the first blank. The teacher anticipated that students would choose 68 (the ones are equal and therefore the tens need only be subtracted) or 89 (no regrouping necessary). After discussing solutions, the teacher solved the problem using the standard algorithm, and asked students to find mental calculation strategies to solve it, and to record their thinking in their journals. The post-lesson discussion clarified the point that by solving the problem first, the teacher was de-emphasizing answer-getting and helping to develop students' ability to decompose numbers. Many participants found this approach unusual and retained the idea that the math in a lesson can happen <u>after</u> students have found the answer. Several survey responses sum up the reasons why participants felt this lesson was useful to observe:

> "This lesson was most informative to me because it was the one that I felt best illustrated a successful example of the teacher and students together doing mathematics, and recording the story of the mathematics they did together on the board and student notebook. It illustrated for me an excellent example of a teacher watching students working on a problem, and facilitating the discussion in a way that collectively the class explored the underlying process and number sense involved in subtracting while considering the benefits and restrictions of different methods."

> "The lesson was informative because the teacher modeled excellent practices. The classroom culture was solid, his inter-desk monitoring was thorough, his bansho was clear, and especially his nagirai (sp) beautifully built thinking from simple to sophisticated. It was obvious that he used the data from the students' individual work to inform his discussion points. His pedagogy was enviable, and I respected how he interacted with the students to accelerate their learning. The post-lesson debrief was informative because it highlighted the importance of a good facilitator, who moved the conversation from reflections about the lesson to the pedagogy to the content in an easy and professional way, without taking three hours."

> "The concept of composing and decomposing numbers in elementary school is vital to student understanding properties in their mathematical reasoning. Components of this lesson should be a focus with any elementary team setting up a unit."

> "I especially appreciated how the T linked a student's strategy to the property of additive identity (53+2-(28+2)) that the class had agreed upon prior and posted as an accepted statement to the board. I'm not clear on whether this was part of the intended lesson but it contributed to the development of these ideas across lessons."

In later discussions about the lesson, participants commented on the teachers' patience and his positive classroom social environment where students showed empathy for each other (e.g., about how hard math is). Dr. Takahashi also elaborated on several aspects of the pedagogy used, for example the instructor's use of the horizontal form of the math sentence (rather than the standard vertical form used in the U.S.) as a way to support algebraic reasoning, the attempt to lead all students toward a common efficient solution (Rei's solution), and the use of a team teacher to encourage a normally quiet student to persevere and speak up in class.

In addition, this post-lesson discussion helped to demonstrate how student work can be used during a post-lesson discussion. Copies of the board work were handed out and used during the discussion, helping to support observer's comments. One participant commented:

> "Didn't recognize until today how important the observer's role for observing individual students is – not just about observing teacher but also really looking for evidence of students' understanding."

Day 5, June 29

> **Activities** – Morning reflection time (OUSD group meeting); 5th grade lesson at Tokyo Gakugei University attached school on finding the number of edges of a cube to cut to create a net; taiko drum concert

This morning the OUSD group met to gather their thoughts about what they had learned and how to bring this knowledge back to OUSD. Observing this meeting, it became clear how difficult it is even for those who have previously experienced lesson study to understand it. The OUSD participants were struggling to distinguish lesson study from Japanese education and teaching through problem solving, and were also still unclear on the possible uses for lesson study. Several notable comments included:

- "I'm trying to think about the roles I'm seeing lesson study play – professional development, learning opportunities, means for curriculum vetting, collective ownership for student learning, content development for teachers. Some things are like what I expected and some things are different from what I expected. We've been working on creating new curriculum: it was an a-ha to see that lesson study can be a perfect process for vetting curriculum."
- "I'm confused about what lesson study is: who was on planning team. When I think about advertising this, I think I need a much more specific set of parameters of what lesson study is. What is our vision for lesson study and how to explain it to people? It and the conversation after has been different at all three schools that we've been to."
- "How we've done lesson study is different from what I've seen [here]. We haven't had conversation about what are kids learning – this is an opportunity to formalize the process for finding out what students are learning and pushing their learning. That's what I want to get to. How to get there, I'm not sure about. Each of the lessons opened up the conversation about student learning, pushed our understanding of math and what students are getting out of it – this needs to happen in Oakland."
- "Comparing contexts helps me realize that lesson study is different across professional learning systems. It can be across school site, across sites in a cohort, how a region

showcases their work, how to organize learning for kids. It's also about developing pedagogy."

- "I am still grappling with what is lesson study is… I see that there is a lesson flow, then there is teachers doing the teaching through problem solving process. I really like what Fujii said about the proportion of content vs. pedagogy. How do we use this as a way to raise the level of professionalism for teachers?"
- "I'm not clear on what is lesson study and what is Japanese education. How can you do lesson study without Japanese education? How does that look different? All these teachers knew the problem because it was based on the textbook, but what we do differs classroom by classroom. … Maybe this is a way to set expectations higher than what they are now."

About a third of survey respondents nominated the lesson and post-lesson discussion on this day as the least professionally useful for them. At least one participant was unclear on the mathematics of the lesson (wondering why it is important to know seven cuts are needed to make the net of a cube), and could have benefitted from some mathematical pre-briefing prior to observing the lesson. In contrast, others appreciated the mathematics in the lesson, as noted below:

> "…the mathematics was quintessential. Important in Japan, ignored in US. Masterful setting up the question, just because it looks like there are 7 edges cut, can you prove it? 7th grade, putting understanding of cube "nets" to use. Distinguishing between inductive and deductive reasoning. The post-lesson discussion was torture for some, but i really enjoy the translation (however poor it is) about students telling their stories, and stating their opinions (think legal "opinions") even when those opinions were incomplete opinions (mathematical practice: precision)"

> "…during the lesson observation, I experienced an "ah-ha" moment. It's when the teacher directed the students to answer the question of 7 cuts, after leading them to reaffirm the 11 net patterns. He was asking them to prove that you had to make 7 cuts, based on their knowledge--a mathematical proof. This was the "deepening our understanding" aspect of the lesson. The post-lesson discussion and commentary further revealed the inductive/deductive reasoning of the lesson. The discussions also brought out how we can easily focus solely on instructional practice without consideration of the mathematical content and student learning--seeing the trees but not the forest metaphor. The lesson was deceptively simple; the student thinking was rich in content."

The most common reasons why the lesson study experience this day was difficult for people was because the discussion continued for three hours with poor translation and "was not well organized and too long," "highly critical," and "not related to student learning." Despite these difficulties, the experience highlighted the importance of having a good discussion moderator and having "think time" for lesson observers to collect their thoughts before the discussion, and provided another example of a structure for organizing observations for the post-lesson discussion (the colored strips of paper). One individual's comment sums up many of these ideas:

> "As educators, we know that we need to give students time to reflect before we ask them to share their ideas with others. This is the first lesson study in which this idea was explicitly

required of the teachers as well. The teachers wrote down aspects of the lesson they agreed with, things that could be improved, and things they had questions about. This seems like an important practice before beginning any post-lesson discussion. The first school had a similar approach, but only required the reflections to be group reflections. Although I think both approaches are effective, the individual think time seems to be the most promising for helping teachers change their own practice…. We have all agreed that the post-lesson discussion was less-than-stellar. This led me to question the role of the moderator and the perceived purpose of the discussion by participants. It did seem that the same ideas were continually rehashed. Should the moderator have stepped in and focused the discussion?"

Because of the difficulties of the post-lesson discussion, the on-the-bus debriefing at the end of the day was important to help participants understand the intended lesson emphasis on deductive reasoning. Dr. Takahashi pointed out that the teacher tried to introduce inductive reasoning by following students' interest in investigating the number of nets, and this decision changed the direction of the lesson. (Students presented that there are 14 possible nets using inductive reasoning rather than using deductive reasoning to come up with an argument.) Dr. Takahashi also explained that the commentator focused his comments on things the teacher needed to do to develop deductive reasoning (e.g., establish the property of the cube and help students understand how to find the answer of 7 without investigating every case) and on the fact that by selecting the focus of the lesson to "understand and tell story," the instructor did not give enough attention to the other themes for a research lesson and therefore did not accomplish what the school really hopes to accomplish. With this additional debriefing, Dr. Takahashi helped the group reflect on the value of fully understanding lesson goals and the importance of being well-prepared for in-the-moment instructional decisions.

Day 6, June 30

Activities – Panel discussion with OUSD administrator Tucher and Dr. Fujii; presentations on Common Core State Standards (Daro) and Lesson Study (Lewis); Q&A

Few IMPULS participants attended this optional day of activities, but for those who did, the activities provided additional perspectives on both U.S. policy and research (from Daro and Lewis) and on Japanese education, and may have supported program organizers to establish a stronger connection with OUSD administrators. Phil Tucher from OUSD reported his two biggest "a-has" from the program so far were:

"…that the mathematics starts after the answer is already found. And we must learn to read closely what students are doing in their work… When we share curriculum, when we share math content knowledge, and we share teaching strategies, none of them matter unless we have students in front of us. Maybe those students are on videotape, but like Prof. Takahashi has taught me, that's like watching baseball on TV. No peanuts; no beer. It's this work of observing students closely that I think makes the biggest difference."

The subsequent panel discussion with Japanese colleagues allowed participants to listen in on an interchange of ideas around important topics like "How do Japanese teachers nurture younger teachers?"

Day 8, July 2

Activities – Reflection; Q&A; 9th grade lesson at Sengen Lower Secondary School (Tokyo) on multiplication with square roots; afternoon meetings in observation/ write-up groups

Participants met in the morning to reflect on and discuss observations and ask questions about lesson study and the lesson scheduled for the day. Several participants commented on the fact that lesson study implementation looks quite different from place to place, with different structures for the post-lesson discussions and different roles for the moderator. Dr. Takahashi elaborated on the post-lesson discussion structure for the afternoon lesson, informing the group that the research lesson this day was part of a prefecture-supported cross-district initiative designed to support leadership professional development. In this program, the post-lesson discussion follows a discussion protocol where each team member writes and posts three sticky notes (positive elements of the lesson, areas to improve, and suggestions for how to improve). The group then organizes the post-it notes into categories related to their teaching and learning goals, and the moderator uses the categories and notes to organize the debriefing discussion.

Dr. Takahashi described that in his opinion this program can focus too heavily on practical instructional techniques rather than on the quality or content of the lesson and asking "why." This led to a conversation about the Japanese education and teacher preparation system, with participants asking questions about the emphasis on content over procedure in the previous post-lesson discussions, building student content knowledge through discovery learning, and types of teacher certification offered for Japanese teachers. Dr. Takahashi further explained teacher preparation offered by national versus private universities.[6]

While many participants commented on the lack of student engagement during this lesson and the instruction that involved more teacher "showing" than eliciting of and using student thinking, some participants commented on learning about mathematics and about the idea of sequencing instruction by starting with the concrete (a rectangle) and moving toward the abstract (proof), as two comments reflect:

> "I would like to experiment with eliciting students' thinking and spending more time on making sense of the value of square roots ($\sqrt{2}$, $\sqrt{5}$, etc.) at the beginning of the lesson. For example, it will be interesting to experiment whether students can make sense of that $\sqrt{2}$ x $\sqrt{5}$ = $\sqrt{10}$ by estimating $1< \sqrt{2} < 2$ and $2 < \sqrt{5} < 3$ or $\sqrt{2}$ x $\sqrt{5}$ will be more than 3 but less than 4."
>
> "The question was one I had never seen before and will use."

Several participants also commented on the fact that during the debriefing discussion they learned that the teacher developed the lesson by herself, which contradicted their current understanding of the lesson planning process. For example, two participants wrote:

"In lesson study, shouldn't the lesson be developed collaboratively by a group of teachers

[6] The explanation offered was that national universities (e.g., Gakugei) encourage teachers to develop a particular content knowledge expertise, and therefore lesson study programs associated with these schools can focus more on content and supporting teachers to do good *kyouzai kenkyu*. At other private universities, teachers do not have that particular content focus so their lesson study programs often focus more on procedure.

rather than individually by the teacher teaching the lesson?"

"What I was not thrilled with is that it really did seem like no one else on the team had participated in constructing or providing feedback on the lesson plan. Although its good to hear how others teach a topic, it seemed like those conversations should have occurred before, not after, the lesson. I was left wondering at the level of *kyouzai kenkyuu* in this lesson. The teacher kept claiming that she chose the particular ordering because it was in the textbook. But is the textbook always the best?"

Although the translation for this post-lesson discussion was again difficult, participants appreciated the new model for the post-lesson discussion and the international exchange of ideas that occurred after the debriefing. One participant shared the following thoughts:

"I enjoyed the discussions among teachers as they reflected on their observations and build the mind/concept map about what they learnt about the lesson, what the things that need improvement as well as the insights that the mentors shared with Sase sensei. This post-lesson discussion showed a strong feature of mentoring teachers to build the capacity of the novice teachers in carrying out research lessons using lesson study. We also appreciated the fact that the post-lesson discussion allowed us to engage in a discussion with teachers and the team."

Day 9, July 3

Activities – Morning work time (OUSD group meeting); 7th grade lesson on the construction of angle bisectors

The OUSD group met again to discuss what they considered the non-negotiable "key features" of lesson study, based on the observations so far. This bulleted list helps to illustrate what mostly novice learners of lesson study were able to glean about "authentic" lesson study from their program participation in the previous seven days:

- A "plan, do, check, action" process, where the action is what arises from the debriefing discussion. Action is important. [The brochure provided by one of the Tokyo schools helped to make this clear.]
- Explicit structured collaboration, from the district to the teacher; people have specific roles and responsibilities; strong connection between math department and the sites; sites not left on their own and autonomous from the district.
- Research themes include both content and pedagogy.
- Lessons are student-driven; teacher is responsive to students
- Planning and lesson plan includes short term and long-term goals; shared goals among everyone involved.
- Focus is on understanding math for students and teachers. It builds a community of professionals.
- Anticipated student responses as a feature that ties so much together. Lesson design is that students will have multiple responses; teacher's role will be to bring those up. Lesson must deal with content, curriculum, pedagogy, social-emotional goals.

- Anticipated student thinking but discovery learning – teacher has to decide in the moment and go with the lesson. Therefore it is important to have a rationale for why you make the decisions you make.

Although Dr. Takahashi later summarized the lesson on this day as "not well taught," this lesson study opportunity provided another learning opportunity for participants, many of whom were unfamiliar with the mathematics of the lesson ("the lesson plan was incomprehensible to me"). Participants reported learning from the lesson plan showing the seven possible student solution methods, the teachers' tactic of having students describe their thinking process while he did the construction on the board, and especially from the strong post-lesson discussion commentary focused on ideas in the lesson and the mathematics that students will encounter in the future related to the angle bisector content. The commentary again emphasized the importance of knowing the math content trajectory, and participants picked up on the idea in their comments:

> "During that post lesson discussion I realized how important the teacher's understanding of the math is integral to lesson study. It was during that post debrief that I could see how valuable the discussion can be in developing a teachers mathematical content and then by extension developing a better teacher."

> "This was my focus lesson, and I thought that the teacher was exceptional in giving students access to the mathematics and allowing them the opportunity to construct viable arguments and critique the reasoning of others. Nonetheless, the mathematics were challenged heavily by the observers in the post lesson discussion, and that surprised and intrigued me. The idea that seventh graders understood properties of perpendicular lines was something that I don't believe American students have access to that often. Therefore, the mathematics content in the lesson and the post-lesson was fascinating and informative to me."

> "…One of the most interesting aspects of the lesson became apparent in the post-lesson discussion when Dr. Nishimura highlighted how the lesson didn't build very well on previous mathematical activity and knowledge development in which students had been considering constructions from the perspective of sets of points being equidistant from other mathematical objects. This highlighted the careful preparation of the 'knowledgeable other' and how they need to carefully and sensitively stimulate the thinking of the teacher community."

Day 10, July 4

Activities – Summary of previous day's lesson; 3rd grade lesson at Hashido Elementary School (Tokyo) on division with remainder using a word problem context; reflection; farewell reception

During the morning conversation, two key themes about post-lesson discussions arose again: 1) the importance of using a pre-briefing discussion prior to the lesson in combination with the lesson plan to support a good post-lesson discussion and to help novice practitioners think about the lesson; 2) How final commentary may emphasize different things, depending on the purpose of the remarks (e.g., to support the lesson instructor, to encourage schools, to highlight mathematical content).

The lesson this day helped participants observe *kikanshido* (between desk teaching), with the teacher walking among the students to assess their thinking and offer hints as needed. In the post-lesson discussion Professor Fujii pointed out that one notable point in the lesson was the teachers' failure to address the students' misconception of the incorrect answer (6 remainder 2). He emphasized that student ideas should be discussed whether they are correct or not – because incorrect answers often provide a window into the thinking of the students, and allow the learning to proceed to a deeper level. He commented that understanding all the different paths to a solution helps teachers improve. Dr. Fujii's commentary provided another example of how a post-lesson discussion and final commentary can push the learning of the lesson study cycle, and his comments left program participants wondering about mathematics, mathematics instruction, and using student thinking, as the following few comments reveal:

> "How do you follow up with wrong answers that represent genuine mathematical reasoning?"

> "I found that the lesson raised some important ideas about teaching. When the teacher did not adequately address the misconception that the student had, he was in a situation that many of us find ourselves in. I also found that the teacher made a strong effort to engage all of the students and to get many perspectives. The post-lesson discussion flowed well and involved many of the teachers. I liked how Professor Fujii asked the teacher the question about manipulatives vs. pictures to get him and the other teachers to think more deeply about what they were doing. His final commentary was excellent as well. He tries to teach the whole teacher (like the whole child). He focused not just on content or pedagogy but where they intersect, which is key. I left his session with many new ideas to think about."

> "I gained most from the post-lesson discussion for this lesson because there were a number of clear "take-aways" that I could use in my own teaching. First, it forced me to think deeper about when and how students use concrete materials to symbolize their thinking. It also helped me think about how student misconceptions that at first seem completely incorrect can actually be representative of mathematically sound reasoning. In this case, it was the student who answered "6 remainder 2" to the story problem of splitting 16 Jellos into groups of 3. Professor Fuji showed that "6 remainder 2" actually represented 6 x 3 + (-2) and this would be a reasonable response if the context was "16 students sharing tables that seat 3... how many tables are needed for all to sit?" Since the teacher did not pursue a discussion of the student's thinking, this point was never able to surface during the lesson."

Figure 1. Reported Mean Learning about Program Elements (Posttest Rating)

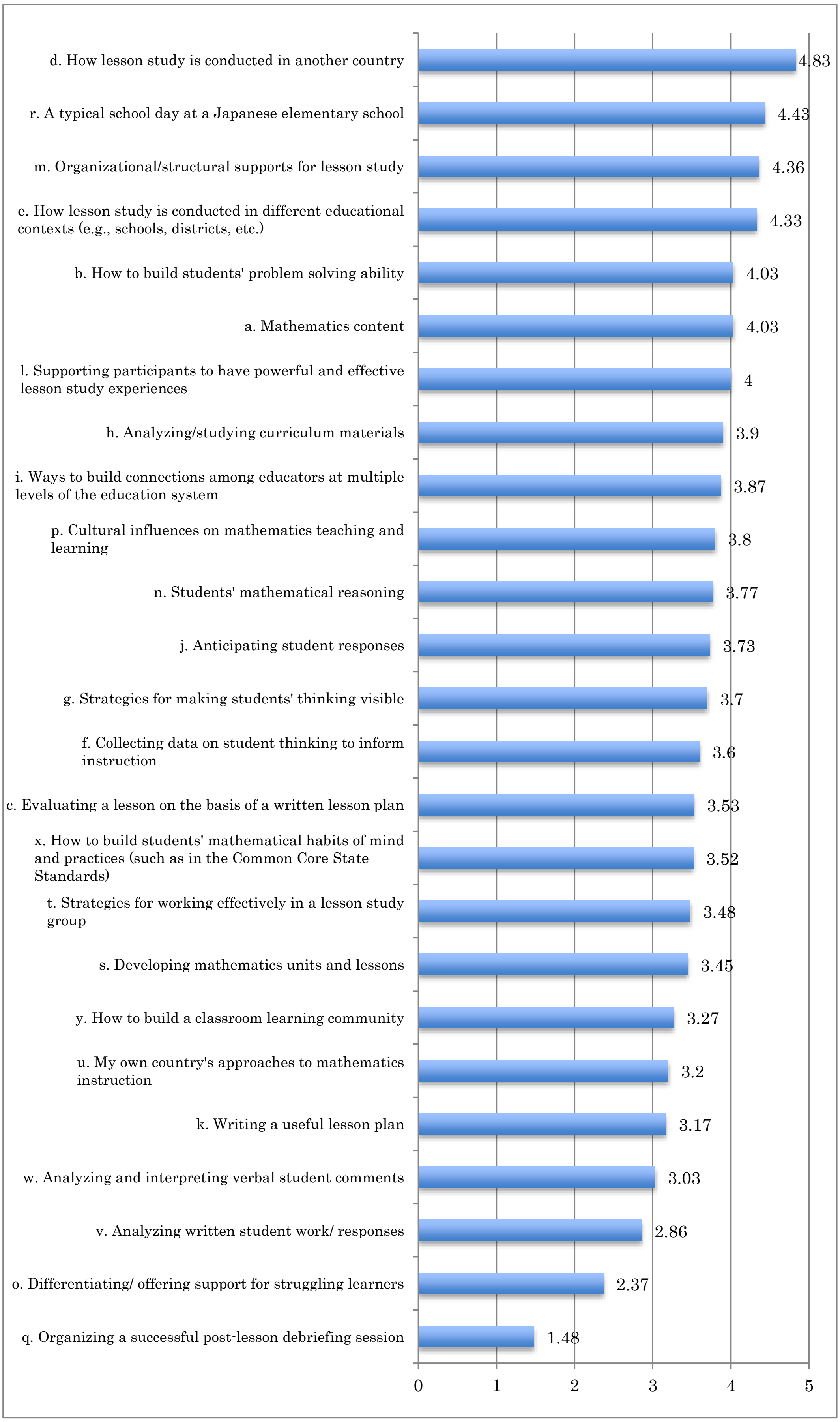
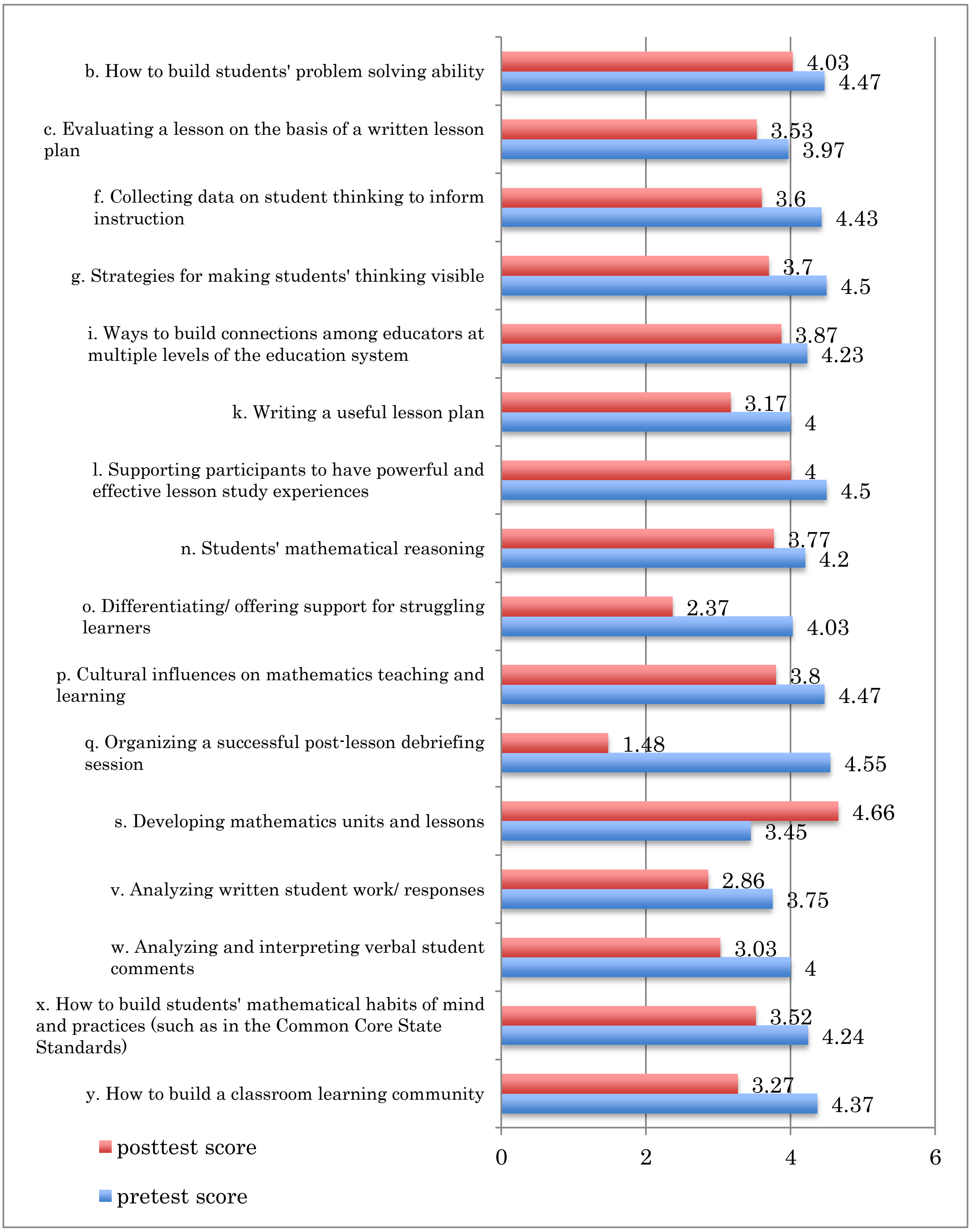


Figure 2. Anticipated and Actual Learning about Program Elements (Statistically Significantly Pretest/ Posttest Differences Only)



Annex ;

(1)List of participants

(2)Lesson Plan

(3)Questionnaire for external evaluation and research

(4)Articles on Japanese news papers

List of Participants for
IMPULS Lesson Study Immersion Program 2012
June 25 - July4 in Tokyo, JAPAN

Name			School/ Department	
1	Ms.	Andrea McGehee	Chavez Elementary Multicultural Academy, IL, U.S.	IMPULS
2	Mr.	Andrew Friesema	Dr. Jorge Prieto Math and Science Academy, IL, U.S.	IMPULS
3	Ms.	Anne Malamud	Grade 4-5 Teacher, Mills College Children's School, CA, U.S.	GER
4	Ms.	Ann-Marie Gamble	Grade 7 Mathematics Teacher, Oakland Unified School District, CA, U.S.	GER
5	Mr.	Barton Dassinger	Principal, Chavez Elementary Multicultural Academy, IL, U.S.	IMPULS
6	Prof.	Belinda P. Edwards	Assistant Professor of Mathematics Education at Kennesaw State University, GA, U.S.	IMPULS
7	Dr.	Catherine Lewis	Senior research scientist, Mills College, CA, U.S.	IMPULS
8	Ms.	Celia Pascual	Elementary Math Specialist, Oakland Unified School District	GER
9	Ms.	Courtney Ortega	District Math Specialist, Oakland Unified School Distric, CA, U.S.	GER
10	Prof.	Colleen Vale	Associate Professor in Mathematics Education at Deakin University, Australia	IMPULS
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13	Ms.	Elizabeth Mondero Torres	Urban Assembly Academy of Government and Law, NY, US	GER
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16	Prof.	Geoffrey Wake	Associate Professor, Nottingham University, U.K.	IMPULS
17	Ms.	Janette Hernandez	Region 2 Executive Officer, Oakland Unified School District, CA, U.S.	GER
18	Mr.	Josha Rosen	K-6 Math Specialist, Dobbs Ferry School District, NY, U.S.	IMPULS
19	Mr.	Joshua Lerner	Chavez Elementary Multicultural Academy, IL, U.S.	IMPULS
20	Ms.	Katherine Wolfe	Alliance Academy, Oakland Unified School District, CA, U.S.	IMPULS
21	Dr.	Kathy Schultz	Dean, School of Education, Mills College, CA, U.S.	GER
22	Prof.	Kelly Edenfield	Carnegie Learning, U.S.	IMPULS
23	Dr.	Kristen Bieda	Assistant Professor, Michigan State University, MI, U.S.	GER
24	Dr.	Linda Kroll	Faculty, School of Education, Mills College, CA, U.S.	GER
25	Ms.	Lisa Lam	Grade 1 Teacher, Oakland Unified School District, CA, U.S.	GER
26	Ms.	Lorelei Nadel	Grade 3 Teacher, Chicago Public Schools,IL, U.S.	GER
27	Dr.	Makoto Yoshida	President, Global Education Resources, NJ, U.S.; Director, Center for Lesson Study, William Paterson University, NJ, U.S.	GER
28	Dr.	Michelle Crillo	Assistant Professor, University of Delaware, DL, U.S.	GER
29	Ms.	Miranda Spang	ASCEND School, Oakland Unified School District, CA, U.S.	IMPULS
30	Ms.	Mdm Lim May Ling Angeline	Master Teacher/Mathematics, Academy of Singapore Teachers, Singapore	IMPULS
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32	Ms.	Nicole Worthey	Head of Maths, Drayton Manor High School, U.K.	IMPULS
33	Mr.	Phil Daro	CCSS co-chair	IMPULS
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38	Dr.	Siew Ling Connie NG	Curriculum, Teaching and Learning Academic Group, NIE, Singapore	IMPULS
39	Mr.	Sun-Chul Kim	Grade 5 teacher, Oakland Unified School District	GER
40	Prof.	Tad Watanabe	Professor of Mathematics Education at Kennesaw State University, GA, U.S.	IMPULS
41	Mr.	Thomas McDougal	Executive Director, Lesson Study Alliance, IL, U.S.	IMPULS
42	Ms.	Tracy Sola	Mathematics Coach, Belmont-Redwood Shores School District, CA, U.S.	IMPULS
43	Ms.	Veronica Chavez	Grade 3 Teacher, Chiaog Public School, IL, U.S.	GER
44	Dr.	Wanty Widjaja	School of Education at Deakin University, Australia	IMPULS

Grade 6　Mathematics Lesson Plan

June 26, 2012(Tue), 5-period
Funabashi Elementary School
Grade 6-4, 31 students
Teacher's Name: Takahiro Kishi

1. Unit:　Area of various shapes

2. Goals of the unit and evaluation criteria

	Interest, Eagerness, and Attitude	Mathematical Way of Thinking	Mathematical Skill	Knowledge and understanding
Goals	Students will try to find the area of circles and to approximate the area of figures around them using graph paper and changing those figures to figures which they have already learned.	Students will be able to think about ways to find area of circles and to approximate the area of figures around them based on already leaned figures.	Students will be able to find the area of circles and to approximate the area of figures around them by counting squares of graph paper and changing them to figures which they have already learned.	Students will understand that to find the area of circles and to approximate the area of figures around them; they can change them to figures which they have already leaned.　In addition, students will have a rich sense of area.
A	Students will become interested in ways to find the area of circles and other figures around them, and they will try to find more accurate area by changing those figures to figures which they have already learned.	Students can think about how to find the area of figures around them by approximating the figures with those they have already learned or dividing the figures into the familiar figures.　In addition, they will think about various ways to find the area of a circle such as changing it to figures which they already learned or by using diagrams and mathematical expression.	Students can find the area of circles and approximate the area of figures around them in various ways changing it to figures which they already learned.	To find the area of circles or to approximate the area of figures around them, students understood to use methods such as area preserving transformation, or figuring out its outline using figures which they already learned.　In addition, students will have a rich sense of area by estimating the area figures around them.

B	Try to find area of the circle and approximate area of figures which can be seen around them counting squares of graph paper and changing it to figures which they have already learned.	Students can think about ways to approximate the area of figures around them in various ways such as grasping the figures as familiar figures and dividing them into figures which they have already learned. And, students can think about ways to find the area of circles by changing them to figures which they have already learned and by using diagrams and expressions.	Can find area of the circle and approximate area of figures around them, counting squares of graph paper and changing those to figures which they have already learned.	Students will understand that to find the area of circles and to approximate the area of figures around them, they can use methods of grasping the outlines as familiar figures and changing them to figures which they have already learned. In addition, students will have a rich sense of area by estimating the area figures around them.

3. With regard to unit

(1) Overview of unit

Students have already learned the center, radius and diameter of the circle in the 3rd grade. And they did mathematical activities such as investigations of circles and drawing of circles. In the 5th grade, they learned and understood the meaning of pi by examining the relationship between diameter and circumference, and they learned to calculate the circumference.

Main purpose of this unit is to learn calculation methods to find the area of circles. As to area of a circle, because a circle is surrounded by curve and therefore unit area can not fit neatly, it is difficult to consider the methods to measure its area.

Parallelogram and triangle which they have already learned in the 5th grade were easier to derive the area formulae by using area preserving transformation and area doubling transformation.

In "1: Area of Circles", students start to think of how many unit area are in there, which is basic idea/method to think about area, and I will make them find the approximate area of circles using graph paper. Then, by transforming circles into figures which they have already learned, they will think of methods to find the area of circles, and derive the formula. Finally, students will summarize the formula as "Radius x Radius x Pi."

In "2: Approximate Area," first, I will make students try to grasp the approximate outline of figures which can be seen around them and estimate their areas. To think and understand approximate area size is important and beneficial in mathematics, so by emphasizing hands-on activities, I will try to create learning opportunities in which students can feel area sizes.

(2) Actual condition of students

As the students moved up to the 6th grade, classes are re-organized and the number of students in a class was reduced from 41 students to 31 students. It makes them concentrate well on studying. Most students study hard, however only certain students answer or speak up in class. Many of them find it difficult to express their own ideas or thoughts.

There is difference on mastery levels among students; some of students have not yet fully mastered the study contents which they have learned before.

With regard to finding area, students can calculate the area of rectangles, triangles, and parallelograms etc. using the formulae. In this unit, I want students to think about how to find the area of circles with interest while being aware of curves of circles.

(3) Method of approaching the primary theme

Theme of Study：Nurture students who express own thoughts and deepen each other's understanding

～Through *neriage* in mathematics lessons～

⇩

Ideal image of students envisioned by the upper grade subcommittee: Students who enjoy group learning (pleasure/enjoyment of thinking, solving, explaining and listening)

⇩

From the actual condition of students, firstly, it is necessary to take enough time to think by themselves. Therefore, I'll take a sufficient individual thinking time for individual students.

① Leaning modes（Individual → Group）

Secure sufficient time to think over individually. At that time, it is significant for students to understand "what the question is" and "how to think". So, I'll show question clearly using concrete/tangible examples and questioning to make point clear and help their consideration.

② Make a place to learn from each other（small learning groups）

Make the time to speak about own thoughts or ideas, which they thought by themselves individually, in small learning groups. These learning groups were purposely-created by the teacher considering abilities and qualities of students. I think that students are able to see objectively and think deeper own ideas or thoughts by listening mutual ideas in small groups (3 to 4 students in one group). After presentation in groups, I'll make the time for them to think another thought based on listening to other students' ideas. This will make the small group presentations and discussion time become more effective activity for students. Hereafter, I hope to apply this presentation experience in small leaning groups to future activity such as putting together their thoughts within group.

③ Devised-method for presentation

To make their thoughts clear, I prepare a worksheet. Using the completed worksheet, students will present their thoughts in small groups. In the group, group members discuss their own ideas and summarize their ideas into a central idea for the group, which is the easiest to understand.

During the lesson, I'll try to talk with individual students to get to know their ideas or thoughts.

After group discussion, if any thought is eliminated as a result of group discussion, I'll prepare the time for those students to explain their ideas by calling on them.

4. Teaching Plan on the Unit

Sub-unit	Number of hours	Learning contents	Evaluation Criteria
Area of circles	1	● Investigate ways to find the area of a circle with a radius of 10cm written on a graph paper.	● Students are thinking about methods to find approximate area.
	2	● To find the area of the circle by transforming it to figures which they have already learned. ● To derive the area formula by transforming the circle into a rectangle.	● Students are applying the area preserving transformation and previously learned area formulae to derive the area formula for circles.
	3	● To derive the area formula for circles by transforming it to shape other than rectangles. ● Discover that the formula for area of circles can be derived based on any kinds of shapes.	● Students understand that it is useful to transform a circle into shapes they already knew to find its area.
	4	● To find the area of circles using the formula. ● To investigate how the circumference and the area change if the diameter is doubled.	● Students can calculate the area of circles using the area formula.
	5	● To find the perimeter and area of figure composed of semicircles, squares, and/or quarter circles.	● Students can apply the formulae they have learned to calculate the area of semicircles and complicated diagrams.
	6	● Investigate the relationship between the central angle and the area of sectors, and find the area of sectors.	● Students can draw and calculate the area of sectors.
Approximate the area	7	● To find the area of irregular shapes by counting squares of graph paper or looking at approximate shape.	● Students can identify parts of irregular shapes to which they can use the area formulae to find the area of shape which is not rectilinear.
	8	● To copy the area on the map onto a graph paper and find its area. ● To find the area by approximating the outline of figure as a basic figure.	● Students try to find the area of lake and prefecture where he or she lives using a map. ● Students can find the area by approximating the outline of a region as a basic figure.

Exercise	9	● To promote better understanding of items they have already learned.	
Quiz (test of student's ability)	10	● Determine students' comprehension of items they have already learned.	● Understand meaning of reduction to common denominator.
	11	● To derive the area formula by transforming circles made of ropes.	● Students derive the area formula based on the area formula for triangles.

5. Lesson plan on May 26

(1) Goals of the Lesson

・ Think about methods to find the area of circles using graph paper

・ To device ways and means to deal with the places that are not complete squares.

(2) Flow of the Lesson

	Learning activity	○Points to remember for teaching	Evaluation criteria
5 minutes - Clarification of question/ task	**1. Comprehend the question** T①: Let's recall methods to find area of figures. What is area? C1 : It is amount of space. T②: That's right. For example, how much is the area of this rectangle? C2 : This is rectangle, so it is possible to find the area by length x width. T③ : Right. In the case of rectangles, why can you find the area by length x width? C3 : A base unit of area is 1 ㎠. So, to find how many 1 ㎠ squares in the rectangle, we multiply how many 1 ㎠ squares fit in the width and the	○Prepare only writing materials on desk. ○Show students rectangle which is written on graph paper.	

	number of layers of 1 ㎠ squares. T④ : It is correct. When you think about area, it is good to think based on 1 ㎠ unit. Then, how can we find the area of this shape? (Distribute worksheet) C4 : Can we change it into shapes which we learned? C5 : I think it is possible to find by counting the number of 1 ㎠ squares. **Try to find approximate area of circles using a graph paper.**	○Display the worksheet which is distributed to students on the TV screen using projector, and explain.	**Interest, Eagerness, and Attitude** Students try to find the area of circles.
10 minutes - Think and solve by themselves	**2. Solve the problem** T⑤ : Let's write down your ideas and thoughts on how much the area of this circle is on your worksheet. After ten minutes, I'll give you time to present and discuss your thoughts in group, so organize your thoughts so that you can explain it to your friends.	○ Tell students to get their ideas in shape to explain clearly using their own words and making use of the circle drawn on the graph paper. ○ Look around the classroom to check students who already organized their thoughts.	**Mathematical way of thinking** Students are thinking of how to find the area approximately. **Mathematical way of thinking** Students are using the basic idea to find the area, which is to decide on unit (㎠) and count its numbers.
10 minutes - Comparative discussion	**3. Explain own idea in the group** T⑥ : Let's present and explain your thought in turns in your group. After everyone presented his or her idea, please choose the clearest idea in the group. Also, find the area of the circle using that idea.	○Instruct to change desk arrangement to make the discussion easier. ○ If a student is still thinking about it, tell him or her to explain the idea which he or she prepared so far. ○Have students make sure that other members of the group understand their explanations.	

15 minutes - Presentation	**4. Discuss some of the ideas** T⑦ : Then, I'll ask you to present the results of your group discussion. Please first group. C6 : We divided circle into quarters. There are 69 1 ㎠ squares in one piece. There are squares that are missing parts, so we thought that 2 of those squares will be about 1 ㎠. Since there were 17 of those squares, the total is 8.5 ㎠. All together, it became 77.5 ㎠. This is area of one quarter of the circle, so 77.5 ㎠ multiplied by 4 equals 310 ㎠. So, we believe the approximate area of this circle is 310 ㎠. C7 : I looked at the part that is outside of the circle. When I counted, there were 14 1 ㎠ squares and 15 squares with missing parts. So like the first group, I thought that 2 of those squares total 1 ㎠. Then total area of those squares became 7.5 ㎠. Sum up these, the total become 21.5 ㎠, and 21.5 ㎠ x 4 equals 86 ㎠. Because total area of the large square is 400 ㎠, 400 ㎠ minus 86 ㎠ equals 314 ㎠.	○ Teacher will project the worksheet and provide additional explanation on the chalkboard as necessary. ○ Put up the worksheets on chalkboard and put the same ideas together. 17÷2＝8.5 69+8.5＝77.5 77.5×4＝310 1×14＝14 15÷2＝7.5 14＋7.5＝21.5 21.5×4＝86 400－86＝314	
5 minutes - Summary	**5. Summarize today's lesson** T⑧ : Please review what was discussed today and write your impression or thoughts and present it. T⑨ : Today, we could find approximate area of the circle with various ways. However, it would be difficult to find the exact area because	・ Remind them of learning content which they tackled today, and encourage them to reflect on it.	

	there is curve. In next class, let's try to find out way to find the exact area of circles.		

(3) Evaluation of today's lesson

・ Were students able to think about the ways to find the area of circles using graph paper?

・ Were students able to devise ideas to count squares of graph paper that were missing some parts?

June 27

Lower Secondary Grade 1 (Grade 7) Mathematics Lesson Plan

Teacher: SAKURAI, Junya

1. Title of Unit: Positive and Negative Numbers

2: About the Unit:

In this unit, students' number world will expand from non-negative rational numbers they learned in elementary schools to the entire rational numbers. Up to this point, not all subtraction problems were solvable; however, in this new number world, all four arithmetic operations are possible all the time. In lower secondary school, solidifying students' understanding of rational numbers is a central focus; therefore, this unit has a particular importance as the foundation of mathematics learning in lower secondary school.

With the introduction of negative numbers, students will learn that subtraction may be changed to addition because of the existence of "a number that will make the sum of 0" (additive inverse). Students have learned that division may be changed to multiplication because of the existence of "a number that will make the product of 1" (multiplicative inverse). Although terms such as "identity" or "inverse" are not a part of the instructional content, it is hoped that students will understand that addition and multiplication have the same structure by recognizing these types of numbers exist with respect to each operation. Such an understanding may lead to students' realization that steps in solving equations -- such as transforming equations in the form, $ax = b$, or changing the coefficient of x to 1 -- utilize the additive and multiplicative inverses. With respect to the four arithmetic operation, representations with number lines will be used to help students make sense of the meaning of the operations and to deepen their understanding.

Positive and negative numbers are also used in our everyday life. Thus, in the unit, we will incorporate activities to identify situations in our lives where positive and negative numbers are used. In addition, by applying positive and negative numbers in problem solving, we want to develop the disposition to seek the merit of using mathematics (positive and negative numbers in this case) such as simplifying calculation and easily grasping the differences from the point of reference.

3. Goals of Unit:

1. Students will be interested in thinking and representing various phenomena mathematically by grasping them using positive and negative numbers and discovering their characteristics and properties. They will also actively use mathematical ideas in reasoning and making judgments as they solve problems.
2. Students will be able to reason clearly and logically and represent phenomena using their knowledge and skills of fundamental patterns and relationships of positive and negative numbers. They can also deepen their understanding by reflecting on their reasoning.
3. Students will master the ability to calculate with positive and negative numbers. They can also use expressions and equations with positive and negative numbers as representation tools and interpret them.
4. Students will understand the meaning and need for positive and negative numbers. They also understand the meaning of the four arithmetic operations with positive negative numbers and master the calculation skills.

4. Evaluation Standards for the Unit:

Interest, Eagerness, and Attitude Toward Mathematics	Mathematical Way of Thinking	Mathematical Skill	Knowledge and Understanding Regarding Numbers, Quantities and Geometrical Figures
Needs for and meaning of positive and negative numbers			
Students will be interested in positive and negative numbers and think about their needs and meaning. They will try to represent various phenomena in their surroundings using positive and negative numbers.	Students will be able to think about how positive and negative numbers may be used by identifying specific situations in which positive and negative numbers are used such as expressing the differences of high temperature between yesterday and today.	Students will be able to represent various phenomena in their surroundings using positive and negative numbers. Students will be able to represent characteristics and directions that are opposite of each other using positive and negative numbers. Students can represent positive and negative numbers on a number line and express their relationships using the equal and inequality signs.	Students understand the need for and the meaning of positive and negative numbers. Students understand the meaning of the size relationship of various numbers (natural numbers, whole numbers, positive and negative numbers), the meaning of positive and negative signs, and the meaning of absolute values.
Meaning of four arithmetic operations and calculations			
Students will be interested in the four arithmetic operations with positive and negative numbers. Students will think about ways to calculate with positive and negative numbers and carry out the calculations.	Students will be able to figure out ways to calculate with positive and negative numbers based on their previous knowledge of calculations. By expanding the range of numbers to include both positive and negative numbers, students will be able to consider addition and subtraction operations from a unified perspective. As a result, students can consider an expression involving both addition and subtraction as a sum of terms with positive and negative terms.	Students will be able to calculate with positive and negative numbers. Students will be able to represent an expression with both addition and subtraction operations as a sum of positive and negative terms.	Students understand ways to calculate with positive and negative numbers. Students understand that addition and subtraction operations can be considered from a unified perspective by expanding the numbers to include positive and negative numbers.
Processing and representing with positive and negative numbers.			
Students will be interested in using positive and negative numbers. They will try to represent and process various phenomena using positive and negative numbers.	Students can examine various phenomena and situations involving changes by using positive and negative numbers to express the differences from the established target value.	Using positive and negative numbers, students will be able to represent and process various phenomena in their surroundings such as determining the arithmetic mean using an estimated mean.	Students understand that some phenomena and situations involving changes may be represented clearly or processed efficiently by using positive and negative numbers.

5. Unit Plan (Total of 28 Lessons):

Sub-Units	# of lessons	
1. Positive and negative numbers	5	28 total
2. Addition and subtraction	8	
3. Multiplication and division	11	
4. Using positive and negative numbers (today's lesson)	1	
5. Projects	2	
Summary of the unit	1	

6. Today's Lesson:

1. ***Date:*** Thursday, June 27, 2012 14:10 ~ 15:00

2. ***Location:***
   Lower Secondary School Attached to College of Education of Yamanashi University, Red Brick Building

3. ***Title:*** Let's split the team into two groups with an equal average height

4. ***Objectives:***
   - Students will learn that the calculation of arithmetic mean will be simplified by using the idea of tentative average and positive/negative numbers.
   - Students will foster the disposition to use mathematics in problem solving by learning about the merits of the idea of tentative average and the use of positive/negative numbers.

5. ***Strategies for helping students develop own questions:***

In order to create combinations that have the equal average height, the average height for each combination must be calculated. By making students experience the tediousness of the process may prompt students to ask, "Is there a way to make the calculation simpler?" Furthermore, by selecting the numbers (heights) so that there are more than one way to split the team into two groups with an equal average height, the need to repeat the process again even after students find one combinations that have the equal average will emerge. That will make it even more likely for students to ask, "Is there a way to make the calculation simpler?" In this lesson, both of these points were considered to create the main task.

While students are working on the task, those pairs who are using the differences between the average height and each data point will be identified and noted. If no such pair is present, those who are focusing on the average value in their attempts. That is because if they are focusing on the average height, the chance is good that they are thinking about the differences between the height and players' heights. By bringing out those students' ideas, it is hoped that the class can experience the merits of using the differences between the average and individual data points as the numbers will become smaller, thus easier to process. By making the merit of using the differences between the average and individual data points a shared understanding, it is aimed to make a connection to the main question for the lesson, "Is there a way to make calculation simpler?"

6. *Flow of the lesson:*

<table>
<tr><th>Step</th><th>Instructional Activity</th><th>Anticipated Students' Responses</th><th>Notes</th></tr>
<tr><td rowspan="2">Introduction</td><td>1. Understand the problem situation (3 min.)

[Problem] There are 10 new members on our school basketball team. In order to evaluate the skill levels of these new members, Mr. Sakuragi who is the faculty sponsor of the team decided to have them play intra-squad games by creating two teams, X and Y. Mr. Sakuragi figured out 3 different ways to create two teams so that the average heights are the same, and they played 3 intra-squad games. How did Mr. Sakuragi made these teams. Here are the heights of the players.

<table>
<tr><td>Name</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td></tr>
<tr><td>Height (cm)</td><td>166</td><td>164</td><td>161</td><td>156</td><td>153</td><td>151</td><td>150</td><td>146</td><td>143</td><td>138</td></tr>
</table>
</td><td></td><td>• Distribute the sheets with the problem.</td></tr>
<tr><td></td><td>• The players are listed in order of their heights, from the tallest to the shortest.</td><td>• Check to see if there is anything that is unclear.</td></tr>
<tr><td>Development</td><td>2. Tackle the problem in pairs. (10 min.)</td><td>1. Distribute the players alternatively from the tallest to the shortest.
Team X: A, D, E, H & I
Average Height 152.8 cm
Team Y: B, C, F, G, & J
Average Height 152.8 cm
2. Pair up the tallest and the shortest and assign them to a team. Then, pair the tallest and the shortest of the remaining players and assign them to the other team. Repeat this process.
Team X: [A, J], [G, H] & E
Average Height 152.8 cm
Team Y: [B, I], [D, G] & F
Average Height 152.8 cm
3. Create a group of 4 players by selecting the two tallest and two shortest players. Assign them to team X. Do the same with the remaining players and assign them to team Y. Assign the tallest of the remaining to team X and the last player to team Y.
Team X: [A, B, J, I] & E
Average Height 152.8 cm
Team Y: [C, D, H, G] & F
Average Height 152.8 cm
4. Distribute the players alternatively starting with the player whose height is closest to the average height of the 10 players, 152.8cm.
Team X: E, G, C, I, & A
Average Height 152.8 cm
Team Y: F, D, H, B, & J
Average Height 152.8 cm</td><td>• Have students sitting next to each other work together. If any student is missing, form a group of 3.
• Distribute graphing calculators, one to each pair.
• For sharing, have each pair write their response on a magnetic white board.
• Identify pairs who are using either the minimum (138) or the average (152.8) as the base and representing other data points by their differences from the base.</td></tr>
</table>

Development	3. Sharing (12 min.)	• Share the solutions 1 ~ 4 above. • It is tedious to calculate the average height of each team every time we try a different way to split the tplayers into two groups. • Is there an easier way to calculate the average?	• Incorrect or incomplete solutions should also be shared. • Ask students "What was challenging as you tried to solve the problems?"
Deepening	4. Look for easier ways (15 min.) [Task] To make two teams with equal average height, is there ways to simplify the calculations?	• Have students share the idea of using a base value and re-organize the table using the differences from the base. a. Using the average for the 10 players (152.8) as the base (0), express all data points using positive/negative numbers. b. Using 153 as the base (0), express all data points using positive/negative numbers. c. Using 150 as the base (0), express all data points using positive/negative numbers. d. Using 138 (minimum value) as the base (0), express all data points using positive/negative numbers. • The numbers become smaller and that makes calculations simpler. • By using positive and negative numbers, the sums become smaller because + and - cancel out and that makes calculation simpler.	• Ask students, "What's the merit of re-organizing the table using a base value?" **"What is the merit of re-organizing the table?" (explore merits)**
Summary	5. Think about the merits of using the idea of tentative average and positive/negative numbers. (10 min.) • Journal writing	• The idea of tentative average makes the numbers smaller and make the calculation simpler. • By using positive and negative numbers, the sum of values become smaller, and the smaller values are easier to calculate.	• Explain the idea of tentative average.

June 28

Grade 3 (Classroom # 1) Mathematics Lesson Plan

Teacher: KOIKE, Kohji

1. Title of the Unit: Let's think about ways to simplify calculations

2. About the Unit:

In this unit, students learn about mental calculations of the sums and the differences of two 2-digit numbers (including those that require regrouping).

This unit is constructed to address the following points mentioned in the *Elementary School Mathematics Course of Study*.

3. The Content of Grade 3
A. Numbers and Calculations
(2) Students will be able to add and subtract accurately and reliably and will further enhance their ability to use those operations appropriately.

Remarks Concerning Content
(2) As for the contents A-(2), (3) students should be able to calculate simple calculations mentally.

In the unit, Better Ways to Calculate, in Grade 2, students learned about calculating 2-digit numbers ± 1-digit numbers by decomposing the numbers using the structures of numbers. In addition, students have already learned how to calculate 2-digit numbers ± 2-digit numbers using the algorithms. In a 3rd grade unit, Let's Take Another Look at the Multiplication Table, students have explored ways to calculate 2-digit numbers × 1-digit numbers like 12 × 4 by the multiplicand, 12, into 8 and 4 or using 10 as the base, 10 and 2. These units have a common theme, "ways to simplify calculation." In this unit, students will study mental computation based on the same theme.

With respect to mental calculation, *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grade 1-6)* makes the following point:

> Simple mental calculations indicated in Remarks Concerning Content A-(2) include addition of 2-digit numbers or subtraction that is their inverse. This type of calculation is often used in daily life and is even a necessary part of the process of multiplication and division. Mental calculations are often used to make estimates in daily life. It is important to consider these kinds of applications when teaching. (pp. 90-91)

As noted here, mental addition and subtraction of 2-digit numbers are used in a variety of everyday situations and students' future study. Therefore, it is important for students to understand the needs for mental calculations.

In addition, study of mental calculations may also enrich students' number sense. For example, thinking with approximate numbers (38 is "about 40") or thinking about a number in relationship to another number (38 as "2 more to make 40") are very important parts of fostering number sense. Therefore, we intend to have students deal with compositions and decompositions of a variety of numbers.

The following two points will be kept in mind as we teach this unit.

First, the same calculation may be done in different ways mentally. For example, consider ways to calculate 48 + 36 mentally. The following methods are shown in the textbook (*New Elementary School Mathematics, 3A*, by Tokyo Shoseki, p. 60):

48 + 36	48 + 36	48 + 36
		Think of 48 as 50
40 8 30 6	30 6	Think of 36 as 40
40＋30＝70	48＋30＝78	50＋40＝90
8＋ 6＝14	78＋ 6＝84	2＋ 4＝ 6
70＋14＝84		90－ 6＝84

There are other ways to calculate the sum. For example, "48 is 2 from 50, so decompose 36 to 2 and 24, and calculate: 48+2 = 50, 50+34=84." The value of the study of estimation is to think about ways to calculate using various properties of operations. Therefore, while acknowledging each student's method, we want to help each student identify more efficient approaches on his or her own while exploring a variety of ideas.

The fact that a variety of methods are possible means that students will have opportunities to encounter many different ideas. Thus, by sharing each other's ideas, students have opportunities to interpret other students' reasoning processes. Through these opportunities, students may discover new ideas of their own or re-examine their own ideas with new insights. Therefore, it is important to establish an opportunity for students to examine and understand a variety of ideas.

Another point to be kept in mind as we teach this unit is to design the lessons so that children can imagine how mental calculations may be utilized in everyday situations. One way to experience the merits of mental calculation is for students to identify situations in their everyday life or in their study where mental calculation is being used and actually use the ideas from this unit in those situations. Therefore, a goal of this unit is for students to develop a disposition to actively use the ideas they learn in this unit in mental calculation they do in their everyday situations while helping them realize mental calculations are used in many situations in their daily living.

3. Goals of the Unit:

* As we set up the goals of the unit, we kept the following points in mind so that the goals will reflect mathematics teaching practices that are based on the foundation of the career education.

- In "Goals of the Unit," list both goals of the subject matter (mathematics) and goals of the career education. This is done so that the goals of the unit and the goals of the career education may be compared and linked with each other. Moreover, by listing the goals in parallel, it may be easier to see the relationships to the goals in other subject areas.
- Since Moral Education is the foundation of every subject matter, how it is related is shown.

Mathematics & Moral Education	**Career Education**
• Students will understand mental calculation of addition/subtraction of 2-digit numbers, and they will be able to utilize the mental calculation reasoning in everyday situations and in schools.	<Ability to establish human relationships> ○ Student can listen to other people's ideas and try to understand each other. [Communication Ability]
• Students will acknowledge the merits of mental calculation and try to use it in everyday situations and in schools. [Interest, Eagerness, and Attitude] • Students will be able to think about ways to calculate mentally by looking at numbers flexibly such as decomposing them or using approximations, and they can represent their reasoning processes. [Mathematical Way of Thinking • Students will be able to mentally add or subtract two 2-digit numbers. [Mathematical Skill] • Students understand mental calculation strategies for addition and subtraction by making use of the structures of numbers and properties of operations. [Knowledge and Understanding]	<Ability to make decisions> ○ Student will be able to select and use an efficient approaches from a variety of ideas. [Selection Ability] ○ Student can reflect on what they have previously learned, explore possible solution approaches, and solve the problem at hand. [Problem Solving Ability] * The goals in the boxes are the images of "ideal students" emphasized by lower/intermediate/upper grade groups and particularly appropriate for this unit.
<Moral Education> 1-(2) Complete things you must perform responsibly.	

4. About Research Theme:

In order to raise students' "ability to think coherently by anticipating and to represent their ideas," it is helpful to give students tasks in which students experience disequilibrium and experiment with many different ideas. That is because such tasks will generate the process to examine what they have learned previously and investigate ways to use their prior knowledge as a starting point to solve the given tasks.

However, as I look at the students in this particular class, or 3rd grade students in general, many students cannot represent their thoughts. It is not rare that students cannot reach a conclusion because they were unable to summarize their thoughts.

Therefore, in this unit the emphasis are placed on the following. First, students should make explicit their ideas on "for what purpose" and "what strategy will be used." Then, students will be prompted to organize their thoughts by using words such as "first" and "next," or making a numbered list. By doing so, own ideas can be expressed more clearly and others can interpret them more easily. It is also possible that by making one's thought and the thinking process clearer, students' desire for future learning may be heightened.

In studying mental calculation, because mental manipulation of numbers is necessary, it is usually not desirable to have students record their methods in notebooks. However, in order to devise a faster and simpler methods of mental calculation, it is necessary to express one's own thought processes so that they may be compared with other ideas. It is through that examination, students can identify more efficient method. Therefore, in this lesson, we will incorporate an activity in which students will represent and examine mental calculation processes. The goal is to help students develop more efficient methods of mental calculation through this careful examinations.

(1) Children who "think coherently by anticipating and represent their ideas"
In this unit, it is hoped that students will exhibit some of the following.

- Estimating the sum by approximating one or both of the addends.
- Thinking about different ways to calculate by applying properties of operations.
- Representing their strategies to simplify calculation in equations, diagrams and words.
- Recognizing the merits of other students' ideas and try to use them.

(2) Teaching that will heighten students' "ability to think coherently by anticipating and to represent their ideas"

① Consider ways to pose the learning task
In the "Grasp" stage of the lesson, pay close attention to students' ideas, discuss them as a class so that students can understand the task. Ideas like "Can we use what we have learned previously?" "What is different from what we have learned so far?" and "What idea may be useful as the starting point?" can help students anticipate how the solution may be developed. On the other hand, "The answer is about ..." suggest students are anticipating the results. By sharing students' ideas as a whole class, encourage each student to tackle the task on his or her own with clear vision toward a solution.

② Make students record their calculation methods in their notebooks
Instead of just writing the results of mental calculations in their notebooks, students will be required to write down their thinking processes. Students will be encouraged to use phrases such as "because ~," "in order to ~," and "at first I did ~, then~" so that the purposes and the processes are clearly recorded. Students should also use equations and expressions to represent their thinking processes, not just in words.

③ Set up opportunities where students can share their ideas with classmates
When we try to communicate own ideas to others, we reflect on our ideas. In order to help students think about "how can I more effectively communicate my idea to my friends" and "in what order should I describe my ideas," we will set up opportunities where students can share their ideas with each other.

④ Set up an activity in which students may interpret other's ideas and utilize them
It is intended that students will try to think about the meaning of expressions and equations included in other students' solutions and to represent ideas presented verbally with equations and expressions. In this way, we want students to use multiple representations to express each other's ideas. In addition, in order to help students identify an efficient mental calculation strategy, an activity to use other students' ideas will be incorporated.

5. Unit Plan (total of 2 lessons):

	Main Task	Evaluation Criteria	Career Education Perspectives
1	Students will individually think about ways to mentally calculate 48+36. Students will share and analyze their strategies.	• Students will acknowledge the merits of mental calculation and try to use it in everyday situations and in schools. [Interest, Eagerness, and Attitude] • Students will be able to mentally calculate the sums with two 2-digit numbers by making use of the structures of numbers and properties of operations [Mathematical Skill]	**Student will be able to select and use an efficient approaches from a variety of ideas. [Selection Ability]** **Student can reflect on what they have previously learned, explore possible solution approaches, and solve the problem at hand. [Problem Solving Ability]**
2	Students will individually think about ways to mentally calculate 53 - 28. Students will share and analyze their strategies. **[today's lesson]**	• **Students will be able to think about ways to calculate mentally by looking at numbers flexibly such as decomposing them or using approximations, and they can explain their reasoning processes. [Mathematical Way of Thinking]** • Students will be able to mentally calculate the differences of two 2-digit numbers. [Mathematical Skill]	**Student can listen to other people's ideas and try to understand each other. [Communication Ability]**

6. Today's Lesson:

1. ***Goal of the lesson:***
   Students will be able to think about ways to calculate mentally the differences of two 2-digit numbers and explain their ideas.

2. ***The aim from the career education perspective:***
   Student can listen to other people's ideas and try to understand each other. [Communication Ability]

3. ***Date:*** Thursday, June 28, 2012, 1:50 - 2:35 pm (5th period)\

4. ***Location:***
   Oshihara Elementary School, Showa Township Schools, Grade 3 (Room 1)

5. ***Intent of the lesson:***
   Students can tackle mental calculation of the differences of two 2-digit numbers in this lesson by reflecting on the discussion of mentally calculating the sums of two 2-digit numbers in the previous day's lesson. Students will be thinking about ways to mentally calculate by decomposing numbers based on different properties of operations and using approximate numbers. It is hoped that students will be able to take advantage of their learning in the previous lesson. Therefore, in today's lesson, the emphasis is placed on representing own ideas using equations, expressions, diagrams, and words, and organizing their mental calculation processes clearly. For this purpose, it is necessary that students must

interpret other's ideas and compare and contrast with their own ideas. During the whole class discussion, the activity of interpreting equations and expressions and representing ideas expressed in words using equations and expressions. Then, by having students reflect on their own ideas in light of other's ideas, we hope to help students identify more efficient (faster and simpler) ways of mental calculation.

6. *Flow of the lesson:*

Step	Contents & Learning Task	Points of Considerations/ Materials	Evaluation
Grasp (5 min.)	1 Understand the task. $\square$ - 28 (1) Anticipate. ① Enter one of the numbers, 89, 53, and 68, in the box. • It is easy to calculate with 89. • With 68, the numerals in the ones place are the same, so calculation is easy. • With 53, calculation is more complicated because we must regroup. ② Check the answer for 53-28 by calculating it using the subtraction algorithm. Let's think about ways to mentally calculate 53 - 28.	• Ask for the reasons why calculation is easy or complicated. Help students develop ideas for tackling the learning task. • Confirm that mental calculation of 53-28 is more complicated.	
Explore (12 min.)	2 Think about ways to mentally calculate. (1) Find the difference mentally. a) Decompose 53 into 50 and 3 50 - 28 = 22, 22 + 3 = 25 b) Decompose 28 into 20 and 8 53 - 20 = 33, 33 - 8 = 25 c) Add 5 to 53 58 - 28 = 30, 30 - 5 = 25 d) Approximate 28 as 30 53 - 30 = 23, 23 + 2 = 25 e) Add 2 to both 53 and 28 (53 + 2) - (28 + 2) = 55 - 30 = 25 (2) Record the calculation processes in notebooks.	• Encourage students to think about their strategies for adding 2-digit numbers mentally. • Remind students to use diagrams (arrows, segments to show how numbers are split), words, and equations.	Can students think about ways to simplify the calculation? Can they represent their ideas in words, equations/expressions, and words? (Check students' notebooks) [Mathematical Way of Thinking]

Deepen (20 min.)	3 Whole class discussion (1) Share mental calculation processes. • Explain ideas represented by equations and expressions. • Represent explanations given verbally using equations and expressions. (2) Think about efficient processes. (3) Use the processes that were judged to be efficient and solve 83 - 15.	• Acknowledge and support orderly explanations that incorporate words like "first," and "next." • Clarify what idea contributes to the efficiency. • Help students realize that the solutions c, d, and e used properties of operations to eliminate the need for regrouping.	**Are students trying to understand each other's ideas? [Communication Ability]**
Summarize (8 min.)	4 Summarize (1) Pose a problem in a shopping situation [(amount you have) - (price) = (amount left)] and have students solve it mentally. (72 - 48) (2) Have students write a journal entry. • I learned a simple mental calculation process by listening to my classmates' ideas. I want to use it when I go buy something. • I was able to write down and explain my ideas clearly.	• Give a daily situation where mental calculation may be used. • Make suggestions so that students can incorporate the following in their journal entries. o How did your mental calculation processes changed. o What made explanations easier to understand.	Were the students able to do 2-digit minus 2-digit subtraction by mental calculation? [Skills] (Check students' notebooks)

7. ***Evaluation:***
   - Were students able to think about and explain ways to simplify mental calculation processes to find the difference of two 2-digit numbers?

7. Evaluation from the career education perspective throughout the unit:

In this unit, the evaluation of communication ability, selection ability, and problem solving ability will be primarily through students' notebooks and in-class discussion. Within each lesson, situations will be set up so that students can explicitly think about a specific ability. For example,

- Acknowledge and share with the whole class any student who exhibited the intended career education abilities.
- During the journal writing time, suggest students to reflect on the career education abilities.

References

1. 新しい算数３上　教師用指導書　指導編(2010)　pp.92-95　東京書籍
2. 新しい算数３上　教師用指導書　研究編(2010)　pp.66-67, pp.188-195　東京書籍
3. 文部科学省(2008)「小学校学習指導要領解説　算数編」　東洋館出版社
4. 中村享史(2002)「『書く活動』を通して数学的な考え方を育てる算数授業」pp.11-12　東洋館出版社
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7. 新算数教育研究会(2010)「講座　算数授業の新展開３　第３学年」pp.54-61　東洋館出版社

Translator's Note

- References 1 & 2 above refers to the textbook. An English translation of this textbook series may be purchased from Global Education Resources (www.globaledresources.com).
- Reference 3 is from *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grades 1 - 6)*, a Ministry of Education document that provides detailed explanations of the Course of Study. It is available online at
  http://e-archive.criced.tsukuba.ac.jp/data/doc/pdf/2010/08/201008054956.pdf
- Other references are from books that are available only in Japanese.

Grade 5 Mathematics Lesson Plan

June 29, 2012 Period 5
Grade 5 Classroom 4; 39 students
Teacher: TAKAHASHI, Takeo

1. Name of the Unit: Let's investigate solid figures

2. Goals of the Unit

- Students will try to investigate properties of cubes and cuboids (rectangular prisms) based on their previous study of geometric figures. [Interest, Eagerness, and Attitude]
- Students will think about properties of cubes and cuboids by focusing on constituent parts of solid figures. [Mathematical Way of Thinking]
- Students will be able to draw nets of cubes and cuboids. [Mathematical Skill]
- Students will know the numbers of edges, vertices and faces of cubes and cuboids. In addition, students will understand the parallel and perpendicular relationships among faces and edges. [Knowledge and Understanding]
- Students will understand the concept and properties of prisms. [Knowledge and Understanding]

3. About the Unit

Students have been studying about the basic solid figures. In the Grade 1 unit, "Let's Play with Shapes," students examined solid figures intuitively through observations and investigations of the features of concrete materials. In Grade 2 unit, "Shapes of Boxes," students discovered relationships between plane figures and rectangular prisms by copying the faces of boxes and building boxes using rectangles and squares. They have also explored properties of cubes and cuboids by focusing on their constituent parts of faces, edges and vertices.

In this lesson, students will first clarify the concept of cubes and cuboids by observing the shapes of faces in cubes and cuboids. Then, they will deepen their understanding of the characteristics of cubes and cuboids. As students examine cubes and cuboids, it is natural for them to notice perpendicular and parallel relationships of faces and edges. Therefore, this lesson will also enrich students' spatial sense.

With respect to cubes and cuboids, students have learned about their sketches and nets, parallel and perpendicular relationships of faces and edges, and their constituent parts. In addition, students have learned that there are 11 different nets of a cube.

In this lesson, based on students' previous study, students will think about the reason why 7 edges must be cut in order to open a cube into a net. I believe that students' understanding of the constituent parts of cubes will be deepened by thinking about the reason why the number of edges to be cut to open a cube must be 7. What follows are anticipated students' reasoning.

[Solution 1] Figure 1 shows a result of cutting open a cube. This net is composed of 6 squares, and by connecting these 6 squares at 5 appropriate locations, a net of a squares we can make different nets of a cube. For these 5 locations, 10 of the 24 sides of the squares are used. Therefore, there are 24-10=14 sides are left. Since each pair of these 14 sides will form an edge of a cube, $14 \div 2 = 7$ is the number of edges of a cube that must be cut to open a cube.

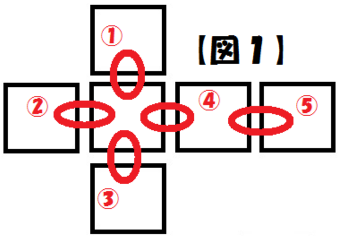

Figure 1

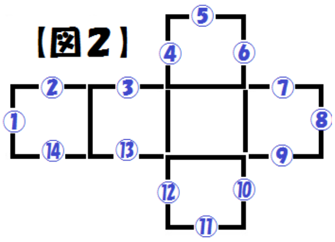

Figure 2

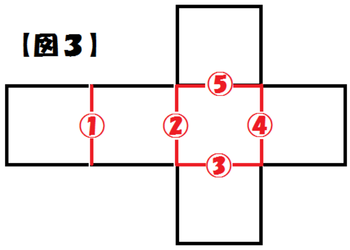

Figure 3

[Solution 2] I anticipate many students will use this reasoning. When a cube is cut open, it will match one of the 11 possible nets of a cube. In these nets, as shown in Figure 2, there are 14 sides of squares that will become edges of a cube when the nets are folded to make a cube. These 14 sides will be paired up to make edges of a cube. Therefore, the answer is 14 ÷ 2 = 7. The main difference of this solution from Solution 1, is the question, whether or not there will always be 14 sides of squares will be left unconnected. However, this question may be answered by focusing on the relationship between the number of items in a line and the number of spaces in between.
[Solution 3] This reasoning may be as popular as Solution 2. There are 12 edges in a cube. Of those edges, 5 edges still remain in a net of a cube. Therefore, the answer is 12 - 5 = 7.

4. Relationship to the Research Theme

The mathematics group has set this year's research theme as "nurturing students who can think on their own, express their ideas, and learn from each other." We have been focusing on the aim of developing "mathematical ways of thinking" in our students and conducting *kyozai kenkyuu* with "coherence of content" in mind. In addition, starting this year, we began to focus on students expression using diagrams such as number lines. By doing so, we want to attempt to develop students "mathematical ways of thinking" as they learn from each other. With this research in mind, we will briefly discuss the idea of "ability to deepen own understanding and give an account of own ideas" that relates to the research theme.

○ About "ability to deepen own understanding and give an account of own ideas"

In the mathematics group, by "giving an account of own ideas," we are imagining a student going back and forth between "what I am thinking" and "diagrams representing my thinking" as necessary while explaining his or her own thinking.

Thus, "deepen own understanding and give an account of own ideas" means for students to renew and modify what they have learned previously while explaining their ideas using those contents. For example, think of the various properties of operations. At first, they are just knowledge, for example, if the divisor becomes 10 times, the quotient will become 1/10. However, while studying the division by decimal numbers, this idea becomes an important method to transform the divisor into whole numbers (which they have previously learned). Or, in the situations involving inversely proportional relationships (constant product, i.e., □×△=A), this property may be considered as the reason why an quantity becomes 1/10 as much when the other quantity becomes 10 times as much. In other words, we consider students' understanding is "deepened" whey they can use an idea in a context that is different from the context in which the idea was learned originally. Therefore, perhaps we can say that to "deepen own understanding and give an account of own ideas" in mathematics means that students can "use what they have learned previously as appropriate in situations and make coherent explanation with conviction."

Furthermore, with respect to the four perspectives on "research methods" suggested by the Research Committee below, we will focus on the first two ideas.

◎ Designing a learning environment in which students will feel a desire to or a necessity to think.
◎ Designing a learning environment which may promote thinking that will be connected to real life situations.
○ Designing a lesson that can deepen students' understanding
○ Ways of expressing ideas.

This is because we believe that students will learn to ask questions if they are placed in situations where they feel they want to think, or they must express their thinking, and if everyone in the situation, including the teacher, learn from each other. Furthermore, in such situations, students will be solving problems using what they have learned previously, in other words, it is a real-life situations.

5. Unit Plan (1 lesson)
Topic Lesson 1 lesson (Today's lesson is 1 of 1)

6. Instruction of the Lesson
(1) Goal of the Lesson

○ Students will deepen their understanding of characteristics and properties of cubes by examining and understanding the reason why 7 edges must be cut in order to open a cube into a net.

(2) Flow of the Lesson

Step	Anticipated students' actions	○ Points of considerations ☆ Evaluation (Evaluation method)
Grasp	Show a cube and 2 or 3 different nets of a cube for demonstration. **How many edges of a cube do we need to cut to open it to be a net?** • 6 edges • 7 edges • 8 edges	○ Do not let students actually cut open a cube. ○ Remove the nets once students understand what it means to open a cube.
Plan	● Actually verify. • It looks like we need to cut 7 edges always.	
Explore	**We discovered that if we cut 7 edges of a cube, we can open it to make different nets. Why do we always have to cut 7 edges to make any net?**	

<table>
<tr>
<td>Sharing & Discussion</td>
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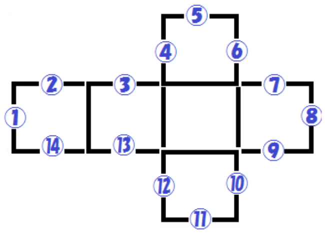
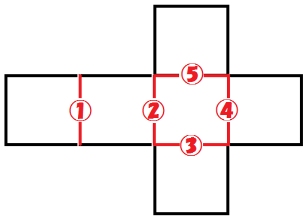
● Please explain why you think we need to cut 7 edges. (After students wrote their ideas in their notebooks, have them share their ideas.)

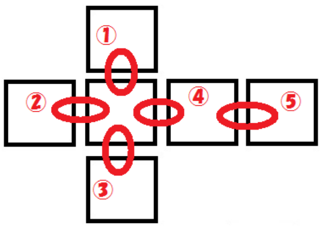
- It's because I tried to imagine opening a cube in my head.
- When we fold a net to make a cube, 2 sides from 2 squares will match up to make an edge of a cube. Since there are 14 sides in a net (along the perimeter), we will need to cut 14 ÷ 2 = 7 edges.



- In a cube, there are 12 edges, but 5 of them are still left intact in a net. Therefore, 12-5=7 edges must be cut.



- A net of a cube is made up of 6 squares. In 6 squares, there are 24 sides, but we need to use 10 of them to connect the squares to make a net because 2 sides from 2 square together will make an edge of a cube. So, if we take away 10 from 24, we know that there are 14 sides of the squares are still left. Since 2 sides make an edge, we need to cut 14 ÷ 2 = 7 edges.



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○ While students are writing their ideas in their notebooks, monitor their ideas and think about the order in which the ideas are to be shared.

☆ Are students thinking about the number of edges to be cut with their own reasons? (Notebooks, oral presentations)

☆ Are students making connections to the number of faces and edges of a cube? (Notebooks, oral presentations)

○ As students cut open the cubes they made themselves. tell them to think carefully so that they will not end up with the same net.

○ To make it easier to see how sides of the squares in a net will match up to form an edge, have a permanent marker to mark the sides.

○ Depending on how the lesson plays out, ask students how many edges of an octahedron must be cut to open it to make a net.

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<td>Summarize</td>
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- Write a journal entry.

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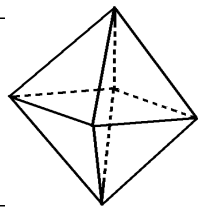

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July 2

Lesson Plan for Mathematics

Period5, Monday, July 2, 2012
13：30～14：20
Grade 9, Classes3 and 4, 32 Students (Standard leveled class)
Instructor: Sase Miwako (Head teacher), Sengen Junior High School, Fuchu-city
Place: Room of Grade 9, Class 4 (3rd floor)

1 Title of the Unit

Square Roots

Chapter 2 Section 2 “Caluculation of Expressions with Square Roots”

Text Book *Mirai e Hirogaru－Mathematics3－* Keirinkan

Sub-material *Study Note for Mathematics 9th Grade* Seishinsha

2 Goals of the Unit

Students will be able to calculate expressions with square roots. Furthermore, students will deepen their understanding of square roots by thinking of ways of calculation and try to express or examine specific situations using square roots.

3 Evaluation Criteria

view point	A. Interest, Eagerness, and Attitude	B. Mathematical Way of Thinking	C. Mathematical Skill	D. Knowledge and Understanding
Evaluation Criteria of the Unit	Students will be interested in calculations of expressions with square roots, and learn with high motivation.	Students can think of ways of calculating and transforming expressions with square roots based on the meaning of square roots.	Students can calculate expressions with square roots and can do transformations such as rationalization.	Students understand the ways of calculations and the steps of transformation of expressions with square roots.

Specific evaluation criteria for instructional activities	① Students can think about ways to multiply and divide square roots. ② Students can transform expressions with square roots ,using calculations of square roots, and they try to think of ways of calculating values of other square roots using the approximate value of a certain square root. ③ Student will be interested in calculations of expressions with square roots, and try to solve problems.	① Students will examine multiplication and division of square roots and understand that the calculations may be carried out in the same way as multiplication and division of integers. ② Students can think about expressing the square root of a number in the forms of √a or a√b and transformations such as rationalizing denominator based on the meaning of square roots, and they can use those ideas as appropriate. ③ Students can think about the calculations of sums, differences, and products of expressions with square roots by using the ideas of expansion and factoring of expressions.	① Students can multiply and divide square roots ② Students can simplify radical expressions and can rationalize the denominator, and they can calculate values of other square roots using the value of a certain square root. ③ Students can calculate sums, differences, and products of expressions with square roots using the distributive property and multiplication formulae.	① Students understand ways of multiplying and dividing square roots ② Students understand the meaning of rationalizing denominators and the steps to transform them. ③ Students understand the ways of calculating expressions with square roots.

4 About Teaching

(1) About the Unit

In the seventh grade, students learned about positive and negative numbers and deepen their understanding about numbers. In the ninth grade, students will be introduced to square roots and expand the range of numbers to include irrational numbers. They will learn about the meaning, ways of expression, and ways of calculating with square roots. Building on that understanding, students will be using square roots to solve quadratic equations and to determine missing lengths using the Pythagorean Theorem.

In this unit, students will find out what will happen with the four arithmetical operations with square roots. It can be reasoned that $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ but $\sqrt{a}+\sqrt{b}\neq\sqrt{a+b}$ by calculating the approximate values. Students will deepen their understanding of square roots by verifying and explaining these facts while further developing the ability to examine and explain phenomena logically. In the previous unit, students learned about expressing square roots using the radical sign, √. In calculating expressions with square roots, $4\times\sqrt{3}$ can be expressed as $4\sqrt{3}$ and $4\sqrt{3}+2\sqrt{3}$ as $(4+2)\sqrt{3}$. In this way, numbers with radical signs may be considered in the same way as letters. It is important that students understand the merit of expressing irrational numbers concisely using radical sign. Furthermore, students should appreciate the fact that, in calculations and examinations of phenomena, radical expressions may be treated in the exact same way as when working with numbers and literal expressions which has already been learned. Moreover, a care should be taken so that students will cont consider the radical sign as just a sign. √2 and √2+1 are both specific quantities. In order to help students to develop the awareness that square roots are also numbers, approximate values of square roots may be used to compare them with integers and to locate them on a number line as necessary.

【Grade 7】

Positive and Negative numbers
- ○Four arithmetic operations with positive and negative numbers

【Grade 8】

Calculations of expressions
- ○Addition and subtraction of expressions
- ○Multiplication and division of monomials

【Grade 9】

Expansions and factorizations of expressions
- ○Multiplication and division of expression
- ○Multiplication formulae
- ○Prime factorization
- ○Factorization of expressions

→

【Grade 9】

Square roots
- ○Square roots
- ○Values of square roots
- ○Rational numbers and irrational numbers
- ○Multiplication and division of square roots
- ○Calculating expressions with square roots

→

【Upper Secndary School】

10th Grade: Math I
- ○Real number
- ○Quadratic equations
- ○Data analysis --- distribution and standard deviation

↓

【Grade 9】

Quadratic equations
- ○How to solve quadratic equations
- ○Quadratic formula

Pythagorean theorem
- ○Pythagorean theorem
- ○Using the Pythagorean theorem

(2) About Students

At this school, only one of the four weekly mathematics lessons is held in students' own homerooms. For the other 3 lessons, 2 homerooms are split into 3 groups based on their achievement levels (Basic・Standard・Advanced). This class is the Standard group, but there is a wide range of academic achievement; while some students are good at mathematics, about 20 percent of students feel they are weak in mathematics. Students try very hard and work with problems very diligently, however many students tend to get stuck when more difficult problems are posed or in situations where they need to explain why something happens. Thus, a major goal in this class is to foster mathematical ways of viewing and thinking. In this unit, students will learn about calculations of irrational numbers for the first time. Thus, the way to calculate with irrational numbers will be carefully explained to promote mastery. However, in order to avoid making the lessons just calculation practices, students will be asked to explain "why" so that their investigation will become more in-depth.

(3) About the Materials

First, we will create a multiplication expression with square roots by tying to find the area of a rectangle with the length of $\sqrt{2}$ cm and the width of $\sqrt{5}$ cm. Since it is the first time to think about calculation with numbers that have radical signs, the question of whether or not the method of multiplication with rational numbers can be applied in the same way will be the starting point. Students can predict that $\sqrt{2}\times\sqrt{5}=\sqrt{10}$ by thinking about the multiplication of $\sqrt{4}\times\sqrt{9}$, which is actually a product of rational numbers. The class will be led to the general characteristic of multiplication, $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$, by thinking about about reasons why the above prediction is true.

We have already been doing things such as finding approximate values of square roots using a calculator and squaring square roots. When the students are asked to think about how to calculate the expression, $\sqrt{2}\times\sqrt{5}$, it is most likely that students will try to use these two ideas. When approximate values are used, there will be a difference between $\sqrt{2}\times\sqrt{5}$ and $\sqrt{10}$. I will help the students understand that explaining this way isn't enough to show that the prediction is correct in general. Therefore, we need to think about the product of square roots by squaring the expression. At that time, students need to be careful that $a^2=10$ means that $a=\pm\sqrt{10}$, and if "a" is a positive number, the answer will only be $\sqrt{10}$. Students will explain and share the ways of their thinking with the whole class, think and come up with a solution with the whole class, and will make sure that in multiplication of square roots, you just have to put a radical sign around the product of the numbers inside. Finally at the end, students will complete practice problems to make sure of the way of calculating.

5 Unit Plan (Eight lessons)

Sub-Unit	Goals	Content of Study・Learning Activity	# of Classes	Evaluation Criteria (Ways of Evaluation)
Multiplication and division of expressions with square roots	○Students will discover the properties of products and quotients of square roots.	○Think about the product, $\sqrt{2}\times\sqrt{5}$, by calculating the approximate values and verifying by squaring the expression.	4 (Today's lesson is # 1 of 4)	A－① A－② B－① B－② C－① C－② D－① D－② (Observation・Notes・Individual
	○Students can transform square roots into different forms by using the properties of products and quotients of square roots.	○By using the property of products, simplify radical expressions. Also, perform transformations such as bringing the number outside the radical sign inside by squaring it and rationalizing the denominator.		

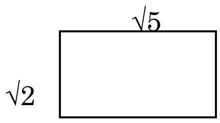
	○Students can find the approximate value of a square root by looking at the approximate value of another square root.	○Determine the approximate value of square roots by using transformations of square roots. Also, think about what will happen to the approximate value of a square root when the number inside the radical sign becomes 10 times, 100 times… .		support during lessons・Small tests)
Calculating Expressions with Square Roots	○Students will understand that the sum of expressions with square roots can be simplified by using the distributive property if the numbers inside the radical sign are the same.	○Verify that √a+√b≠√a+b by squaring the expressions or by using counterexamples. Also, think about the sums (and the differences) of expressions with square roots in the same way as calculations of literal expression by using the distributive property.	3	A－③ B－③ C－③ (Observation・Notes・Individual support during lessons・Small tests)
	○Students can calculate the sums, differences and products of expressions with square roots by reasoning in the same way as calculations of literal expressions.	○Think about various ways of calculating expressing with square roots and calculate using the distributive property or the expansion formulae.		
Practice	○Review and practice what was learned.		1	

6　This lesson　（１／８）

(1)　Goal of this lesson

Students will discover and understand ways to multiply square roots.

(2)　Flow of the lesson

Duration	Learning activities and activities	Points of considerations	Evaluation criteria (Ways of Evaluation)
Introduction 5 min.	Goal of this lesson Let's think about the product of square roots  	Check the goal of this lesson To help students imagine a specific rectangle, draw it on the board.	

Development ① 30min	T：What is the expression to calculate the area of the rectangle? S：√2×√5		
	T：The area of a rectangle can be calculated by length×width so here, the expression will be √2×√5. So what is the answer? S：√10	Write the expression on the black board and make the students guess the answer.	
	T：Why do you think so? S：Because 2×5＝10. S：Just because. T：In multiplication of integers, 2×5＝10, but this is a multiplication of numbers with the radical signs, √. Can we really do it the same way as with integers? S：Yes. S：Maybe not.	Make students answer “why they think so”. It is likely that many students will answer "Just because.” Since this is the first time for students to calculate products with numbers inside radical signs, I will make students think if it is really correct or not.	
	T：How about trying a different expression like√4×√9? S：√4＝2 and √9=3 so √4×√9＝2×3＝6. When you express 6 using a radical sign, it becomes √36, so √4×√9＝√36. The number inside becomes the product of the numbers inside.	Make students reason by analogy of product of rational numbers written as expressions with the radical sign	
	T：If you think of it the same way, it seems like √2×√5＝√10. Are you sure √10 is correct? S：Yes. S：Maybe not. T：Then let’s think about it more in detail.		
		Confirm the task and write it on the black board.	A－① (Individual support during lessons)
	T：Let’s think what kind of numbers √2 and √5 were. You may use your calculator. S：(Think about how to verify and write it down in the notes.)	Give time to think on their own. Walk around the desks and give hints to students who are stuck.	

	T：(Hint) What was the approximate value of $\sqrt{2}$? T：(Hint) What happens when you square the expression, $\sqrt{2}\times\sqrt{5}$?	To the students who could figure out by using the approximate values, give them the next hint and let them think of a general explanation.	
	T：If you were able to find out, get together with people around you and share and explain your ideas to each other. S：(Explain each others' ideas and come up with a solution) S：(Write the idea on black board)	Choose a few students and have them write their ideas on the black board so the whole class can verify each others' ideas	
	T：Let's explain how you verified. S①：When you think of the approximate values, $\sqrt{2}=1.414$ and $\sqrt{5}=2.236$, so $\sqrt{2}\times\sqrt{5}=3.161704$. Since $\sqrt{10}=3.162$, $\sqrt{2}\times\sqrt{5}=\sqrt{10}$	(Compare the approximate values)	
	S②：When you square $\sqrt{2}\times\sqrt{5}$… $(\sqrt{2}\times\sqrt{5})^2$ $=(\sqrt{2}\times\sqrt{5})\times(\sqrt{2}\times\sqrt{5})$ $=\sqrt{2}\times\sqrt{5}\times\sqrt{2}\times\sqrt{5}$ $=(\sqrt{2})^2\times(\sqrt{5})^2$ $=2\times5$ $=10$	(Thinking based on the meaning of square roots)	B－① (Speaking in front)
	The square root of 10 may be positive or negative, but since this is a positive number, $\sqrt{2}\times\sqrt{5}=\sqrt{10}$. T：From the above explanation, can we say that $\sqrt{2}\times\sqrt{5}=\sqrt{10}$? S：Yes. T：Then can we say $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$ in general? S：Yes.	Verify that "If you square a number and get 10, then the number is a square root of 10" by reminding them what they learned previously.	
	T：We can think of the general case in the same way. The product of square root will be $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$. You just have to calculate the product of the numbers inside and put it inside a radical sign.	Generalize and summarize it as the property of the product of square roots.	

Development② 10min	T : Now let's try some practice problems. S : (Write the problems and the answers in their notes.)	Re-affirm that you just have to multiply the numberd inside and put it in a radical sign. Since (2) is a product of a positive number and a negative number, the answer becomes a negative number. Some students may write√－30 so this needs to be carefully watched.	D－① (Notes)
Conclusion 5min	T : We learned that the product of square roots is the square root of the product of the numbers inside, that is, √a×√b=√a×b. In the next lesson, we will be learning about quotients of square roots and about the products of a number with a radical sign and one without radical sign.	Review the important points on the black board and announce what will be learned in the next lesson.	

(3) What to observe (Evaluation points for the actual lesson)

①Theme of the group: "Planning materials from which students can feel satisfaction and fulfillment"

Through the lesson, were the students able to understand the calculation of product of square roots and have the sense of accomplishment by understanding why it works out like that?

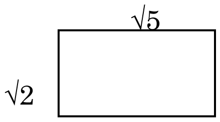
②Was the content for this lesson something that the students could work with high motivation?

③Were the students able to explain √2×√5＝√10 using good reasons?

7　Board writing plan for today's lesson

Date　Multiplication of Square Roots

How many cm^2 is the area of a rectangle with the length of $\sqrt{2}$cm and the width of $\sqrt{5}$cm?



$\boxed{\text{Can we say } \sqrt{2}\times\sqrt{5}=\sqrt{10}?}$

(With different numbers)

$\sqrt{4}\times\sqrt{9}$

$=2\times3$

$=6$

$=\sqrt{36}$

(Using approximate values)

Compare $\sqrt{2}=1.414$、

・・・

$\sqrt{10}=3.162$

$\sqrt{2}\times\sqrt{5}=\sqrt{10}$

(If we square)

$(\sqrt{2}\times\sqrt{5})^2$

=

=

=10

$\sqrt{2}\times\sqrt{5}=\sqrt{10}$

Date　Multiplication of Square Roots

For positive numbers a and b,

$\boxed{\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}}$

Practice

(1)$\sqrt{3}\times\sqrt{7}$　　(2)$\sqrt{5}\times(-\sqrt{6})$　　(3)$\sqrt{2}\times\sqrt{8}$

Seating Chart（Class 3, 4　Course B）

Sato	Fukushima	Takahashi	Kurajima	Tashiro	Nishioka
Narioka	Horiuchi	Ishikawa	Sarai	Yajima	Imai
Fukumoto	Miyagami	Ichikura	Tokuta	Ooi	Ozawa
Matsuno	Abukawa	Ganbara	Hongo	Ozaki	Sato
Matsumoto	Kuroda	Kimura	Yoshida	Onozawa	Sugimoto
Miyata		Sanpei			

Research Theme：

Designing lessons that enhance the quality of mathematical activities

7th Grade Lesson Plan for Mathematics

Date：July 3rd, 2012(Tuesday) 14：20～
Class：Grade 7 Class C（20Boys・20Girls）
Instructor：Koganei Junior High School KABASAWA, Kouichi

１．Title of Unit　Plane Figures

２．Theme　Construction of Bisectors of Angles

３．Goals of the Unit

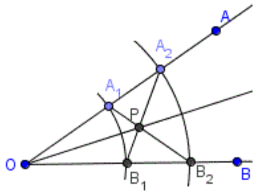
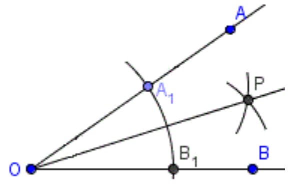
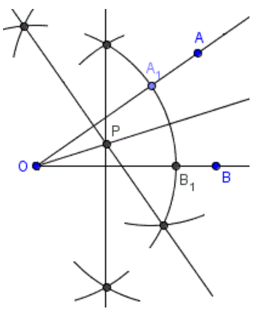
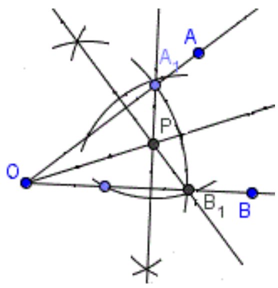
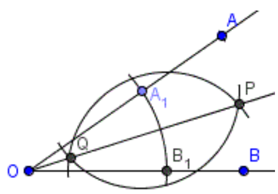
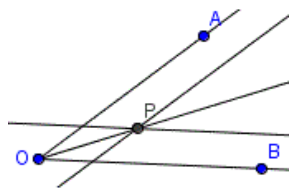
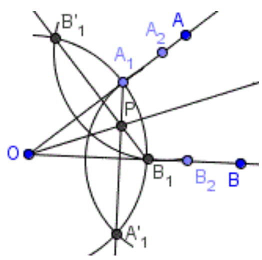
- Students will be able to construct bisectors of angles using points of symmetry.
- Students will be able to explain the steps of construction indicating the center of circle, the radius, and the two points through which a straight line passes.
- Students will deepen their understanding about thinking behind each method and about bisectors of angles through examination of various ways of construction.

４．Unit Plan

Construction of Regular Hexagons
Set of points that are equidistant from a given point
Set of points that are equidistant from two given points（Perpendicular bisector）
Set of points that are equidistant from three given points
Set of points that are equidistant from a given line（Construction of parallel lines, transformation of angles, construction of perpendicular lines）
Consolidation of basic construction ①（Basic construction, organizing the terms）
Set of points that are equidistant from a pair of given lines (Construction of bisectors of Angles) ←Today's lesson
Set of points that are equidistant from three given lines; Construction of a perpendicular line that passes through a point on the line; Construction of tangents
Various construction
Transformations of figures

５．Flow of the lesson

Steps of instruction	Students' anticipated responses	○Points of considerations ☆Evaluation Criteria
Introduction： Presentation of problem ・"What can you say about a set of points that are equidistant from two given lines?" ・"Construct the bisector of an angle.	・It will be the bisector of an angle. ・It will be an axis of symmetry.	○ Re-consider the set of points that are equidistant from two given lines as the bisector of an angle. ☆Students try to re-examine the results of construction and try to express them using words.

Development : Independent problem solving ・Ask students to demonstrate their constructions on the black board. ・Compare and contrast various methods of construction and discuss them.	・Sharing and discussion of various methods of construction (Examples of students' anticipated responses) (I)  (II)  (II) is the method of construction written in the text book. (III)  (IV)  (V)  (VI)  (VII) 	○ Have students explain the method of construction in words, and the teacher constructs on the black board according to the explanation. ○ Have students discuss characteristics of each method, good points about them, and points in common. ○Confirm especially about the points below ・Excess lines ・Position of the point of intersection ・How to pick the center and the radius ○ When taking up the construction method II in the whole class, it is expected that students will construct point P differently. We will discuss whether all of the points can be looked at as the same. ○The construction method of II will be taken up in the whole class. Discuss 2 or 3 more other methods and compare and contrast. ☆Students are able to construct in their own methods. ☆Students are motivated to work on construction and try to think of different methods.
Development : Compare and contrast ・"Are there ways of looking at various construction methods as the same?"	• They are all constructing congruent triangles (or other figures). ・They are taking a point on each side of the angle of the bisector. (Simple expressions such as "I did the same thing on this side and the other side," is also expected.) ・They are based on the fact that the circle is	○When discussing what makes these methods the same, display the circle that has been implicit in students' presentations. ○If any of these ideas do not come from the students,

	line-symmetric. ・They constructed several pairs of points that are positioned symmetrically from the angle of bisector (the axis of symmetry) and constructed 2 lines using those points. Then, they found the point of intersection of the 2 lines.	the teacher will, after giving time to think, organize these ideas and summarize.
・"If we look at the points we summarized, can't we try other methods of construction?"	・By drawing the whole circles instead of just arcs, select other combinations of symmetric points on the circle to construct the bisector. ・Construct the bisector using other symmetric points.	○ If there is extra time, construct the bisector using other methods. ○ Think about other methods of construction not to find a better method but to help students experience the merit of the idea that are in common with all of the methods of construction.
Summary 「Let's summarize what we learned from today's lesson.」	・We used the symmetry around the angle bisector. ・We used the fact that the segments connecting points that are symmetric around the axis of symmetry will intersect on the axis of symmetry. ・Each method can be explained in various ways but there is only one way to construct the angle bisector.	○ If there is enough time, have students write what they thought about today's lesson and share.

Grade 3 Mathematics Lesson Plan

Wednesday, July 4, 2012, 5th Period
Teachers: HONOBE, Koh (Home room teacher for Classroom 1)
SEKI, Satoe (Home room teacher for Classroom 2)
ARASHI, Genshu (Support teacher for mathematics lessons)

Research Theme: Mathematics learning that nurtures students who can use what they have learned ~ Through activities to express own thinking ~

Prior knowledge and ways of thinking we want students to use	Strategies to heighten students' ability to use what they learned	Ways of observing and thinking to nurture
○ Division calculation ○ Division that uses basic multiplication facts.	○ Developing problems that might heighten students' motivation and interest. ○ Displaying the contents that have been learned.	○ Analogical thinking Thinking about similar situations and use them to reason in a novel situation ○ Integrated thinking Identifying and summarizing the essential commonality among various situations

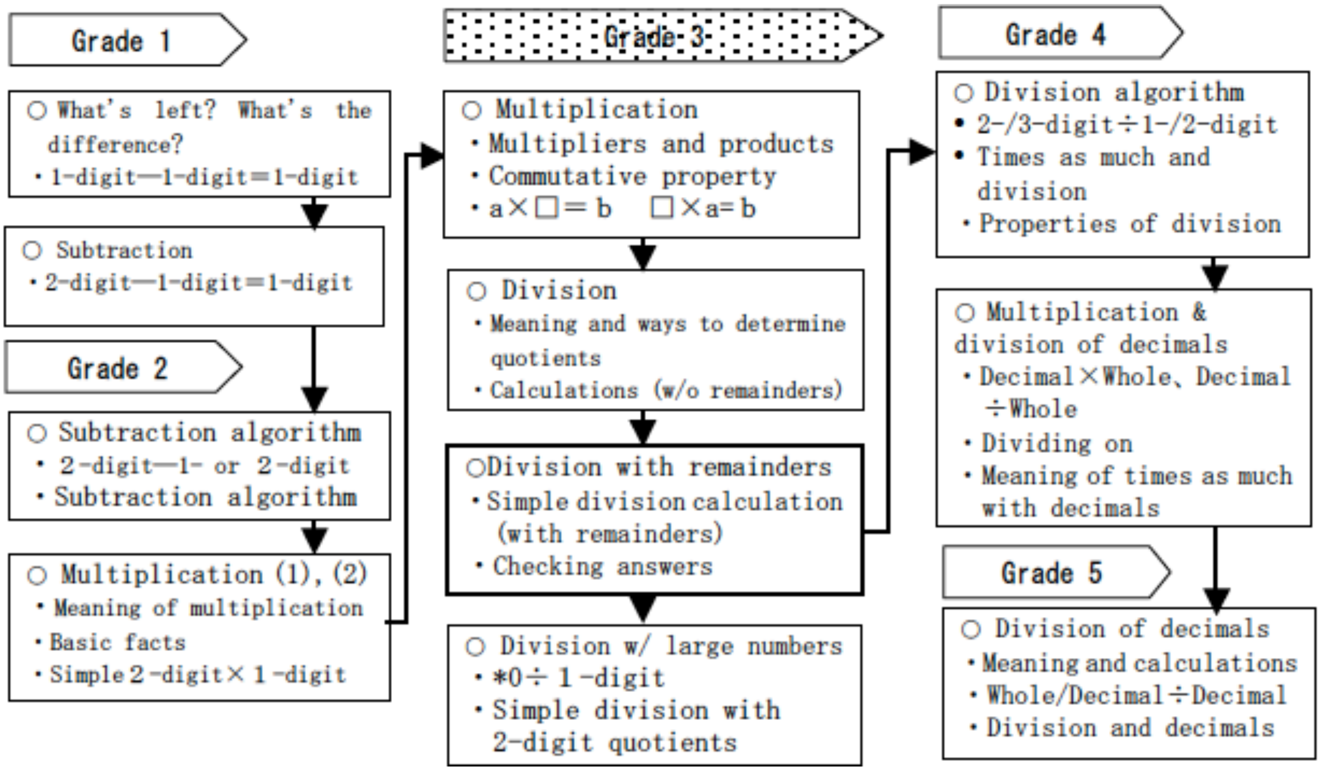
1. Name of the Unit　　Division with remainders

2. Goals of the Unit and Evaluation Criteria

○ Students will understand division situations with remainders and deepen their understanding of the division operation. Furthermore, students will be able to use what they learned.

Interest, Eagerness, and Attitude	Students try to think about the meaning and ways to calculate division in situations where there are remainders based on division situations without remainders.
Mathematical Way of Thinking	Students can think about division situations with and without remainders in an integrated way, and they can represent the meaning and ways of calculating division with remainders using concrete materials, drawings, and expressions/equations.
Mathematical Skill	Students will be able to calculate division with remainders, and they can determine the quotients and the remainders.
Knowledge and Understanding	Students will deepen their understanding of the division operation by knowing the meaning of the remainders and the relationship between the divisor and the remainder

3. Flow of the content



4. State of the students

(1) Small group instruction for mathematics

At the beginning of this school year, new homerooms were created and 2 homeroom teachers and a support teachers for mathematics were assigned. This is the first time students experience mathematics instruction in reduced-size groups (instead of in their own homerooms). The students are separated into 3 groups of more or less equal achievement level.

There is not much difference in students' achievement at this point, yet. All students engage in mathematics lessons eagerly raising their hands and speaking up during lessons. We can sense students' desire for "I want to understand" as they continue to persist when they make mistakes. They write quite a bit in their journals, too. However, there are students who are still trying to master how to take notes, or those who cannot maintain their concentration and look bored from time to time.

From the result of a survey conducted in April, we learned that the number of students who do not feel comfortable with mathematics is not small. According to the survey, 65% and 28% of the students indicated they liked mathematics or somewhat liked mathematics, respectively, while 7% stated that they disliked mathematics somewhat. Thos students who indicated they liked mathematics seem to feel that mathematics is "useful," "enjoyable," and "happy when problems are solved successfully." On the other hand, those who disliked mathematics indicated that mathematics is "hard" and "time consuming." From another survey item, we learned that 47% of students enjoy being able to share their "own ideas." Thus, it appears many students want to publicly share their ideas and the level of interest in participating class discussion is high.

It is our desire to maintain the high level of students' eagerness/desire and their willingness to engage in mathematics learning. In order to do so, we need to identify and provide appropriate support in the areas where students are not comfortable. Moreover, during a lesson, we need to acknowledge individual students' ideas and make them shared understanding of the whole class.

(2) The results and the analyses of the readiness test

Problem	Correct (all questions)	Incorrect
(1) In 12 ÷ 4, which is the "divisor" and which is the "dividend"?	46	16
(2) Multiplication equations with missing numbers (e.g., 3 x [ ] = 15, [ ] x 4 = 24) (4 questions)	59	3
(3) To calculate the following, which facts (e.g. 4's facts, 7's facts) do you use? (5 questions)	53	9
(3) Calculate and find the answers (e.g., 21 ÷ 3, 6 ÷ 1). (5 questions)	55	7
(4) Calculate 0 ÷ 7.	61	1
(5) There are 32 cookies. If you place 8 cookies to a bag, how many bags will there be? (equation and answer)	61	1
(6) Calculate 38 ÷ 6. (division with a remainder - not yet learned)	18	44

(Number of Students)

(1) "divisor" and "dividend" [Translator's note: The original Japanese words used are more student-friendly, not the technical terms. The same is true for "multiplicand" and "multiplier" below.]

Many students do not yet understand these terms. Although the terms "multiplicand" and "multiplier" were used in the discussion of multiplication in Grade 2, students' understanding of these terms are not that strong. It may be an indication that although these students can perform calculations correctly, there are still a significant number of students who lacks the essential understanding of mathematical expressions.

(2) Multiplication equations with missing numbers

Almost all students completed these questions successfully. They tend to engage in these types of problems eagerly in everyday lesson.

(3) Using the basic multiplication facts to calculate division

Most students understand that they need to apply the basic multiplication facts to calculate division. However, when students are asked individually, "why can we use the basic multiplication facts?" many students appeared perplexed. Thus, the reality appears to be that many students can calculate mechanically but lack an understanding. Therefore, before the first lesson in the unit, we decided to set up Lesson 0 in which division without remainders will be reviewed.

(4) Division of 0

Almost all students seem to understand.

(5) Word problem with division

Almost all students answered the question correctly. However, in Grade 3, students only need to divide the larger number in a word problem by the smaller number, if there is an indication that the word problem is "division problem," then there is no need for students to interpret the problem. We suspect a significant number of students are solving these types of problems mechanically. Therefore, even though the question was a word problem, it might not indicate much about the students' ability to think and reason.

(6) Division with remainders

This is the focus of the current unit, and the students have yet to learn it formally. Those students who answered correctly labeled "remainder" clearly, indicating they have learned this content at home or *juku's*. However, for 38 ÷ 6, some students responded 6 + 2, and their responses were counted as correct. Another students responded "6·2." These responses seem to indicate the students' disposition to use what they have previously learned. Therefore, we would like to teach lessons that can take advantage of this type of reasoning.

5. Plans to increase students' ability to use their prior learning

i. Designing the lesson opening that will heighten motivation and interest

Problems that may heighten students' motivation and interest

Many problems of "division without remainders" and "division with remainders" to help students sense the enjoyment of partitioning many things. In addition, to make connections to students' everyday life, we chose to use the problem context involving sharing of snacks. It is hoped that such a problem would make it easier for students to relate to the problem situations and become motivated and interested in the lesson.

Setting up the learning environment in which students feel safe and confident in applying their prior learning

As we studied division in May of this year, many students appeared to engage in the lessons with comfort because they felt confident that they can do the calculations. However, as noted earlier, many of them are using memorized facts mechanically and unable to answer the question, "why can we use the basic multiplication facts?" Therefore, before the first lesson of the unit, Lesson 0 is set up to review "using the basic multiplication facts to division calculations without remainder." Through this lesson, it is hoped that students understand the meaning of the use of the basic multiplication facts in division. Moreover, we will begin the lesson with divisions without remainder with the numbers like 12 and 15 so that students feel confident that calculations can be completed. Then, a division problem with a remainder will be displayed and students will be asked "How is this problem different from the others?"

ii. Designing mathematical activities that increase the ability to express own ideas

① Setting up a problem situation that will promote a variety of thinking

In the beginning, a mixture of "divisions without a remainder" and "divisions with remainders" will be displayed, and the students' thinking will be broadened. Then, as the lesson progresses, students' understanding will be gradually deepened and focused. Then, at the end of the lesson, an extension task will broaden students' thinking once again. In addition, actually packages of pudding will be used so that students can connect the problem solving situation with their everyday experiences.

② "Today's learning" - deepening understanding through reflecting on the day's learning

At the end of a lesson, students have been writing journals, "today's learning." At first, their entries were short, but gradually, more students started including specifics. For example, in the addition unit, a number of students were writing about the differences from what they learned in Grade 2. Other students wrote entries that anticipated future learning. For example, some wrote, "Since I can now regroup twice with addition, I want to learn to regroup in many places with subtraction." Thus, we can see that students are deepening their understanding through journal writing.

6. Unit Plan (10 lessons, today's lesson is the 2nd of the 10 lessons)

	Goals	○Learning Activity	◇ Prior learning to be used • Design to nurture the ability to use [Evaluation]
0	<Review of prior learning> Students will understand and be able to explain how to find the answers for division without remainders.	○ Discuss what we already know about division. ○ Write the explanation of how to find the answer for division without remainders (15 ÷ 3) in the notebook using diagrams and expressions/equations. ○ Remember the relationship between multiplication and division.	◇ Answers for division problems may be found by using the basic multiplication facts. [Interest, Eagerness, and Attitude] Students are thinking about and trying to explain ways to find the answers for division without remainders.

		□÷△=○ ⇔ △×○=□ ○ Write the appropriate expression/equation and think about the meaning of each number.	[Understanding and Knowledge] Students understand ways to find the answers for division without remainders. Also, students understand the relationship between multiplication and division.
❶ Division with remainders [6 lessons]			
1	Students will be able to calculate division with remainders using concrete materials (counters).	○ Students will write expressions/equations for division problem situations with remainders. ○ Discuss the meaning of remainders.	◇ Division means to create equal sized groups (meaning of division). ◇ Reasoning through manipulation (based on the manipulations, clarify the meaning of a remainder). [Interest, Eagerness, and Attitude] Students are trying to find the answers for division with remainders using concrete materials. [Mathematical Skill] Students will be able to calculate the answers for division with remainders using concrete materials.
2	**Students will think about ways to find the answers for division with remainders and explain them in notebooks.** **Students will understand that the answers for division with remainders can be found by using the basic multiplication facts.** **Today's Lesson**	**○ Think about ways to find the answers for 16 ÷ 3 and record them in notebooks.** **○ Share students' ideas (using diagrams, the basic multiplication facts).** **○ Sort various division expressions into those with remainders and those without remainders.**	**◇ How to find quotients.** **• Have an extension problem, "How can we change division with remainders so that there will be no remainders?"** **[Mathematical Way of Thinking] Students will find ways to find the answers for division with remainders and record them in notebooks.** **[Knowledge and**

			Understanding] Students understand that division with answers can also be found by using the basic multiplication facts.
3	Students will understand the relationship between the divisor and the remainder.	○ Investigate the relationship between the divisor and the remainder in the case of 13÷4. ○ By changing the dividend 14, 15, ..., think about the relationship between the divisor and the remainder.	◇ Reasoning based on manipulation of concrete materials (using the meaning of remainders as the basis). [Knowledge and Understanding] Students understand that the remainder is less than the divisor.
4	Students will understand that the division operation can be applied to partitive (fair sharing) situations.	○ Think about the appropriate expression for partitive division situations and how to find the answer by making sense of the situation. ○ Discuss their ideas and summarize ways to calculate partitive division.	◇ Calculation methods for quotitieve (measurement) division without remainders. ◇ Reasoning with mathematical expressions. [Mathematical Way of Thinking] Students are thinking about ways to calculation partitive division with remainders and explaining their methods using concrete materials, diagrams and/or expressions (or equations).
5	Students will understand how to check the results of calculation for division with remainders. They will also develop the disposition to check their calculations routinely.	○ Think about ways to check the answers for calculation of division with remainders. ○ Practice checking calculations.	◇ How to check the answers for division without remainders. ◇ Reasoning with diagrams (students are trying to capture phenomena and situations using diagrams)

6	Students will practice calculations, including division with remainders.	○ Calculation practice, including checking the results.	◇ Practice for calculation of division without remainders. [Mathematical Skill] Students can calculate division with remainders and check the results of their calculations.
❷ Problems that require students to think about the meaning of remainders. [2 lessons]			
7 8	Students will deepen their understanding of remainders in situations.	○ Understand problem situations and write and calculate the appropriate expressions. • Situations in which Quotient +1 will be the answer • Situations in which Quotient will be the answer	◇ How to calculate division without remainders ◇ Reasoning with diagrams ◇ Reasoning with mathematical expressions (expressing phenomena and situations using mathematical expressions) • To help students make sense of situations, have them first draw diagrams. [Knowledge and Understanding] Students understand how to deal with quotients and remainders depending on the problem situations
❸ Summary [2 Lessons]			
9	Students will solve problems applying what they have learned.	○ Complete "Power Builder."	[Mathematical Skill] Students can solve problems using what they have learned.
10	Students will consolidate and master what they learned.	○ Complete "Mastery Problems" and calculation practice problems.	[Knowledge and Understanding] Students have mastered what they have learned.

7. Today's lesson (Lesson 2 of 10)

(1) Goals

- Students will think about ways to find answers for division situations with remainders and explain their methods in their notebooks.
- Students will understand that they can also use the basic multiplication facts even when there are remainders.

(2) Flow of the lesson

	○ Learning activities T: *Hatsumon* and instruction C: Anticipated responses	◇ Prior learning to be used • Design to improve students' ability to use what they have learned [Evaluation] ◆ Ways of observing and thinking to be mastered
Understanding the task/Plan	Understand the problem Look at several division expressions and figure out which ones may involve remainders. T: From these expressions, let's find division expressions that will give us remainders. Can someone pick one? C: This one. T: Why did you select this one? C: Just because (intuitive)/it is not in the basic multiplication table, etc T: Let's think about this more carefully. T: (Select 16 ÷ 3) Does this look like the one we studied yesterday? Recall what they did (use counters to separate them into groups). T: Do we need counters to calculate all of these expressions? It's a little tedious, isn't it? So, let's think about ways to find the answers without using counters. T: Let's think about ways to find the answer for 16 ÷ 3 using what we have learned so far. Write down your methods in your notebooks so that you can share how you found the answer later. C: If it were 15 ÷ 3, I can use the multiplication facts, but ...	• By displaying many expressions, increase students' motivation. 15 ÷ 3, <u>16 ÷ 3</u>, <u>17 ÷ 3</u>, 8 ÷ 4, <u>13 ÷ 4</u>, <u>26 ÷ 4</u>, 25 ÷ 5, <u>43 ÷ 6</u>, 56 ÷ 7, 81 ÷ 9 (5 division without remainders and 5 division with remainders) • Help students make connections to the previous lesson. Have the display materials from the previous lesson ready. • Help students anticipate a simpler method than using concrete materials (counters) to increase their interest in the task.

Independent Problem Solving	Students will think about ways to find the answer for 16 ÷ 3 and record their methods in their notebooks. C1: Draw a diagram representing how they manipulated concrete materials (counters). Write an explanation in words. C2: Expressions/equations, words, and diagrams to explain. C2-1: 3 x 5 = 15, 15 + 1 = 16 (16 - 15 = 1) C2-2: 3 + 3 + 3 + 3 + 3 = 15, 15 + 1 = 16 C2-3: 3 x 1 = 3 3 x 2 = 6 3 x 3 = 9 3 x 4 = 12 3 x 5 = 15, 15 + 1 = 16 [Translator's Note: In Japan, the number in each group is written first.] C3: Cannot get started. C4: Student can calculate but cannot explain. Just write "16 ÷ 3 = 5, remainder = 2."	◇ Division without remainders can be calculated using the basic multiplication facts. • On the display board in the classroom, create a section in which what we have studied will be highlighted. Display in the section, "15 ÷ 3 can be calculated using the 3's multiplication facts." • With C3, remind him/her that we used counters in the previous lesson. Have him/her draw a diagram to represent the method with the counters. Then, ask him/her if the method can be represented using mathematical expressions and equations. • With C4, remind him/her how we calculated division without remainders (15 ÷ 3). • It does not matter at this point how students figure out the remainder or record their answers. [Mathematical Way of Thinking] Students think about ways to find the answers for division with remainders and write their explanations in their notebooks.

Discussion	Share their own ideas and listen to other students' ideas. (Have C1, C2-1, C2-2, and C2-3 share their ideas.) T: Let's listen carefully how your friends figured out the answer and thinking about the similarity to and the difference from your own idea." C: All of them have the answer of "5 remainder 1." C: C2-1 and C2-3 are doing the same thing. T: Can someone else explain the method of C2-1? C: ~ T: How did we find the answers for 15 ÷ 3? C: We used the 3's multiplication facts. C: It's the same for 16 ÷ 3. (T: But, we don't have 16 in the 3's multiplication facts. Why is it possible to use the 3's multiplication facts to find the answer for 16 ÷ 3?) Write the equations and learn how to record and read the answers. T: The calculation you did is written as "16 ÷ 3 = 5 remainder 1." When there is a remainder, we say that it is not divisible. When there is no remainder, we say it is divisible.	◆ Reasoning to unify ideas; Reasoning with analogy "Calculation for division with remainders can be carried out using the same method for division without remainders." • Help students become interested in other students' ideas by comparing and contrasting with their own ideas. • By identifying the similarities, help students unify various ideas. • By asking students to explain other students' ideas, provide more opportunities for students to make public presentations. • In case students' understanding is insufficient, have additional questions ready.)

Application problems and Summary	Sort expressions into "divisible" and "not divisible" (record in the notebooks). T: OK, let's use this method to calculate division to sort many division expressions. Let's first select division expressions that can be solved using the 3's multiplication facts. Then, those that can be solved using the 4's multiplication facts. ~ T: Now, let's sort these division expressions into "divisible" and "not divisible." (T: What do we need to change to make 16 ÷ 3 so that it will be divisible?) Write "Today's learning." T: Let's write "Today's learning."	[Mathematical Way of Thinking] Students understand that division with remainders can also be calculated using the basic multiplication facts. • Have an extension question ready. • Before students write their journal entries, remind them what today's task was. In addition, encourage students to write the details of "how they thought."

2012 Japan Trip - Initial Survey

Response ID:" 2 Data

2.

1. Please enter the unique participant ID number emailed to you.

3.

2.
Please describe your experiences with lesson study to date, including:

a. Number of years you have been involved in lesson study;

b. Content area (e.g., math, English/ language arts) of lessons you have experienced;

c. Number of times you have observed and participated in lesson study;

d. Whether these experiences were within your home country or in another country.

4.

3. What do you think are the strengths/ benefits of using lesson study in your local context(s) (e.g., district, school, university setting)?

5.

4. What do you think are the challenges to using lesson study in your local context(s)?

6.

5. Please describe how your current organizational contexts use lesson study for educational improvement.

7.

6. Please describe how you hope to use lesson study for educational improvement in your current organizational contexts after this trip.

8.

7. To what extent do you expect to learn about each of the following during the immersion trip to Japan?

	1	2	3	4	5
a. Mathematics content					
b. How to build students' problem solving					
c. Evaluating a lesson on the basis of a written lesson plan					
d. How lesson study is conducted in another country					
e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)					
f. Collecting data on student thinking to inform instruction					
g. Strategies for making students' thinking visible					
h. Analyzing/studying curriculum materials					
i. Ways to build connections among educators at multiple levels of the education system					
j. Anticipating student responses					
k. Writing a useful lesson plan					
l. Supporting participants to have powerful and effective lesson study experiences					
m. Organizational/structural supports for lesson study					
n. Students' mathematical reasoning					
o. Differentiating/ offering support for struggling learners					
p. Cultural influences on mathematics teaching and learning					
q. Organizing a successful post-lesson debriefing session					
r. A typical school day at a Japanese elementary school					
s. Developing mathematics units and lessons					
t. Strategies for working effectively in a lesson study group					
u. My own country's approaches to mathematics instruction					
v. Analyzing written student work/ responses					

w. Analyzing and interpreting verbal student comments					
x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)					
y. How to build a classroom learning community					

9.

8. Please select and rank in order of importance the <u>five</u> items from the previous question that you believe will be <u>most professionally useful</u> for you <u>within the next year</u>.

	1st Most Useful	2nd Most Useful	3rd Most Useful	4th Most Useful	5th Most Useful
a. Mathematics content					
b. How to build students' problem solving ability					
c. Evaluating a lesson on the basis of a written lesson plan					
d. How lesson study is conducted in another country					
e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)					
f. Collecting data on student thinking to inform instruction					
g. Strategies for making students' thinking visible					
h. Analyzing/studying curriculum materials					
i. Ways to build connections among educators at multiple levels of the education system					
j. Anticipating student responses					
k. Writing a useful lesson plan					
l. Supporting participants to have powerful and effective lesson study experiences					
m. . Organizational/structural supports for lesson study					
n. Students' mathematical reasoning					
o. Differentiating/ offering support for struggling learners					
p. Cultural influences on mathematics teaching and learning					
q. Organizing a successful post-lesson debriefing session					
r. A typical school day at a Japanese elementary school					
s. Developing mathematics units and lessons					
t. Strategies for working effectively in a lesson study group					
u. My own country's approaches to mathematics instruction					
v. Analyzing written student work/ responses					
w. Analyzing and interpreting verbal student comments					
x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)					
y. How to build a classroom learning community					

10.

9. Four teachers were discussing the way they believe mathematics is learned by students. To their surprise, no two of them agreed on the principal way mathematics is learned, although each suggested that intellectual processes were necessary.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about the way mathematics is learned. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "To learn mathematics, students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."
SUSAN: "The most important thing is reasoning. If students can reason logically and can see how one mathematical idea relates to another, they will understand what is taught."
BARBARA: "The primary thought process in learning mathematics is memory. Once students have the facts and rules memorized, everything else falls into place."
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures–right or wrong–and discover things for themselves, they will understand the mathematics and how it is used."

Please write about your view of how students learn mathematics.

11.

10. Four teachers were discussing the role of problem solving in students' learning of mathematics. To their surprise, no two of them agreed on the role of problem solving in mathematics learning.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about problem solving in mathematics. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "Problem solving is like any other skill in mathematics. Students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."
SUSAN: "The most important thing in problem solving is to develop logical reasoning. Problem solving helps students learn to reason logically and can see how one mathematical idea relates to another. Thus it helps them understand mathematics."
BARBARA: "Students should first master the prerequisite facts and skills of mathematics before they are assigned problem solving. Problem solving should emphasize the application of these facts and skills to real life situations."
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures–right or wrong–and discover things for themselves, they will understand the mathematics and how it is used."

Please write about your view of the role of problem solving in students' learning of mathematics.

12.

11. Four teachers were discussing the teaching of problem solving in their mathematics classes. To their surprise, no two of them agreed on the teaching of problem solving in mathematics.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own teaching of problem solving in mathematics (or your thinking about it). You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "I present mathematics problem solving as a set of steps that can be mastered and practiced like any other skill in mathematics. Students have to practice, practice, and practice. It' s like playing a musical instrument–they have to practice until they have it down pat."
SUSAN: "Problem solving is the most important goal of mathematics for me. I try to include some problem solving in every lesson to help students develop their logical reasoning."
BARBARA: "The prerequisite knowledge and skills of mathematics must be mastered first. Problem solving takes too much time to include except maybe once a week or once every two weeks."
DENISE: "I sometimes use a problem solving situation to introduce a concept. The students can explore, make conjectures, and discover relationships. That is what mathematics problem solving is about."

Please write about your view of the teaching of problem solving in mathematics.

13.

12. Please indicate how well each of the following statements describes your current attitude. (Circle ONE for each statement.)

	1	2	3	4	5	6
a. I enjoy learning about mathematics.						
b. I have learned a lot about student thinking by working with colleagues.						
c. I have strong knowledge of the mathematical content taught at my grade level.						
d. I have good opportunities to learn about the mathematics taught at different grade levels.						
e. I think of myself as a researcher in the classroom.						
f. I have learned a great deal about mathematics teaching from colleagues.						

g. I am always curious about student thinking.						
h. By trying a different teaching method, teachers can significantly affect a student's achievement.						
i. I am interested in the mathematics taught at many grade levels.						
j. I would like to learn more about the mathematical content taught at my grade level.						
k. Working on mathematics tasks with colleagues is often unpleasant.						
l. I find it useful to solve mathematics problems with colleagues.						
m. . Japanese mathematics teaching approaches are not likely to be useful outside of Japan.						

14.

13. Please indicate your current position (Check ALL that apply.)

15.

14. How many years of teaching experience do you have?

16.

15. Please list any grades to which you have ever taught mathematics.

l

17.

16. Please add any comments or feedback you have about this survey.

2012 Japan Trip - Post Survey

Response ID:" 5 Data

2.

1. Please enter the unique participant ID number emailed to you.

3.

2. After the trip, what do you now think are the strengths/ benefits of using lesson study in your local context(s) (e.g., district, school, university setting)?

4.

3. What do you think are the challenges to using lesson study in your local context(s)?

5.

4. Please describe how you hope to use lesson study for educational improvement in your current organizational contexts after this trip.

6.

5. How much did you learn about each of the following during the immersion trip to Japan?

	1	2	3	4	5
a. Mathematics content					
b. How to build students' problem solving					
c. Evaluating a lesson on the basis of a written lesson plan					
d. How lesson study is conducted in another country					
e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)					
f. Collecting data on student thinking to inform instruction					
g. Strategies for making students' thinking visible					
h. Analyzing/studying curriculum materials					
i. Ways to build connections among educators at multiple levels of the education system					
j. Anticipating student responses					

k. Writing a useful lesson plan					
l. Supporting participants to have powerful and effective lesson study experiences					
m. Organizational/structural supports for lesson study					
n. Students' mathematical reasoning					
o. Differentiating/ offering support for struggling learners					
p. Cultural influences on mathematics teaching and learning					
q. Organizing a successful post-lesson debriefing session					
r.A typical school day at a Japanese elementary school					
s. Developing mathematics units and lessons					
t. Strategies for working effectively in a lesson study group					
u. My own country's approaches to mathematics instruction					
v.Analyzing written student work/ responses					
w.Analyzing and interpreting verbal student comments					
x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)					
y. How to build a classroom learning community					

7.

6. Please select and rank in order of importance the five items from the previous question that you believe will be most professionally useful for you within the next year. (Please select only five items to rank order.)

	1st Most Useful	2nd Most Useful	3rd Most Useful	4th Most Useful	5th Most Useful
a. Mathematics content					
b. How to build students' problem solving ability					
c. Evaluating a lesson on the basis of a written lesson plan					
d. How lesson study is conducted in another country					
e. How lesson study is conducted in different educational contexts (e.g., schools, districts, etc.)					
f. Collecting data on student thinking to inform instruction					
g. Strategies for making students' thinking visible					
h. Analyzing/studying curriculum materials					
i. Ways to build connections among educators at multiple levels of the education system					
j. Anticipating student responses					
k. Writing a useful lesson plan					
l. Supporting participants to have powerful and effective lesson study experiences					
m. . Organizational/structural supports for lesson study					

n. Students' mathematical reasoning					
o. Differentiating/ offering support for struggling learners					
p. Cultural influences on mathematics teaching and learning					
q. Organizing a successful post-lesson debriefing session					
r. A typical school day at a Japanese elementary school					
s. Developing mathematics units and lessons					
t. Strategies for working effectively in a lesson study group					
u. My own country's approaches to mathematics instruction					
v. Analyzing written student work/ responses					
w. Analyzing and interpreting verbal student comments					
x. How to build students' mathematical habits of mind and practices (such as in the Common Core State Standards)					
y. How to build a classroom learning community					

8.

7. Please review the following three problems, and provide your ratings again after the trip.

Four teachers were discussing the way they believe mathematics is learned by students. To their surprise, no two of them agreed on the principal way mathematics is learned, although each suggested that intellectual processes were necessary.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own thinking about the way mathematics is learned. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "To learn mathematics, students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."
SUSAN: "The most important thing is reasoning. If students can reason logically and can see how one mathematical idea relates to another, they will understand what is taught."
BARBARA: "The primary thought process in learning mathematics is memory. Once students have the facts and rules memorized, everything else falls into place."
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures–right or wrong–and discover things for themselves, they will understand the mathematics and how it is used."

Please write about your view of how students learn mathematics.

9.

8. Four teachers were discussing the role of problem solving in students' learning of mathematics. To their surprise, no two of them agreed on the role of problem solving in mathematics learning.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with

your own thinking about problem solving in mathematics. You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "Problem solving is like any other skill in mathematics. Students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."
SUSAN: "The most important thing in problem solving is to develop logical reasoning. Problem solving helps students learn to reason logically and can see how one mathematical idea relates to another. Thus it helps them understand mathematics."
BARBARA: "Students should first master the prerequisite facts and skills of mathematics before they are assigned problem solving. Problem solving should emphasize the application of these facts and skills to real life situations."
DENISE: "Exploring is the key to learning mathematics. If students explore problem situations, make conjectures–right or wrong–and discover things for themselves, they will understand the mathematics and how it is used."

Please write about your view of the role of problem solving in students' learning of mathematics.

10.

9. Four teachers were discussing the teaching of problem solving in their mathematics classes. To their surprise, no two of them agreed on the teaching of problem solving in mathematics.

You have a total of 100 points. Allocate the points to the position(s) below according to the strength of agreement with your own teaching of problem solving in mathematics (or your thinking about it). You may distribute the points in any size increments. You may assign all 100 points to a single position and 0 to the remaining positions.

MARY: "I present mathematics problem solving as a set of steps that can be mastered and practiced like any other skill in mathematics. Students have to practice, practice, and practice. It's like playing a musical instrument–they have to practice until they have it down pat."
SUSAN: "Problem solving is the most important goal of mathematics for me. I try to include some problem solving in every lesson to help students develop their logical reasoning."
BARBARA: "The prerequisite knowledge and skills of mathematics must be mastered first. Problem solving takes too much time to include except maybe once a week or once every two weeks."
DENISE: "I sometimes use a problem solving situation to introduce a concept. The students can explore, make conjectures, and discover relationships. That is what mathematics problem solving is about."

Please write about your view of the teaching of problem solving in mathematics.

11.

10. Please indicate how well each of the following statements describes your current attitude. (Circle ONE for each statement.)

	1	2	3	4	5	6
a. I enjoy learning about mathematics.						

b. I have learned a lot about student thinking by working with colleagues.						
c. I have strong knowledge of the mathematical content taught at my grade level.						
d. I have good opportunities to learn about the mathematics taught at different grade levels.						
e. I think of myself as a researcher in the classroom.						
f. I have learned a great deal about mathematics teaching from colleagues.						
g. I am always curious about student thinking.						
h. By trying a different teaching method, teachers can significantly affect a student's achievement.						
i. I am interested in the mathematics taught at many grade levels.						
j. I would like to learn more about the mathematical content taught at my grade level.						
k. Working on mathematics tasks with colleagues is often unpleasant.						
l. I find it useful to solve mathematics problems with colleagues.						
m. . Japanese mathematics teaching approaches are not likely to be useful outside of Japan.						

12.

11. Please select the research lesson and post-lesson discussion that you feel was most professionally informative for you.

12. Please explain why you selected this lesson and post-lesson discussion. What about the lesson and post-lesson discussion was informative for you?

13. Copy of

13. Please select the research lesson and post-lesson discussion that you feel was least professionally informative for you.

14. Please explain why you selected this lesson and post-lesson discussion as the least professionally informative for you. What specifically was missing?

14.

15. What changes to the trip itinerary might have helped to deepen your own learning about lesson study and mathematics teaching and learning?

15.

16. In what ways will you draw on the colleagues you met and worked with on this trip in the future?

16.

17. Please add any comments or feedback you have about this survey.

山梨日々新聞　　2012 年 6 月 29 日
The Yamanashi Nnichinishi Shinbun　　June 29, 2012

4ヵ国の教員招き研究授業

押原小　日本独自の取り組み注目

昭和町の押原小で28日、米国、英国など4カ国の教員を招いた研究授業が行われた。同校によると、大勢の教員が見学する研究授業は日本独自の取り組みで、海外から注目されているという。

海外の教職員を招いた研究授業＝昭和・押原小

押原小を訪れたのは、米国、英国のほか、オーストラリアとシンガポールの教育関係者計44人。3年1組で行われた算数の研究授業を見学した後、授業の内容を検討する研究会に出席した。

授業では2桁の引き算について、児童がそれぞれの暗算方法を発表。米国のシカゴ授業研究連合会長で、元数学高校教員のトーマス・マクドガルさん(50)は「米国では、児童に指示通りの計算をさせることがほとんど。子どもの考えを引き出す素晴らしい授業だった」と評した。

押原小によると、国内で行われている研究授業を参考に、最近では海外でも「授業の質が向上できる」と実践する動きが出ているという。

日刊工業新聞

Business & Technology Daily News

教育の現場力学ぶ

米の数学教師らが視察

◎…「日本の現場力を持ち帰りたい」と意気込むのは、米ミルズ大学（カリフォルニア州オークランド）のキャサリン・C・ルイス研究教授。シンガポール、豪州のほか、トヨタＵＳＡ財団の支援を受けて、オークランドからは数学教師14人が来日し、都内の小中学校の現場を視察した（写真）。

◎…製造業のＱＣサークル活動と同様に、日本の教育現場では「授業研究」という小集団活動が生きている。「米国では教師間で教え方を検証して、改善していく場がほぼない」と明かす。

◎…米国は国内統一カリキュラムの完全実施を２０１４年に控える。改革案を授業に落とし込んでいくのは現場の教師の力だ。「授業研究は授業のマネジメントだけでなく、教材の改良や教師自身のプロ意識の醸成に結びついている。学ぶことは多い」と収穫は大きそうだ。（立川）

12年7月6日21面

＜July 6, 2012＞

授業研究再認識を

ルイスさん

▽…「米国は教育改革がうまくいかないが、日本で改革を実行できるのはなぜか」と問いかけるのは、来日した米ミルズ大学（カリフォルニア州）研究教授のキャサリン・ルイスさん。授業研究にその秘密があるとか。

▽…授業研究では教師が授業を観察し合い、教え方や教育効果を議論する。教師自身が成長し続けるための仕組みだ。「教育現場で子どもの反応を見ながら、改革案を検証し、改良しながら実装できる」と指摘する。

▽…ただ「外国から見ると素晴らしさが分かるが、日本の教師が授業研究の有用性を十分認識していない。このままでは失ってしまう可能性もある」と心配する。

12年7月11日19面

＜July 11, 2012＞

日本の授業に学ぶ

ダロさん

▽…「米国では数学を学ぶことよりも答えを出すことが優先されてきた」と振り返るのは、このほど来日した米ピアソン財団（ニューヨーク市）シニアフェローのフィル・ダロさん。米国統一カリキュラム（ＣＣＳＳ）の執筆を主導した。

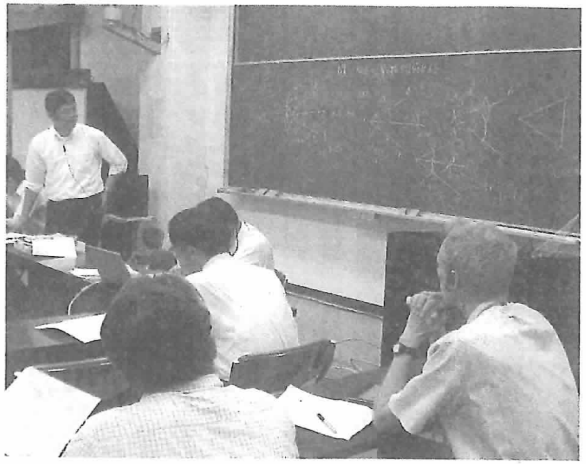
▽…「米国の教師は、『この問題の解き方を身に付けさせるために、どう教えたらよいか』と考えるが、日本では『数学を教えるためにこの問題をどのように使えばよいか』と考える」という。

▽…ＣＣＳＳが完全実施される２０１４年までに、現場の教師の意識を変えていく必要がある。「日本の授業を研究すると学ぶことは多い」と期待をかける。

12年7月13日21面

＜July 13, 2012＞

「教育新聞」　2012年8月13日（月）1面、3面　掲載記事
The Education Newspaper　August 13, 2012



東京学芸大学附属小金井中学校で、アメリカを中心とした算数・数学教育の教員と研究者が参加する国際セミナーが開催された。日本の授業と校内研修に学ぶ機会となった。　（関連記事3面）

校内研修

「ベリー・グッド！」

教材研究

米国教員らが数学授業を参観
全米統一カリキュラムに向け

アメリカを中心とした算数・数学教育の教員と研究者が、日本の授業と校内研修に学ぶ国際セミナーが、さきごろ、東京学芸大学附属小金井中学校で行われた。

セミナーは、算数・数学の授業改善を目指した同学数学科教育学研究室主催による「国際算数・数学授業改善のための自己向上機能を備えた教員養成システム開発」（国際算数・数学授業研究プロジェクト）に伴って開催されたもの。

プロジェクトでは、国内外の研究者が協力しながら、大学での教員養成や現職教員の研修・研究を一層充実化するためのシステム構築を目指して研究を深めている。

そんなプロジェクトを踏まえ、国際セミナーでは、アメリカを中心にオーストラリア、シンガポールなどの教員、研究者44人が参加。

アメリカからは、アメリカ統一カリキュラム「Common Core State Standards（CCSS）」の算数・数学に関する執筆をリードしたピアソン財団のフィル・ダロ上席研究員と、授業研究を通したカリキュラム具現化プロジェクトの責任者で、マイルズ・カレッジのレッスン・スタディー研究プロジェクトのキャサリン・ルイス局長が参加。

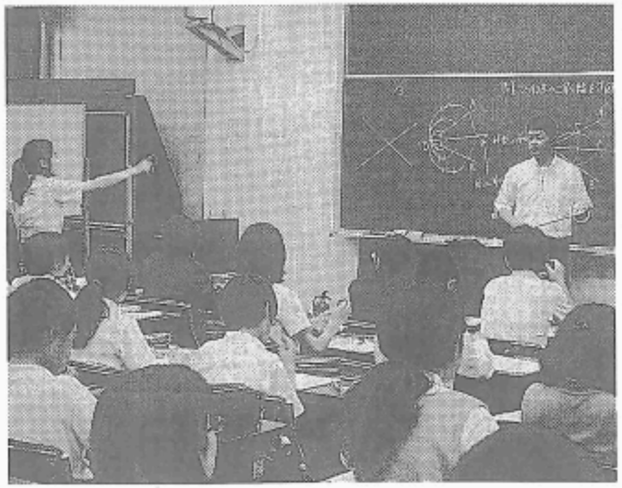
国の統一カリキュラムが施行されていなかったアメリカだったが、2010年、幼稚園から高校までの算数・数学と英語で統一カリキュラムが発表され、現在、45州と首都ワシントンDCが正式にCCSSを採択。2014年の全米での完全実施に向け、どのように授業を具現化していくか、学習評価をどうするかなどの課題に取り組む中で、国として統一的な学習指導要領や教科書を持つ日本の教育に学ぼうと、協議を深めた。

各国からの参加教員は、東京都内と山梨県内の各小・中学校での算数・数学授業と授業後の校内研修などを参観し、意見交換などを行った。公開校のひとつ、東京学芸大学附属小金井中学校では、1年生数学科の単元「平面図形」の中の「角の二等分線の作図」の授業を公開。

課題は「∠AOB」の図をもとに「角の二等分線を作図しよう」を考えたもの。課題を投げ掛けられた生徒たちは角の二等分線を見いだすためにコンパスを使って2直線から等しい距離にある点の集合を見つける方法を検討し、それぞれの解き方を板書していった。また公開授業後は、「数学的活動の質の向上」と題し、授業展開や生徒の反応などについて協議を進めた。

参観したアメリカの教員、研究者からは、「子どもの反応を予想した授業協議や互いの授業を見合うことを可能にする日本の校内研修に良さを感じた」「校内研修をぜひとも取り入れたいが、学校単独での実施は可能でも複数の学校同士が結び合って実施するのが困難」など、日本の校内研修制度の良さを学ぶ一方で、制度的な連携体制がむずかしい状況にあるという現実も聞かれた。

東京学芸大学国際算数数学授業研究プロジェクトのEメール＝info@impuls-tgu.org



角の二等分線を作図する授業を参観する国際セミナー参加教員

「日本教育新聞」　2012年8月27日（月）９面
Japan Educational Press　August 27,2012

東京学芸大が国際算数・数学プロジェクト

日本の「授業研究」を海外へ

米・英などの教員来日し視察

授業のテーマは「角の二等分線の作図」。生徒たちはそれぞれ作図の方法を考え、「これだ」と思うものを発表していく。教員は生徒が発言した作図方法を黒板に書き写し、それぞれを比較検討しながら、角の二等分線の意味を伝えていった。

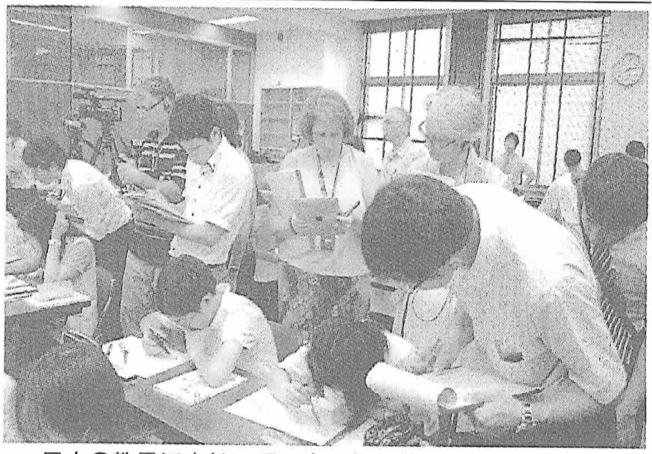
この夏実施された、東京学芸大学附属小金井中学校1年C組の数学は、「数学的活動の質を高める」をテーマにした研究授業だ。多くの教員が見学して内容を検証するとともに、終了後には教員たちが授業の良かった点、改善すべき課題について議論を交わした。

ここまで紹介した「授業研究」の光景は、日本の学校では日常的なものだ。しかし、教室では約40人の外国人教員がこの様子を見学し、自分たちの教育改善に生かそうと真剣に話を聞いてメモを取っていたことが普段と違っていた。

◇

授業内容や指導技術を高めるため、日本では当たり前のように実施されている授業研究による教員研修。だが、海外ではあまり行われてこなかったという。一方で、アメリカの研究者からは「日本の数学教育のレベルが高い要因の一つに、授業研究がある」という高評価を得ており、同国やイギリス、シンガポールなど、取り組む学校が少しずつ増加してきた。

日本の授業研究の方法を海外に伝えるとともに、国内外の研究者の協力で教員養成や現職教員の研修・研究を一層充実させたい―。そんな思いで研究や国内外での講演会の開催、新しいシステム開発などに取り組んでいるのが、東京学芸大学数学科教育学研究室などが昨年度から実施している「国際算数・数学授業改善のための自己向上機能を備えた教員養成システム開発プロジェクト」（国際算数・数学授業研究プロジェクト）だ。



日本の教員に交じって、来日した外国の教員もメモを取りながら研究授業を参観する

海外の算数・数学科の教員に来日してもらい、日本の小・中学校の研究授業を見てもらう「授業研究イマージョンプログラム」も、プロジェクトの事業の一つ。2年目となる今回は、アメリカ、イギリス、オーストラリア、シンガポールの教員が参加し、東京都内と山梨県の小・中学校7校の研究授業を参観した。

◇

平成22年に幼稚園から高校までの数学について、国の統一カリキュラムが発表されたアメリカ。45州と首都ワシントンDCがこのカリキュラムを採択して26年までの完全実施を目指すなど、学習指導要領に基づいた日本式の数学教育が取り入れられつつあり、授業研究への注目度は高い。

今回のプログラムに参加し、この日の授業研究を見学したカリフォルニア州の男性教員は「より良い授業の方法や教材を考える、教員のリーダーシップの取り方を探るなど、授業研究をさまざまなことに役立てている」と感想を述べた。

イリノイ州シカゴの男性教員は「日本で課題解決型の授業ができる理由の一つに、教員同士で意見を出し合う授業研究があることが分かった。授業力を向上させるための参考になる」と強い興味を示した。

アメリカでは各学校の独立性が強く、国や地方自治体の学校が統一して授業研究を取り入れることは難しいという。それでも、参加教員からは「自分の学校には、日本で見た授業研究による教員研修を取り入れることを検討したい」という声が多く聞かれた。

プロジェクトの代表者である藤井斉亮・東京学芸大学教授は「日本の算数・数学教育、教員の自己向上機能を備えた授業研究のシステムは、先生方が思っている以上に海外で高い評価を得ている」と話す。このことを基に、「日本の教員はもっと自信を持って算数・数学の授業に取り組むとともに、一生課題を持って学び続けて力を伸ばしてほしい」と述べた。